Structure of hadron resonances with a nearby zero of the amplitude

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INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS

40th Course

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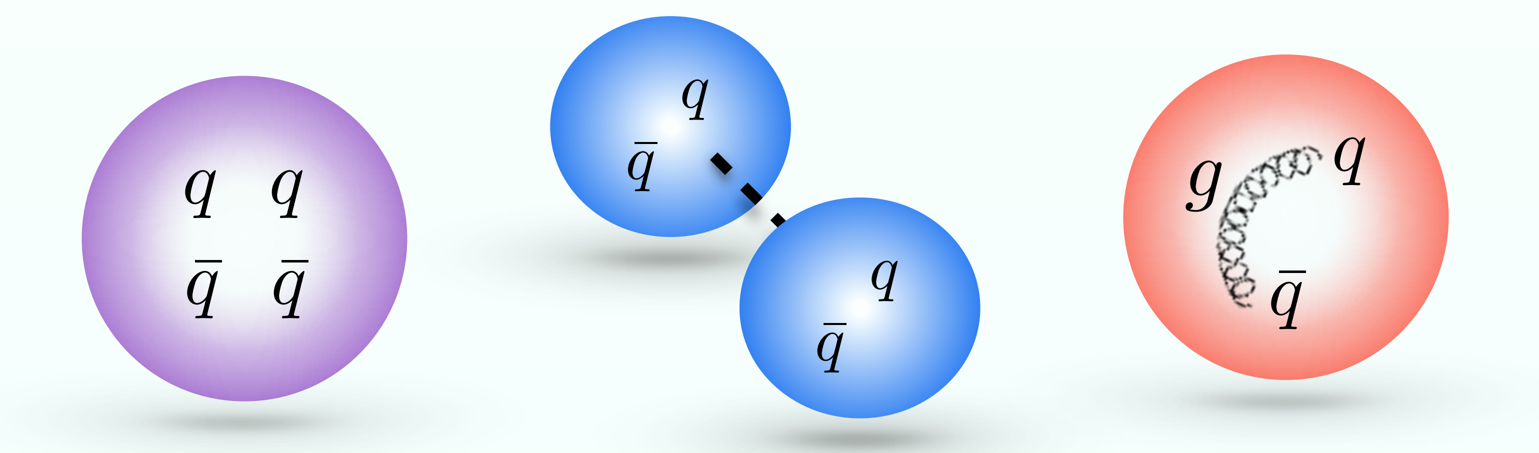
Introduction ~exotic hadrons~

- Exotic hadrons

Hadrons which do not agree with the predictions of the quark model (qqbar, qqq).

More complicated internal structure can be expected.

→
- tetra quark, penta quark
- hadron molecule ⋯



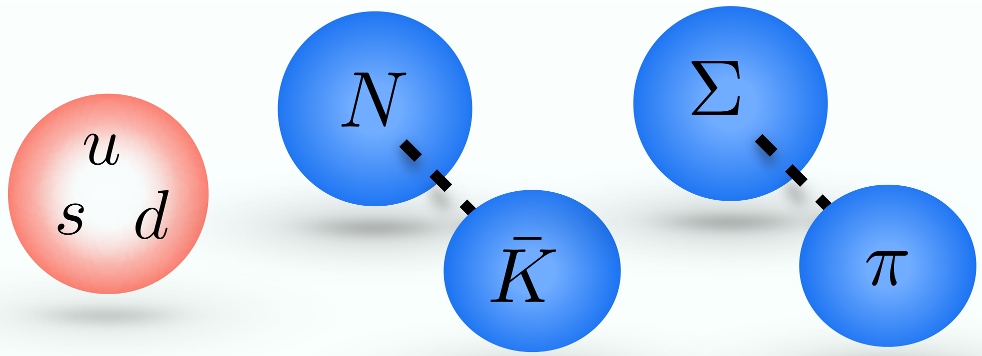
⋯

It is important to reveal the internal structure of exotics because we can acquire knowledge of strong interaction in the hadrons!

Introduction ~exotic hadrons~

$\Lambda(1405)$

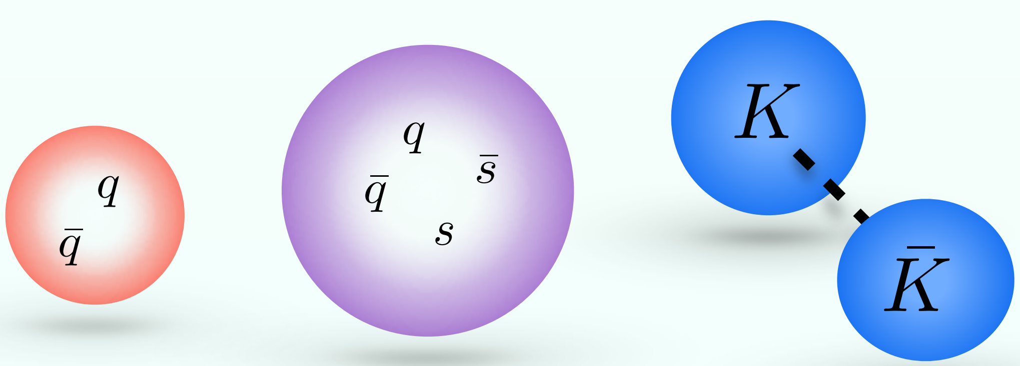
- $J^P = 1/2^-, I = 0$
- lying near $\bar{K}N$ threshold energy



or ⋯

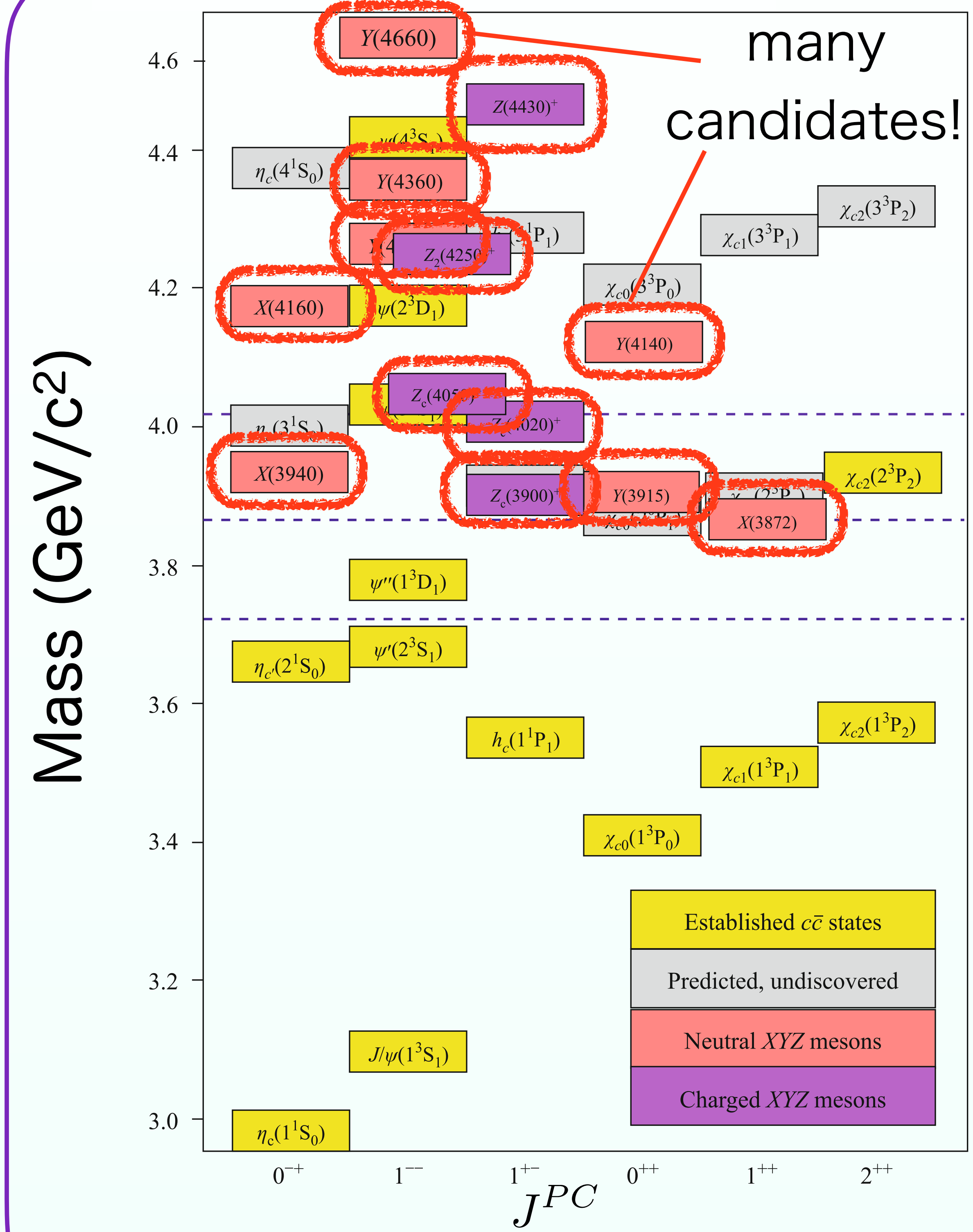
$a_0(980),\ f_0(980)$

- $J^P = 0^+ \quad I = \begin{cases} 1 & \text{for } a_0(980) \\ 0 & \text{for } f_0(980) \end{cases}$
- lying near $\bar{K}K$ threshold energy



or ⋯

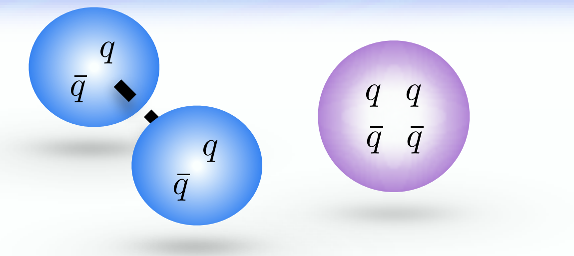
$c\bar{c}$ quarkonium like state



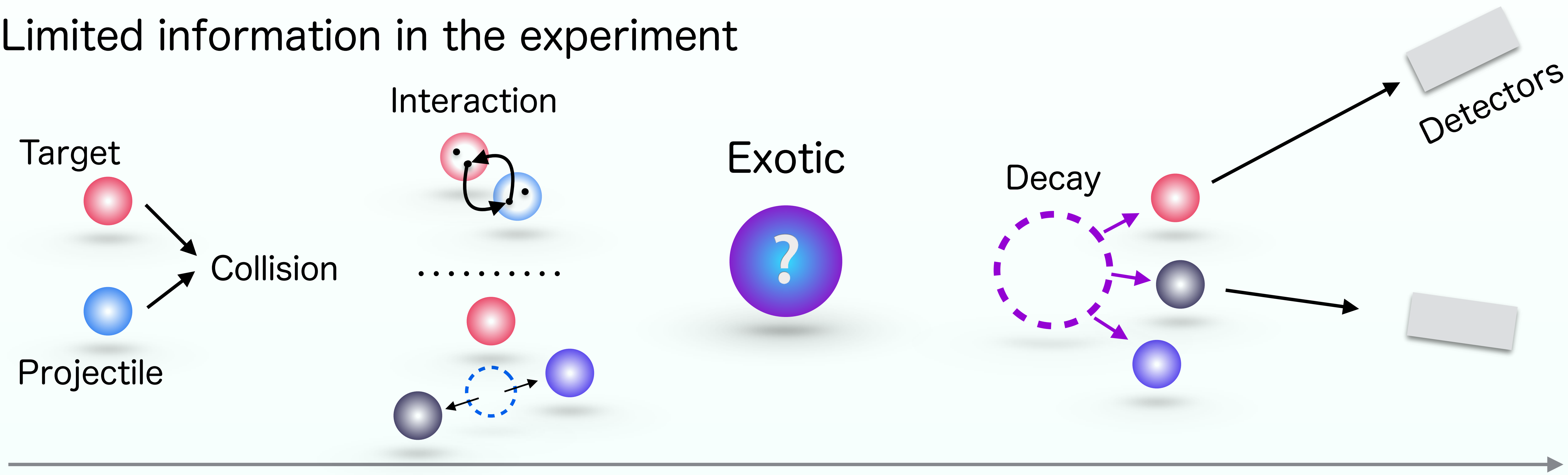
S. L. Olsen, Front. Phys. China. 10, 121 (2015)

