Nonperturbative Vacuum Polarization Effects in One- and Two-Dimensional Supercritical Dirac-Coulomb Systems

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Contents

History of Critical Charge

Point nucleus

- Sommerfeld-Dirac formula ($\hbar = c = 1$):

$$E_{nj} = m_e \left[1 + \left(\frac{Z\alpha}{n - |\kappa| + \left[\kappa^2 - (Z\alpha)^2\right]^{1/2}} \right)^2 \right]^{-1/2}, \ \kappa = \pm(j + 1/2) \ .$$

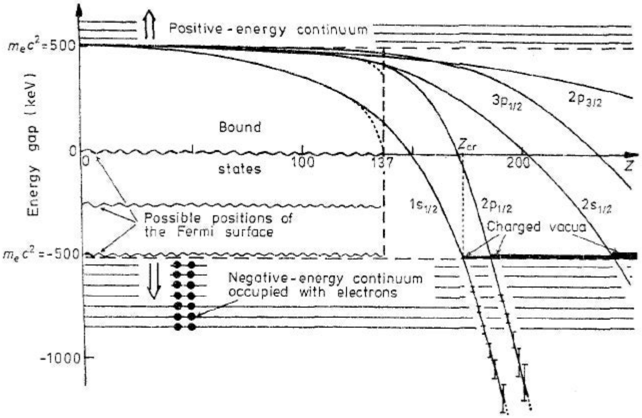
Because of the term $\left[\kappa^2 - (Z\alpha)^2\right]^{1/2}$ the Sommerfeld-Dirac formula crashes for $Z\alpha > |\kappa|$.
Energy is no longer a real value!

Concept of Critical Charge

Finite-size nucleus (or a Coulomb source).

- Critical charge is the charge of the nucleus, for which the discrete level reaches the threshold of the lower continuum. ($E_F = -m_e$)
- The concept of critical charge was first considered by Soviet physicists I. Pomeranchuk, J. Smorodinsky.
  (I. Pomeranchuk, J. Smorodinsky. Phys. USSR 9 97, 1945.)

Dependence of Energy Levels on Z

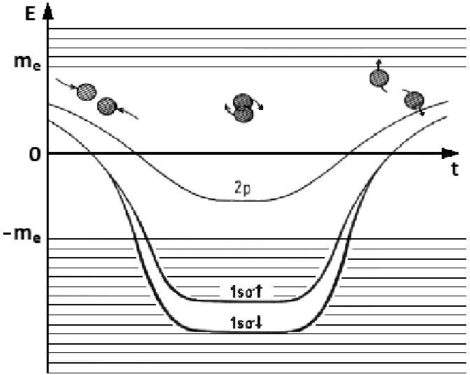


The concept of Charged Vacuum!

Why Should We Investigate Supercritical Systems?

- **What is the status of the supercritical region? (Experiments DRIBs (JINR, Dubna), GSI (Darmstadt) and Argonne National Laboratory (USA) did not give final answers.)**
- **The possibility of continuation the periodic table to the region $Z > 170$ and structure of (hypothetical) superheavy atoms.**
- **FAIR (Darmstadt), NICA (Dubna), Super Heavy Elements Factory (SHE, Dubna).**
- **Superheavy nuclear quasimolecules? What is the influence of magnetic effects?**

Collision of Two Superheavy Nuclei



10^{-21} s – the lifetime of a nuclear quasimolecule.
10^{-19} s – the vacuum positrons formation time.

Vacuum Charge Density ρ_{VP}. The Wichmann-Kroll Method.

The general formula for the vacuum charge density:

$$\rho_{VP}(\vec{r}) = -\frac{|e|}{2}\left(\sum_{E_n<E_F}\psi_n(\vec{r})^\dagger\psi_n(\vec{r}) - \sum_{E_n\geq E_F}\psi_n(\vec{r})^\dagger\psi_n(\vec{r})\right),$$

where E_n and $\psi_n(\vec{r})$ are eigenvalues and eigenfunctions of the corresponding Dirac-Coulomb spectral problem.

