UNIVERSITY OF Southampton
School of Physics and Astronomy

elusives invisiblesPlus
neutrinos, dark matter & dark energy physics

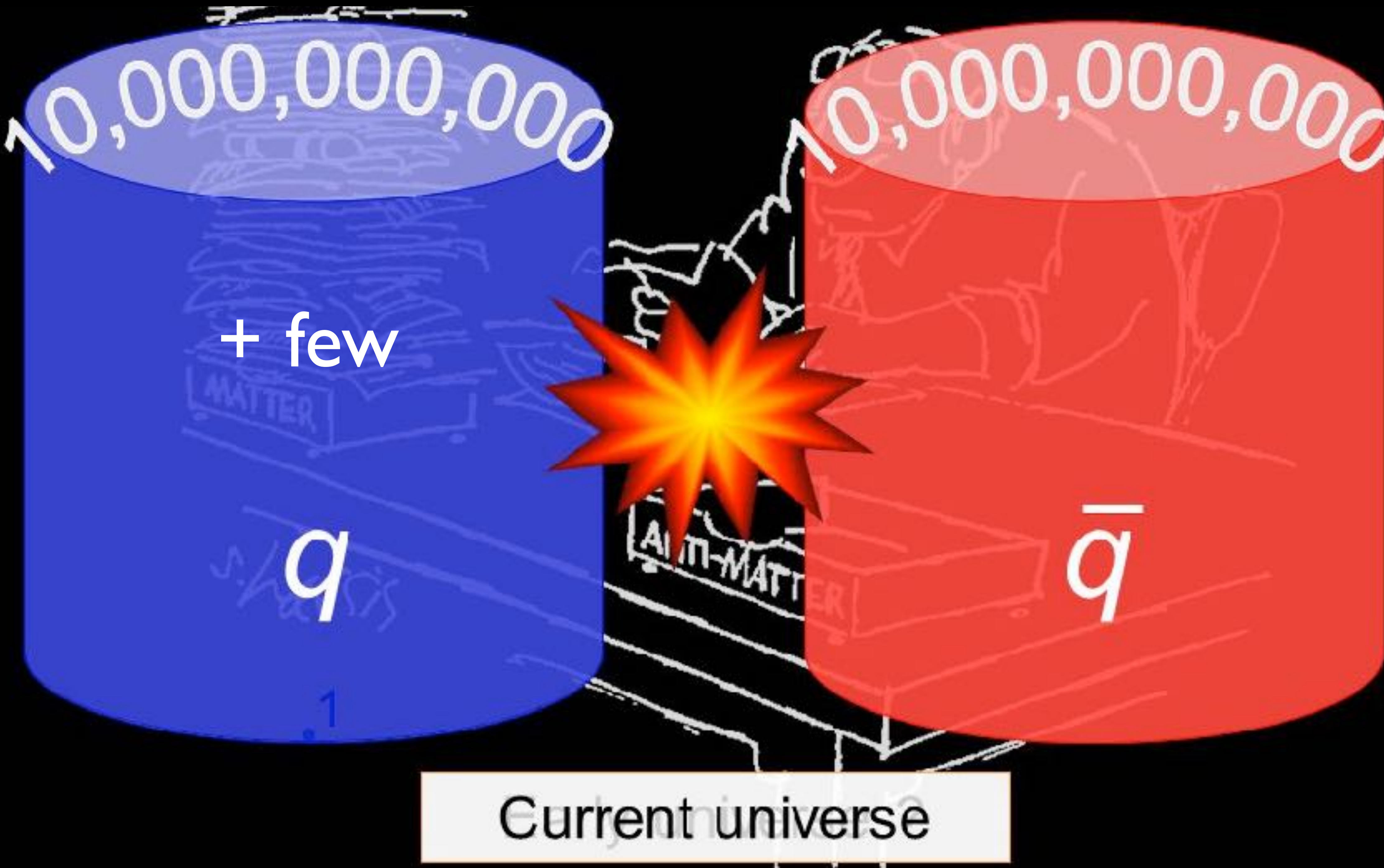
Neutrino Theory

From the review: S.F.King 1701.04413

Prog.Part.Nucl.Phys. 94 (2017) 217

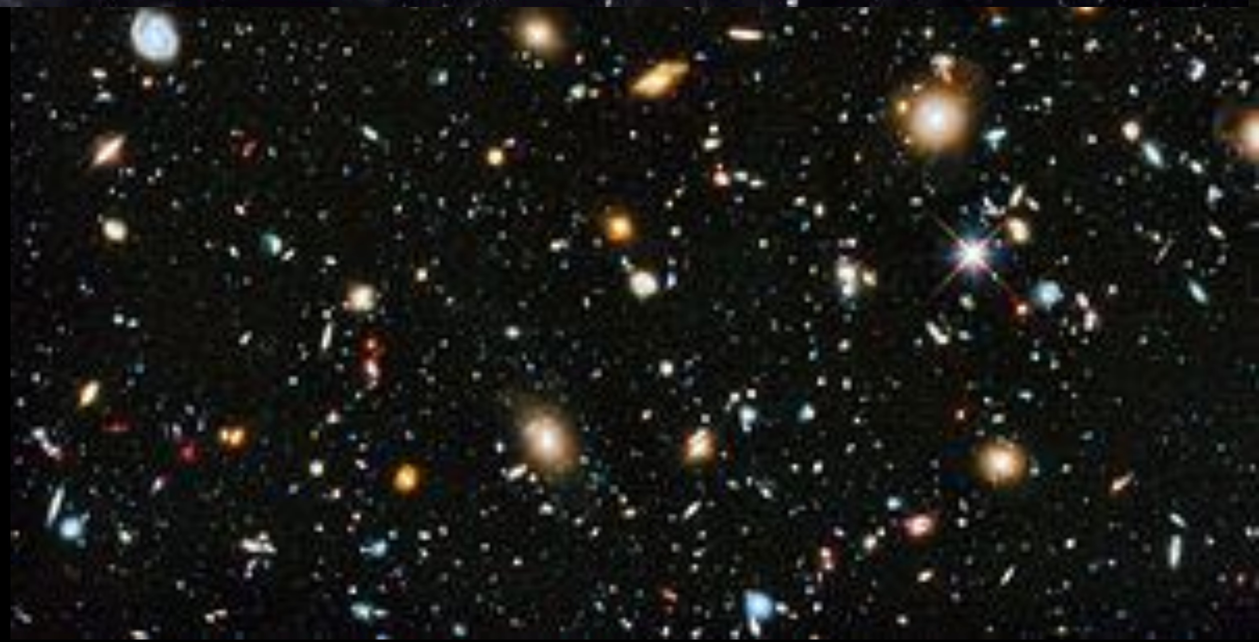
At the end of Inflation the Universe was empty, cold and bare...

After reheating a very slight excess of matter was somehow generated

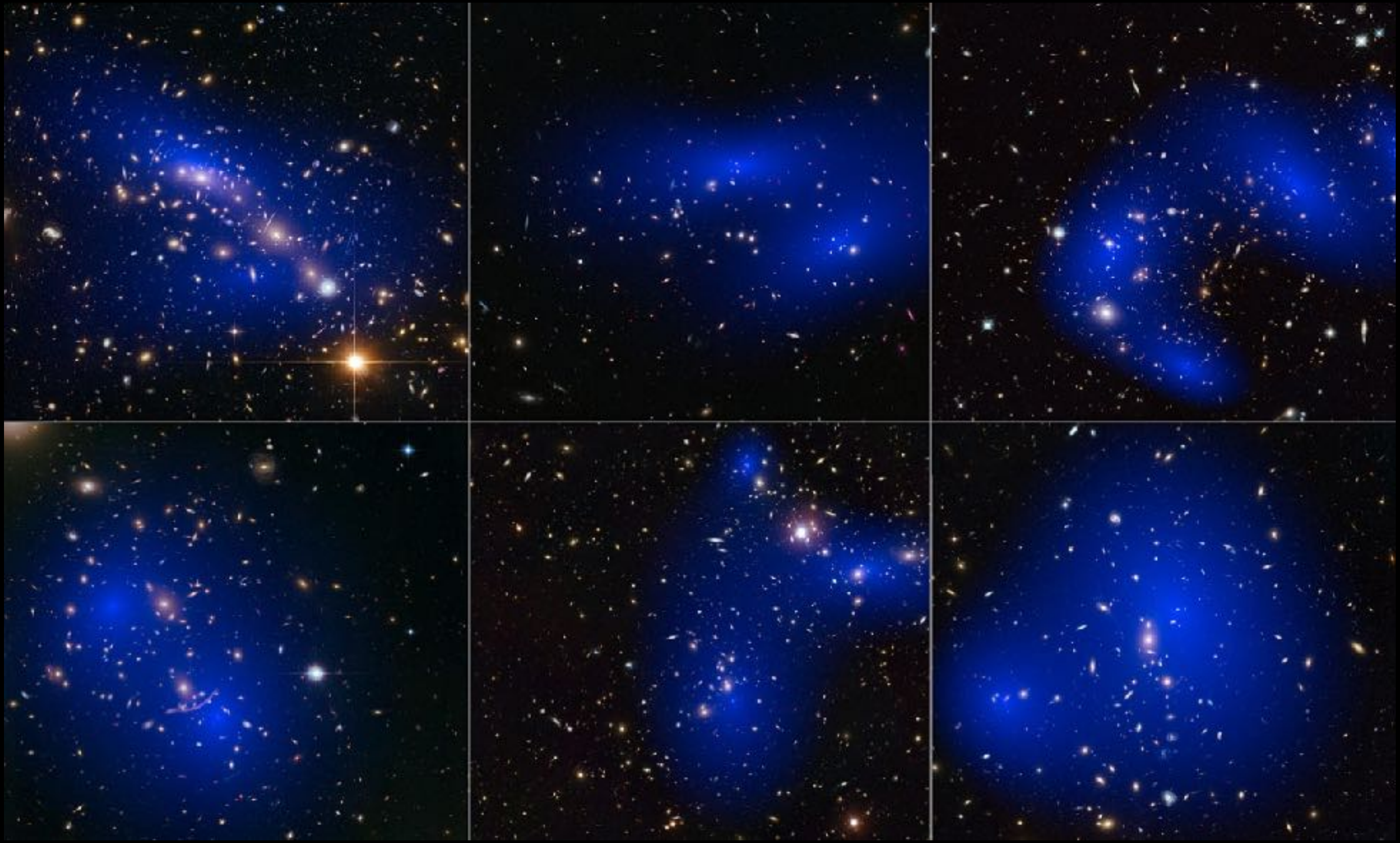


Giving the observed baryon asymmetry of the Universe

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = \frac{n_B}{n_\gamma} \approx 6 \times 10^{-10}$$

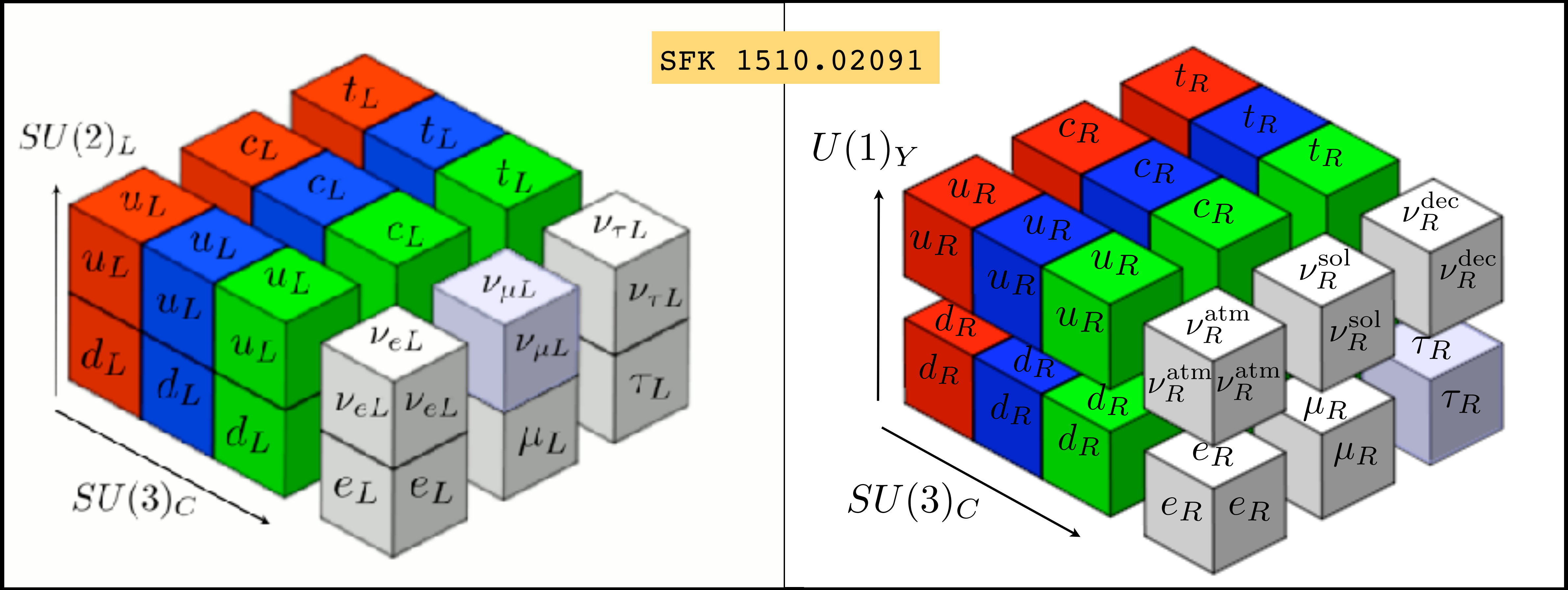


Dark Matter?