Introduction ~Methods to study structure~

- Study on the internal structure



- Difficulty
  - ・Mixing of eigenstates with the same quantum numbers(Spin, Isospin, Parity)
  - ・Limited information in the experiment



- Our Approach
  - Model-independently distinguish the structure directly from the experimental data

Introduction ~Methods to study structure~

- How to extract the structure from the experimental data

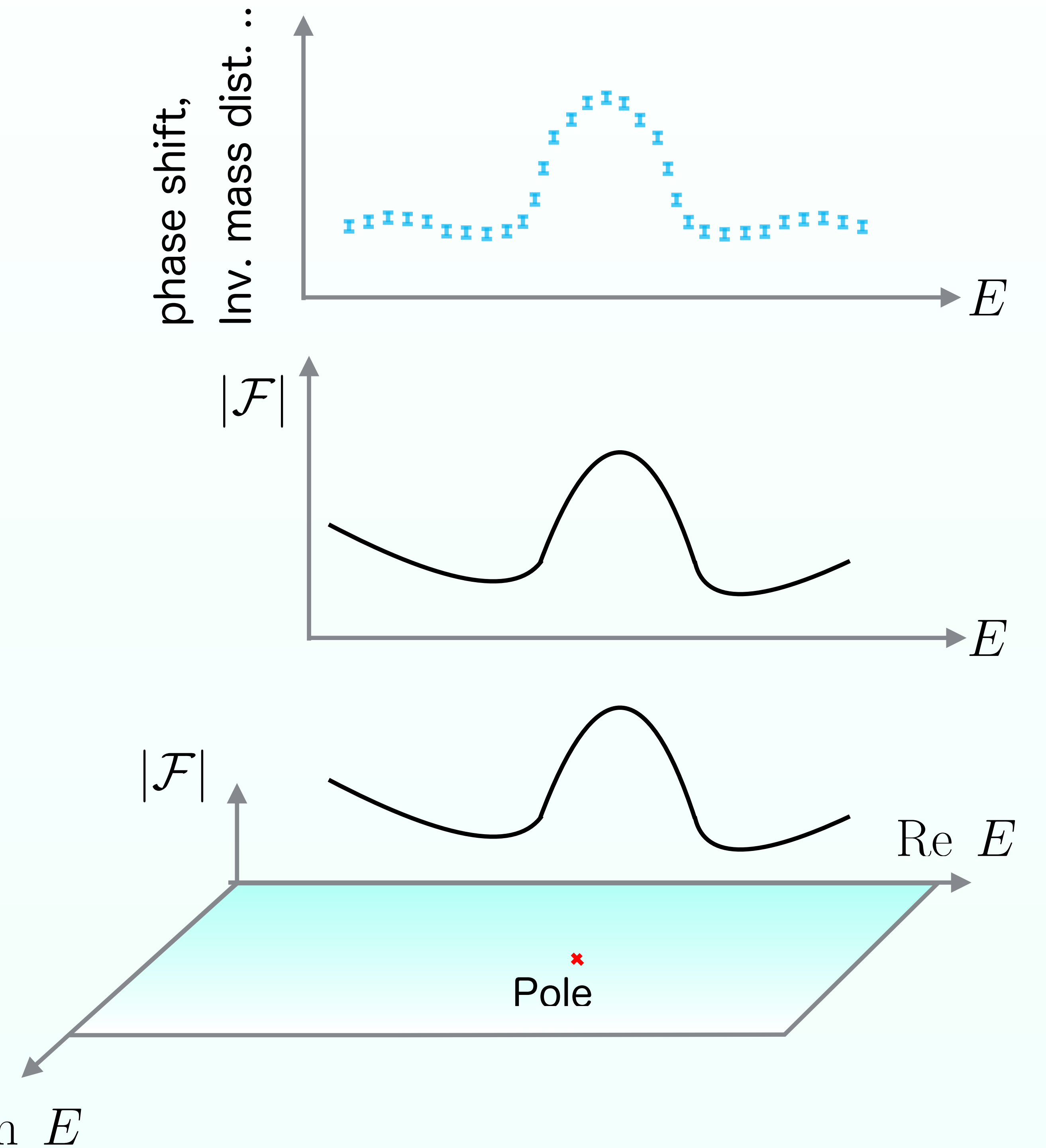
Experimental data

↓ detailed analysis

Scattering amplitude
on the physical real energy

↓ analytical continuation

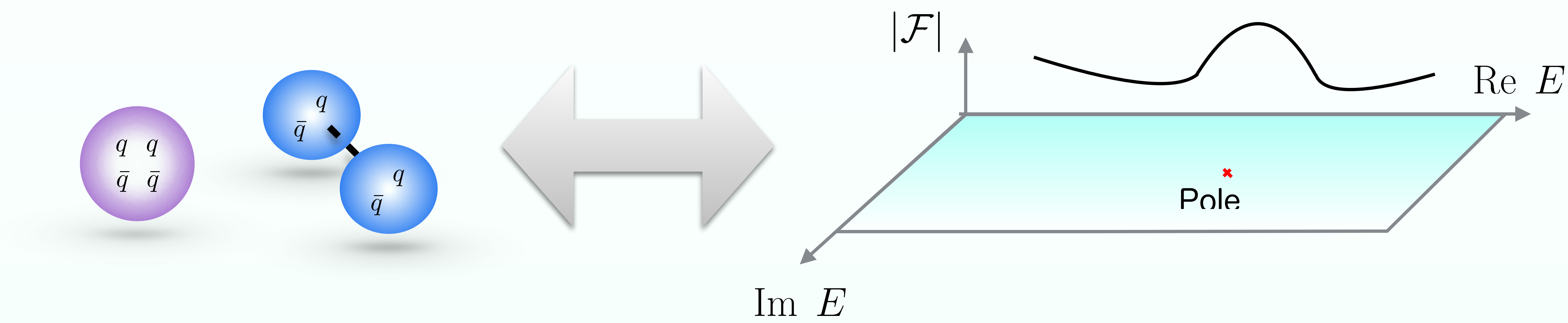
Scattering amplitude
in the complex energy plane
(or pole structure)



Introduction ~Methods to study structure~

How to extract the structure from the experimental data

- Relation between structure and amplitude



- Pole counting method

  Qualitative judgement from the positions of the shadow poles.

  D. Morgan, Nucl. Phys. A543, 632 (1992)

- Evaluation of compositeness

  Quantitative indicator of the amount of the dynamical fraction of the internal structure

  S. Weinberg, Phys. Rev. 137, B672 (1965).

Introduction ~Methods to study structure~

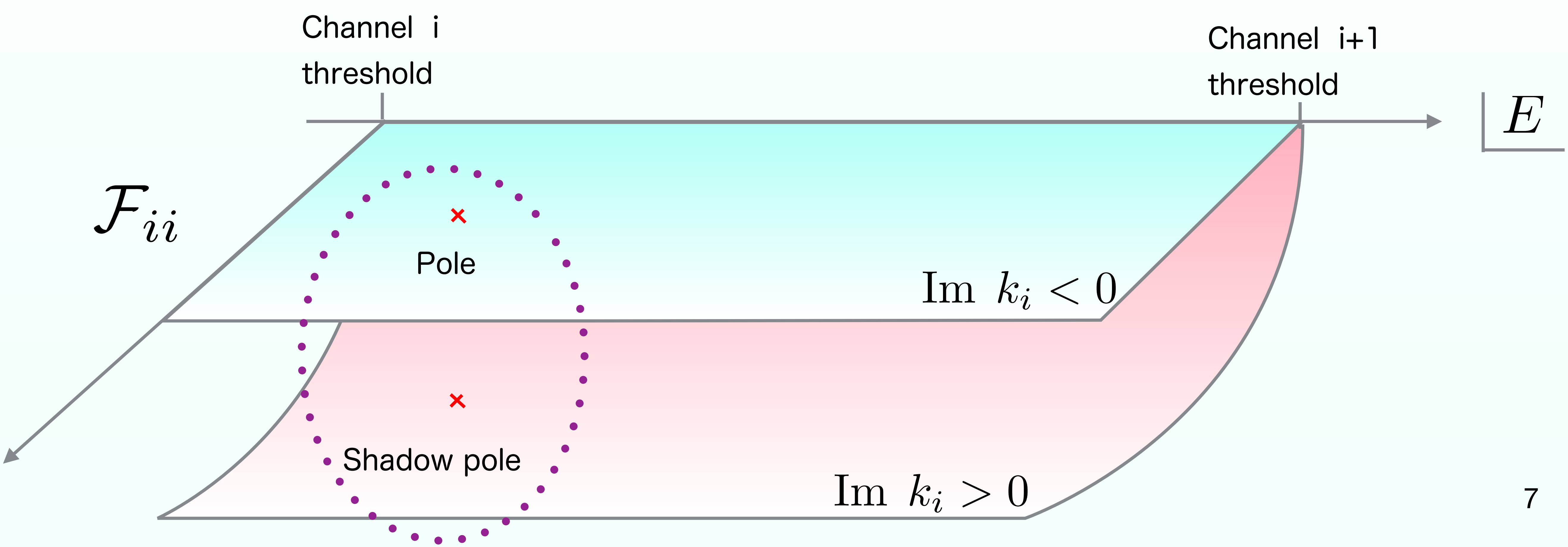
- Pole counting method
  - Qualitative judgement from the positions of the shadow poles.

D. Morgan, Nucl. Phys. A543, 632 (1992)

Does a shadow pole lie around the pole representing the eigenenergy?

Yes —> The focused channel is not the origin of the eigenstate.

No —> The focused channel is the origin of the eigenstate.



Introduction ~Methods to study structure~

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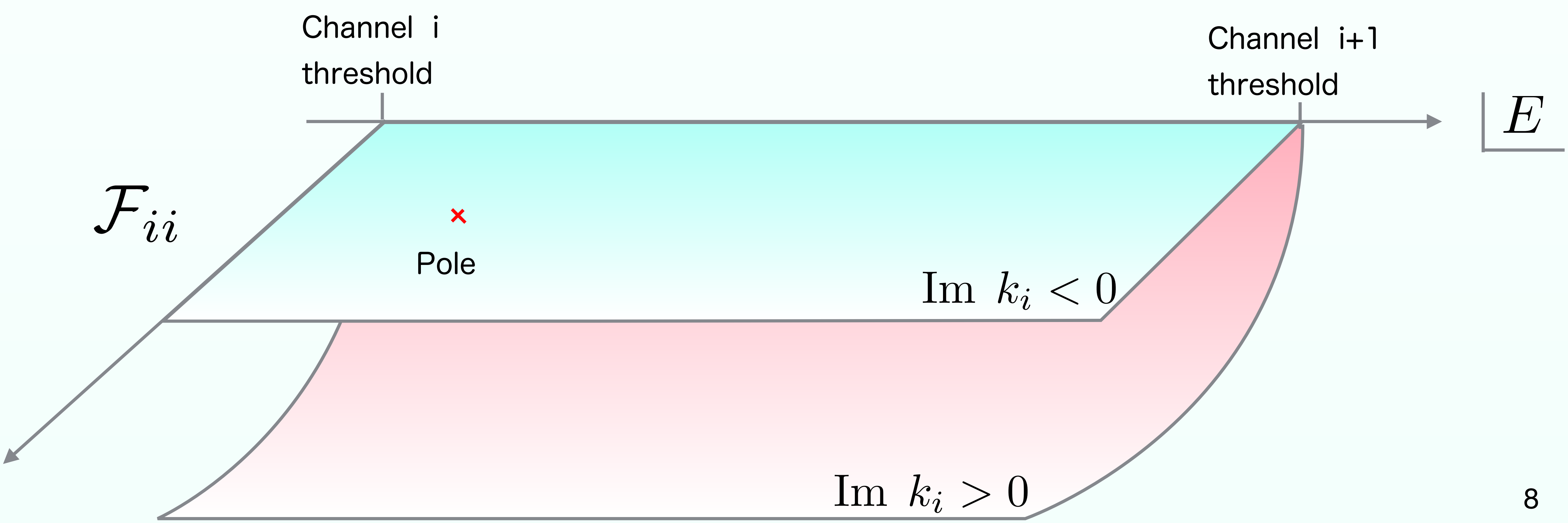


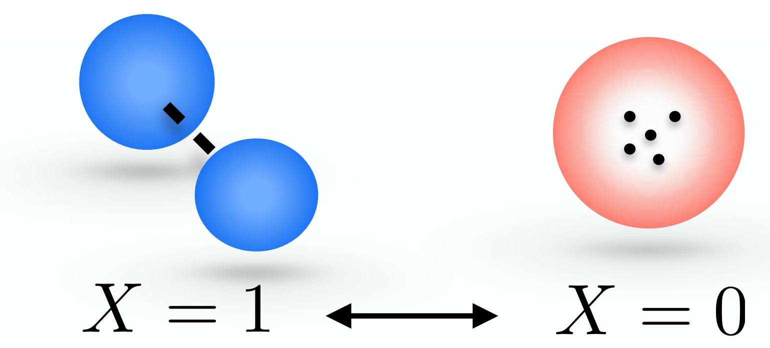
Introduction ~Methods to study structure~

Evaluation of compositeness X (or Weinberg's Z=1-X)

S. Weinberg, Phys. Rev. 137, B672 (1965).

- Quantitative indicator
  - Amount of the dynamical fraction of the internal structure
  - 0 < X < 1 (for stable states)

    —> X can be regarded as the probability.