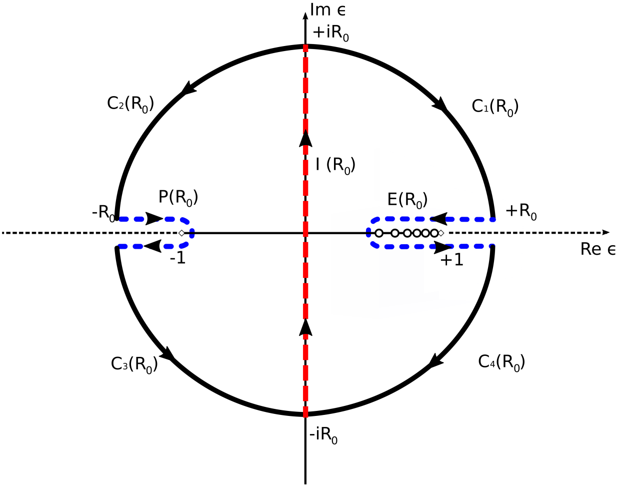
The equation for the Green function:

$$\left(-i\,\vec{\alpha}\,\vec{\nabla} + V(\vec{r}) + \beta - E\right) G(\vec{r},\vec{r}\,';E) = \delta(\vec{r}-\vec{r}\,'),$$

with the formal solution

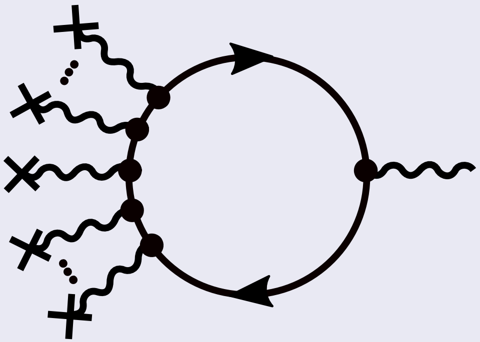
$$G(\vec{r},\vec{r}\,';E) = \sum_n \frac{\psi_n(\vec{r})\psi_n(\vec{r}\,')^\dagger}{E_n - E}\ .$$

Contours of Integration on Energy Complex Plane



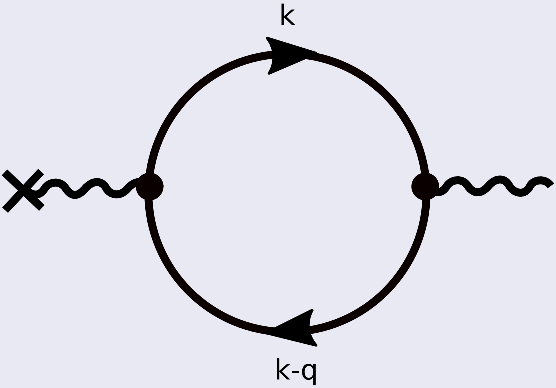
Feynman Diagram

All orders of $Z\alpha$ neglecting virtual photons



About Divergencies

The only divergent Feynman diagram with two external lines.



Calculation formula for ρ_{VP} in 1+1D

$$\rho_{VP}^{ren}(x) = \rho_{VP}^{(1)}(x) + \rho_{VP}^{(3+)}(x) \ ,$$

where $\rho_{VP}^{(1)}(x)$ is the perturbative charge density, obtained by using the renormalized polarization function in 1+1D and

$$\rho_{VP}^{(3+)}(x) = |e| \Big[\sum_{-1 \le E_n < 0} \psi_n(x)^\dagger \psi_n(x) + \\ + \frac{1}{\pi} \int\limits_0^\infty dy \, \mathrm{Re} \left(\mathrm{Tr}\, G(x, x; iy) - \mathrm{Tr}\, G^{(1)}(x; iy) \right) \Big] \ .$$

Calculation formula for ρ_{VP} in 2+1D

$$\rho^{ren}_{VP}(r) = 2\left[\rho^{(1)}_{VP}(r) + \sum_{m_j=1/2,3/2,..} \rho^{(3+)}_{VP,|m_j|}(r)\right] \ ,$$

where $\rho^{(1)}_{VP}(r)$ is the perturbative charge density, obtained by using the renormalized polarization function in 2+1D and

$$\rho^{(3+)}_{VP,|m_j|}(r) = \frac{|e|}{2\pi}\Big[\sum_{m_j=\pm|m_j|}\ \sum_{-1\leq E_n<0} \psi_{n,m_j}(r)^{\dagger}\psi_{n,m_j}(r) + \\ + \frac{1}{\pi}\int_0^{\infty} dy\, \mathrm{Re}\left(\mathrm{Tr}\, G_{|m_j|}(r,r;iy) - 2\,\mathrm{Tr}\, G^{(1)}_{m_j}(r;iy)\right)\Big] \ .$$