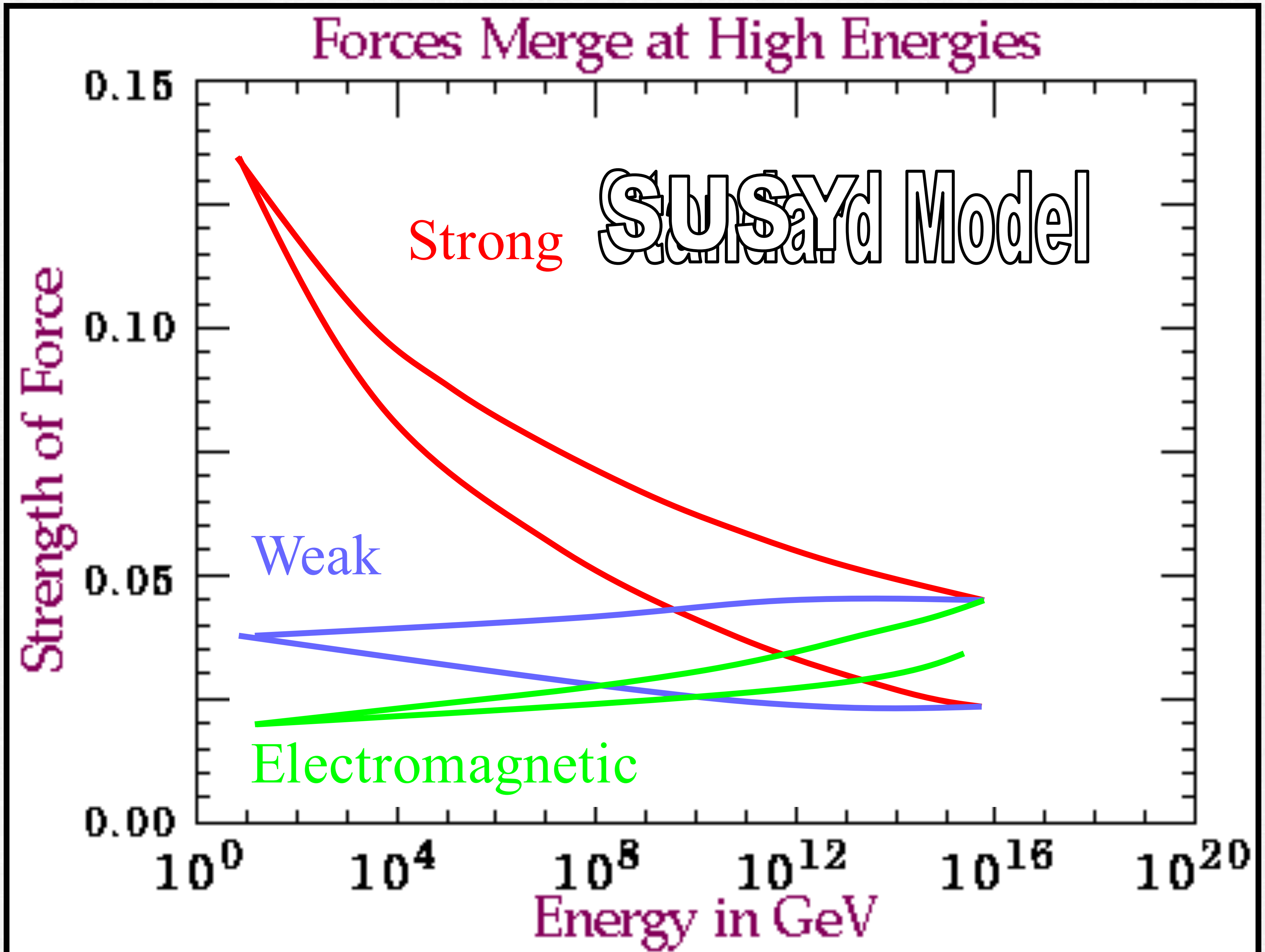
Dark Energy?

The Standard Model

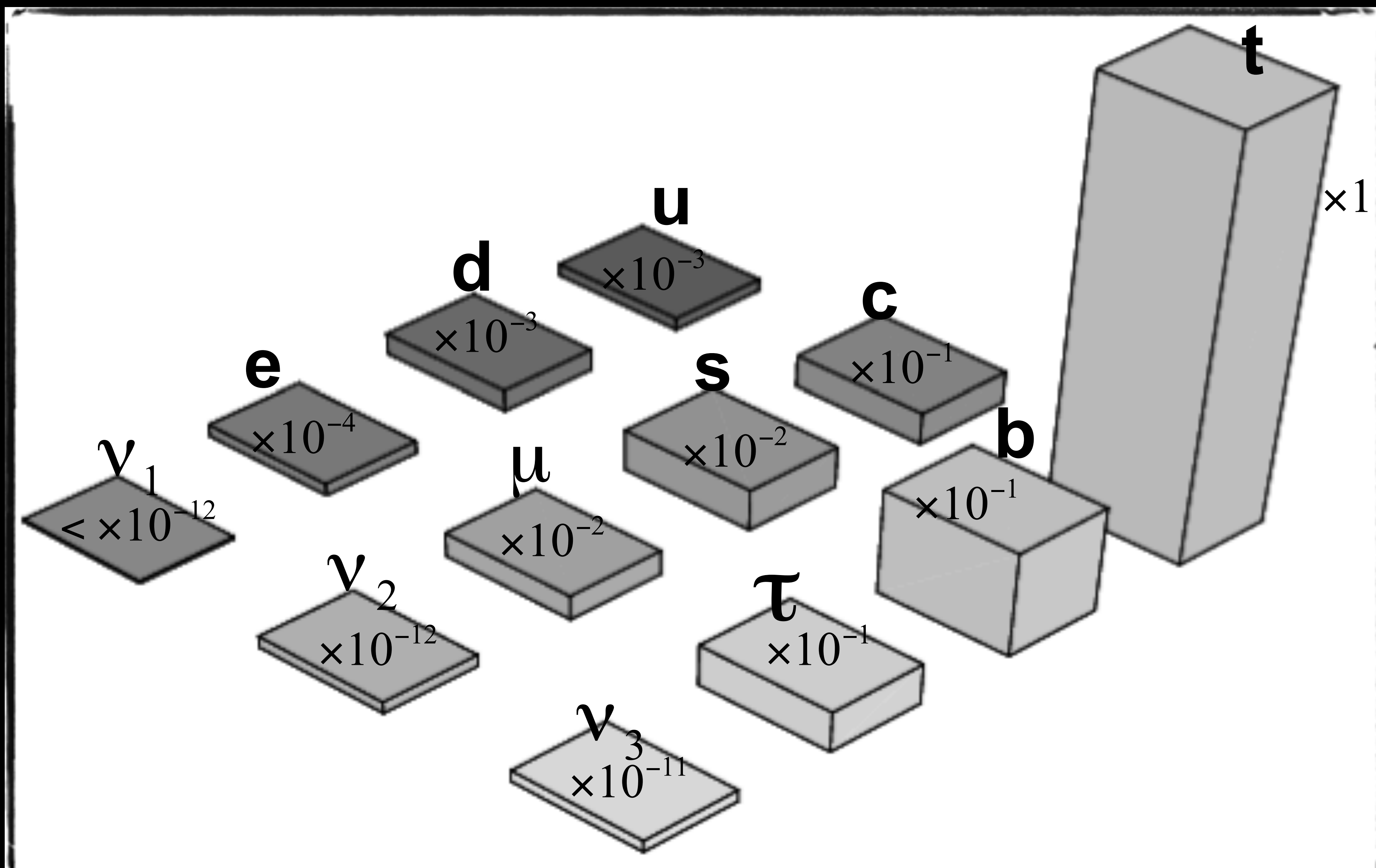


leaves many questions unanswered...

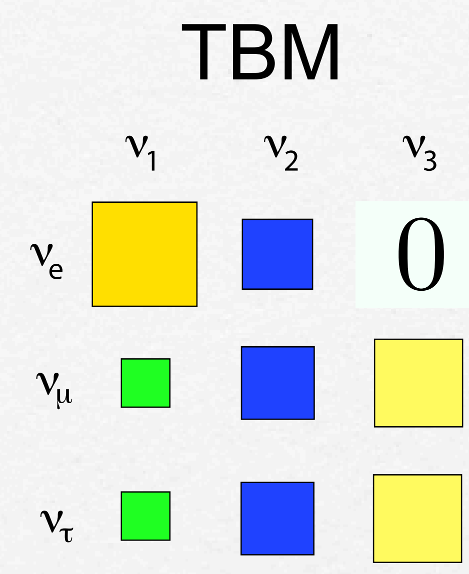
Unification?

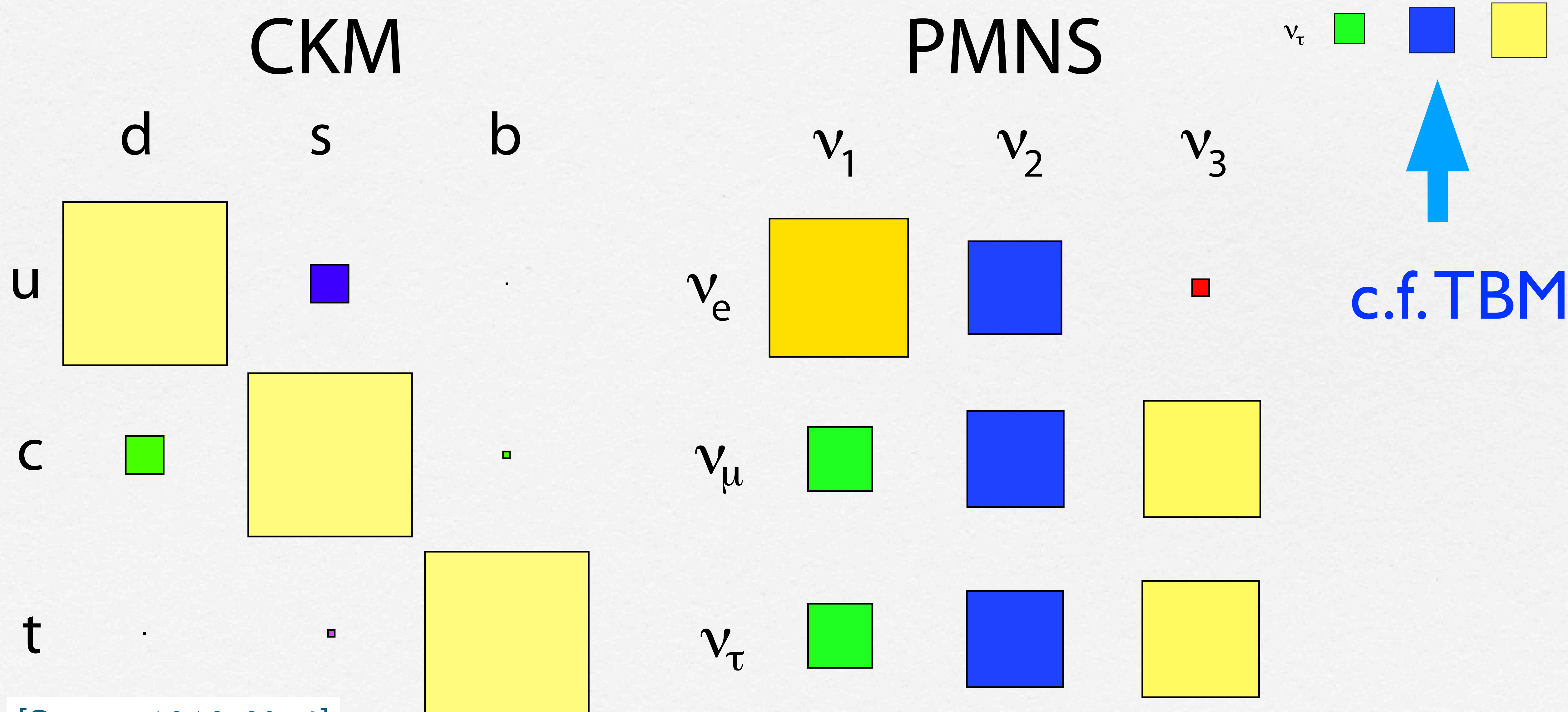


Origin of quark and lepton masses?