- Evaluation method
  - Determination with the eigenenergy and residue of the pole

$$X_i = -g_i^2 G_i'(E_h)$$

$G_i(E)$: loop function of channel i

g_i^2 : residue of eigenstate pole of channel i

T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C 85, 015201 (2012

F. Aceti and E. Oset, Phys. Rev. D 86, 014012 (2012)

T. Sekihara, T. Hyodo, and D. Jido, PTEP 2015, 063D04 (2015)

Z.-H. Guo and J. A. Oller, Phys. Rev. D 93, 096001 (2016)

  - Determination with Weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}(R_{\rm typ}/R)\right\}$$

a_0 ; scattering length

B ; binding energy

$$R = \frac{1}{\sqrt{2\mu B}}$$

S. Weinberg, Phys. Rev. 137, B672 (1965).

Y. Kamiya and T. Hyodo, Phys. Rev. C 93, 035203 (2016)

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017)

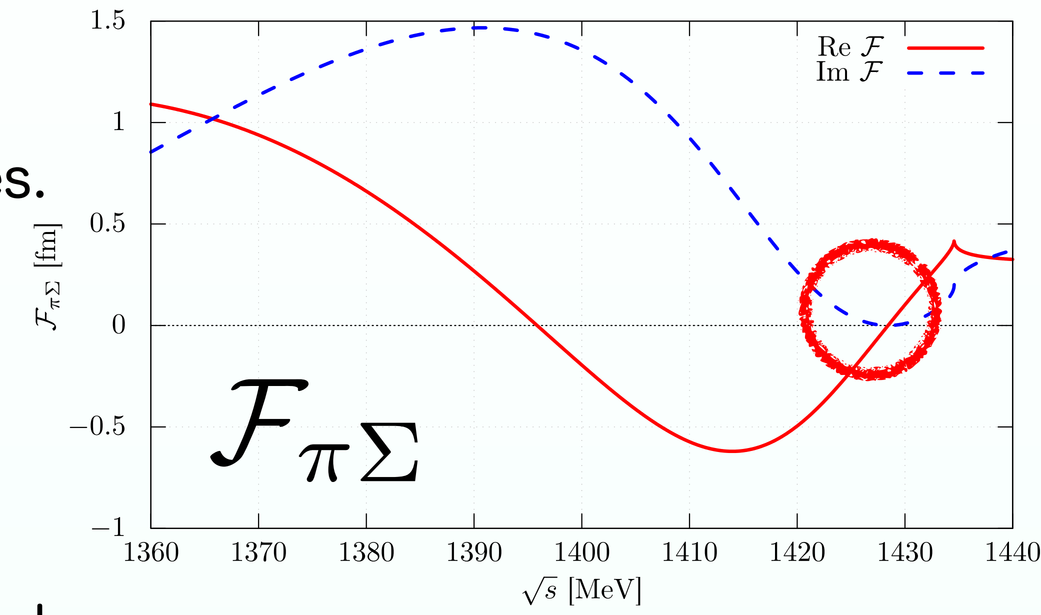
Introduction ~CDD zero~

Castillejo Dalitz Dyson (CDD) Zero

- Energy point where the scattering amplitude F(E) vanishes.

CDD zero : $\mathcal{F}_{ii}(E_C) = 0$

L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. 101, 453 (1956).



- Existence indicates the contribution from outside the model space.

G. F. Chew and S. C. Frautschi, Phys. Rev. 124, 264 (1961).

- CDD zero on $\pi\Sigma$c amplitude

“CDD zero accompanied by nearby $\pi\Sigma$c thresholds performs the crucial role to reproduce the mass and width of Λc(2595).”

Z.-H. Guo and J. A. Oller, Phys. Rev. D93, 054014 (2016), 1601.00862.

- For a coupled-channel problem, both the existence and position depend on the channel.

c. f. The eigenstate pole lies the same position in the every coupled channel.

→ Can we extract information of the internal structure of the eigenstate from the position of the CDD zero?

ZCL of coupled channel amplitude

To investigate the origins of the eigenstate,
we consider the zero coupling limit of the coupled channel scattering amplitude.

Zero Coupling Limit (ZCL)

Switch off the inter-channel coupling in V_{ij}

$$V_{ij} = \begin{pmatrix} V_{11} & V_{12} & \cdots & V_{n1} \\ V_{12} & V_{22} & \cdots & V_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ V_{n1} & V_{n2} & \cdots & V_{nn} \end{pmatrix} \to \begin{pmatrix} V_{11} & 0 & \cdots & 0 \\ 0 & V_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_{nn} \end{pmatrix}$$

Poles and CDD zeros in the ZCL

In the ZCL, the pole exits only in the scattering amplitude of the channel whose interaction is the origin of the eigenstate.

(1) Interaction V_{ii} is the origin of the state. —> The pole remains in F_{ii}.

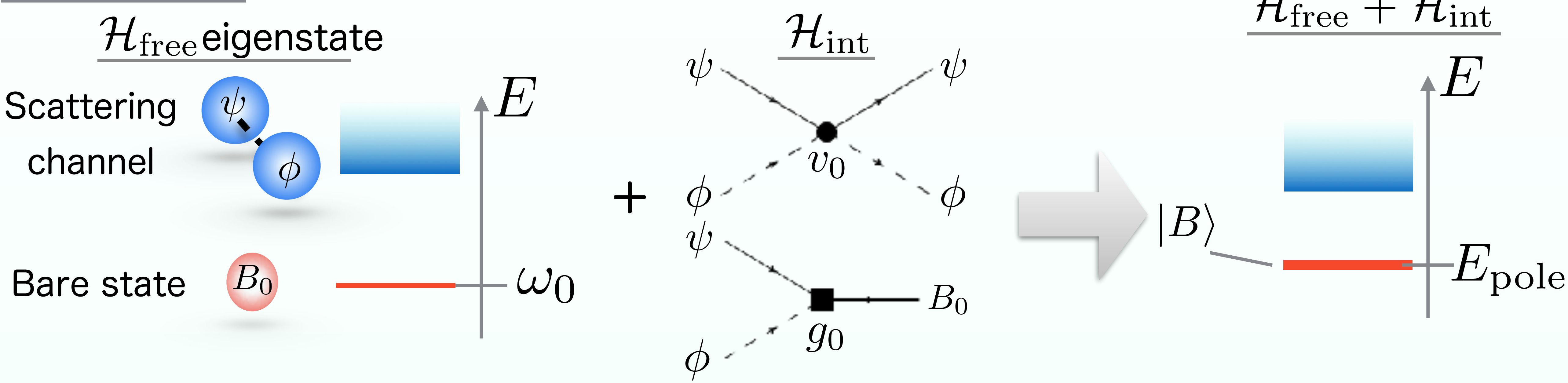
(2) Interaction V_{ii} is not the origin of the state. —> The pole decouples from F_{ii}.

・How about the behavior of CDD zero?

ZCL of coupled channel amplitude

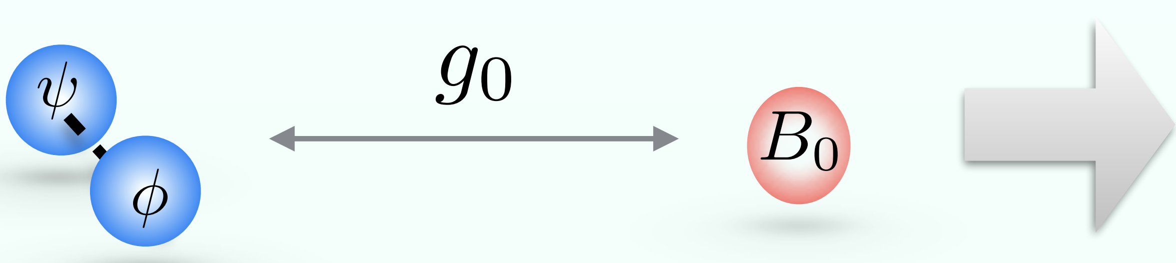
Using non-relativistic field theory as a specific example,
we study the behavior of the pole and CDD zero in the ZCL

Hamiltonian



Zero coupling limit

g_0 gives the coupling between the scattering channel and bare state channel.



ZCL : $g_0 \to 0$

ZCL of coupled channel amplitude

Using non-relativistic field theory as a specific example,
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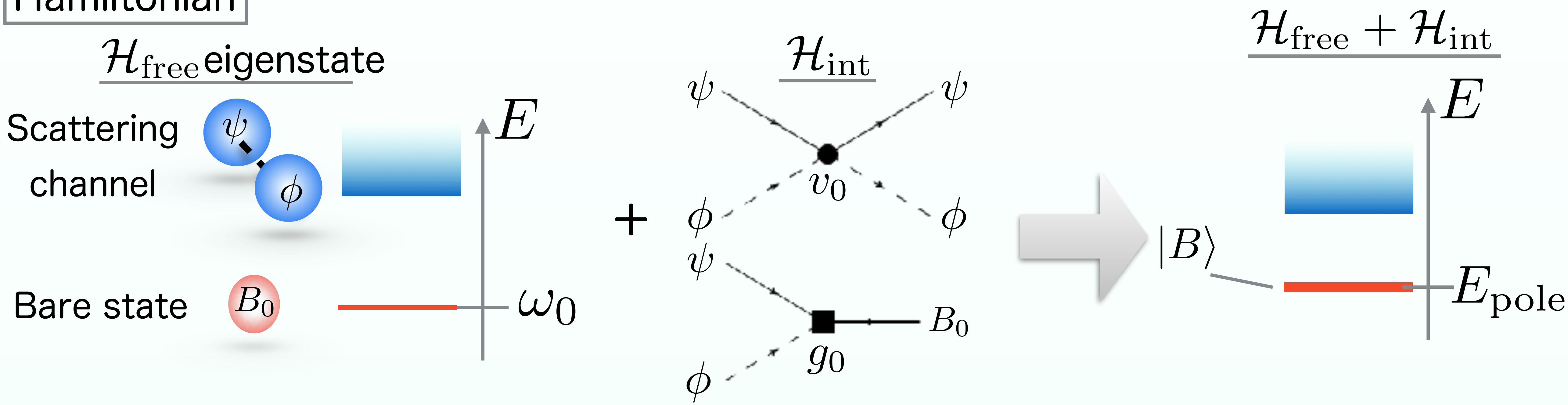
Hamiltonian



T matrix : t(E)