Vacuum Energy E_{VP} in 1+1D

$$E_{VP} = \frac{1}{2}\left(\sum_{E_n<E_F} E_n - \sum_{E_n\geq E_F} E_n\right) ,$$

$$E_{VP} = \frac{1}{2\pi}\int_0^\infty \frac{k}{\sqrt{k^2+1}}\delta_{tot}\, dk + \frac{1}{2}\sum_{-1\leq E_n<1}(1-E_n) ,$$

$$E_{VP}^{ren}(Z) = E_{VP}(Z) + \lambda Z^2 ,$$

$$\lambda = \lim_{Z_0\to 0}\frac{E_{VP}^{(1)}(Z_0) - E_{VP}(Z_0)}{Z_0^2} ,$$

where $E_{VP}^{(1)}$ is the perturbative energy.

Vacuum Energy E_{VP} in 2+1D

$$E_{VP} = \frac{1}{2}\left(\sum_{E_n<E_F} E_n - \sum_{E_n\geq E_F} E_n\right) \ ,$$

$$E_{VP} = \frac{1}{2\pi}\sum_{|m_j|=1/2,3/2,\ldots}\int_0^\infty \frac{k}{\sqrt{k^2+1}}\delta_{tot,|m_j|}\,dk + \frac{1}{2}\sum_{-1\leq E_n<1}(1-E_n) \ ,$$

$$E_{VP}^{ren}(Z) = E_{VP}(Z) + \lambda Z^2 \ ,$$

$$\lambda = \lim_{Z_0\to 0}\frac{E_{VP}^{(1)}(Z_0)\delta_{|m_j|,1/2} - E_{VP}(Z_0)}{Z_0^2} \ ,$$

where $E_{VP}^{(1)}$ is the perturbative energy.

Parameters for Numerical Calculations

1+1D

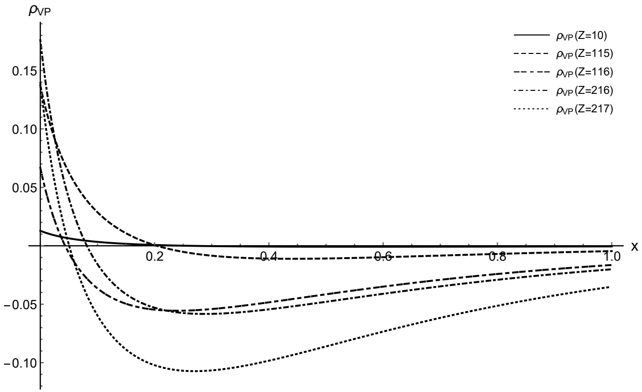
$$V(x) = -\frac{Z\alpha}{|x| + a}\ , \quad a > 0\ .$$

2+1D

$$V(r) = -Z\alpha \begin{cases} \frac{1}{R}, & r < R\ , \\ \frac{1}{r}, & r \geq R\ . \end{cases}$$

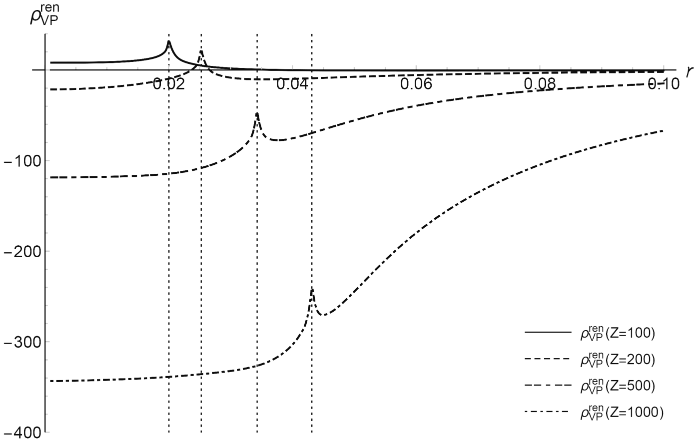
$$R = R(Z) \simeq 1.2(2.5Z)^{1/3}\ \mathbf{fm}.$$

Vacuum Charge Density for 1+1D with $a = 0.1$



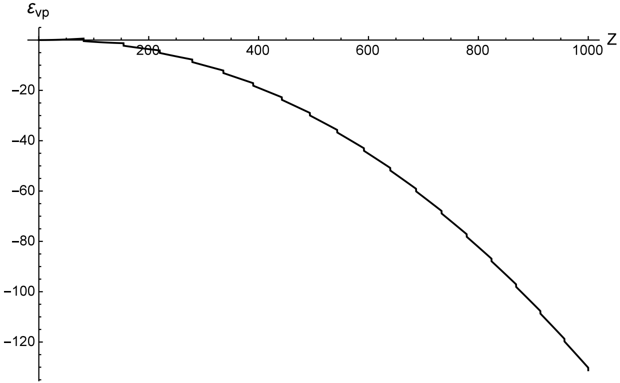
A. Davydov, K. Sveshnikov and Y. Voronina, International Journal of Modern Physics A 32, 1750054 (2017)