Origin of quark and lepton mixing?





[Stone, 1212.6374]

Simple Lepton Mixing Ansatze

$$\theta_{13} = 0^o \quad \theta_{23} = 45^o$$

- **Bimaximal** $$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 45^o$$

- **Tri-bimaximal** $$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P \quad \theta_{12} = 35.26^o$$

- **Golden ratio** $$U_{GR} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P$$

$$\phi = \frac{1+\sqrt{5}}{2} \qquad \tan\theta_{12} = \frac{1}{\phi} \qquad \theta_{12} = 31.7^o$$

Tri-maximal Mixing

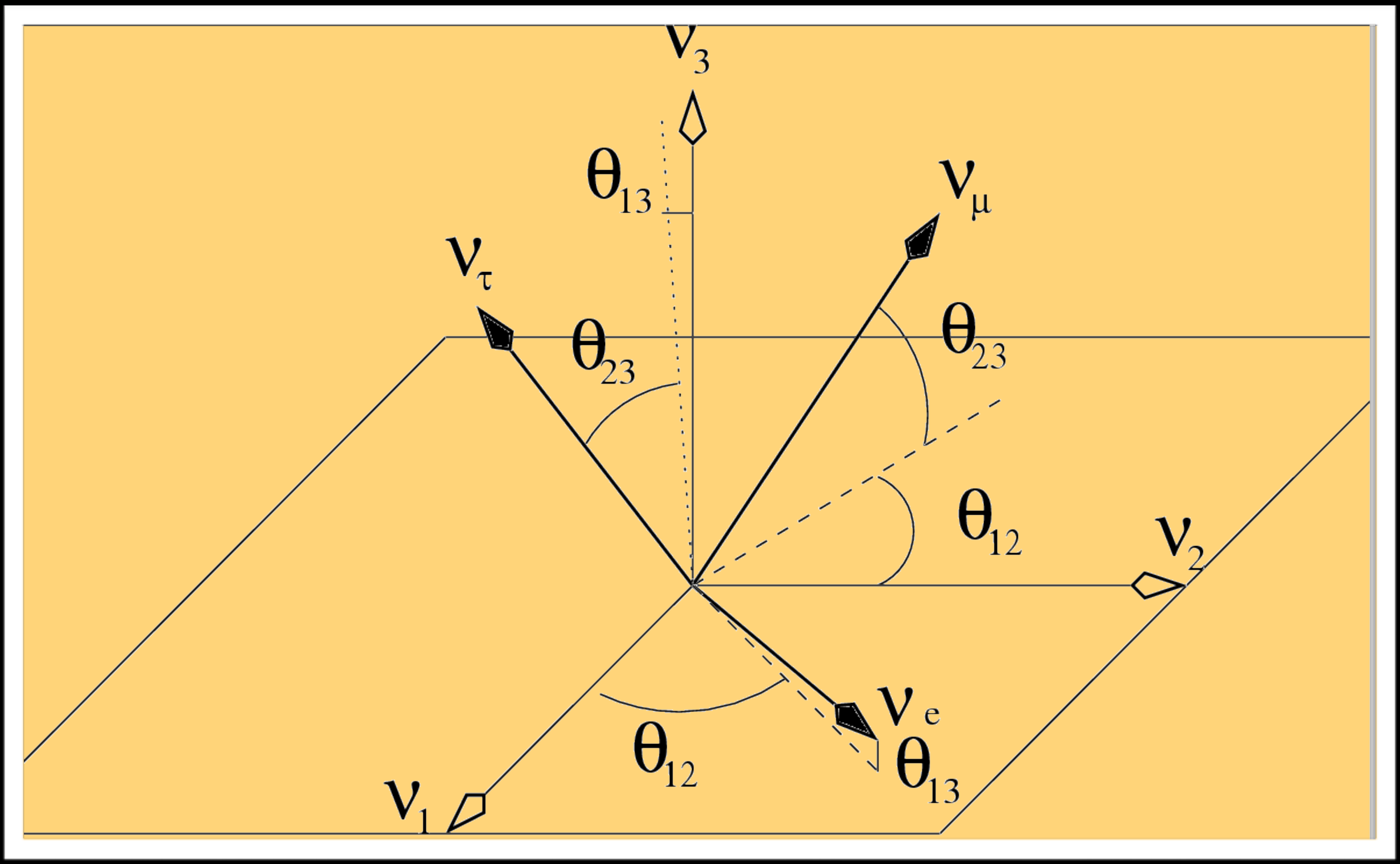
$$U_{\mathrm{TM1}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & - & - \\ -\frac{1}{\sqrt{6}} & - & - \\ \frac{1}{\sqrt{6}} & - & - \end{pmatrix}$$

$$U_{\mathrm{TM2}} \approx \begin{pmatrix} - & \frac{1}{\sqrt{3}} & - \\ - & \frac{1}{\sqrt{3}} & - \\ - & -\frac{1}{\sqrt{3}} & - \end{pmatrix}$$

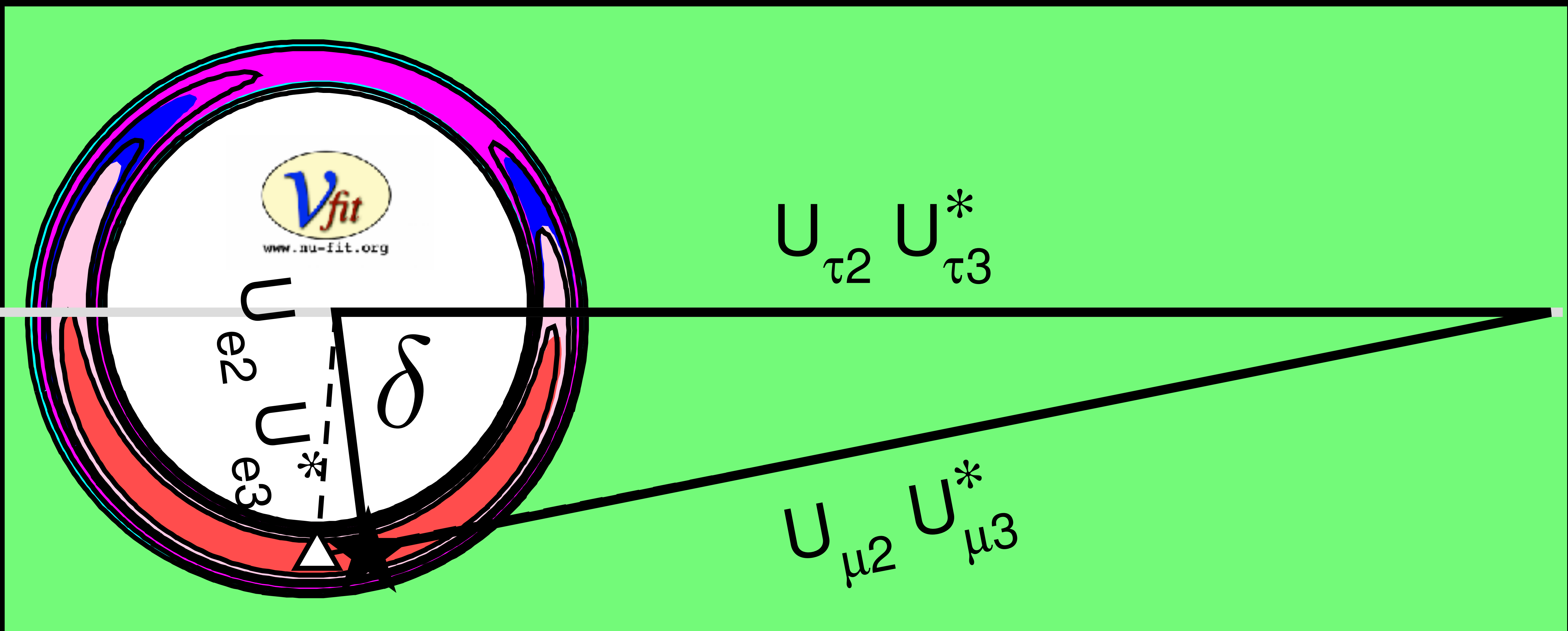
Allows for non-zero reactor angle

Implies sum rule relations between PMNS parameters

The Lepton Mixing Angles



The oscillation observable CP Violating Phase



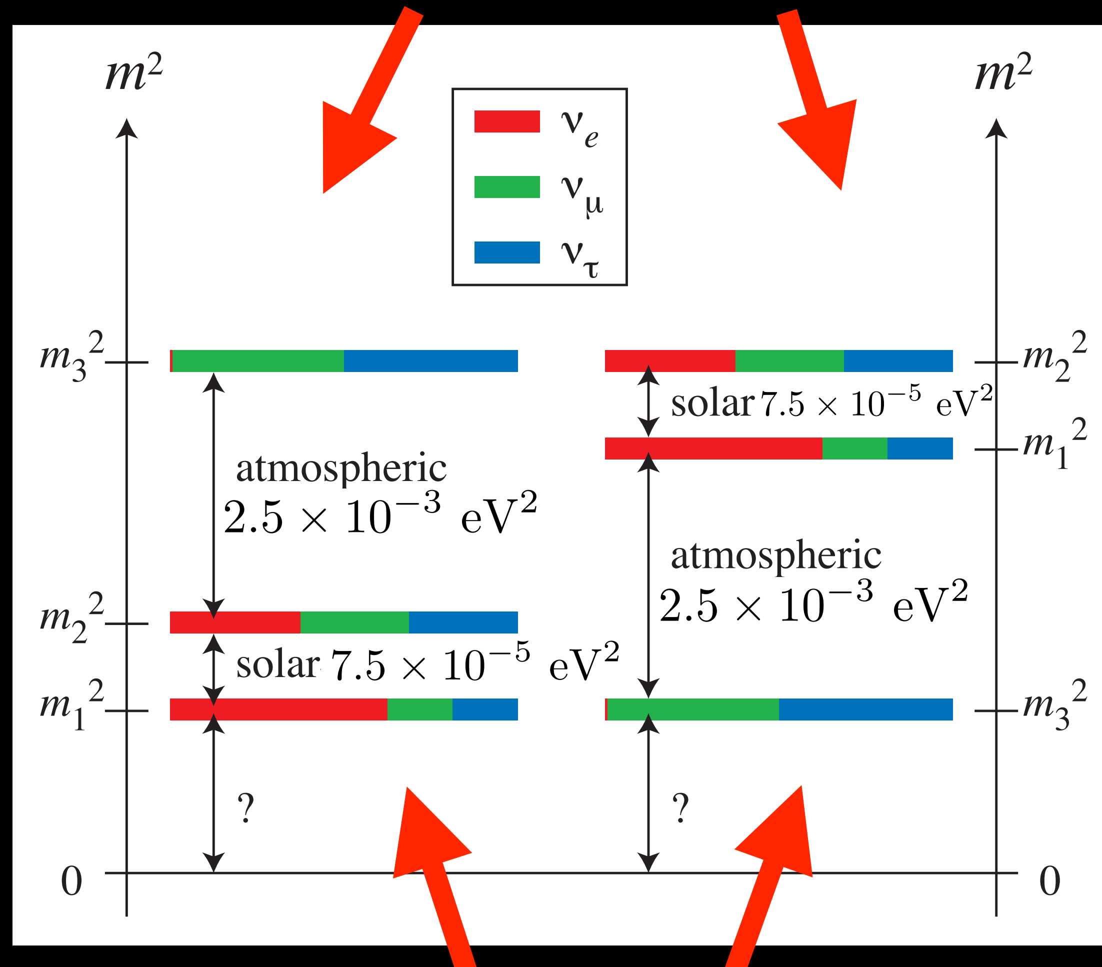
Quark vs Lepton mixings

	θ_{12}	θ_{23}	θ_{13}	δ
Quarks	13° $\pm 0.1^\circ$	2.4° $\pm 0.1^\circ$	0.2° $\pm 0.05^\circ$	70° $\pm 5^\circ$
Leptons	34° $\pm 1^\circ$	45° $41^\circ \pm 1^\circ$ $50^\circ \pm 1^\circ$	8.5° $\pm 0.15^\circ$	-90° $\pm 50^\circ$

Global fits Concha talk

Open Questions

- Is CP violated in the leptonic sector? (Probably)
- Is the atmospheric angle in first or second octant?
- Neutrino mass: NO or IO ?