Lippmann-Schwinger Eq. —>

$$t(E) = \frac{(E-\omega_0)v_0 + g_0^2}{(E-\omega_0)(1-v_0G(E)) - g_0^2G(E)}$$

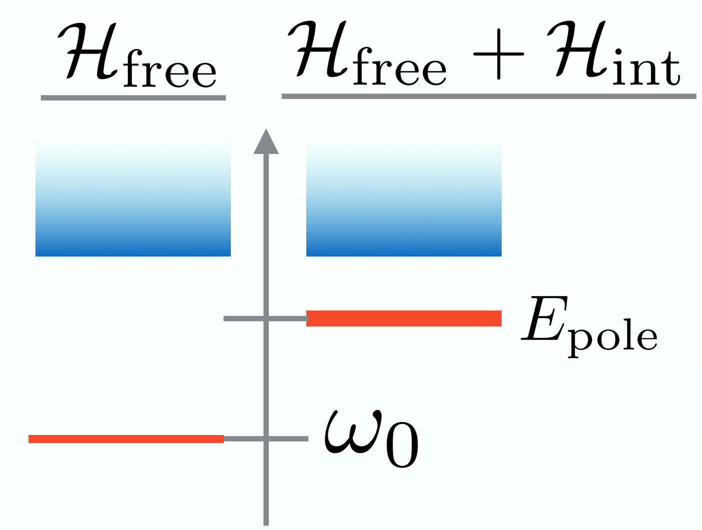
Numerator $= 0$ → CDD zero : $E_C = \omega_0 - g_0^2/v_0$

Denominator $= 0$ → Pole : $(E_{\rm pole}-\omega_0)(1-v_0G(E_{\rm pole})) - g_0^2G(E_{\rm pole}) = 0$

ZCL of coupled channel amplitude

Behavior of pole in ZCL

Pole : $(E_{\text{pole}} - \omega_0)(1 - v_0 G(E_{\text{pole}})) - g_0^2 G(E_{\text{pole}}) = 0$



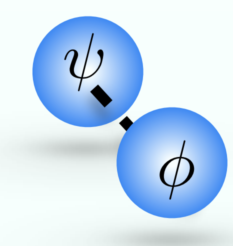
ZCL

(g_0 —> 0)

Pole : $(E_{\text{pole}} - \omega_0)(1 - v_0 G(E_{\text{pole}})) = 0$

Two scenarios about the behavior of the eigenstate pole

Scenario 1 : $1 - v_0 G(E_{\text{pole}}) \to 0$



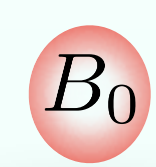
includes only the dynamical information

—> ψ ϕ dynamical origin

Scenario 2 : $E_{\text{pole}} \to \omega_0$

Eigenenergy moves toward the mass of bare state.

—> Bare state (B_0) origin



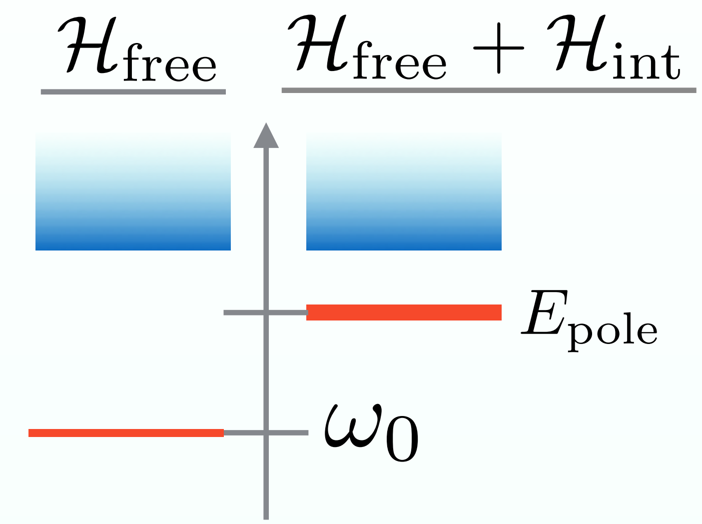
ZCL of coupled channel amplitude

Behavior of CDD zero in ZCL

CDD zero : $E_C = \omega_0 - g_0^2/v_0$

↓ ZCL (g_0 —> 0)

CDD zero : $E_C = \omega_0$



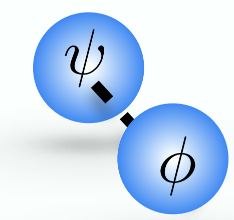
⇒ In any cases, CDD zero moves toward the mass of bare state(ω_0).

(The movement is independent of that of the pole)

ZCL of coupled channel amplitude

Pole : $(E_{\rm pole}-\omega_0)(1-v_0G(E_{\rm pole}))-g_0^2G(E_{\rm pole})=0$

CDD zero : $E_C=\omega_0-g_0^2/v_0$



Scenario 1

If the interaction of the scattering channel $\psi\,\phi$ is the origin of the eigenstate,

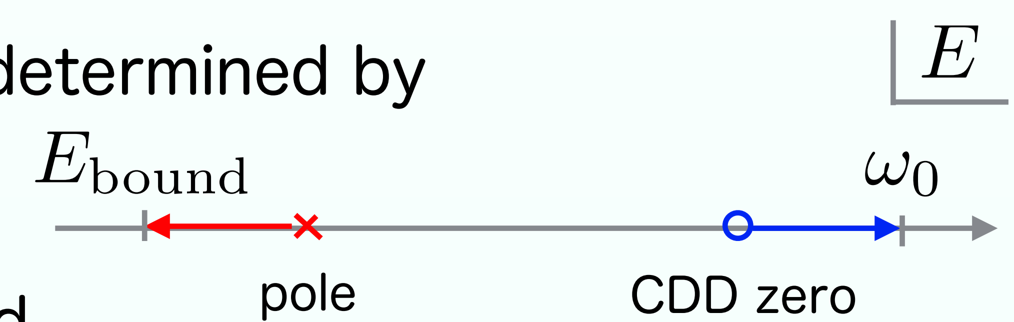
⇔ the eigenenergy of the state in the ZCL (g_0 —> 0) does not depend on the bare energy: $E_{\rm pole}\nrightarrow\omega_0$

- Pole

  moves toward the binding energy $E_{\rm bound}$ determined by

  $$1-v_0G(E_{\rm bound})=0$$

  ↔ The eigenstate is dynamically generated.



- CDD zero

  $E_C\to\omega_0$ (The movement is independent of $E_{\rm bound}$)

⇒ The position of CDD zero is independent of that of the pole.

ZCL of coupled channel amplitude

Pole : $(E_{\rm pole}-\omega_0)(1-v_0G(E_{\rm pole}))-g_0^2G(E_{\rm pole})=0$

CDD zero : $E_C=\omega_0-g_0^2/v_0$

Scenario 2

B_0

If the bare state B_0 is the origin of the eigenstate,

⟺ the pole moves toward the bare state energy ω_0 in the ZCL (g_0—>0) and vanishes in the exact ZCL (g_0=0).

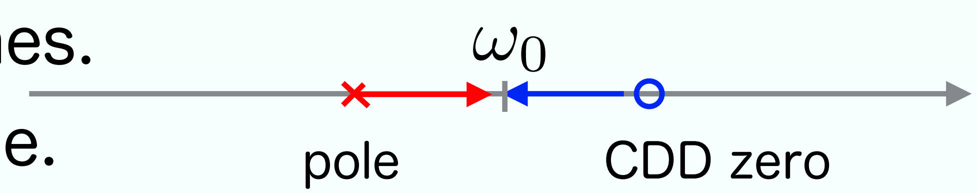
- Pole

  $E_{\rm pole}\to\omega_0$

  In this limit, the residue of the pole vanishes.

  —> The pole decouples from the amplitude.

E



- CDD zero

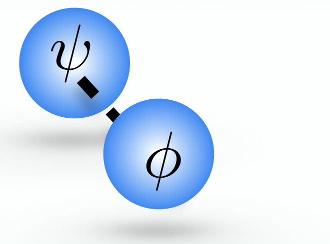
  $E_C\to\omega_0$ (The behavior is the same as the Scenario 1)

The pole and CDD zero encounter with each other at E=ω_0.

⇒ The pole and CDD zero cancels out with each other to decouple from the scattering amplitude.

ZCL of coupled channel amplitude

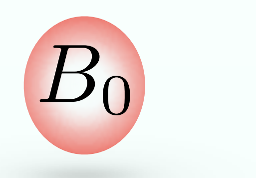
Scenario 1 (Dynamical origin)

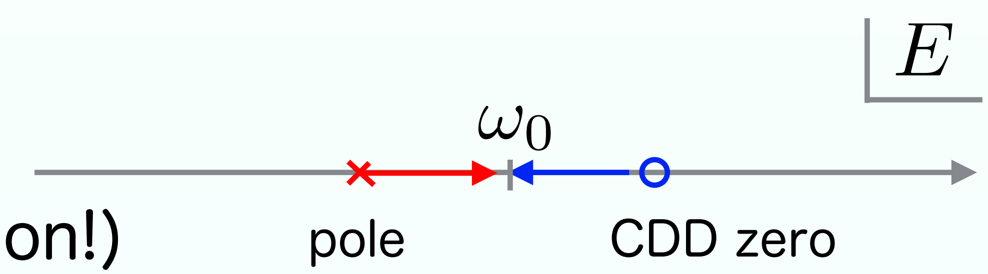


- $1 - v_0 G(E_{\rm pole}) \to 0$ in ZCL
- $E_C \to \omega_0$ (The movement is independent of that of pole)

The position of CDD zero is independent of that of the pole.

Scenario 2 (Bare state origin)





- $E_{\rm pole}, E_C \to \omega_0$ in ZCL (Same destination!)
- The residue of the pole vanishes in the exact ZCL (g_0=0).

—> The pole decouples from the amplitude.

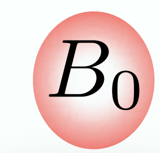
The pole and CDD zero cancels out with each other to decouple from the scattering amplitude.

ZCL of coupled channel amplitude

Pole : $(E_{\text{pole}} - \omega_0)(1 - v_0 G(E_{\text{pole}})) - g_0^2 G(E_{\text{pole}}) = 0$

CDD zero : $E_C = \omega_0 - g_0^2 / v_0$

Scenario 2



If the bare state B0 is the origin of the eigenstate,

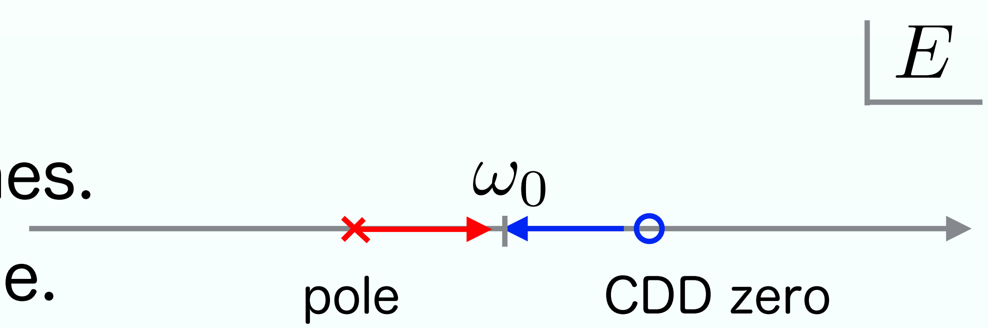
⇔ the pole moves toward the bare state energy ω_0 in the ZCL (g_0—>0) and vanishes in the exact ZCL (g_0=0).

- Pole

  $E_{\text{pole}} \to \omega_0$

  In this limit, the residue of the pole vanishes.

  —> The pole decouples from the amplitude.



- CDD zero

  $E_C \to \omega_0$ (The behavior is the same as the Scenario 1)

The pole and CDD zero encounter with each other at E=ω_0.

⇒ The pole and CDD zero cancels out with each other to decouple from the scattering amplitude.

