Coulomb dissociation of ^{34}Na and its relevance in nuclear astrophysics

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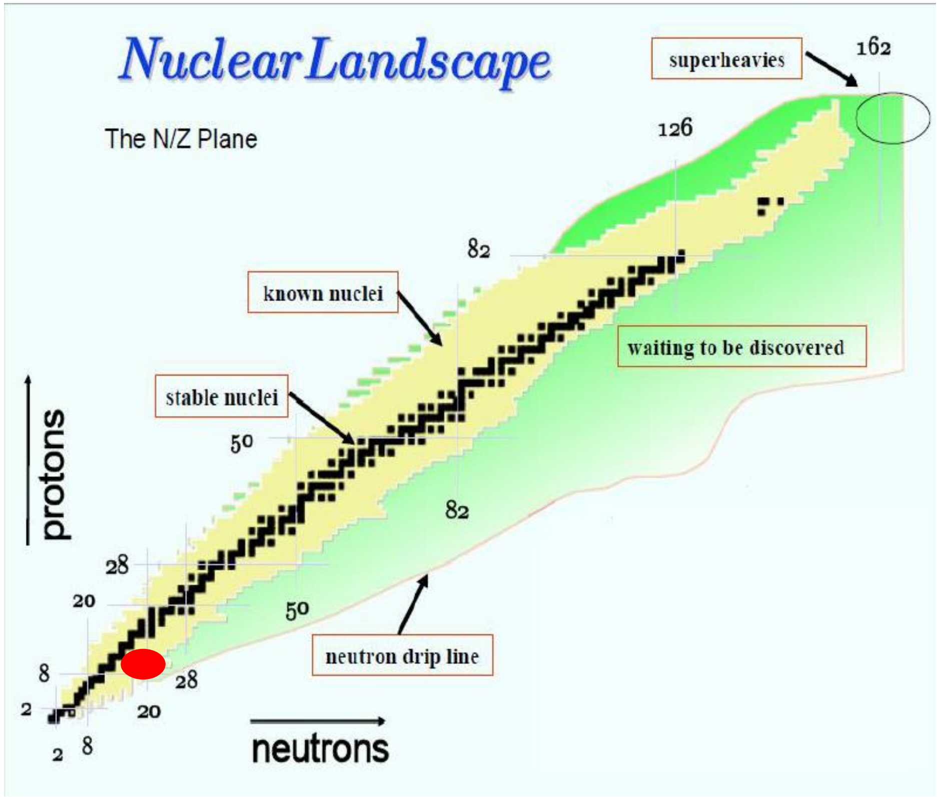
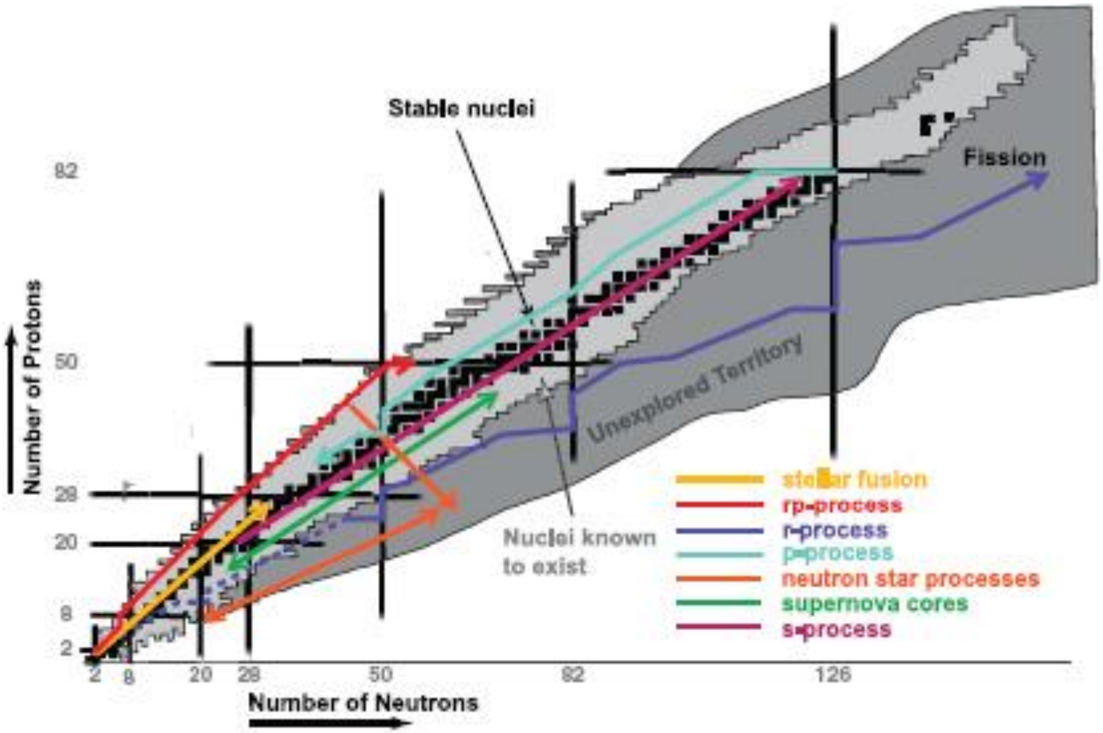
Introduction



The Segré chart!

- The origin & abundances of different nuclei?
- Why unstable gaps between isotones and isotopes?
- Where does fission set in?
- Appearance or disappearance of magic numbers?

Introduction…



The nucleosynthesis processes and their regions in the nuclear chart!

Figure credits: Arcones et al., Prog. Part. Nucl. Phys. **94**, 1 (2017).

The r-process: Enigmatic. Excellent reason to study drip nuclei!

- Stellar fusion: Mainly stable nuclei via *pp*-chains, CNO cycle, etc.
- s-process: slow neutron capture.
- r-process: rapid neutron capture.
- p-process: proton capture.
- rp-process: rapid proton capture.

- BBFH, Rev. Mod. Phys. **29**, 547 (1957).
- Cameron, Pub. A. S. P. **69**, 201 (1957).
- Rolfs, and Rodney, *Cauldrons in the Cosmos* (University of Chicago Press), 1988.
- Wallerstein *et al.*, Rev. Mod. Phys. **69**, 995 (1997).

The r-process and neutron drip line!

Near drip line nuclei important for *r*-process pathway!!

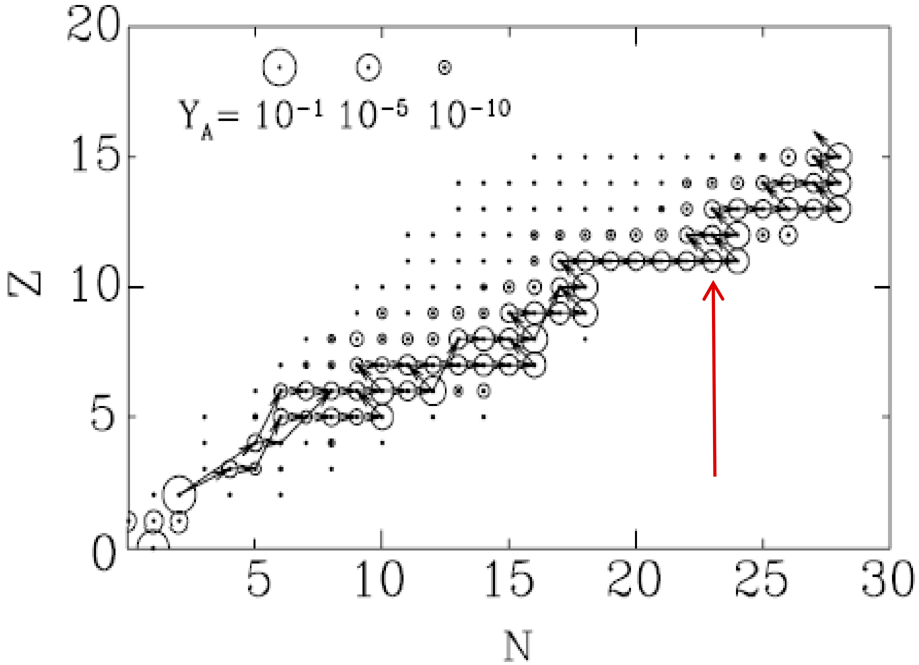
To confirm a nucleus' role as a progenitor of seed nuclei in stellar plasma.

At eqb. Temperature, T_9 = 0.62, competition between α-capture and n-capture!

α-capture > n-capture
⇒ Isotopic chain formation breaks!!

n-capture > α-capture
⇒ Nucleogenesis towards neutron drip line!!

We compare rates of ^{33}Na(n,γ)^{34}Na & ^{33}Na(α,n)^{36}Al reactions!