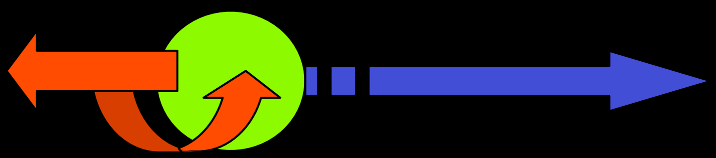


- Lightest neutrino mass?

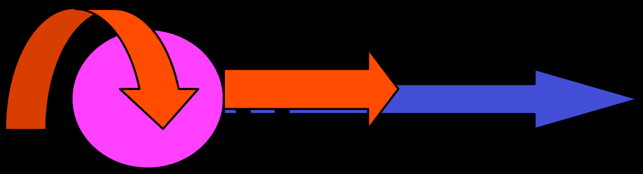
Are neutrino masses Dirac or Majorana?

SFK 0712.1750

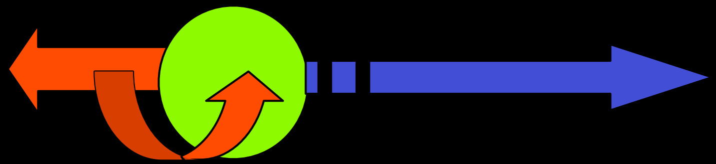
ν_L — Dirac — ν_R



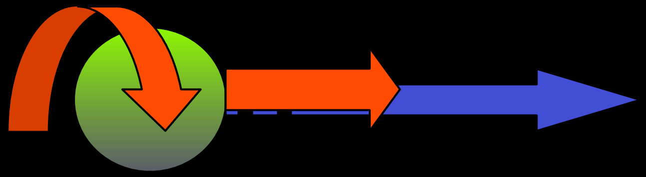
$$m_{\mathrm{D}}\bar{\nu}_L\nu_R$$



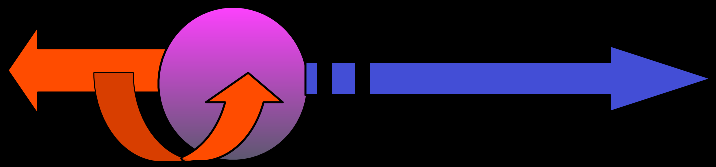
ν_L — Majorana — ν_L^c



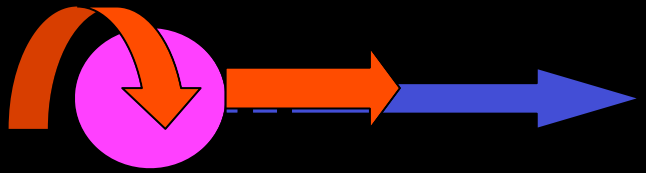
$$m_{\mathrm{L}}\bar{\nu}_L\nu_L^c$$



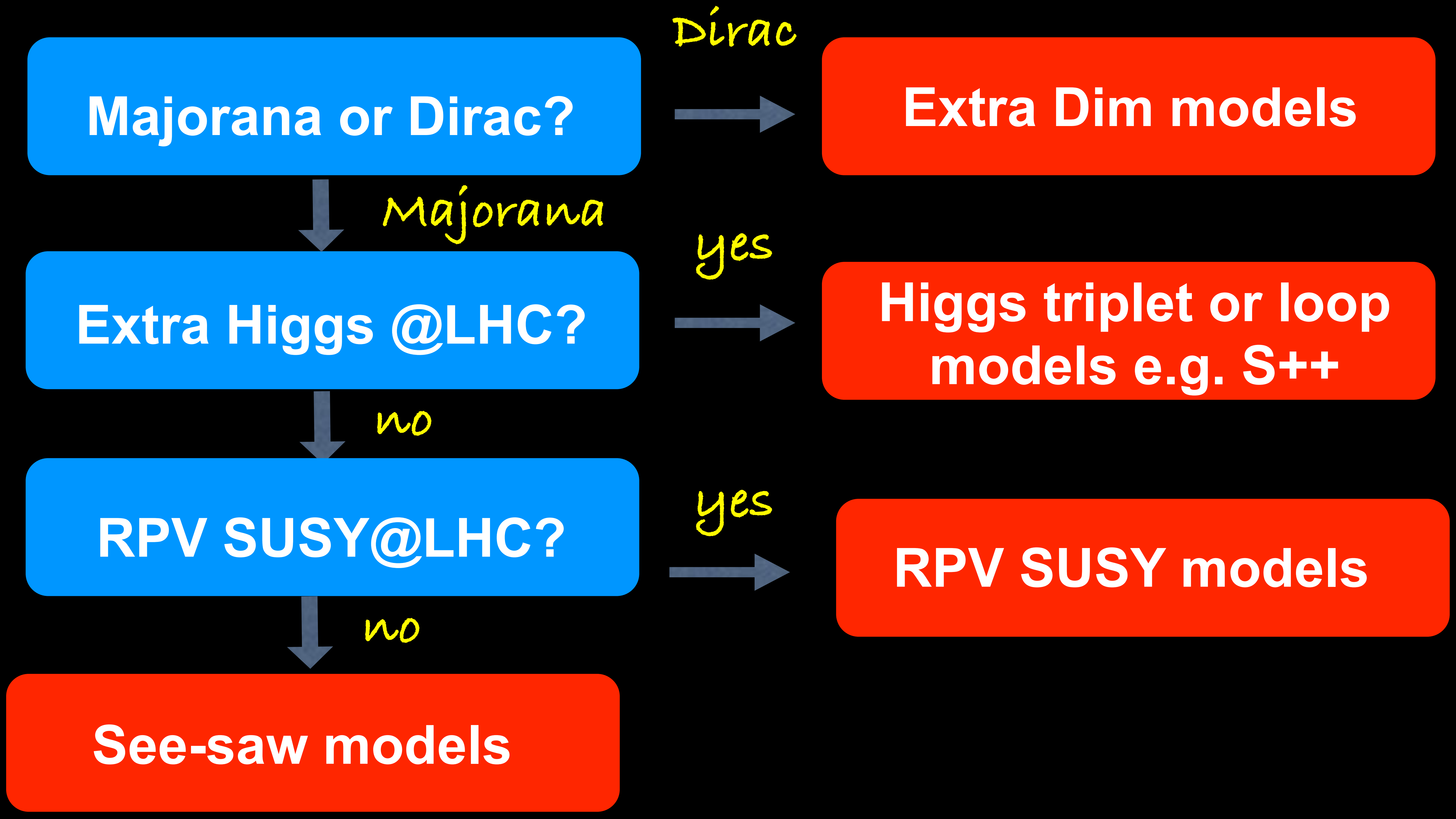
ν_R^c — Majorana — ν_R



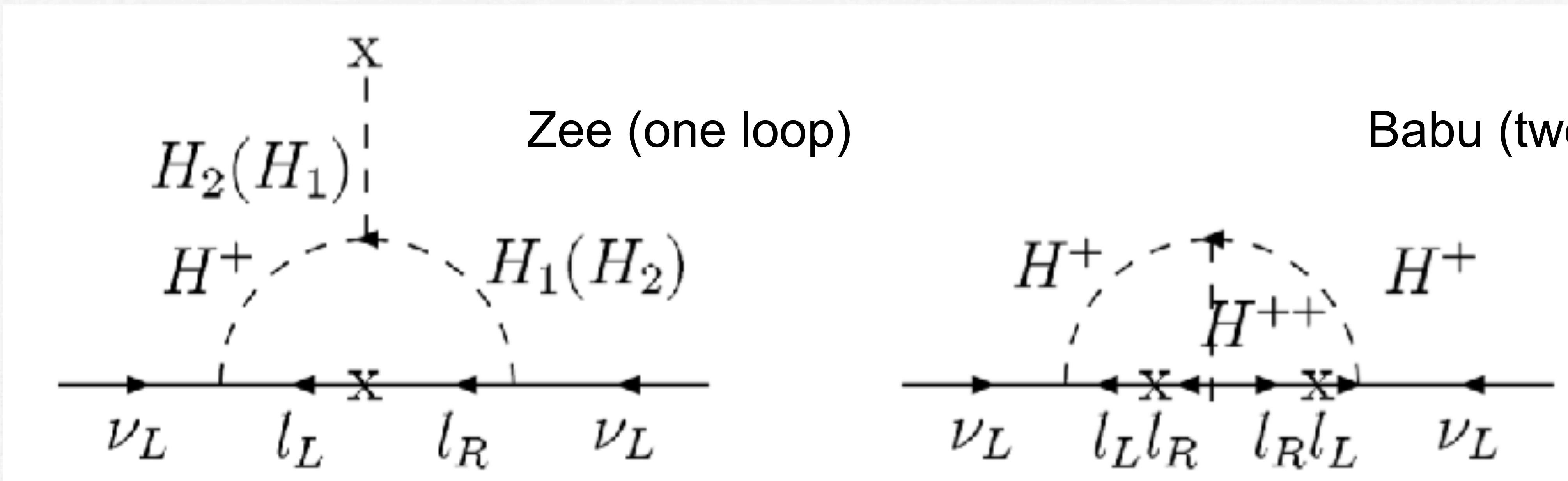
$$M_{\mathrm{R}}\bar{\nu}_R^c\nu_R$$

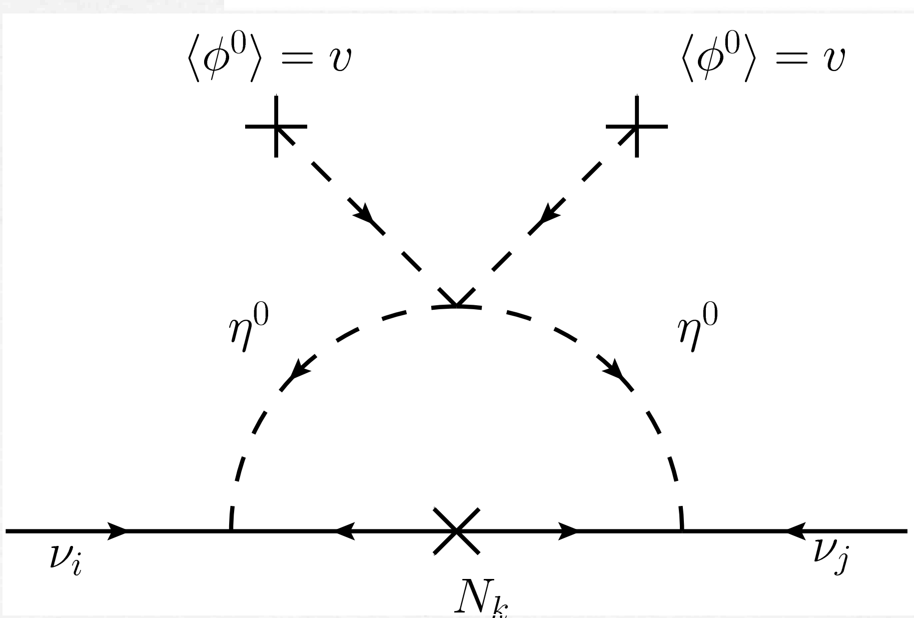


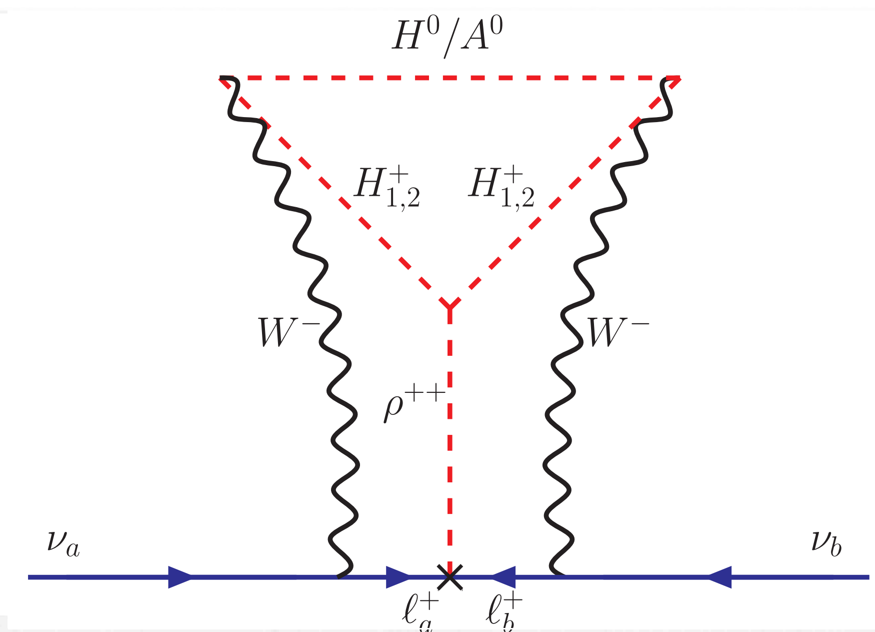
Roadmap of neutrino mass



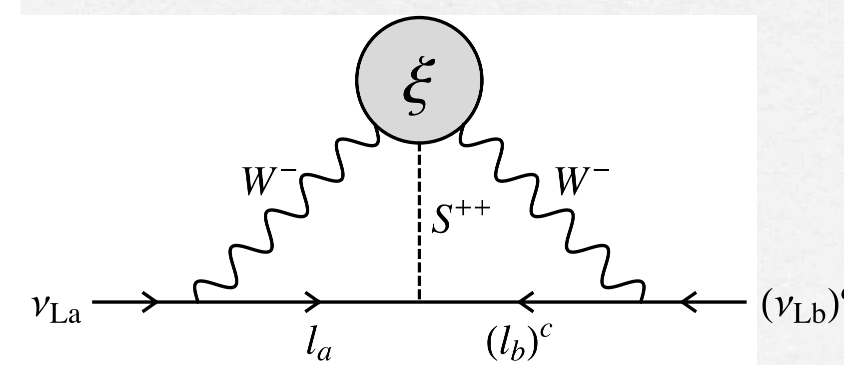
Loop Models of Neutrino Mass







SFK, Merle, Panizzi



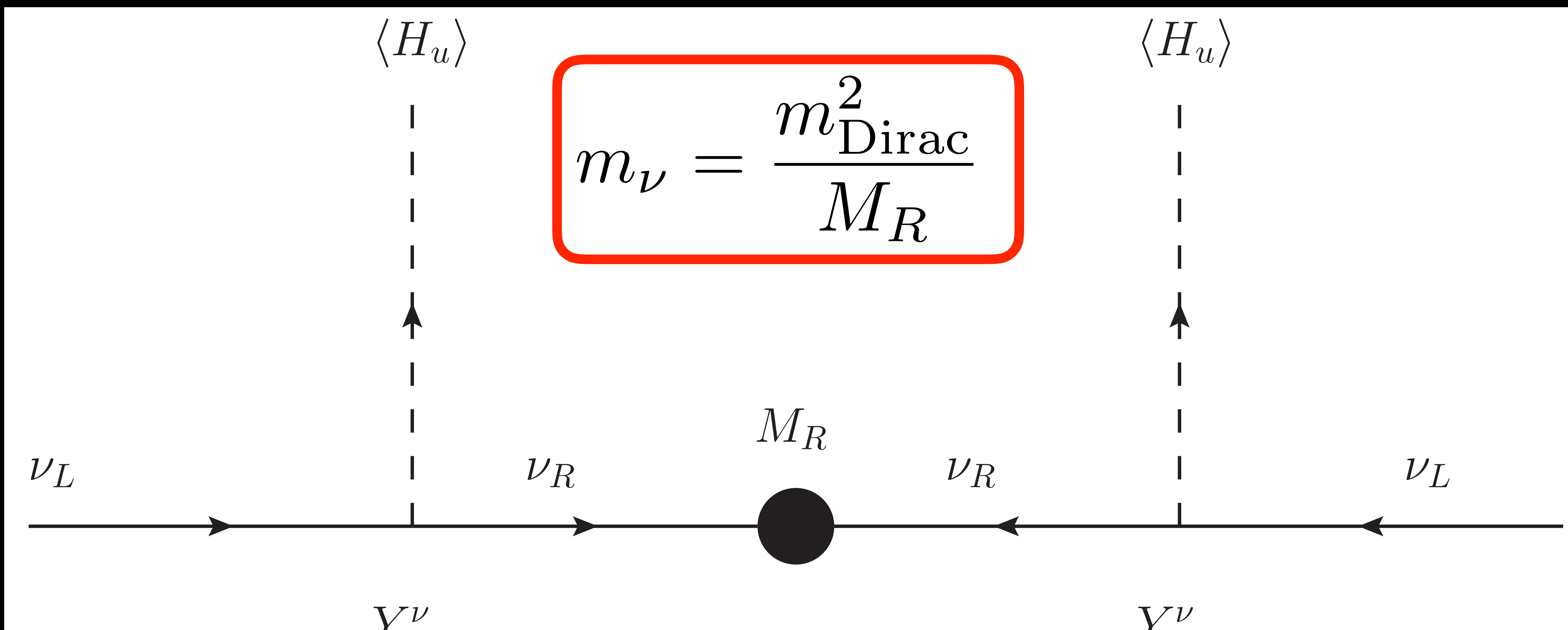
Scotogenic model

Cocktail model

Effective theory

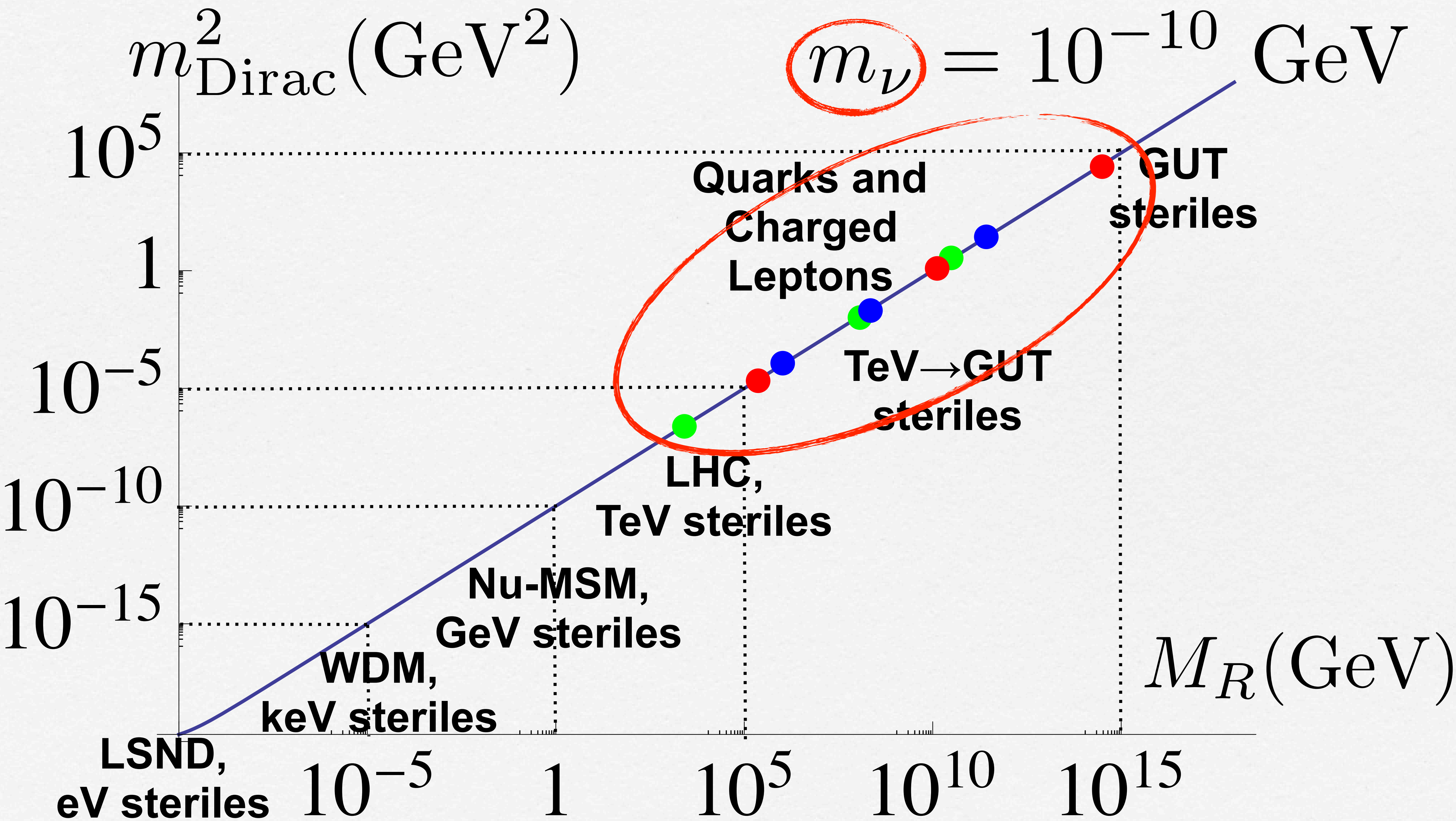
Minimal Type I seesaw

$$m_\nu = \frac{m^2_{\mathrm{Dirac}}}{M_R}$$



Seesaw formula → $m^2_{\text{Dirac}} = m_\nu M_R$