Vacuum Charge Density in 2+1D with $R(Z)$



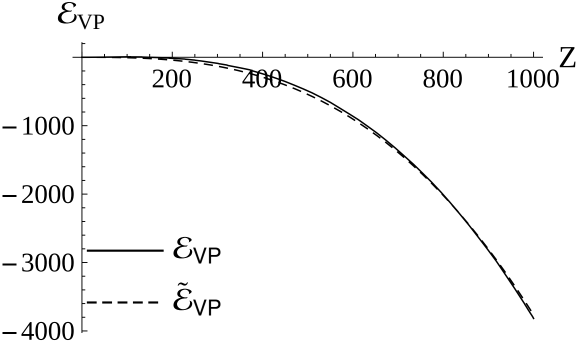
A. Davydov, K. Sveshnikov and Y. Voronina, International Journal of Modern Physics A 33, 1850004 (2018)

Vacuum Energy in 1+1D with $R_0 = 0.1$



$$E_{VP}^{1+1D} \sim -\frac{Z^2}{R_0} \ .$$

Vacuum Energy in 2+1D with $R(Z)$



$$E_{VP}^{2+1D} \sim -\frac{Z^3}{R_0} \ .$$

A. Davydov, K. Sveshnikov and Y. Voronina, International Journal of Modern Physics A 33, 1850005 (2018)

Vacuum Energy in 3+1D with $R(Z)$

$$E_{VP}^{3+1D} \sim -\frac{Z^4}{R} \ .$$

The vacuum energy competes with the repulsive energy of the Coulomb source.

$$E_{VP}^{3+1D}(Z^*, R(Z^*)) + \frac{{Z^*}^2\alpha}{2R(Z^*)} \simeq 0 \ ,$$

$$Z^* \simeq 3000 \ .$$

Conclusions

- **Such behavior of the vacuum energy in the overcritical region confirms once more the correct status of the assumption of the neutral vacuum transmutation into the charged one, which turns out to be the ground state of the electron–positron field in such external supercritical fields, and hence of the spontaneous vacuum positron emission, which should accompany the creation of each subsequent vacuum shell due to the total charge conservation.**

Conclusions

- **In 3+1 dimensions the vacuum energy behaves $\sim -Z^4/R$, and therefore it becomes competitive with the classical electrostatic energy of the Coulomb source!**

P.S.

- **The possibility of the nuclear quasimolecule formation.**
- **Magnetic effects.**

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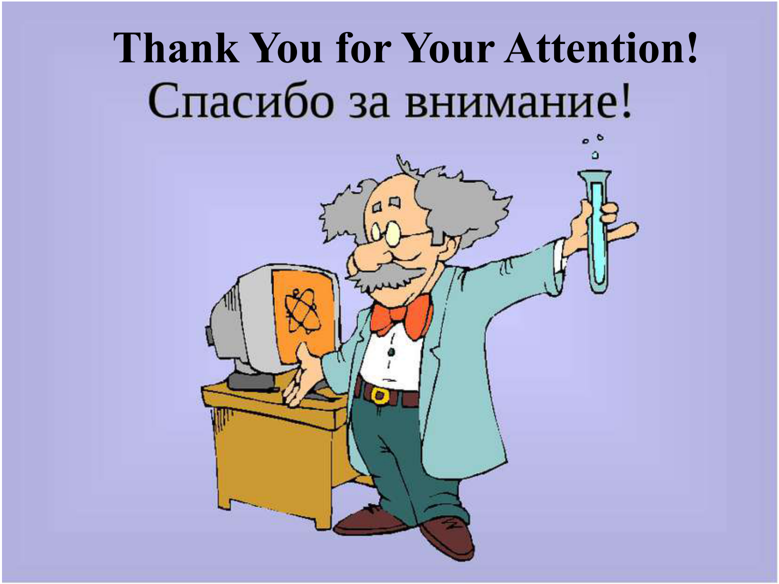
Thank You for Your Attention!
Спасибо за внимание!