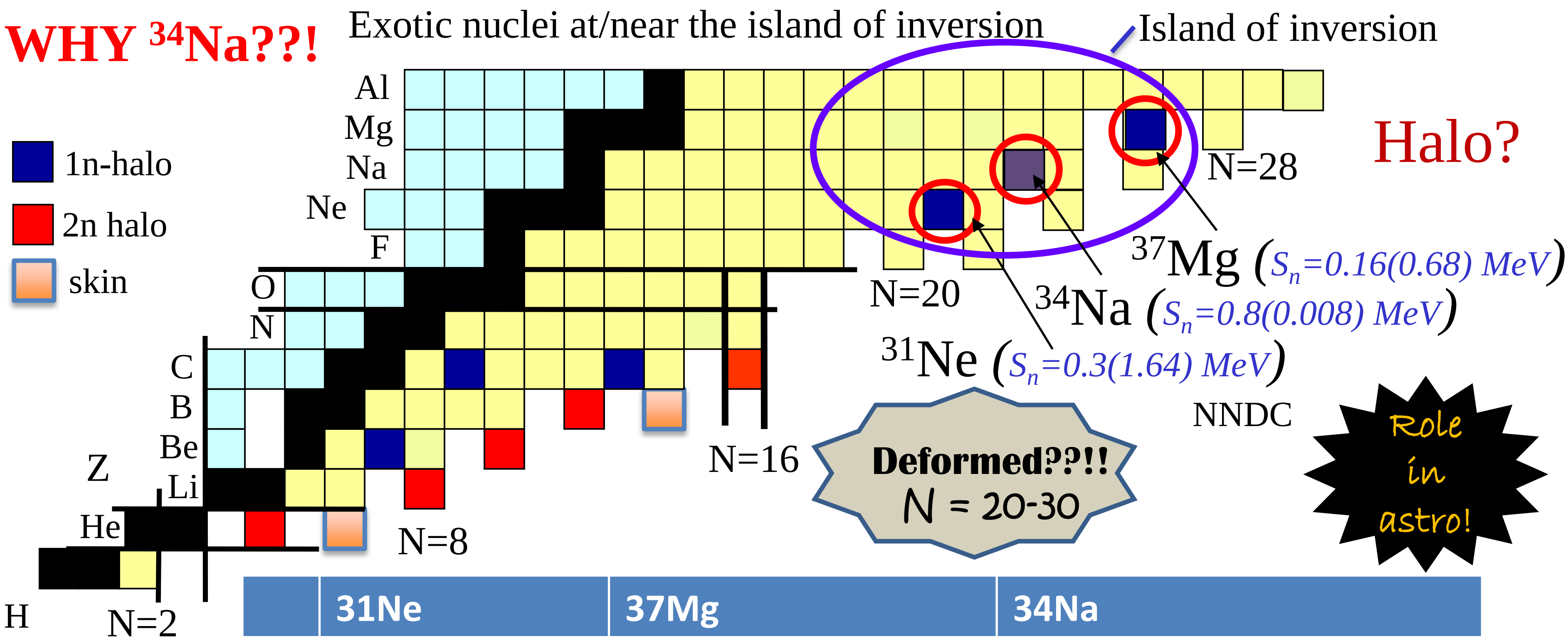
Terasawa M, *et al.*, ApJ **562,** 470 (2001).

^{33}Na(n,γ) important to ^{34}Na, ^{35}Na abundance!

^{33}Na(n,γ)^{34}Na via CD!
^{33}Na(α,n)^{36}Al via Hauser-Feshbach theory (NON-SMOKER code).
Rauscher, At. Data Nucl Data Tables **79**, 47 (2001): http://nucastro.ord/nonsmoker.html.

The curious case of ^{34}Na!

WHY ^{34}Na??!

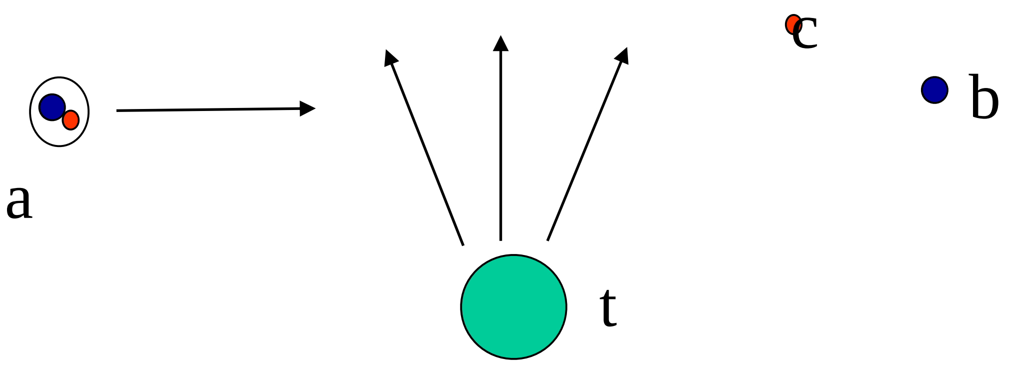


	31Ne	37Mg	34Na
S_n	***0.06(0.41) MeV*** *Gaudefroy L, et al. PRL 109 (2012) 202503.* ***0.29(0.05) MeV*** *Shubh et al. NPA 922 (2014) 99.*	$0.22^{+0.12}_{-0.09}$ MeV *Kobayashi N, et. al., PRL 112 (2014) 242501.* ***0.35(0.06) MeV*** *Shubh et al. NPA 939 (2015) 101.*	***0.17(0.50) MeV*** *Gaudefroy L, et al. PRL 109 (2012) 202503.*
J^π	***3/2⁻*** *T. Nakamura PRL 112 (2014) 142501.*	***3/2⁻ or 1/2⁻*** *Kobayashi PRL 112 (2014) 242501.* ***5/2⁻ or 3/2⁻ or 1/2⁺*** *Watanabe S, et al., PRC 89 (2014) 044610.*	***1/2⁺ or 3/2⁻ or 7/2⁻ (Valence neutron)*** *Gaudefroy L, et al. PRL 109 (2012) 202503. Fortune H T and Sherr R. PRC 87 (2013) 057308. Doornenbal P, et al., PTEP 2014 (2014) 053D01.* (2⁻⁻)

Coulomb dissociation and the finite range distorted wave Born approximation (FRDWBA) theory:

Why and how??

Coulomb dissociation: an elegant method to study nuclear halos!!



The triple differential cross-section:

Phase space factor

$$\frac{d^3\sigma}{dE_b d\Omega_b d\Omega_c} = \frac{2\pi}{\hbar v_{at}} \rho \sum_{lm} |\beta_{lm}|^2$$

Reduced transition matrix

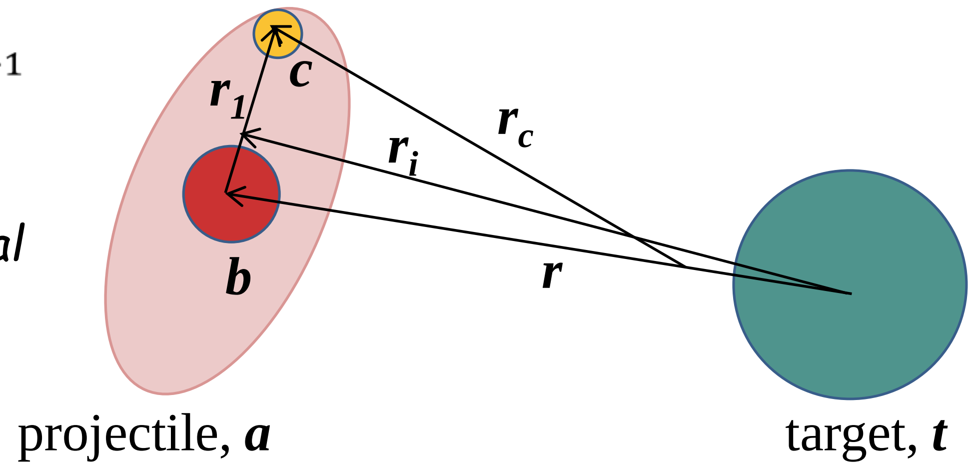
The reduced transition matrix under the FRDWBA theory:

$$\hat{l}\beta_{lm}^{FRDWBA} = \iint d\boldsymbol{r}_1 d\boldsymbol{r}_i \chi_b^{(-)*}(\boldsymbol{q}_b, \boldsymbol{r}_b)\chi_c^{(-)*}(\boldsymbol{q}_c, \boldsymbol{r}_c)V_{bc}(\boldsymbol{r}_1)\phi_a^{lm}(\boldsymbol{r}_1)\chi_a^{(+)}(\boldsymbol{q}_a, \boldsymbol{r}_i)$$

Chatterjee R, Banerjee P, and Shyam R, Nucl. Phys. A **675,** 477 (2000).

$$V_{bc}(\boldsymbol{r}_1) = V_{ws} f(r_1) - \beta_2 R V_{ws} \frac{df(r_1)}{dr_1} Y_2^0(\hat{\boldsymbol{r}}_1)$$

$$f(r_1) = \left[1 + \exp\left(\frac{r_1 - R}{a}\right)\right]^{-1}$$



V_{ws} = Spherical depth of WS potential
$R = r_0 A^{1/3}$; r_0 = radius parameter
a = diffuseness parameter

β_2 = DEFORMATION!!!