$m_\nu = 10^{-10}$ GeV



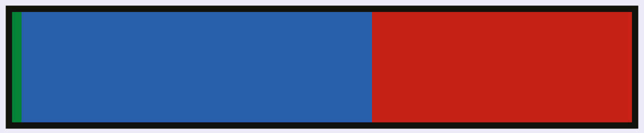
Sequential dominance

SFK'98

Simple way to deal with 3 RHN

normal ordering

$$m_3 \sim 50 \text{ meV}$$



$$\frac{m^D_{\rm atm} m^{D\;T}_{\rm atm}}{M_{\rm atm}}$$

$$\gg$$

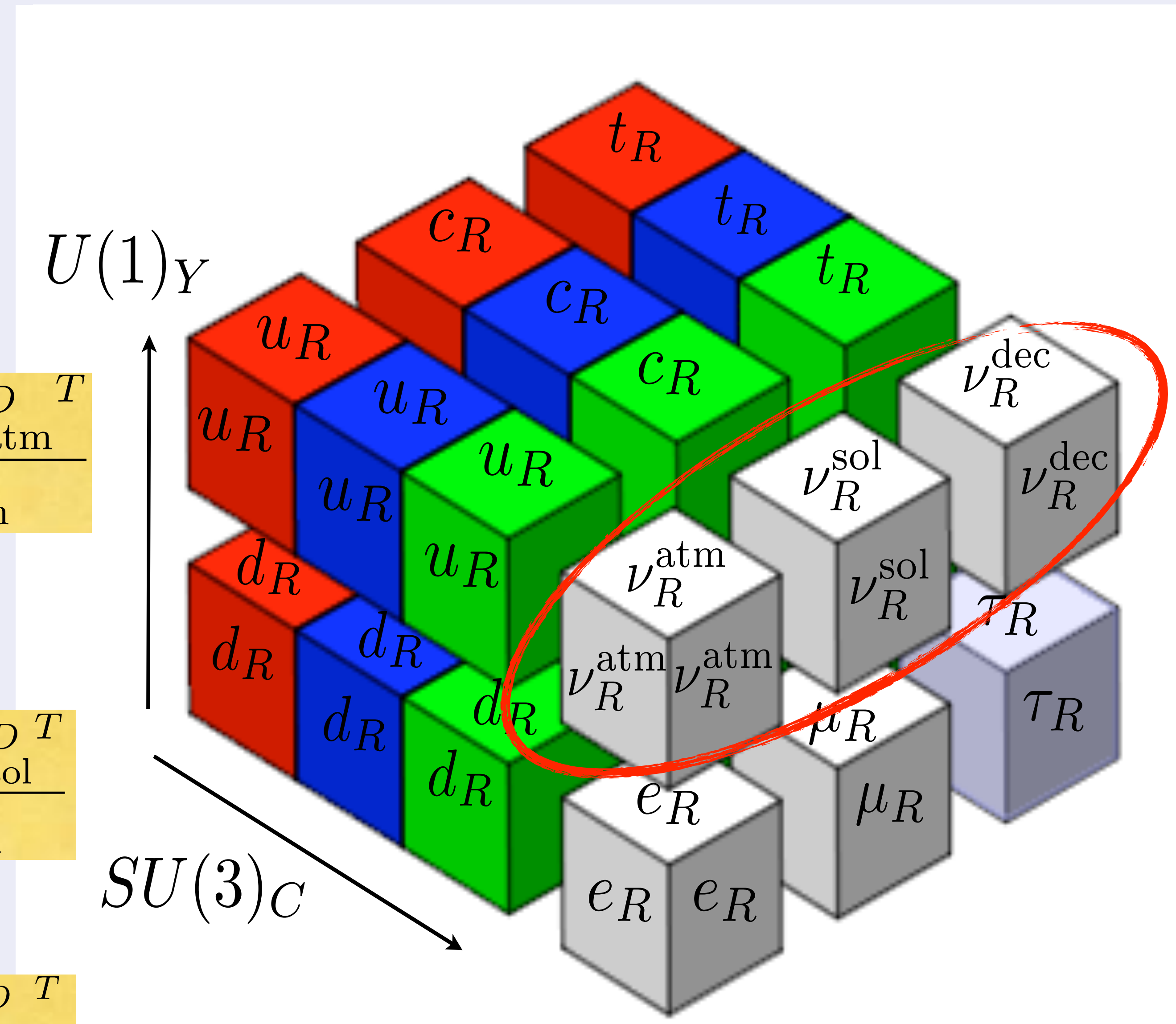
$$m_2 \sim 9 \text{ meV}$$

$$\frac{m^D_{\rm sol} m^{D\;T}_{\rm sol}}{M_{\rm sol}}$$

$$\gg$$

$$\frac{m^D_{\rm dec} m^{D\;T}_{\rm dec}}{M_{\rm dec}}$$

$$m_1 \ll m_2$$



Littlest Seesaw

SFK
1304.6264,
1512.07531

- Two right-handed neutrinos (RHN) $\nu_R^{\rm atm}$ $\nu_R^{\rm sol}$
- Diagonal $M_R = \begin{pmatrix} M_{\rm atm} & 0 \\ 0 & M_{\rm sol} \end{pmatrix}$ completely decoupled $\nu_R^{\rm dec}$
- Diagonal (m_e, m_μ, m_τ)
- Constrained Sequential Dominance (CSD3):

$$m_D = \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix}$$

Type A

or

$$m_D = \begin{pmatrix} 0 & b \\ a & b \\ a & 3b \end{pmatrix}$$

Type B

Enforced by symmetry (see later)

SFK,Luhn
1607.05276

The Littlest Seesaw

Low energy neutrino mass matrices after seesaw:

$$m^{\nu}_{\mathrm{LSA}} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix},$$

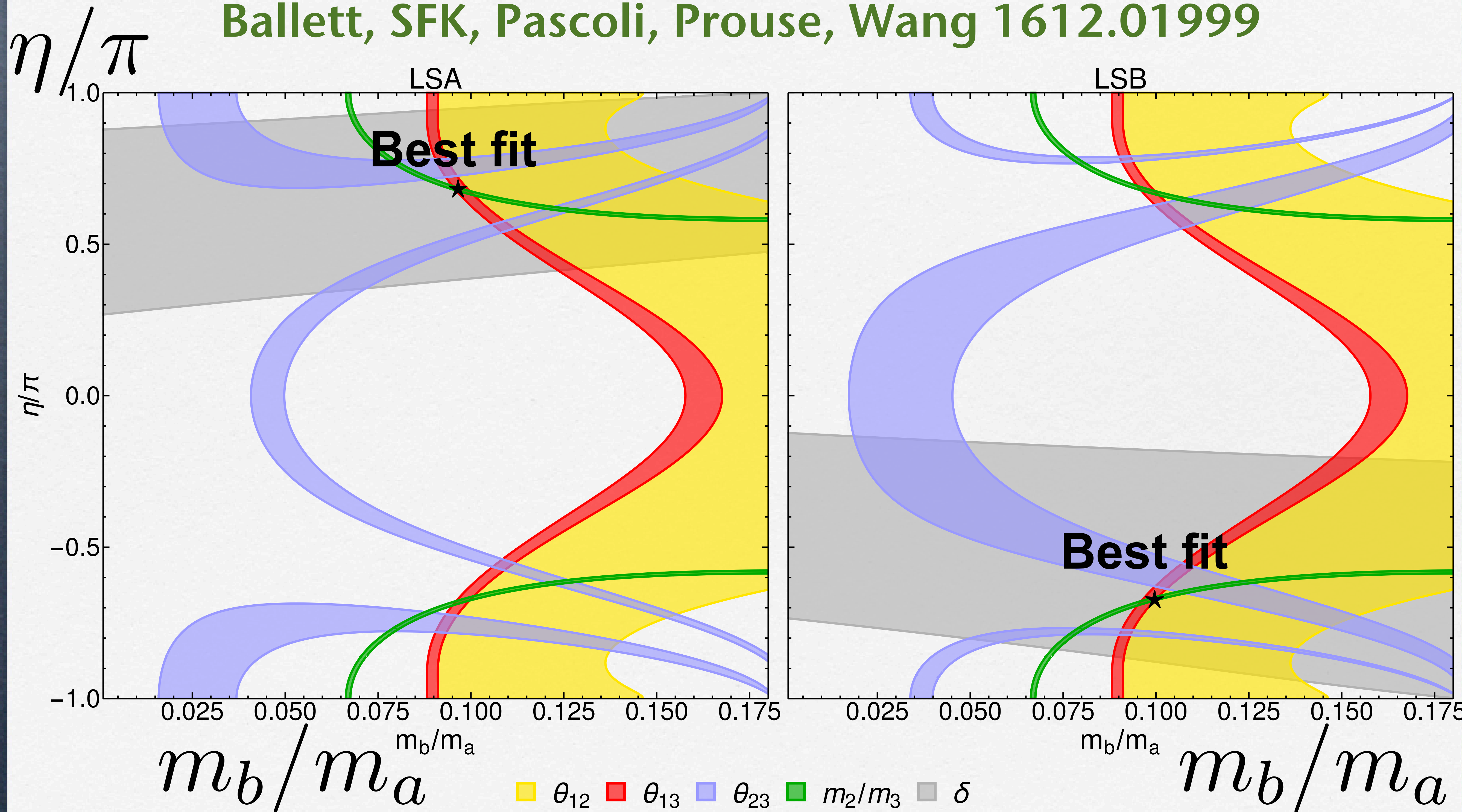
$$m^{\nu}_{\mathrm{LSB}} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{pmatrix}.$$

SD m_a>>m_b predicts NO with m_1=0

Depends on 3 parameters: ma, mb, eta

Littlest Seesaw vs current data

Ballett, SFK, Pascoli, Prouse, Wang 1612.01999



Best Fit LS Predictions

NO with $m_1=0$

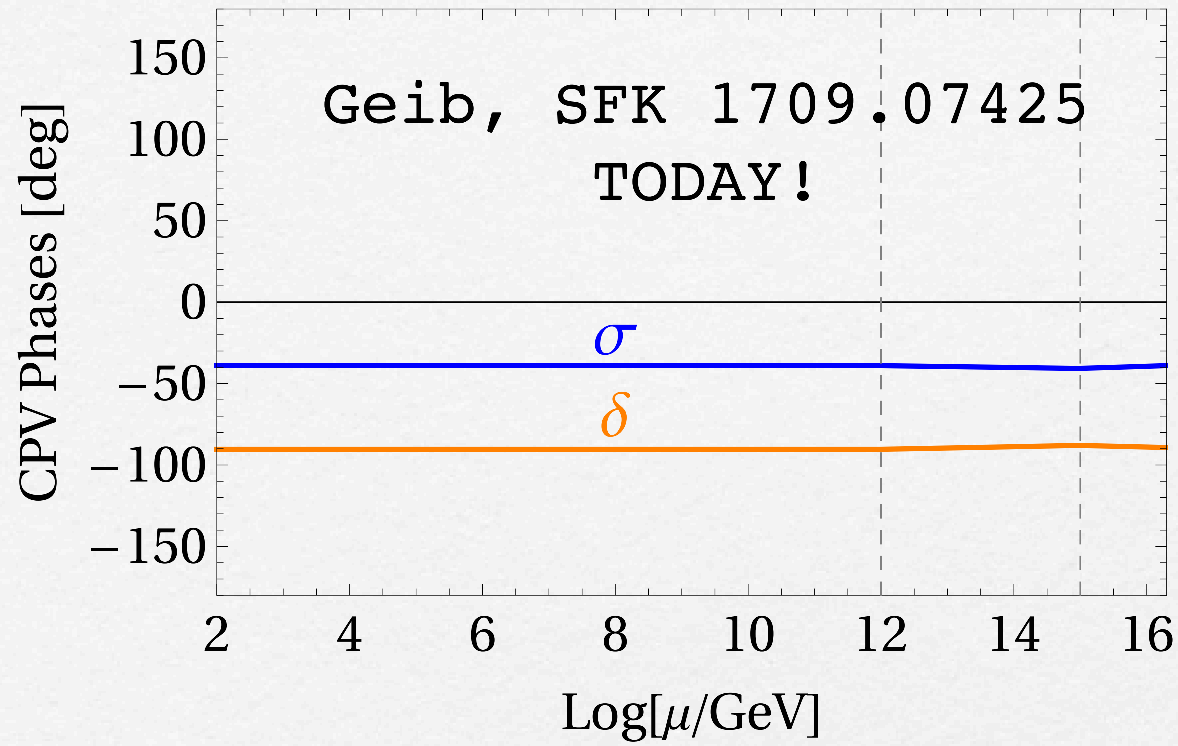
	LSA		LSB		NuFIT 3.0
	η free	η fixed	η free	η fixed	global fit
m_a [meV]	27.19	26.74	26.95	26.75	
m_b [meV]	2.654	2.682	2.668	2.684	—
η [rad]	0.680π	$2\pi/3$	-0.673π	$-2\pi/3$	
θ_{12} [°]	34.36	34.33	34.35	34.33	$33.72^{+0.79}_{-0.76}$
θ_{13} [°]	8.46	8.60	8.54	8.60	$8.46^{+0.14}_{-0.15}$
θ_{23} [°]	45.03	45.71	44.64	44.28	$41.5^{+1.3}_{-1.1}$
δ [°]	-89.9	-86.9	-91.6	-93.1	-71^{+38}_{-51}
Δm^2_{21} [10^{-5}eV^2]	7.499	7.379	7.447	7.390	$7.49^{+0.19}_{-0.17}$
Δm^2_{31} [10^{-3}eV^2]	2.500	2.510	2.500	2.512	$2.526^{+0.039}_{-0.037}$
$\Delta\chi^2$ / d.o.f	4.1 / 3	5.6 / 4	3.9 / 3	4.5 / 4	—