Origin of eigenstate and nearby CDD zero

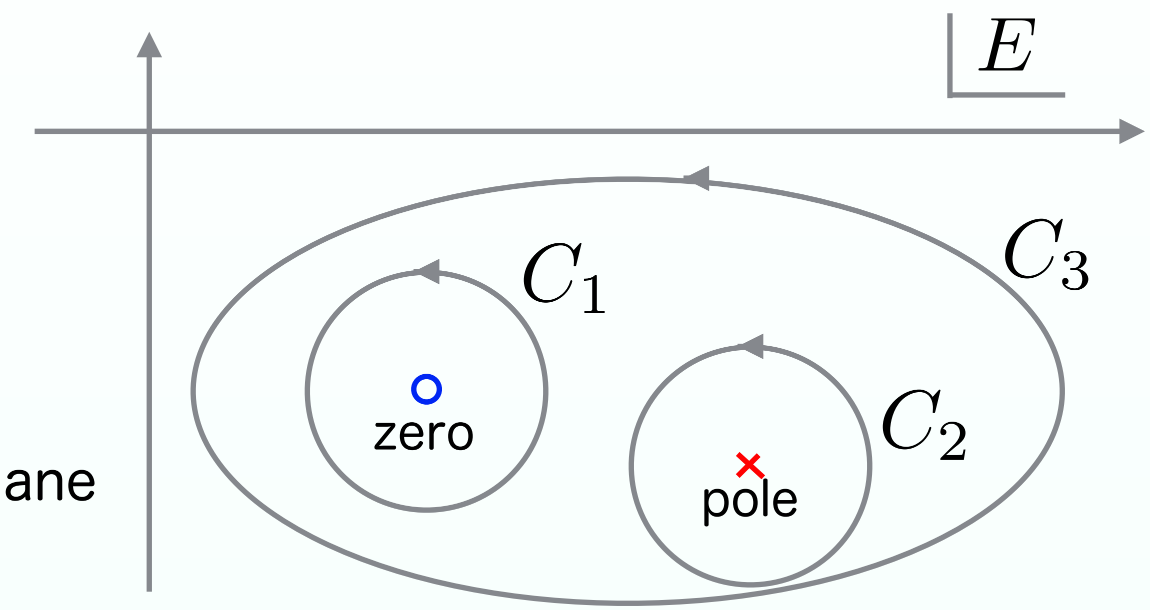
- Principle of argument of scattering amplitude

$$n_C = \frac{1}{2\pi}\oint_C dz \frac{d}{dz} \arg \mathcal{F}(z)$$

F(z) : Partial-wave scattering amplitude

C : Closed integration path in the complex energy plane

(No poles and zeros lie on Path C)



$$\bullet\ n_C = (\#\text{ of CDD zeros in } \mathcal{C}) - (\#\text{ of poles in } \mathcal{C}) \in \mathbb{Z}$$

$$n_{C_1} = 1$$
$$n_{C_2} = -1$$
$$n_{C_3} = 0$$

Origin of eigenstate and nearby CDD zero

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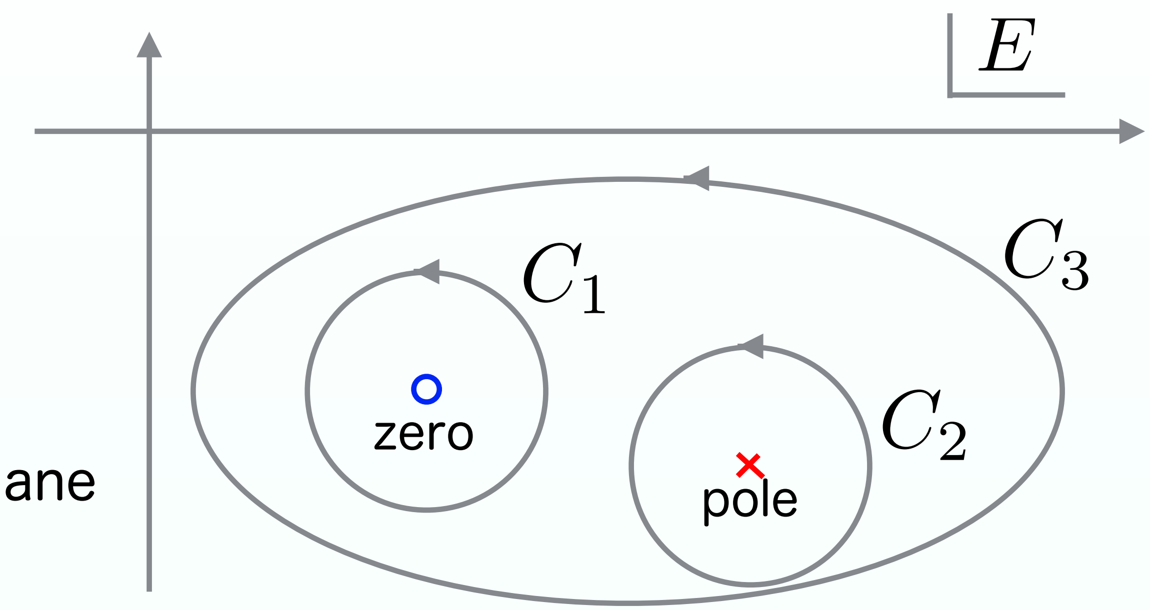

- $n_C = (\#\text{ of CDD zeros in } \mathcal{C}) - (\#\text{ of poles in } \mathcal{C}) \in \mathbb{Z}$

$n_{C_1} = 1$

$n_{C_2} = -1$

$n_{C_3} = 0$

- Topological invariant ($\pi_1(U(1)) \cong \mathbb{Z}$)

—> nc is invariant under the continuous variation of amplitude (e.g. ZCL).

- Sudden vanishment of a pole or zero (nC : ±1 —> 0) is prohibited.
- The pair annihilation of a pole and a CDD zeros does not change nc.

Pole and CDD zero must encounter with each other to decouple from the scattering amplitude.

Origin of eigenstate and nearby CDD zero

- Behavior of pole of scattering amplitude $F_{ii}(E)$ in channel i in the ZCL:

  Interaction of channel i V_{ii} is origin of the state —> Pole remains in F_{ii}.

  Otherwise —> Pole decouples from F_{ii}.

- To decouple from the amplitude $F_{ii}(E)$, pole must meet CDD zero.
- Pole and CDD zero move continuously in the continuous change of amplitude.

Origin of eigenstate and nearby CDD zero

- Behavior of pole of scattering amplitude $F_{ii}(E)$ in channel i in the ZCL:

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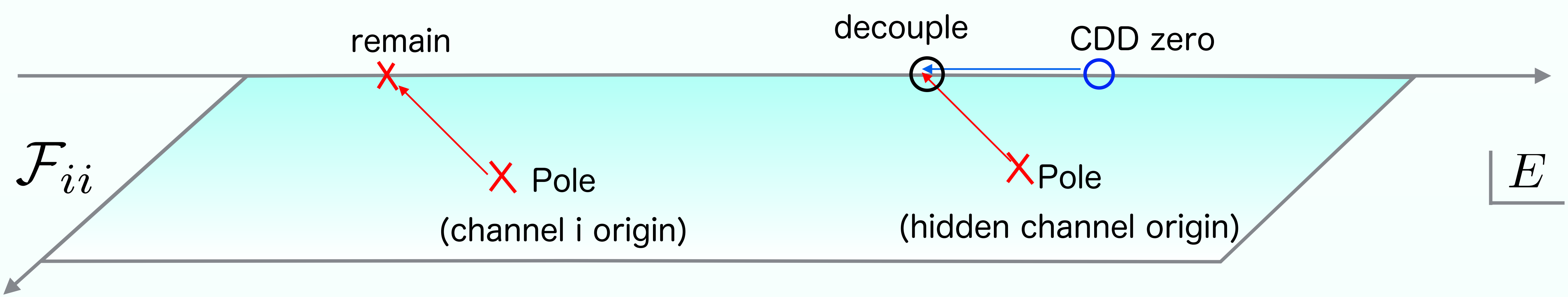
Taking the ZCL limit



Origin of eigenstate and nearby CDD zero

- Behavior of pole of scattering amplitude $F_{ii}(E)$ in channel i in the ZCL:

  Interaction of channel i V_{ii} is origin of the state —> Pole remains in F_{ii}.

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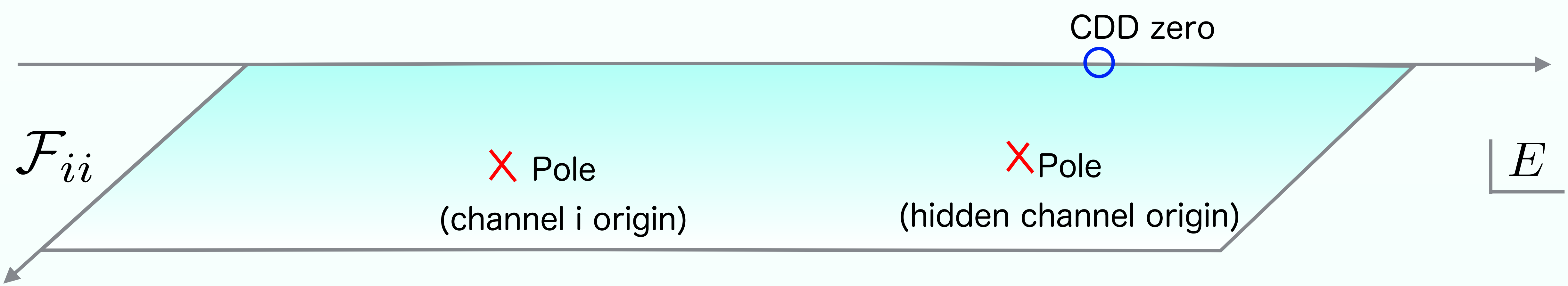
- To decouple from the amplitude $F_{ii}(E)$, pole must meet CDD zero.
- Pole and CDD zero move continuously in the continuous change of amplitude.

Before taking the ZCL limit



Origin of eigenstate	Near the pole in F_{ii}
Channel i	No nearby CDD zero
Not channel i (Hidden channel)	Nearby CDD zero

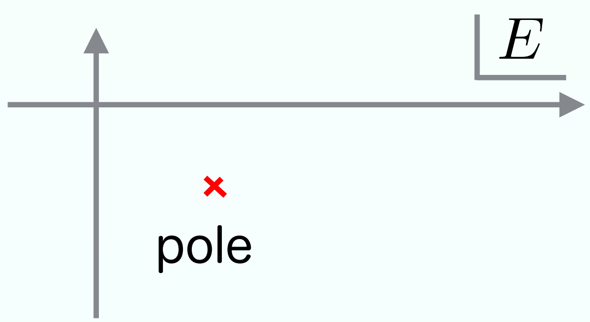
Origin of eigenstate and nearby CDD zero

Origin of eigenstate	Near the pole in F_{ii}
Channel i	No nearby CDD zero
Not channel i (Hidden channel)	Nearby CDD zero

This relation is useful to investigate the origin of the eigenstate.

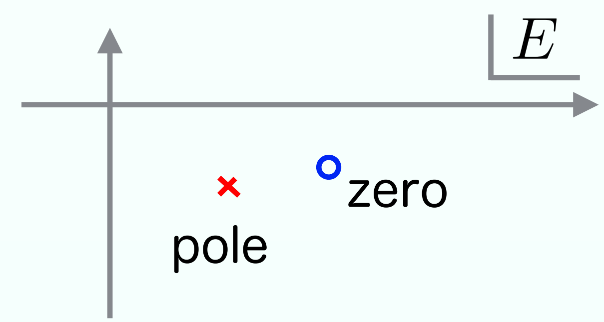
Y. Kamiya and T. Hyodo, Phys. Rev. D, 97, 054019 (2018).

(1) CDD zero does not lie around the pole.



The eigenstate is dynamically generated in channel i.