After Local Momentum Approximation,

For a neutron,

$$\chi_c^{(-)*}(\boldsymbol{q}_c, \boldsymbol{r}_c) = e^{-i(\boldsymbol{q}_c, \boldsymbol{r}_c)}$$

Structure part! *Includes deformation!!*

$$\hat{l}\beta_{lm} = \int d\boldsymbol{r}_1 e^{-i\boldsymbol{W}.\boldsymbol{r}_1} V_{bc}(\boldsymbol{r}_1) \phi_a^{lm}(\boldsymbol{r}_1) \int d\boldsymbol{r}_i e^{-i\delta \boldsymbol{q}_c.\boldsymbol{r}_i} \chi_b^{(-)*}(\boldsymbol{q}_b, \boldsymbol{r}_i) \chi_a^{(+)}(\boldsymbol{q}_a, \boldsymbol{r}_i)$$

Dynamics part!

with, $\mathbf{W} = \gamma \mathbf{q}_c - \alpha \mathbf{K}$
and $\mathbf{K}$ = Local momentum!

Can be evaluated analytically in terms of the Bremsstrahlung integral. ☺

Shubhchintak and Chatterjee R, Nucl. Phys. A **922,** 99 (2014).

Applications to astrophysics

For a **single multipole dominated reaction,**

But where is the Astrophysics application?

FRDWBA!!

$$\sigma^{\pi\lambda}_{\gamma,n} = \frac{E_\gamma}{n_{\pi\lambda}} \frac{d\sigma}{dE_{bc}}$$

Bertulani C A, and Baur G, Phys. Rep. **163,** 299 (1988).

Achieved by invoking the principle of detailed balance relating the photodisintegration cross-section with its "radiative capture" counterpart via:

$$\sigma_{n,\gamma} = \frac{2\hat{j}_a^{\,2}}{\hat{j}_b^{\,2}\hat{j}_c^{\,2}} \frac{k_\gamma^2}{k_{bc}^2} \sigma^{\pi\lambda}_{\gamma,n}$$

$$\langle \sigma(v_{bc}) v_{bc} \rangle = \sqrt{\frac{8}{\pi \mu_{bc} (k_B T)^3}} \int_0^\infty dE_{bc} \sigma_{(n,\gamma)}(E_{bc}) E_{bc} \exp\left(-\frac{E_{bc}}{k_B T}\right)$$

Then, reaction rate,

$$R = N_A \langle \sigma(v_{bc}) v_{bc} \rangle$$

Rolfs, and Rodney, *Cauldrons in the Cosmos* (University of Chicago Press), 1988.

Results and discussion

Singh G., Shubhchintak, and Chatterjee R, Phys. Rev. C **94,** 024606 (2016).

$$^{34}\text{Na} + {}^{208}\text{Pb} \xrightarrow{100\ \text{MeV/u}} {}^{33}\text{Na} + {}^{1}\text{n} + {}^{208}\text{Pb}$$

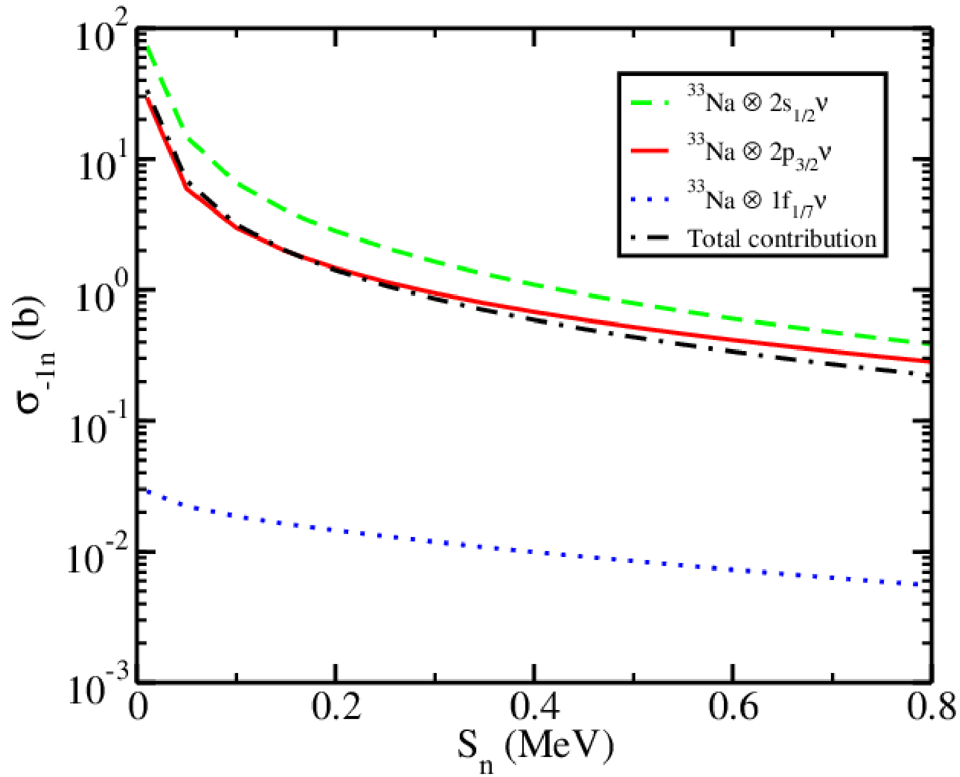
$$\frac{d^3\sigma}{dE_b d\Omega_b d\Omega_c} = \frac{2\pi}{\hbar v_{at}} \rho \sum_{lm} |\beta_{lm}|^2$$

$\sigma_{-1n} \sim 1/S_n$ dependence?

Predominantly ***E1*** transition?

Chatterjee R, Fortunato L and Vitturi A, Eur. Phys. J. A **35**, 213 (2008).

Indirect method applicable!!!



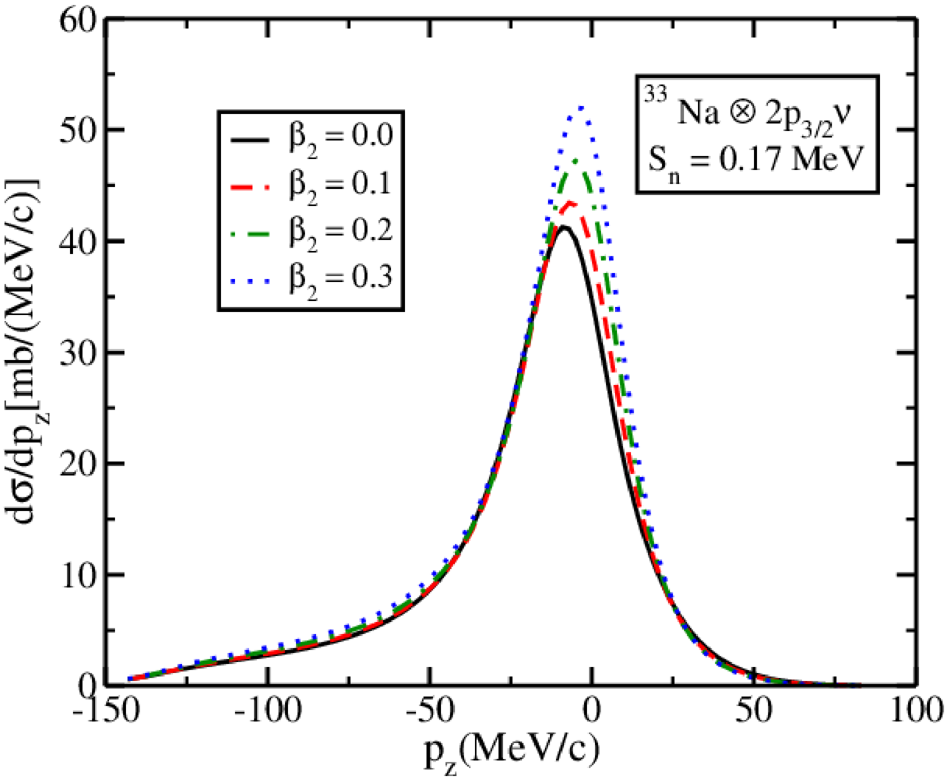
Total one-neutron removal cross-section

Can be compared with EXPERIMENTAL DATA!!!

Results and discussion

Singh G., Shubhchintak, and Chatterjee R, Phys. Rev. C **94,** 024606 (2016).

Parallel momentum distribution



Definitely a HALO!!!

Elegant application of uncertainty principle!

β_2	FWHM (MeV/c)
0.0	36.82
0.1	35.88
0.2	34.76
0.3	33.83

Is ^{34}Na a HALO nucleus??