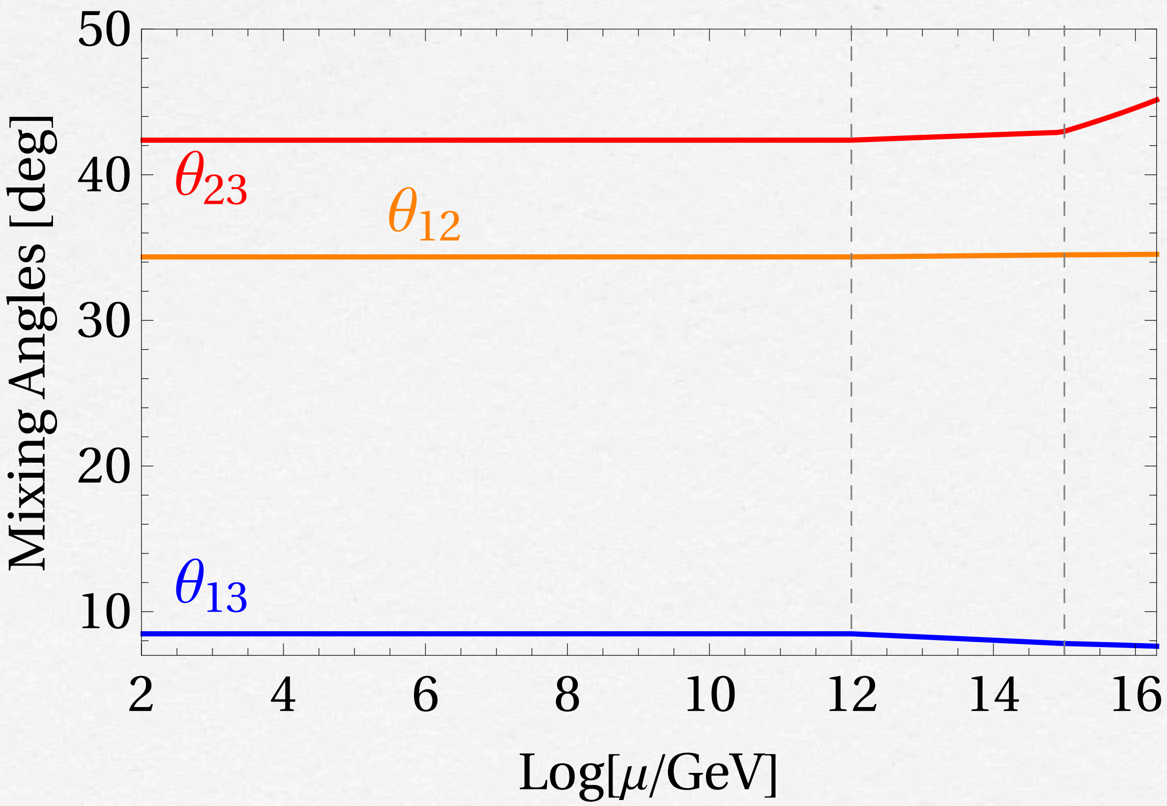
Near maximal mixing

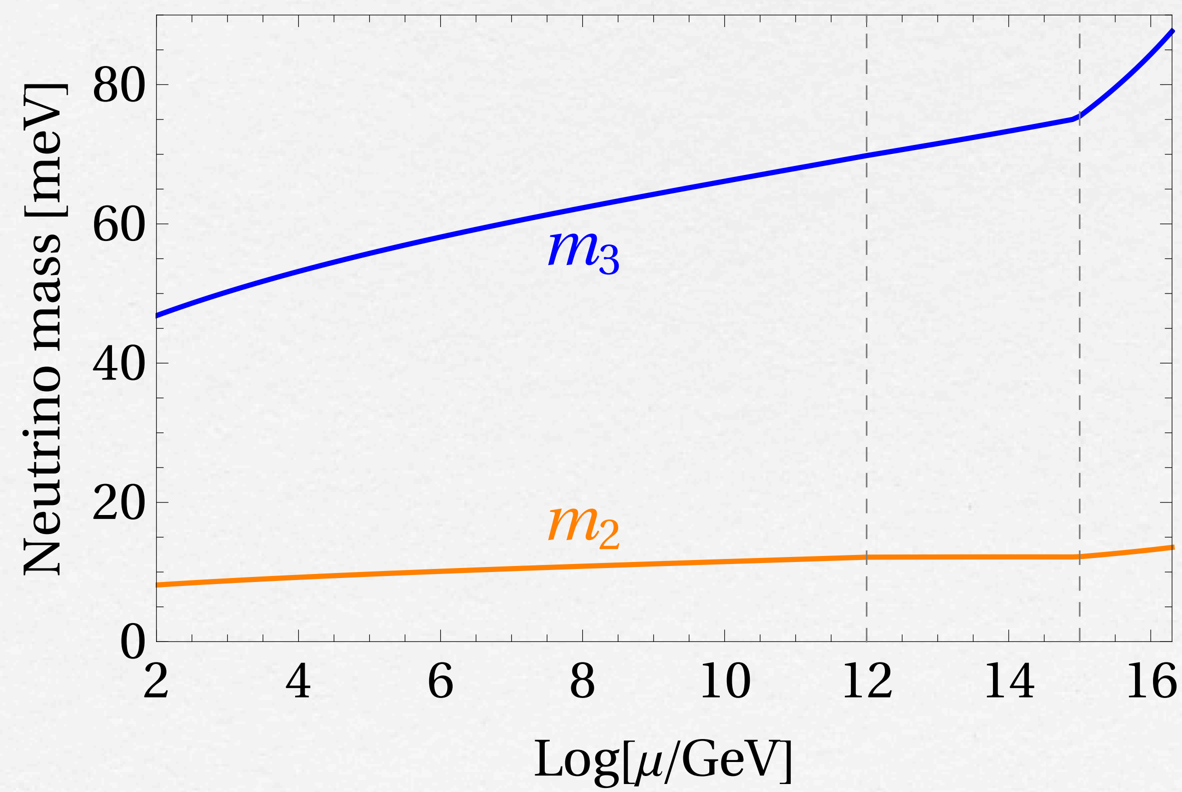
RG Corrections in SM (are large)

SM with $M_{atm} = 10^{15}$ GeV and $M_{sol} = 10^{12}$ GeV

	Λ_{GUT}	M_{atm}	M_{sol}	Λ_{EW}
θ_{13}(deg)	7.62574	7.81215	8.47979	8.4798
θ_{12}(deg)	34.5348	34.4977	34.3575	34.3572
θ_{23}(deg)	45.1425	42.9816	42.3751	42.3744
m_2(meV)	13.537	12.2035	12.1317	8.73113
m_3(meV)	87.6802	75.4657	69.8112	50.2431
δ_{CP}(deg)	–89.2885	–88.0086	–90.3508	–90.3507
σ_{CP}(deg)	–38.9558	–40.649	–38.9917	–38.9917





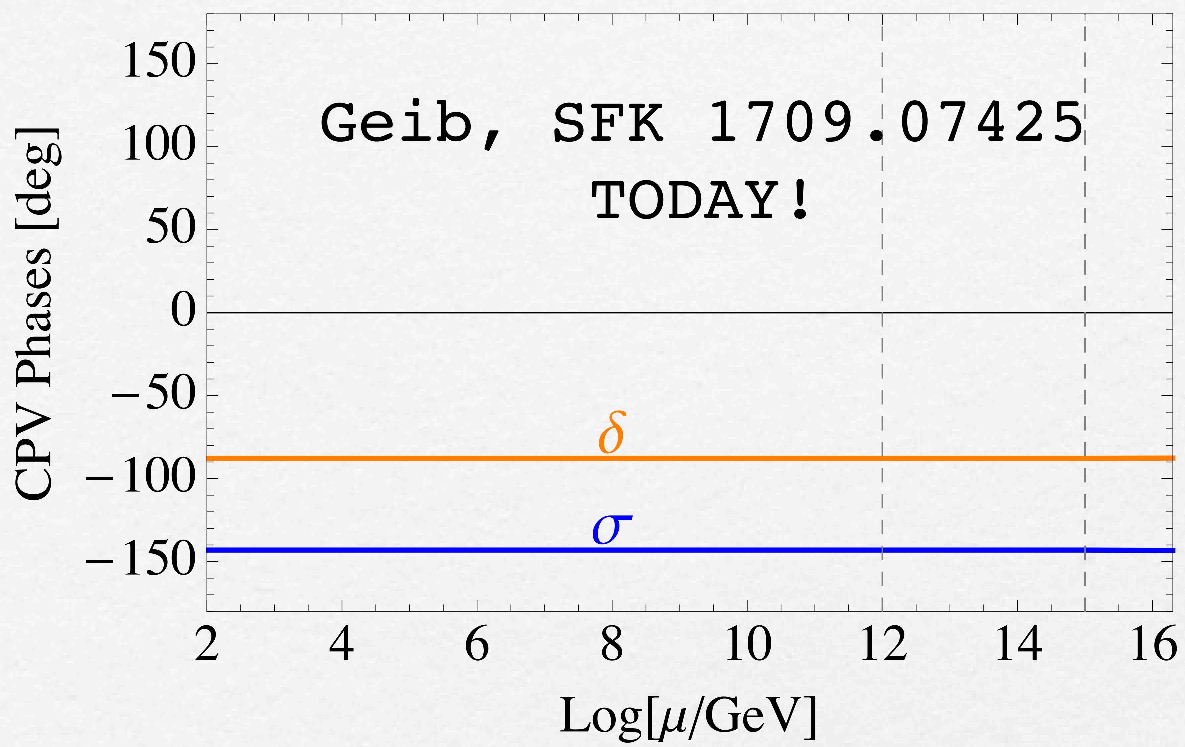


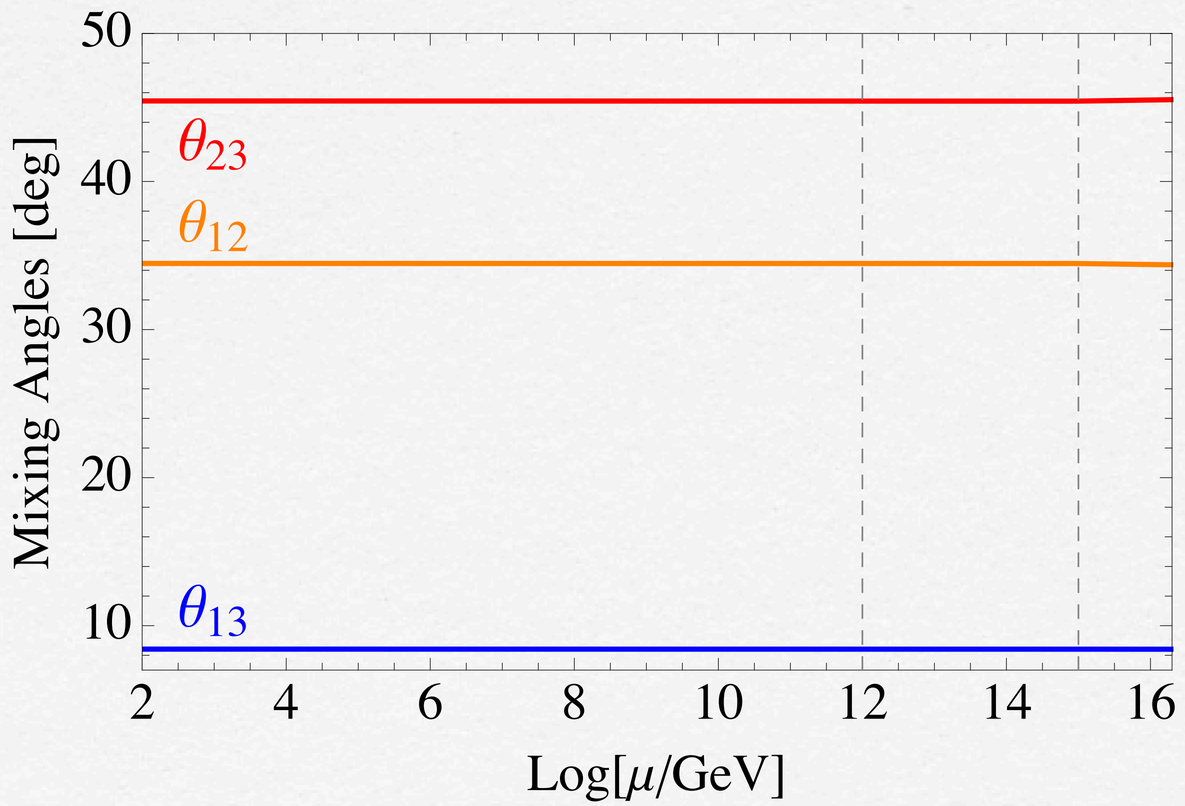
RG Corrections in SUSY (are small)