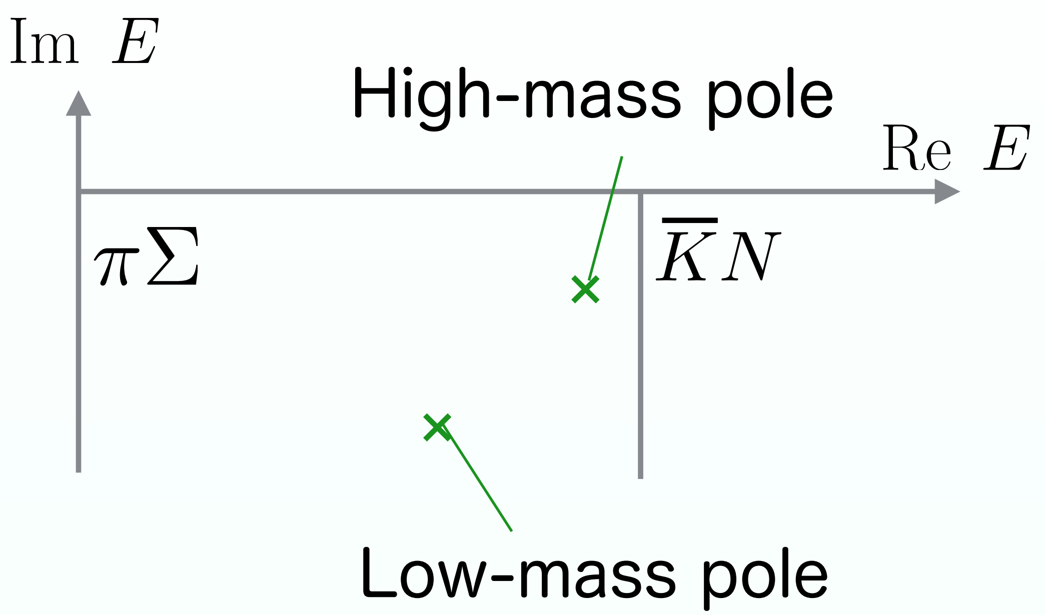
(2) Pole has a nearby CDD zero.



The origin of the eigenstate is not in channel i.

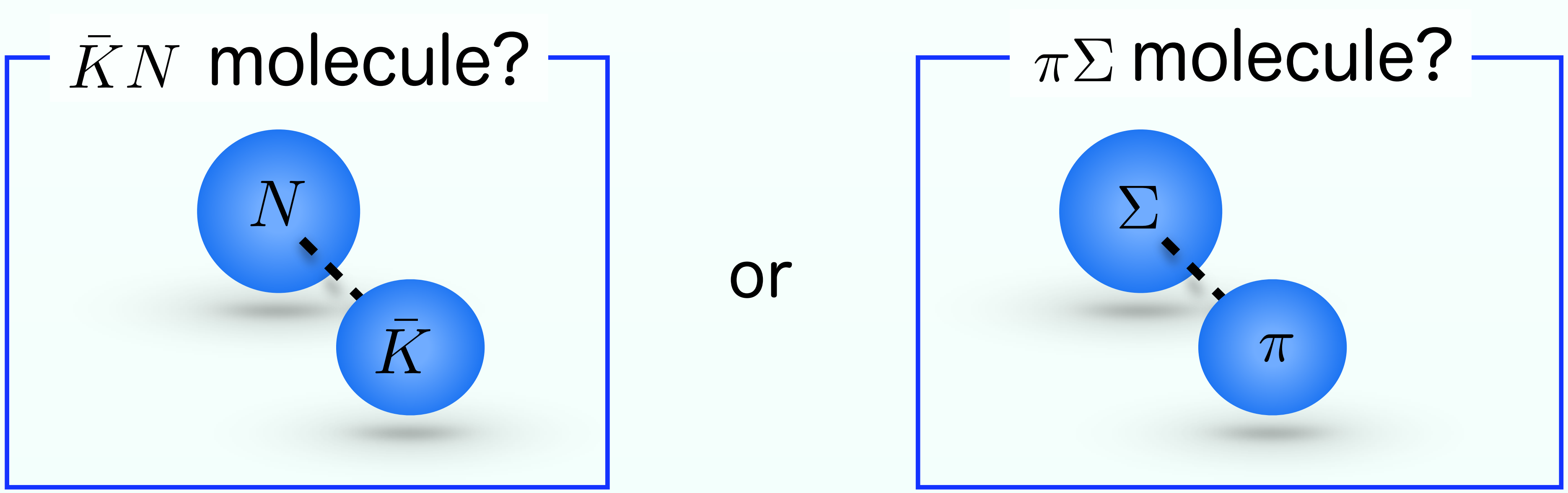
Application to Λ(1405)

- $\Lambda(1405)$ ($I = 0$ $\bar{K}N$ scattering)
  - $J^P = \frac{1}{2}^-$
  - Analysis with chiral dynamics
    —> Two pole structure
    D. Jido et al Nucl. Phys. A 725, 181 (2003)



Recent determinations are tabulated in PDG

C. Patrignani et al. (Particle Data Group), Chin. Phys. C40, 100001 (2016)



Application to Λ(1405)

Effective Tomozawa-Weinberg model

Y. Ikeda, T. Hyodo, and W. Weise, Nucl. Phys. A881, 98 (2012)

- Isospin basis

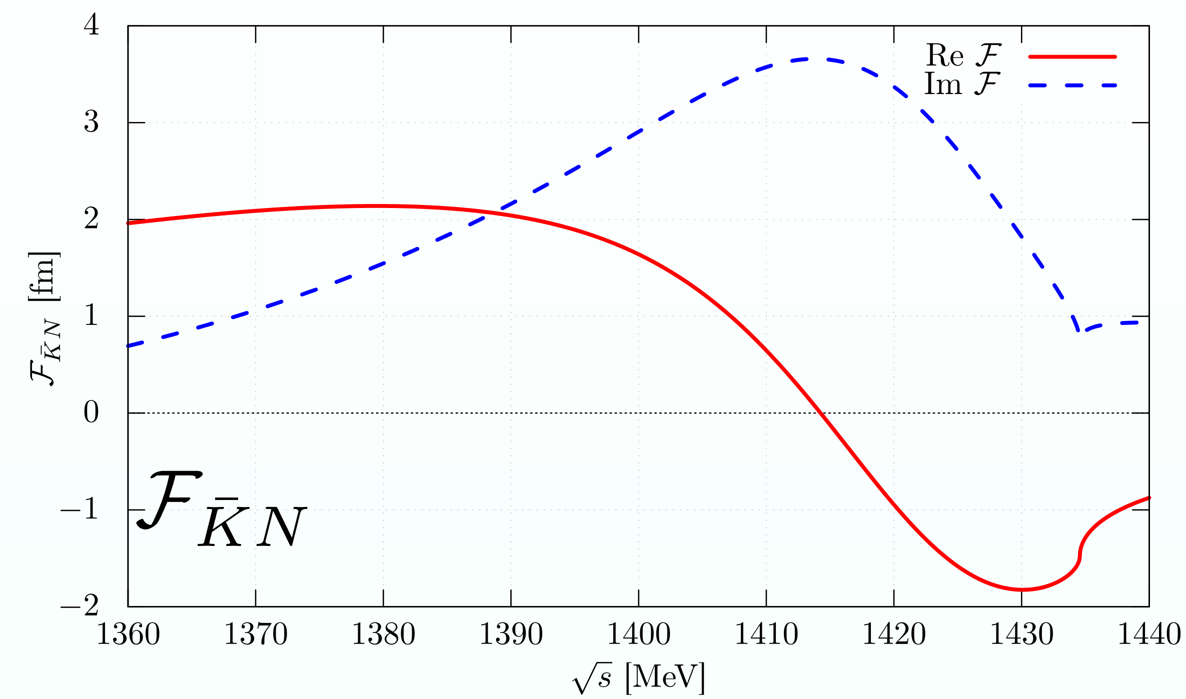
  Coupled channel : $\bar{K}N$-$\pi\Sigma$

- interaction : Tomozawa-Weinberg interaction

- Pole position;