FWHM for normal nuclei ~ 120 MeV/c

while

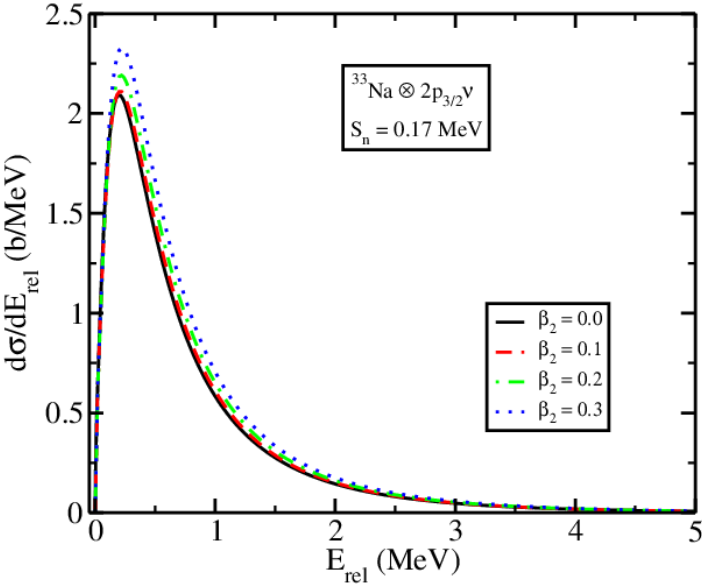
FWHM for ^{11}Be and ^{19}C ~ 44 MeV/c

Chatterjee R, Banerjee P, and Shyam R, Nucl. Phys. A **675**, 477 (2000).

Banerjee P, Thompson I J, and Tostevin J A, Phys. Rev. C **58**, 1042 (1998).

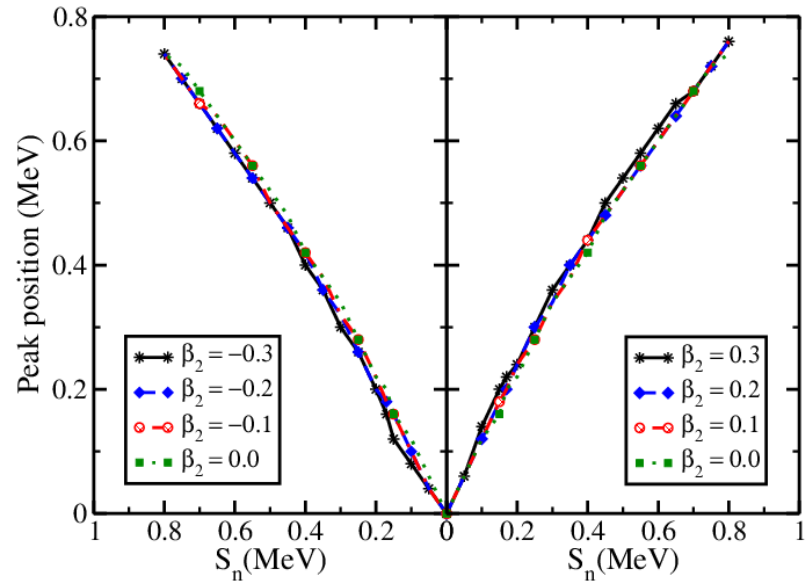
Results and discussion

Singh G., Shubhchintak, and Chatterjee R, Phys. Rev. C **94,** 024606 (2016).



Relative energy spectra

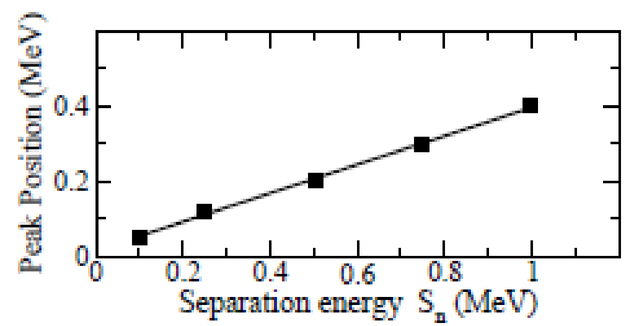
Peak position of relative energy spectrum dependent on separation energy!



Pattern independent of sign of deformation. Scaling valid!!

$\ell_i \to \ell_f$	$dB(E1)/dE_{rel} \propto E_{rel}^{\ell_f+1/2}$ (for small E_{rel})	Peak of $dB(E1)/dE_{rel}$
$s \to p$	$\propto E_{rel}^{3/2}$	$E_{rel} = \frac{3}{5}S_n$
$p \to s$	$\propto E_{rel}^{1/2}$	$E_{rel} \simeq 0.18 S_n$
$p \to d$	$\propto E_{rel}^{5/2}$	$E_{rel} = \frac{5}{3}S_n$
$d \to p$	$\propto E_{rel}^{3/2}$	$E_{rel} = \frac{5}{3}S_n$

Nakamura T and Kondo Y, *Clusters in Nuclei*, Springer Vol. **2** ed. Beck C, (2012) 67.



Chatterjee R, Fortunato L and Vitturi A, Eur. Phys. J. A **35**, 213 (2008).

Results and discussion

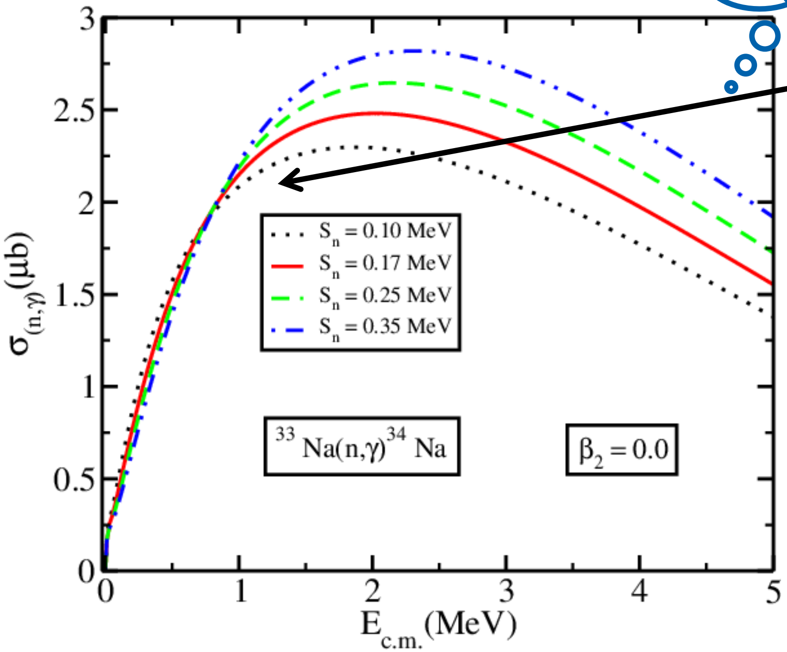
Singh G., Shubhchintak, and Chatterjee R, Phys. Rev. C **95**, 065806 (2017).

Capture cross sections!

Why the flip?!

For $E_{cm} \gg S_n$, as $S_n \uparrow \rightarrow \sigma \uparrow$!

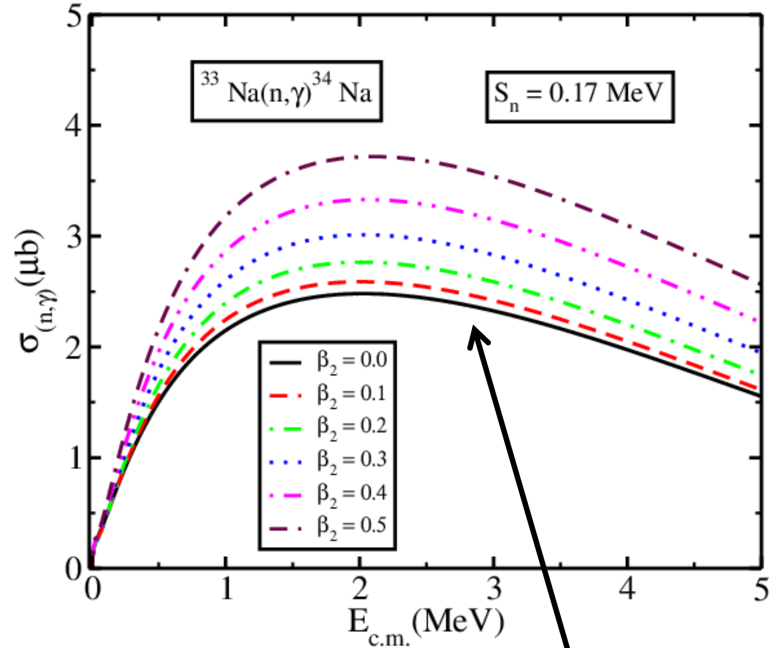
But, for $E_{cm} \approx S_n$, as $S_n \uparrow \rightarrow \sigma \downarrow$!!



Capture cross-section for different one neutron binding energies, S_n.

$$\sigma_{n,\gamma} \propto \left(\frac{k_\gamma}{k_{bc}}\right)^2 \frac{E_\gamma}{n_{\pi\lambda}} \left(\frac{d\sigma}{dE_{bc}}\right)$$

Capture cross-section with different values of β_2.

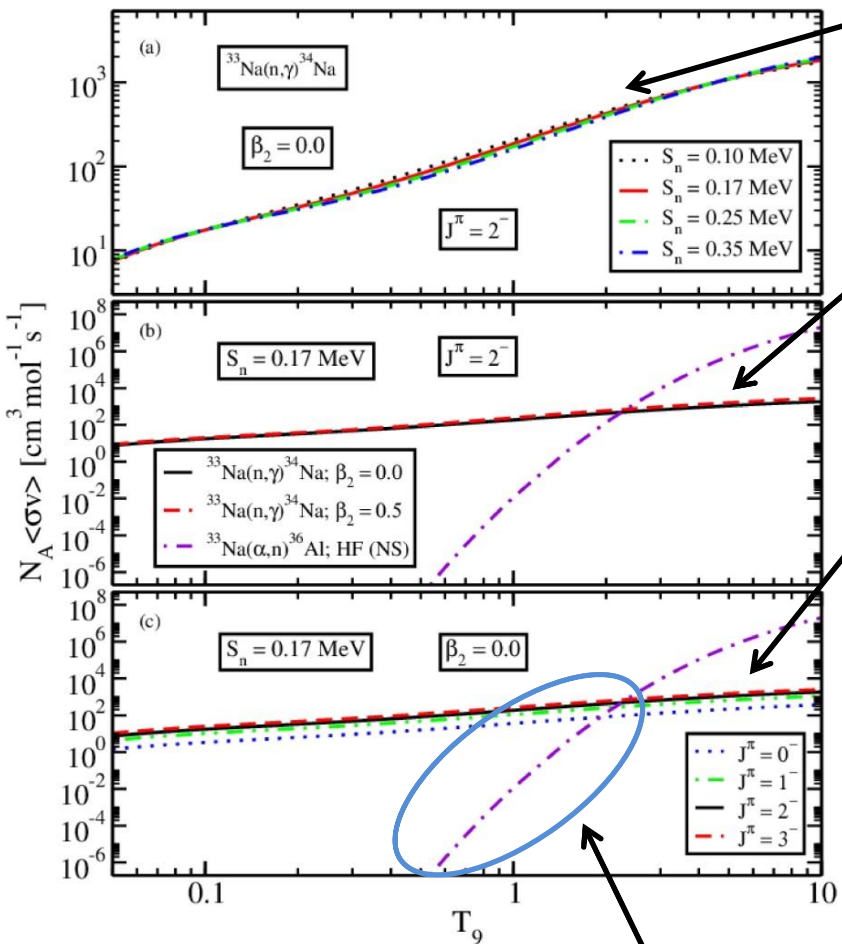


Significant variation with deformation!!

Results and discussion

Singh G., Shubhchintak, and Chatterjee R, Phys. Rev. C **95**, 065806 (2017).

Reaction rates @ $T_9 = 0.62$



Comparison of $^{33}Na(n,\gamma)$ and $^{33}Na(\alpha,n)$.

Reaction rate with various S_n values.

Rate with different β_2.

Rate with J^π

Rates follow cross section patterns!

B.E. increases, rate decreases!!

More deformation, slightly higher rate!!

Higher spin, higher rate!!

n-capture highly dominant over α-capture at the relevant temperature!!

Conclusions

- Various reaction observables were calculated for elastic breakup of ^{34}Na on ^{208}Pb at 100 MeV/u using FRDWBA theory.
- The results from the relative energy spectra showed that peak height varied with the introduction of deformation.
- Going by the trends in this mass region (N = *20-30*) and our results, one speculates that the dominant contribution to ground state of *halo* nucleus ^{34}Na is by $2p_{3/2}$ state.
- Scaling can be used in conjunction with our results to limit the uncertainties in the structural parameters of nuclei.
- Capture cross-sections and reaction rates for formation of ^{34}Na were calculated and it was confirmed that in stellar environment at the equilibrium temperature, neutron-capture dominated the alpha-capture process favoring ^{34}Na.
- ★ This should push the isotopic abundance of Na isotopes towards the neutron drip line.
- Further, we strongly encourage experiments to put our predictions on firmer footing.

Future outlook…

1. Calculations with a fully deformed wave function are desirable.

2. FRDWBA has been applied to medium mass neutron rich nuclei. Its extension to 3-charged particles in the final channel case is desired to explore the proton rich side.

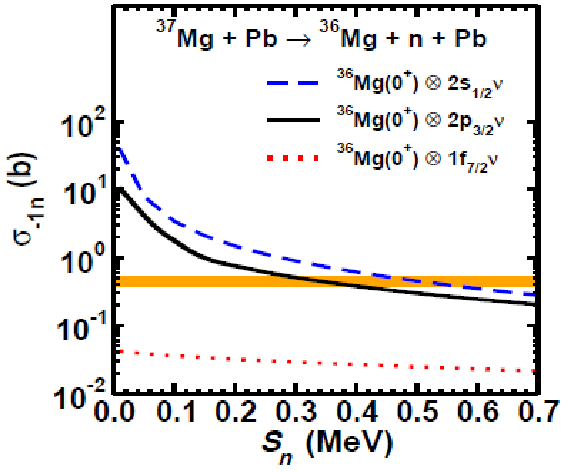
3. Scaling can be used in conjunction with our results to limit the uncertainties in the structural parameters of nuclei.

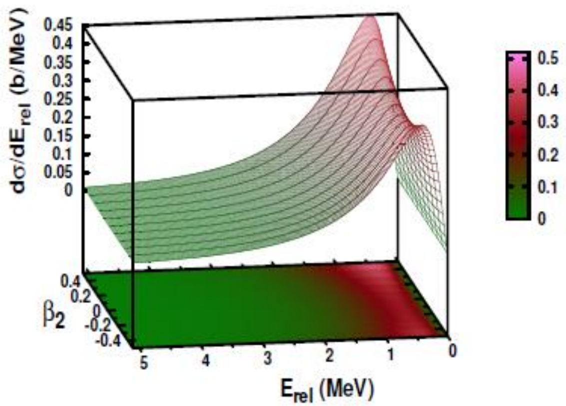
4. There is a need to describe a relativistic reaction theory for future experiments at higher beam energies. An eikonal-DWBA can be a good candidate for the above. It should effectively address the issue of 3-charge particles too.

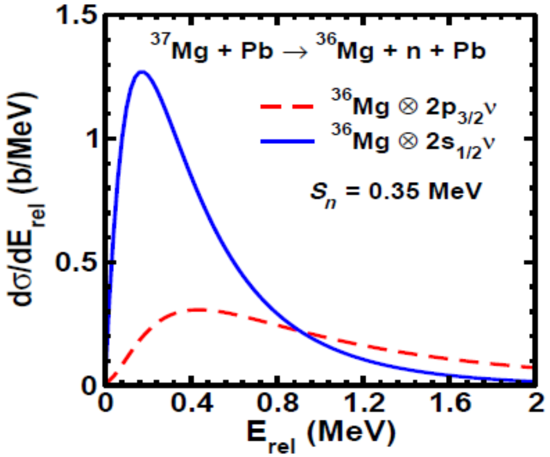
5. Radiative capture reactions also have applications to nuclear reactors.

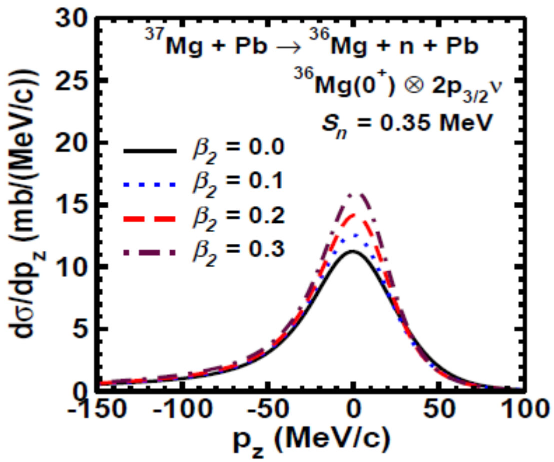
^{37}Mg results

Neelam, Shubhchintak and R. Chatterjee, Phys. Rev. C **92**, 044615 (2015).





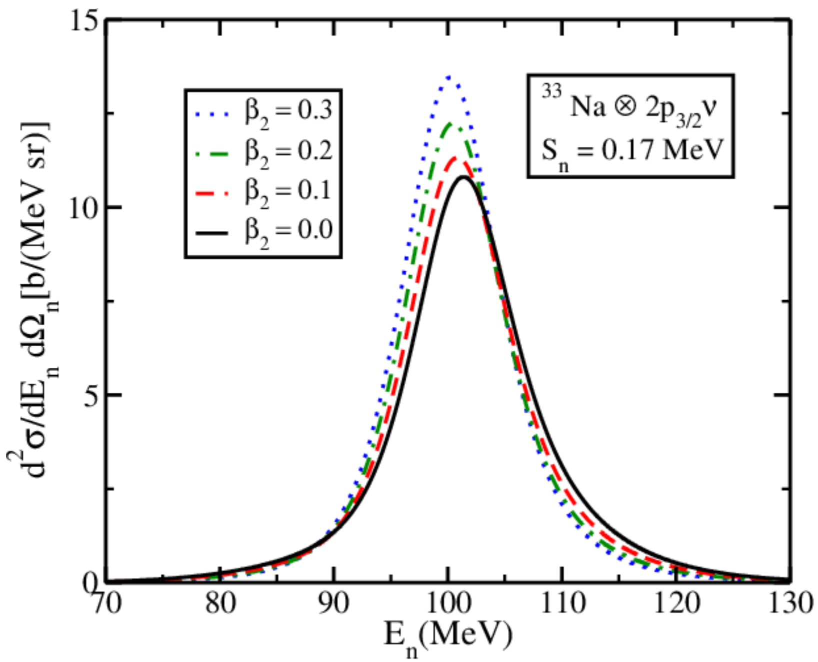




Results and discussion

Singh G., Shubhchintak, and Chatterjee R, Phys. Rev. C **94,** 024606 (2016).