  High-mass pole ; $1423 - 22i$ MeV

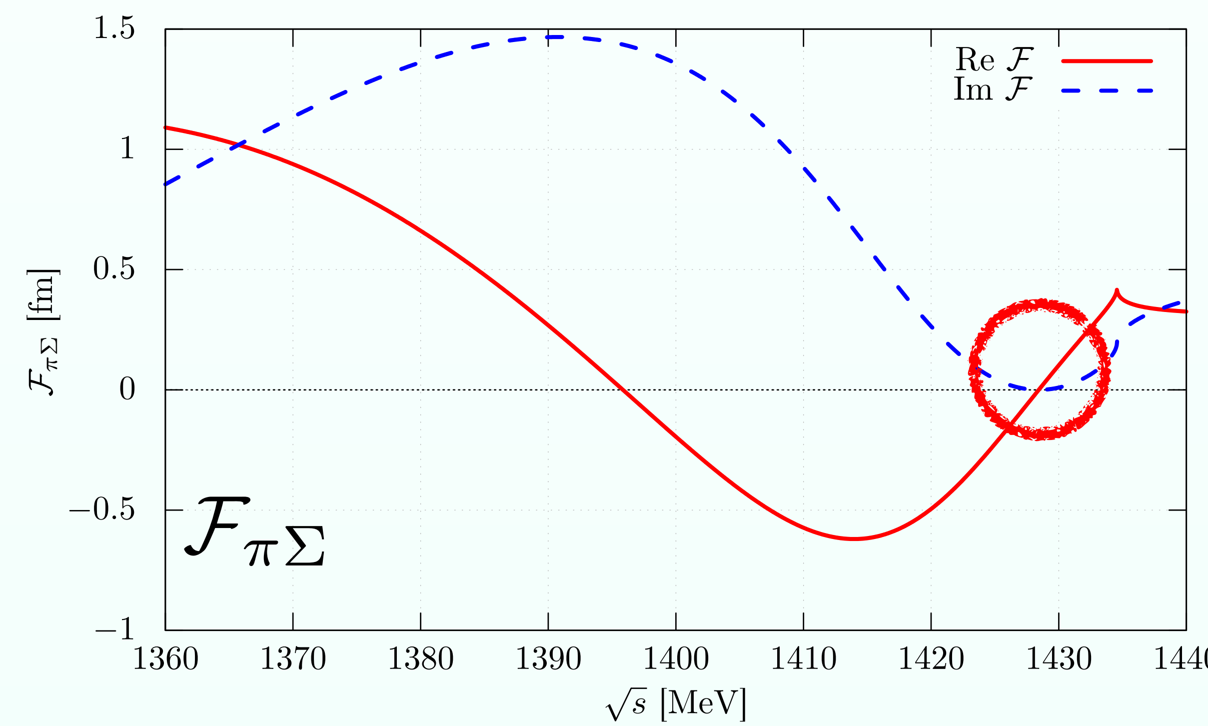
  Low-mass pole ; $1375 - 65i$ MeV



$\mathcal{F}_{\pi\Sigma}$

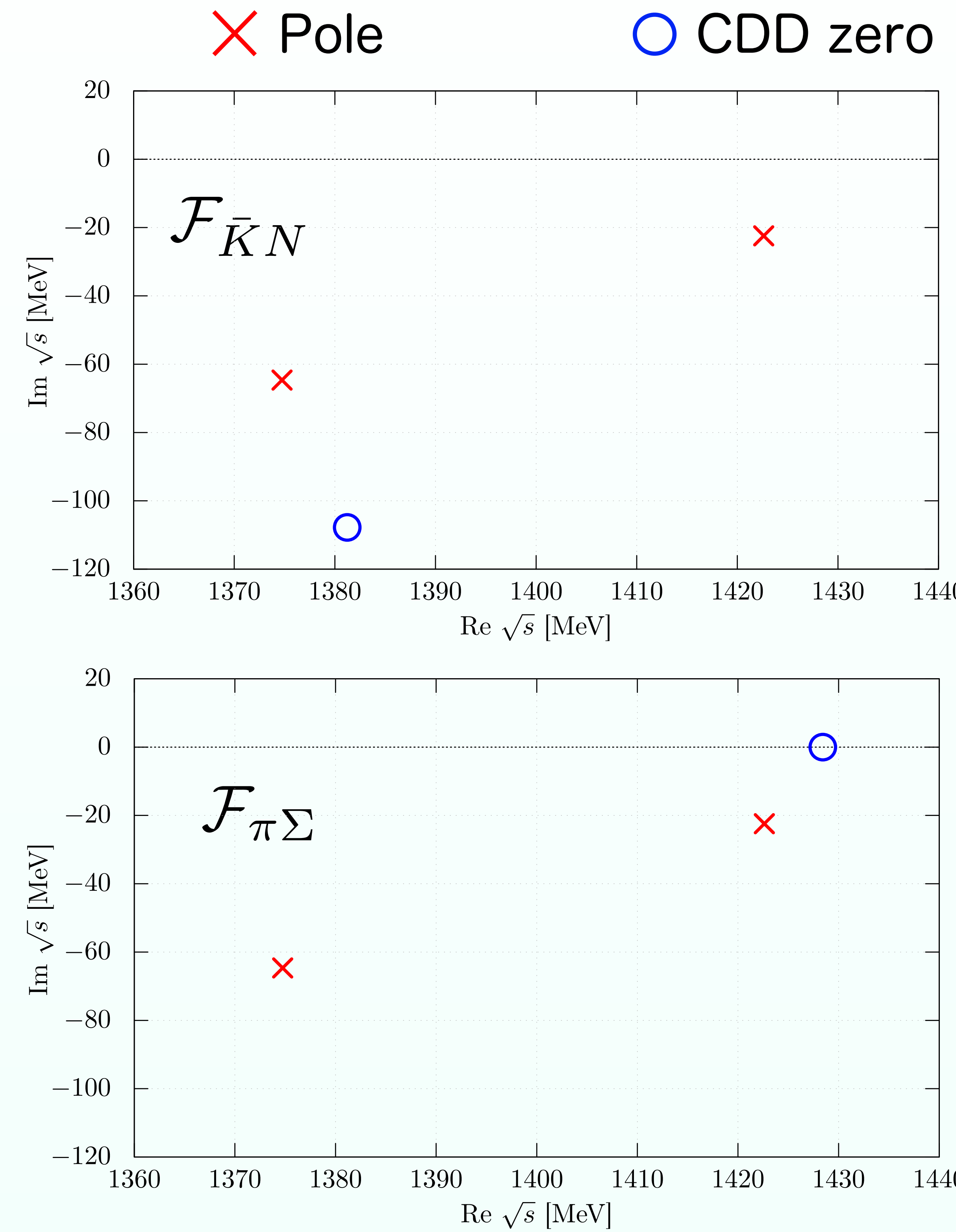
CDD zero lies near high-mass pole.

Y. K. and T. Hyodo, PTEP 2017, 023D02 (2017)



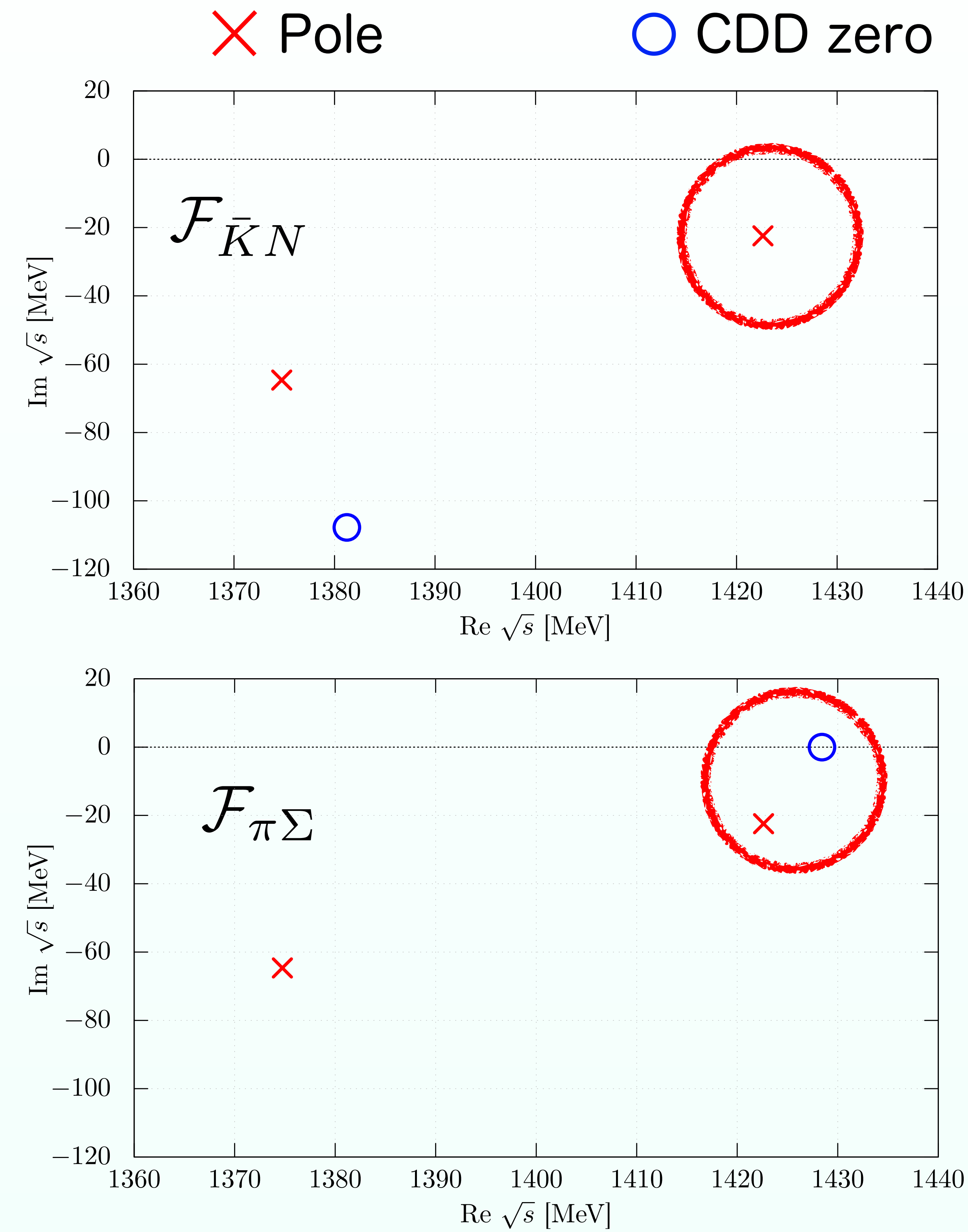
Application to Λ(1405)

Position of poles and CDD zeros



Application to Λ(1405)

Position of poles and CDD zeros



High-mass pole

No nearby CDD zero in $\bar{K}N$ amplitude

Nearby CDD zero in $\pi\Sigma$ amplitude

⇒ Origin is in $\bar{K}N$ channel.

Application to Λ(1405)

Position of poles and CDD zeros

High-mass pole

No nearby CDD zero in $\bar{K}N$ amplitude

Nearby CDD zero in $\pi\Sigma$ amplitude

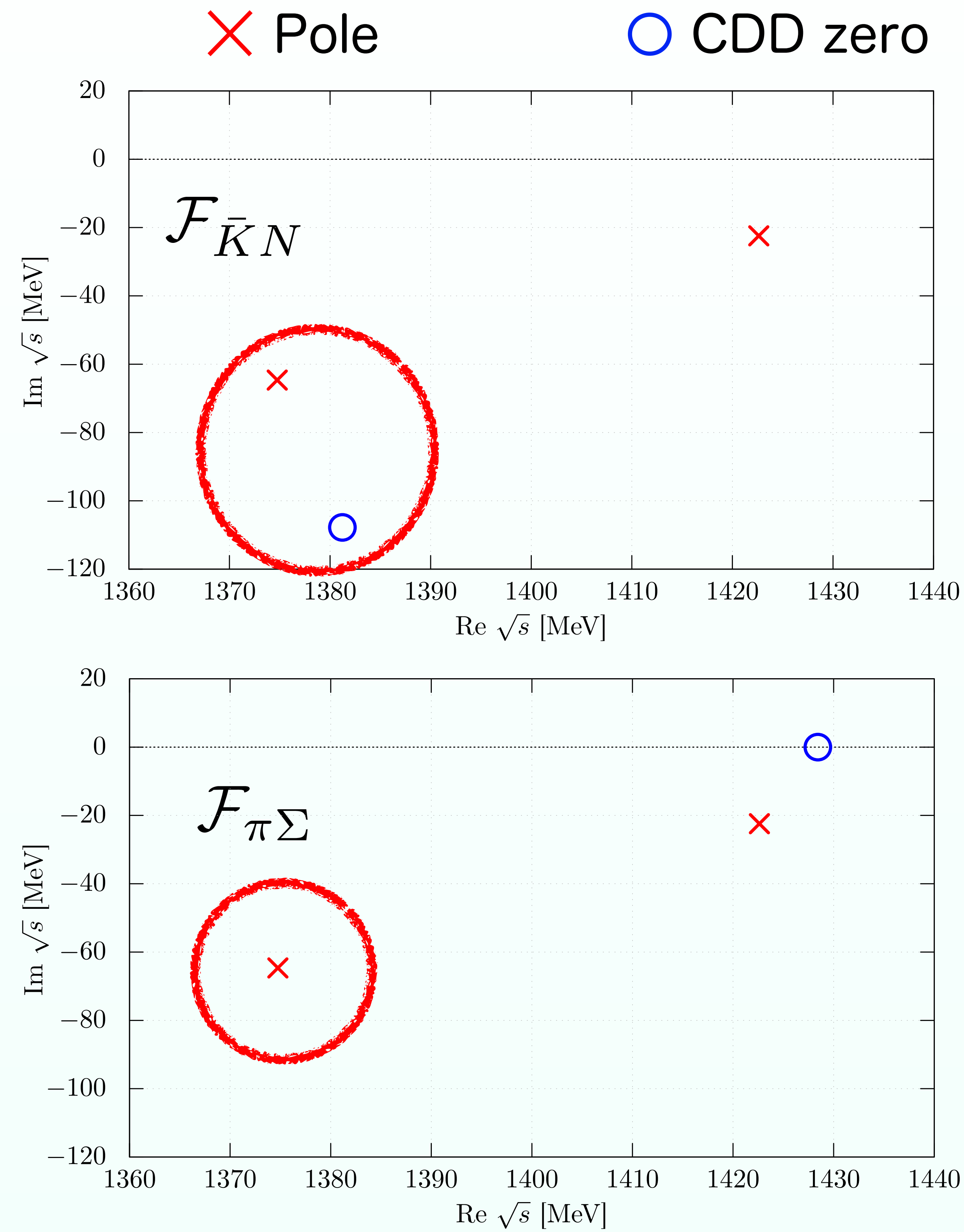
⇒ Origin is in $\bar{K}N$ channel.

Low-mass pole

Nearby CDD zero in $\bar{K}N$ amplitude

No Nearby CDD zero in $\pi\Sigma$ amplitude

⇒ Origin is in $\pi\Sigma$ channel.