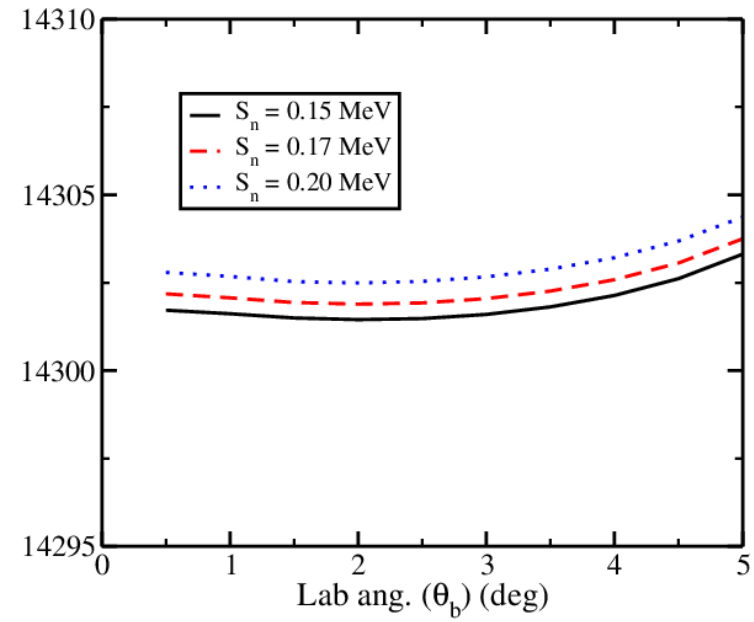
Post acceleration??!



Energy-angular distribution

Peak height increases with deformation.

Average momentum of ^{33}Na fairly constant over very forward angles
→ no effective post-acceleration!



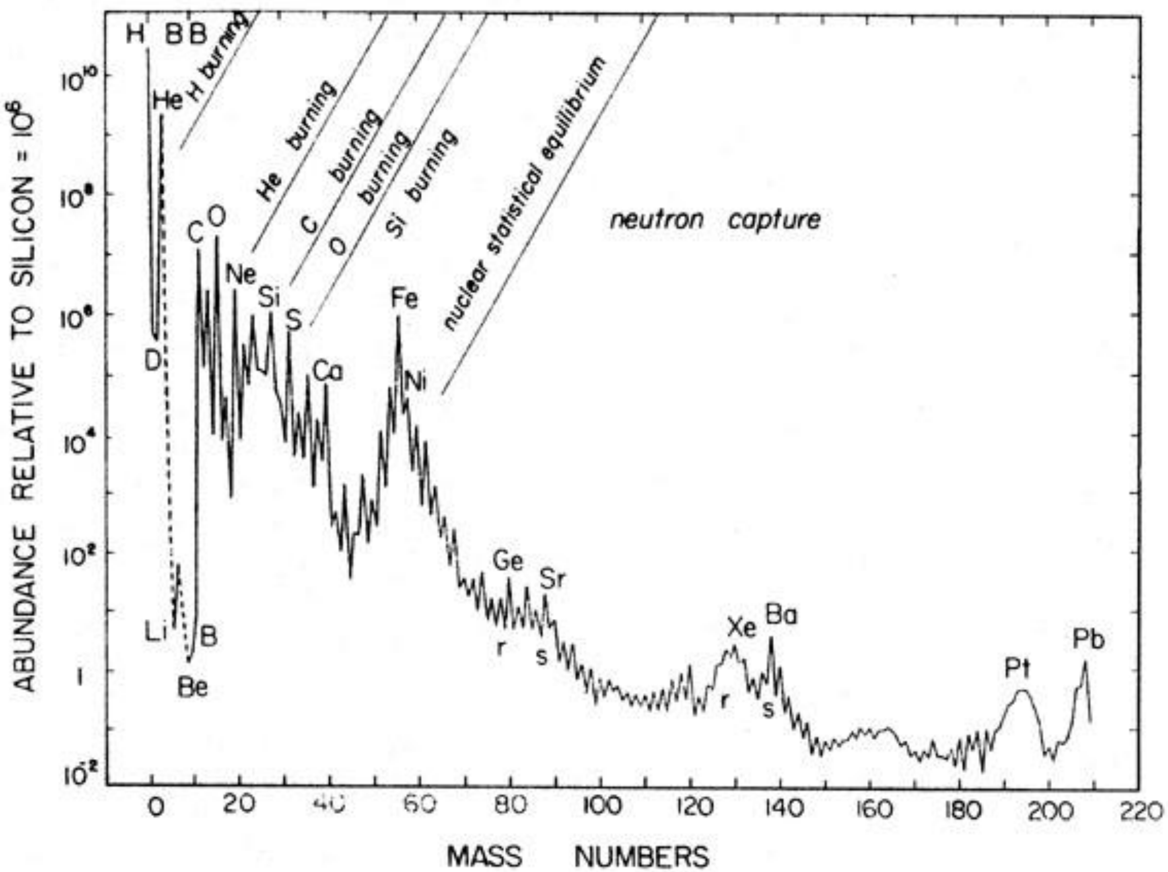
Average momentum

Peaks coincide with beam energy
→ no post acceleration!

Introduction…

- ✓ H: most abundant!
- ✓ H + He ~ 95 – 99 %
- ✓ Cosmological Li problem (in BBN)!
- ✓ C to Ca – successive stages in stellar evolution!
- ✓ NSE @ Fe peak! Explosive nucleosynthesis!
- ✓ Nucleosynthesis mainly by n-capture!

Primordial abundances: age of the galaxy and origin of the Universe!