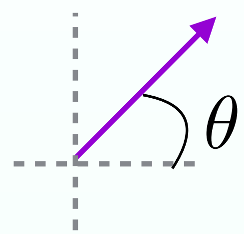


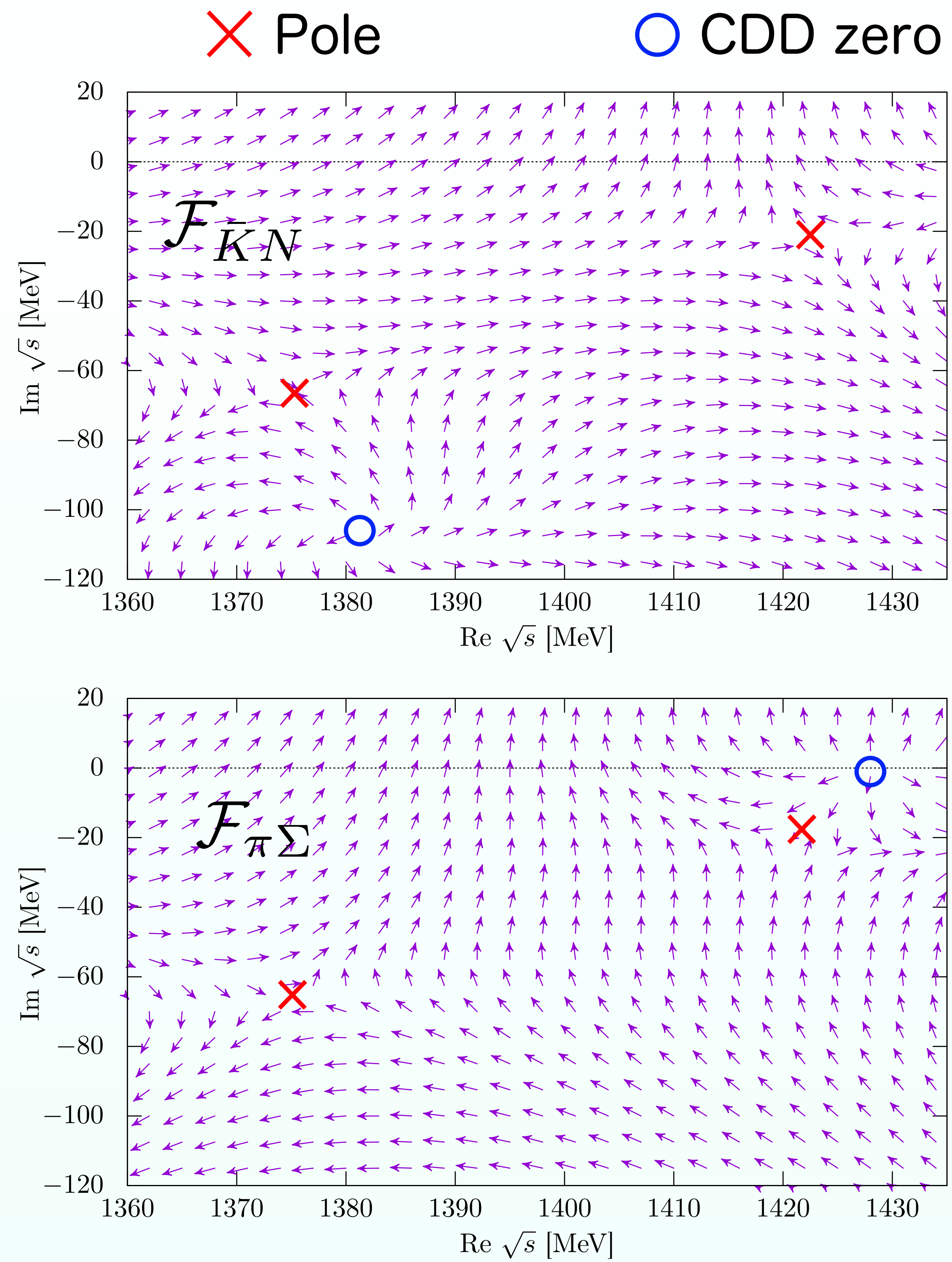
Application to Λ(1405)

Vector map of phase structure of amplitude

The existence of the pole and CDD zero can be confirmed by the phase structure of amplitude.

$$\mathcal{F}(\sqrt{s}) = |\mathcal{F}|e^{i\theta}$$





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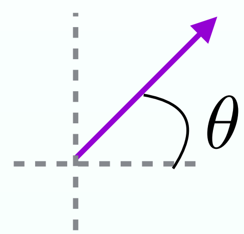
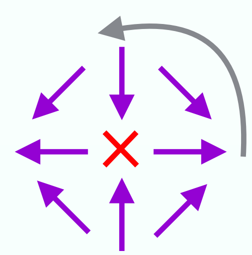
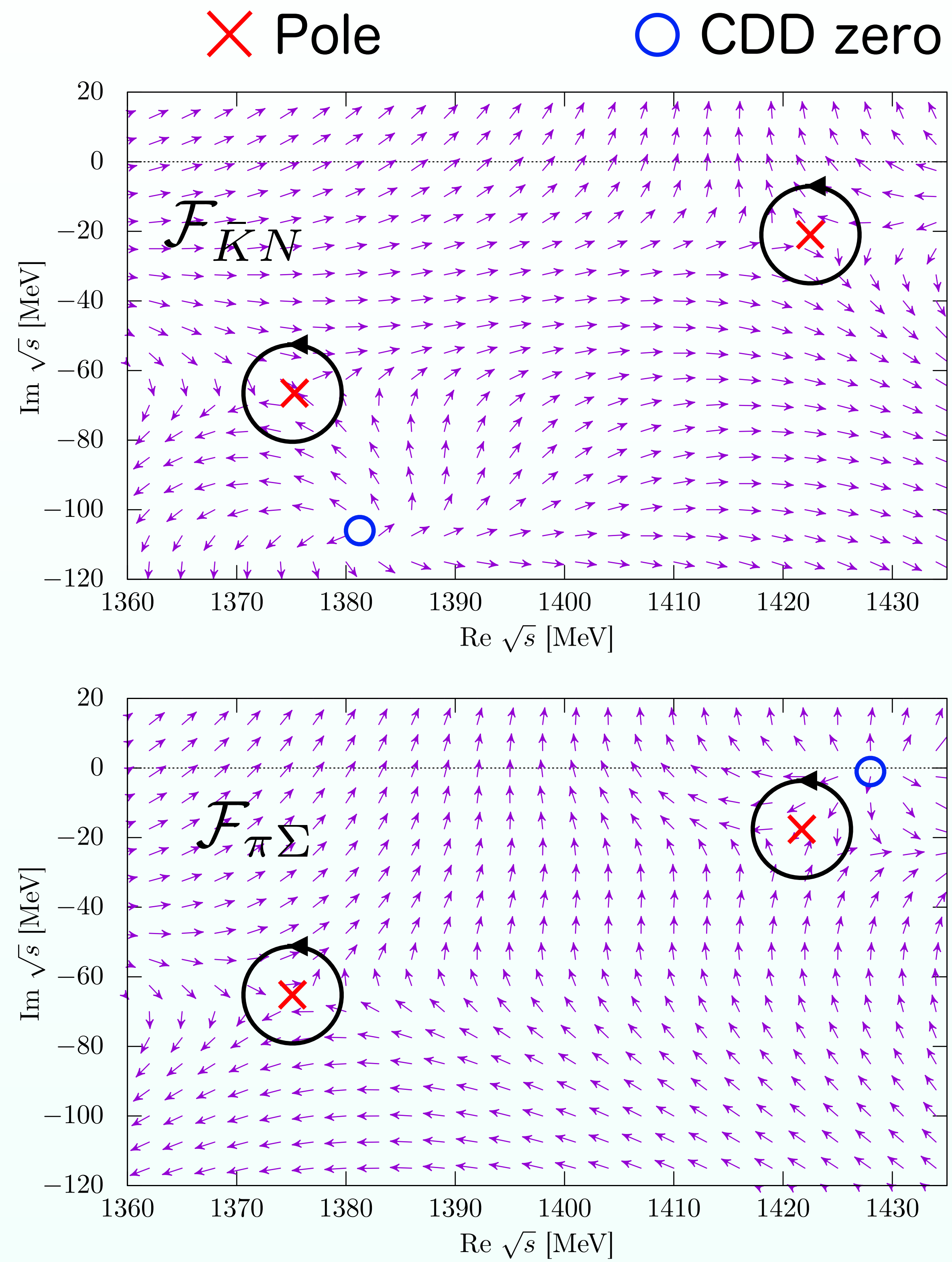


Along contour around pole



The vector of phase turns clockwise.



Application to Λ(1405)

Vector map of phase structure of amplitude

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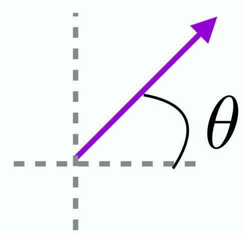
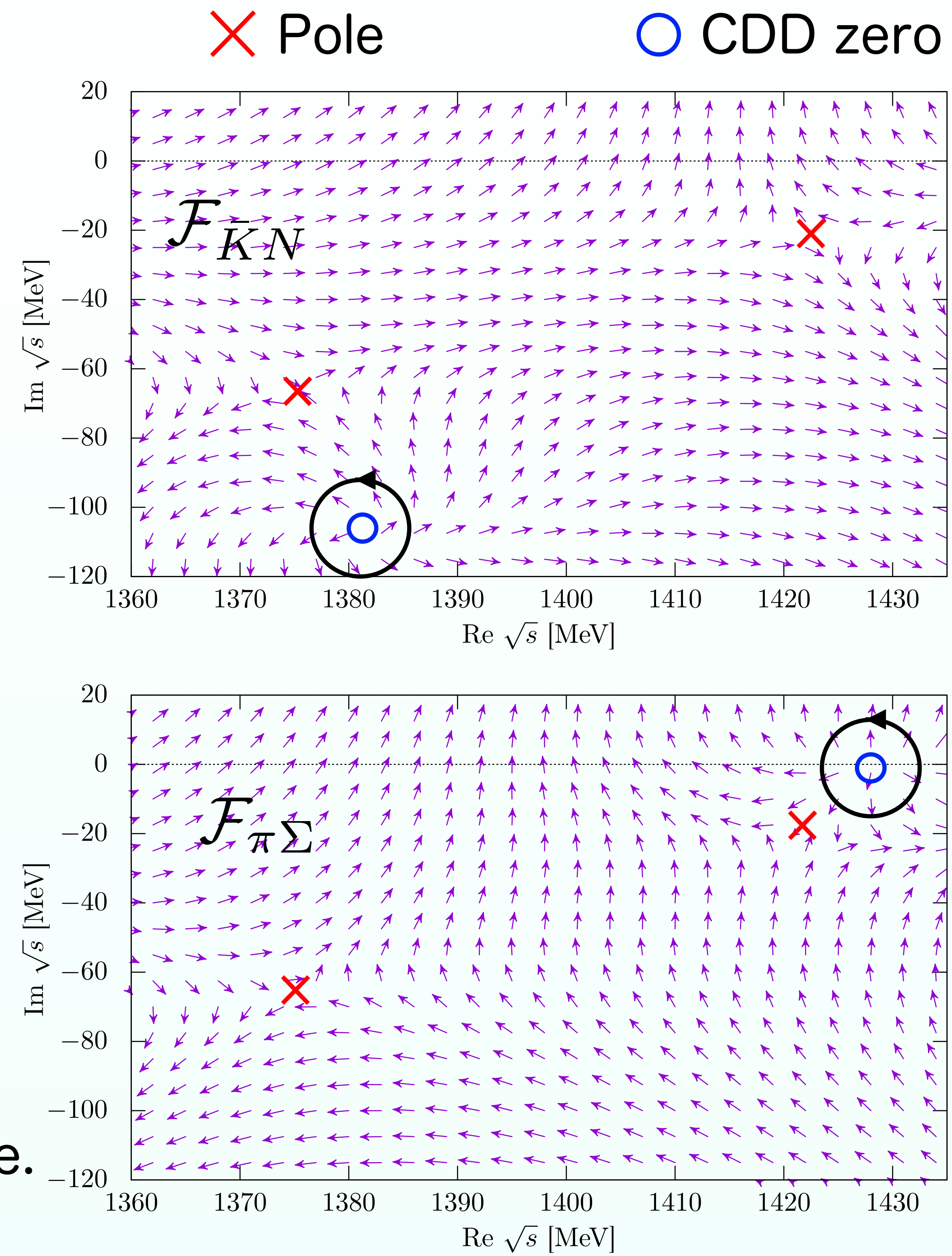
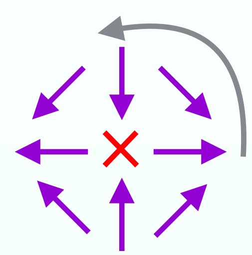


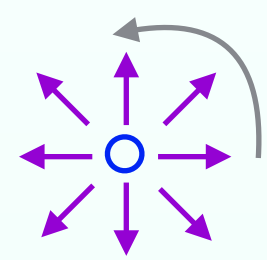


Along contour around pole



The vector of phase turns clockwise.

Along contour around CDD zero



The vector of phase turns counterclockwise.

Conclusion

- The eigenstate pole should decouple from the amplitude in the ZCL, if the eigenstate originates in the other channel.
- We show that the pole must annihilate with CDD zero to decouple.
- New method to study the origin of the eigenstate ;

(1) Pole without a nearby CDD zero —> Dynamical origin.

(2) Pole with a nearby CDD zero —> Origin is the other channel.

- Application to Λ(1405)

Y. Kamiya and T. Hyodo, PRD 97, 054019 (2018).