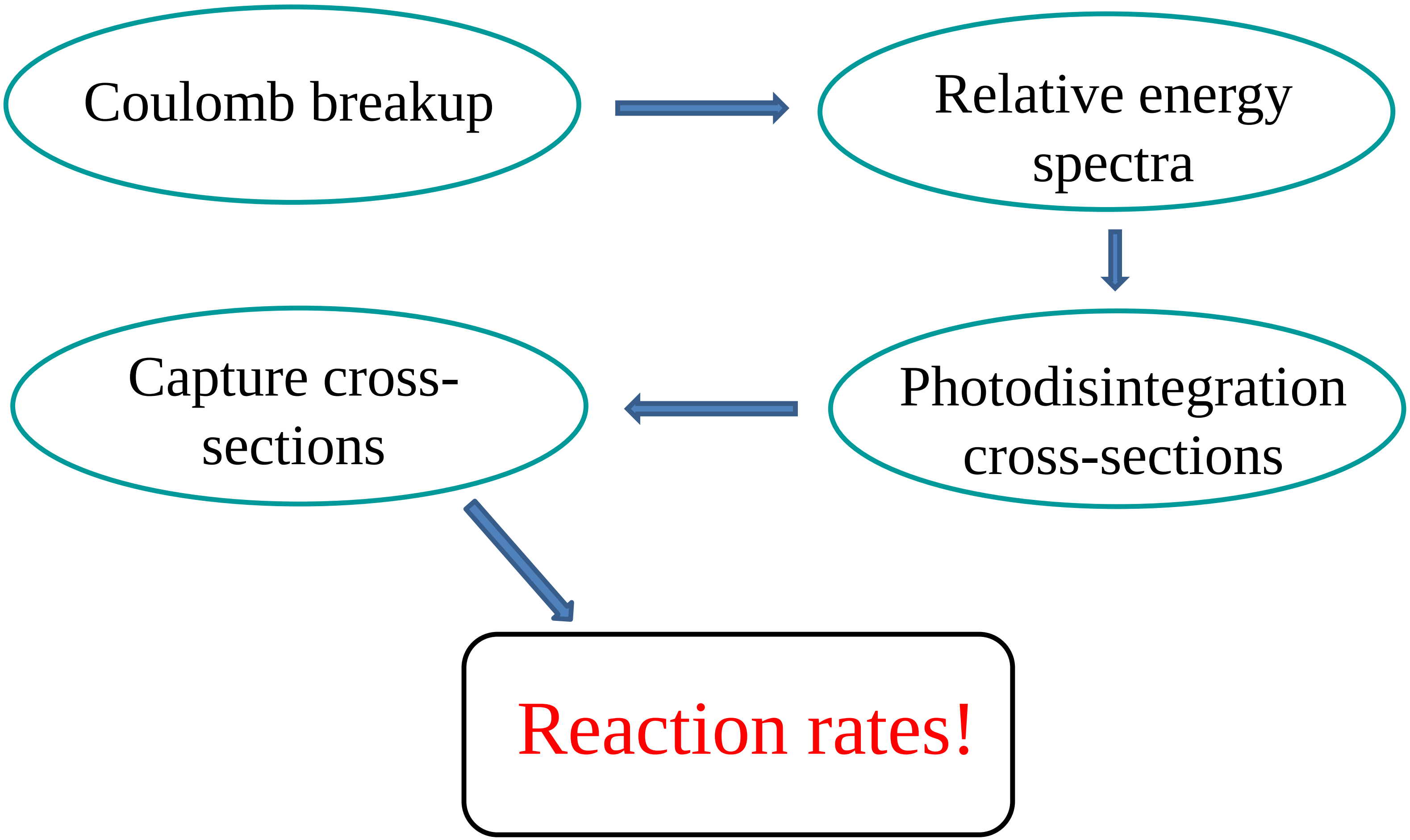


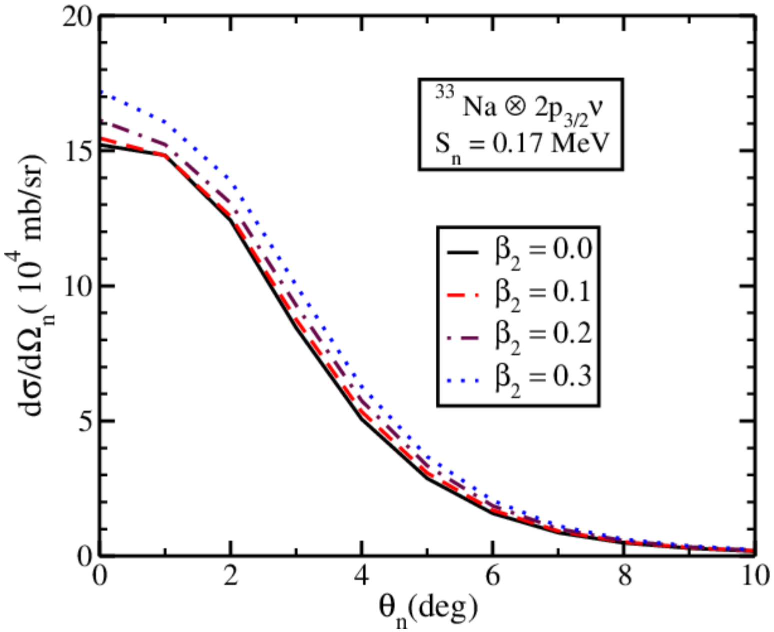
The universal abundance curve!

Figure credits: Cameron A. G. W., *Essays in Nuclear Astrophysics*, ed. C. A. Barnes, D. D. Clayton, and D. N. Schramm (Cambridge University Press, Cambridge), 1982.

- BBFH, Rev. Mod. Phys. **29**, 547 (1957).
- Cameron, Pub. A. S. P. **69**, 201 (1957).
- Rolfs, and Rodney, *Cauldrons in the Cosmos* (University of Chicago Press), 1988.
- Wallerstein *et al.*, Rev. Mod. Phys. **69**, 995 (1997).
- Hou *et al.*, ApJ, **834**, 165 (2017).

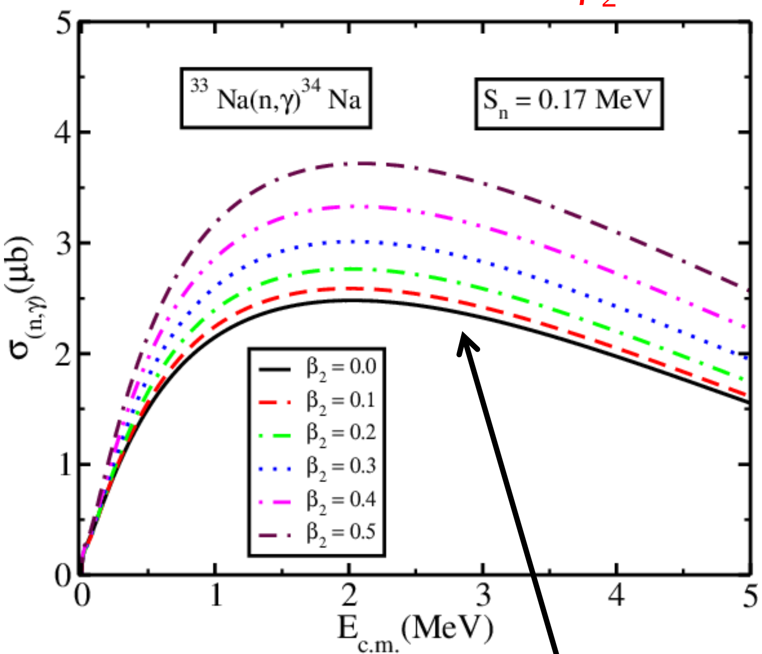
The flow chart!





Angular distribution

Capture cross-section with different values of β_2.



Significant variation with deformation!!

List of publications

In refereed journals:

1. Singh G., Shubhchintak, and Chatterjee R, "*Structural effects of ^{34}Na in the ^{33}Na(n,γ)^{34}Na radiative capture reaction*", Phys. Rev. C **95,** 065806 (2017); *arXiv:* 1706.09687v1 [nucl-th].

2. Singh G., Shubhchintak, and Chatterjee R, "*Elastic Coulomb breakup of ^{34}Na*", Phys. Rev. C **94,** 024606 (2016); *arXiv:* 1607.08055v1 [nucl-th].

In conference proceedings:

1. Singh G., Shubhchintak, and Chatterjee R, "*Structural effects in ^{34}Na*", Proc. DAE-BRNS Symp. On Nucl. Phys. **61,** 356 (2016).

2. Singh G., Shubhchintak, and Chatterjee R, "*Exotic nucleus in the medium mass region: the curious case of ^{34}Na*", Proc. DAE-BRNS Symp. On Nucl. Phys. **60,** 424 (2015).

3. Singh G., and Shubhchintak, "*E1-E2 contribution in ^{12}C(α,γ)^{16}O reaction*", Proc. DAE Symp. On Nucl. Phys. **59,** 622 (2014).

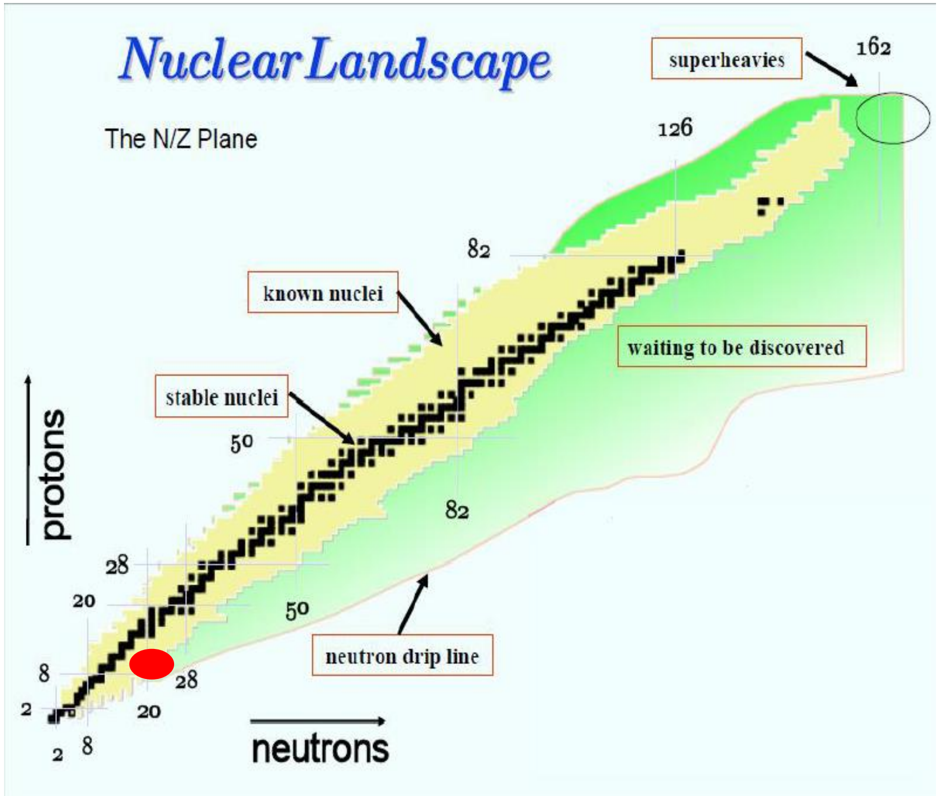
4. Singh G., Shubhchintak, and Chatterjee R, "*Progress towards a relativistic breakup reaction theory*", Proc. DAE Symp. On Nucl. Phys. **58,** 558 (2013).

Theories of breakup reactions!

Theory	Merits	Demerits
DWBA	Derived from exact *T*-matrix. Applicable to breakup/ transfer reactions. Very well studied.	Inelastic excitations are difficult to include in the post-form *T*-matrix.
TDSE	Accurate numerical solutions.	Time consuming. Non-trivial.
Eikonal + extensions (CCE, DEA)	Good results for nuclear dominated reactions. DEA also explains Coulomb part.	Valid at high energies. Simple eikonal blows for Coulomb dominated breakup.
CDCC	Excellent method at low energies, can be done at higher energies as well.	Very long computation time at higher energies; difficult to assess its convergence. Discretization is a problem.

Baye D., *Lecture notes for the MTNR school at Roorkee*, September 2013.
Chatterjee R., Pramana, **75**, 127 (2010).

Landscapes…



The Segré chart!

Figure courtesy: R. Chatterjee, IIT Roorkee.

Landscape of nuclear reactions!

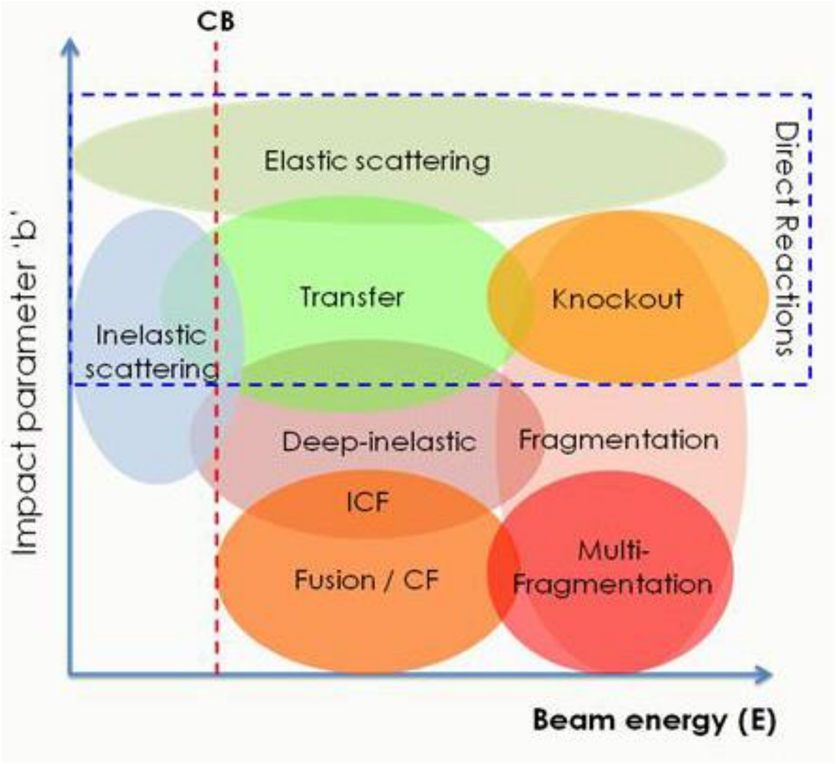


Figure courtesy: IIT Ropar.

WHY ^{34}Na??!

- Calculating the radiative capture cross-section for the formation of ^{34}Na can help predict its role as a seed nucleus/progenitor of seed nuclei in stellar plasma.
- Achieved by invoking the principle of detailed balance relating the photodisintegration cross-section with its radiative capture counterpart via:

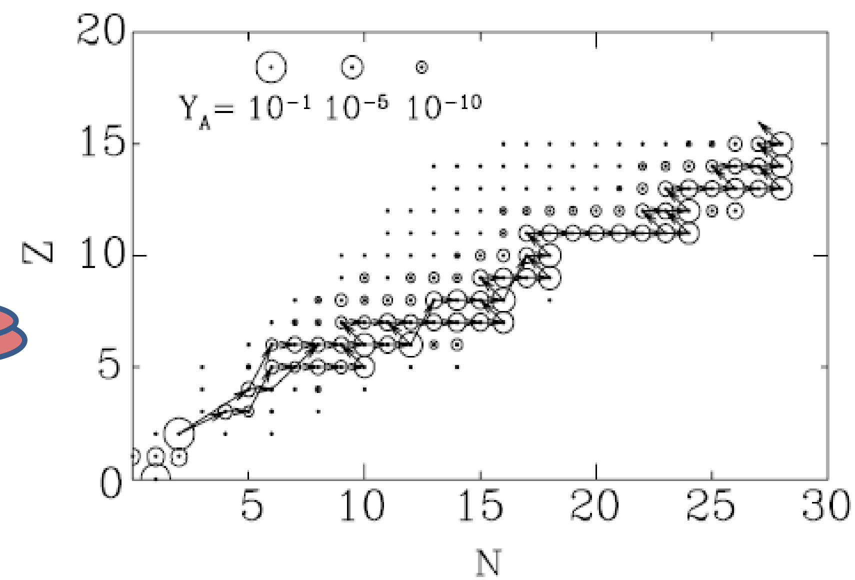
$$\sigma_{n,\gamma} = \frac{2\hat{j}_a^{\,2}}{\hat{j}_b^{\,2}\hat{j}_c^{\,2}} \frac{k_\gamma^2}{k_{bc}^2} \sigma_{\gamma,n}^{\pi\lambda}$$

Further, for a **single multipole dominated reaction,**

FRDWBA!!

$$\sigma_{\gamma,n}^{\pi\lambda} = \frac{E_\gamma}{n_{\pi\lambda}} \frac{d\sigma}{dE_{bc}}$$

Bertulani C A, and Baur G, Phys. Rep. **163,** 299 (1988).



Terasawa M, *et al.*, ApJ **562,** 470 (2001).

Then, reaction rate,

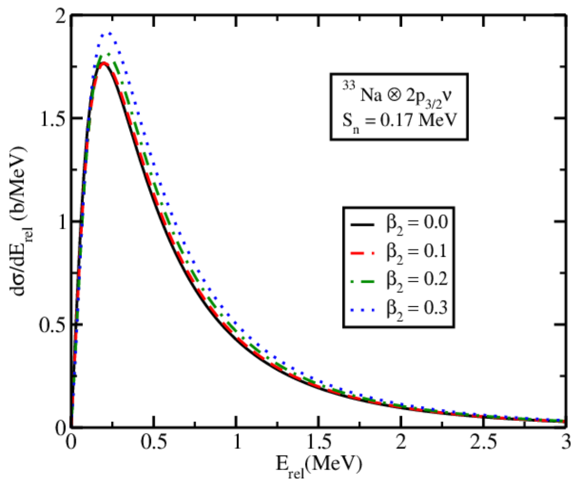
$$R = N_A\langle\sigma(v_{bc})v_{bc}\rangle$$

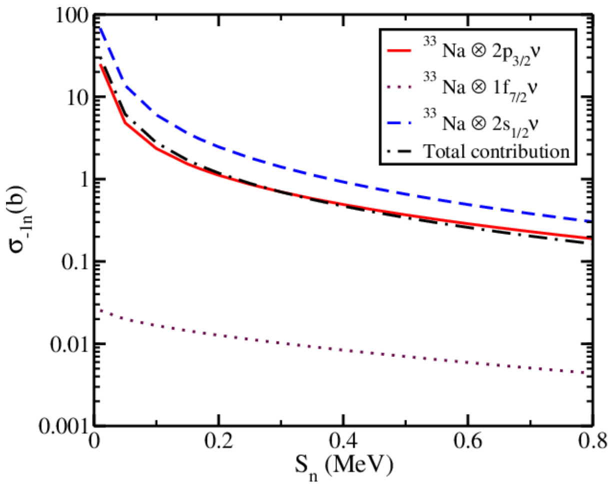
^{33}Na(n,γ) important to ^{34}Na, ^{35}Na abundance!

Results and discussion

$^{34}Na + ^{208}Pb \xrightarrow{100\ \mathrm{MeV/u}} ^{33}Na + ^{1}n + ^{208}Pb$

Relative energy spectrum



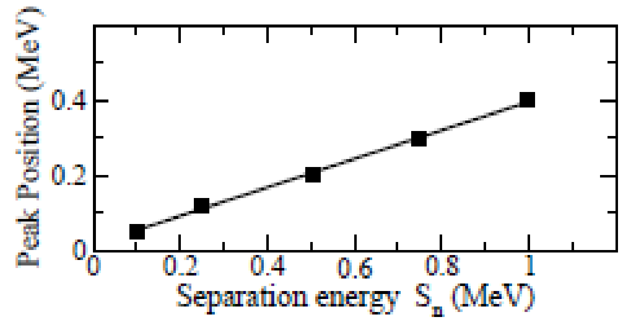


Total cross-section

Peak position of relative energy spectrum dependent on separation energy!

Predominantly ***E1*** transition?

Can be compared with EXPERIMENTAL DATA!!!



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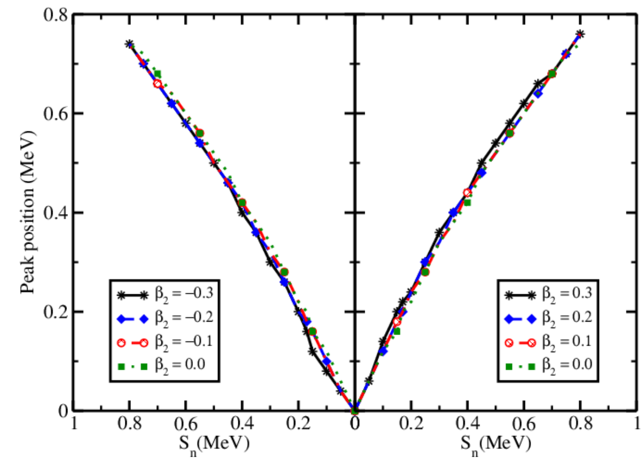

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