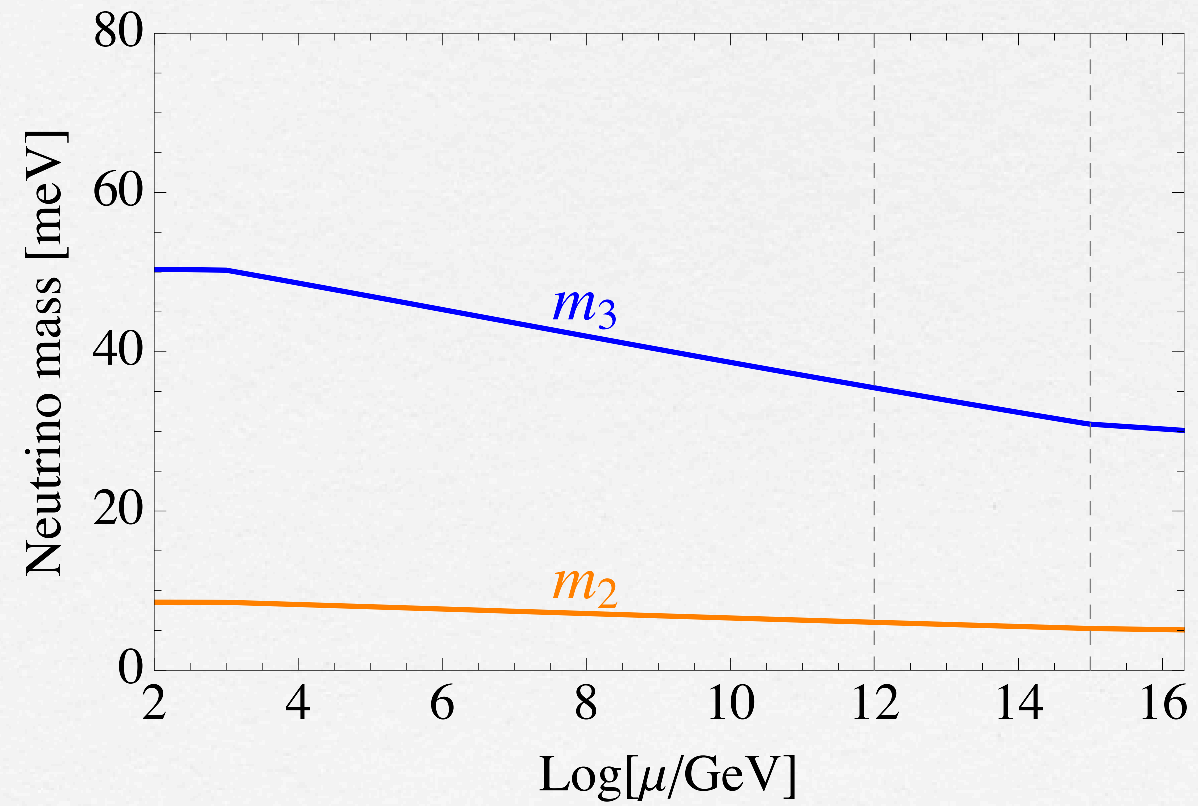
$M_{atm} = 10^{12}$ GeV and $M_{sol} = 10^{15}$ GeV

Case A, $M_{\rm SUSY}$= 1 TeV, tanβ=5, $\bar{\eta}_b$=0.6

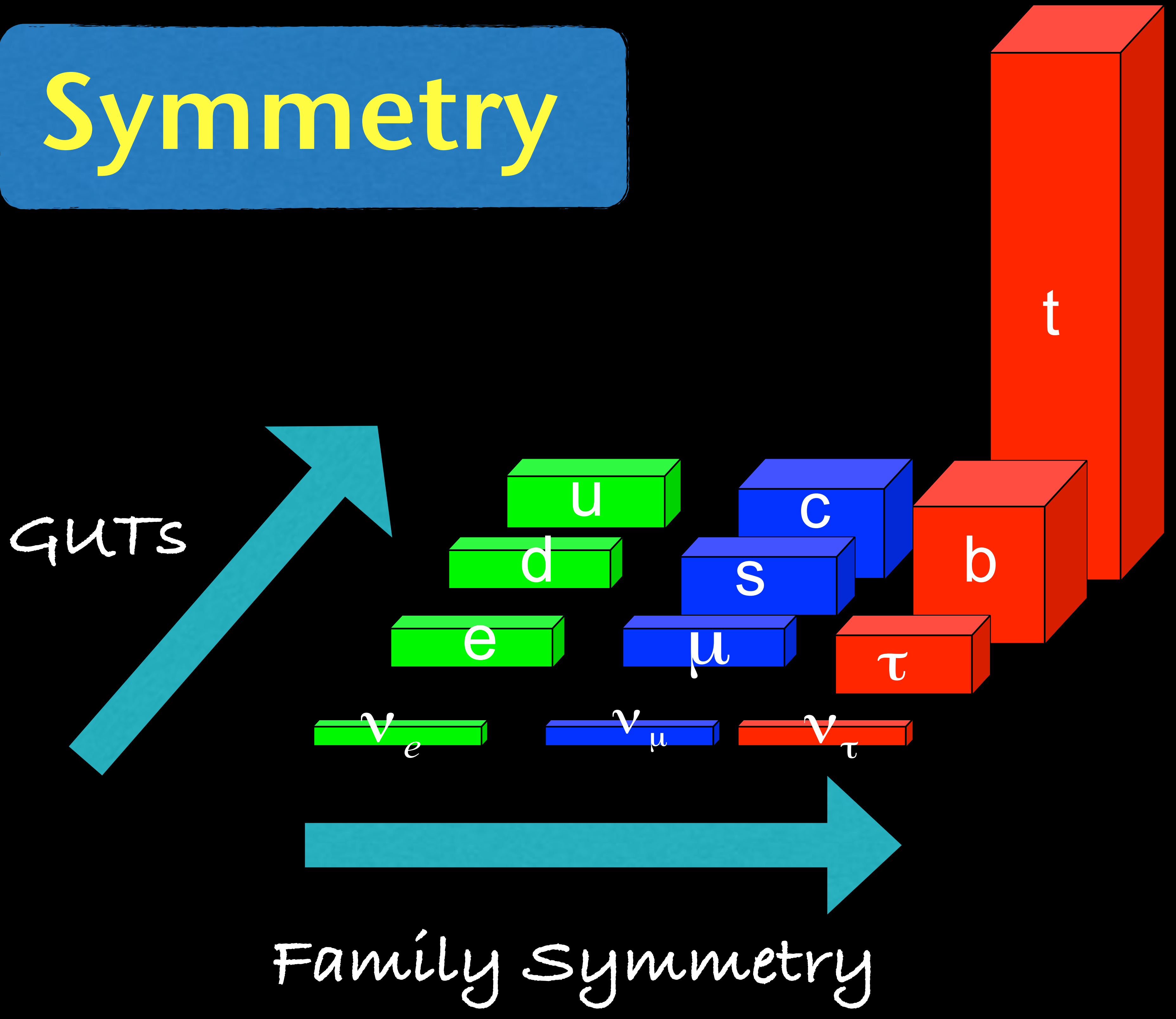
	$\Lambda_{\rm GUT}$	$M_{\rm sol}$	$M_{\rm atm}$	$\Lambda_{\rm EW}$
θ_{13}(deg)	8.41036	8.41346	8.41449	8.41694
θ_{12}(deg)	34.3737	34.4593	34.4613	34.4648
θ_{23}(deg)	45.5262	45.4286	45.4309	45.4401
m_2(meV)	5.06633	5.24637	6.02352	8.53262
m_3(meV)	30.1179	30.9015	35.4702	50.2415
$\delta_{\rm CP}$(deg)	−87.6504	−87.8008	−87.8032	−87.8023
$\sigma_{\rm CP}$(deg)	−143.312	−143.071	−143.067	−143.067



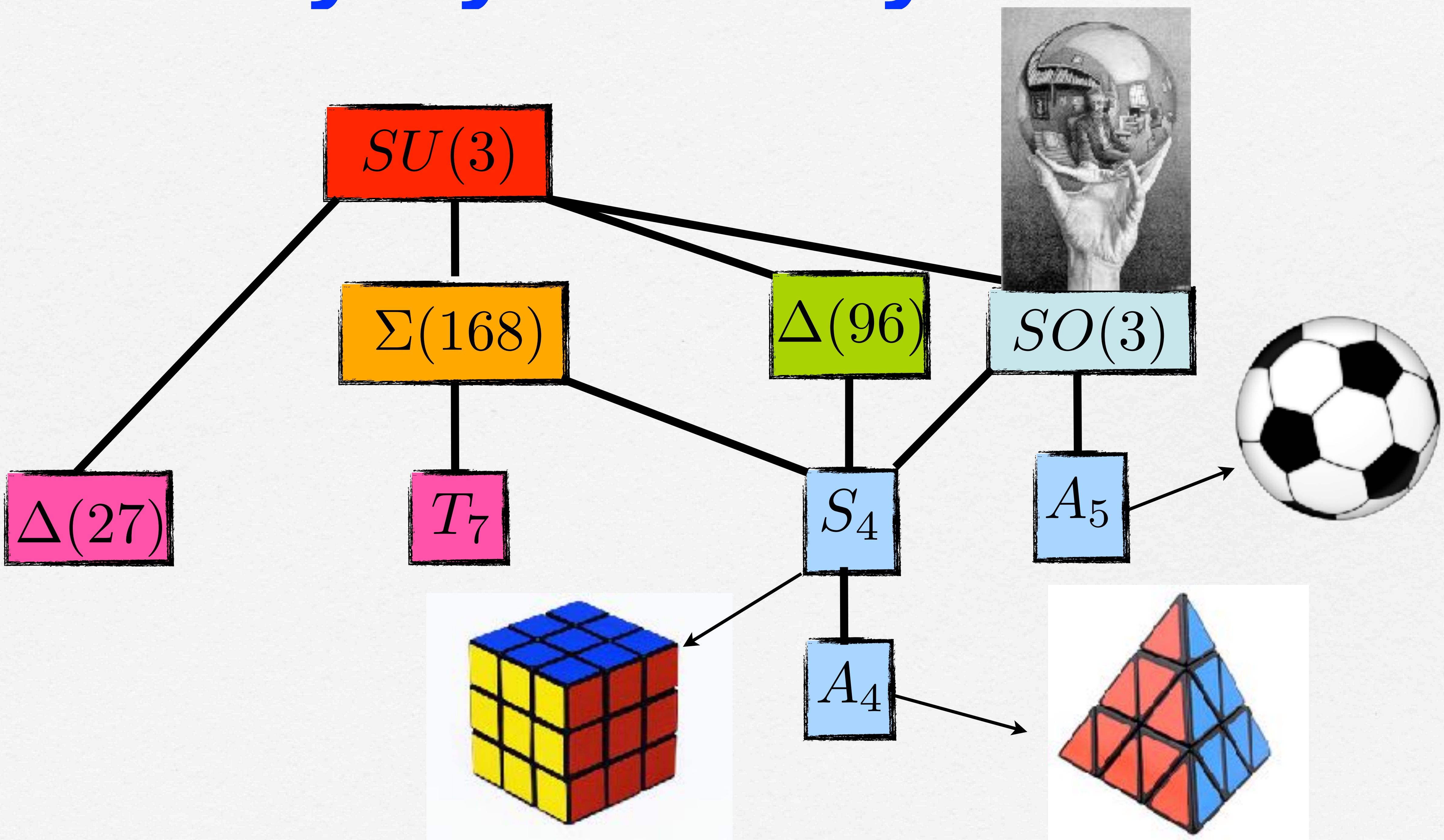




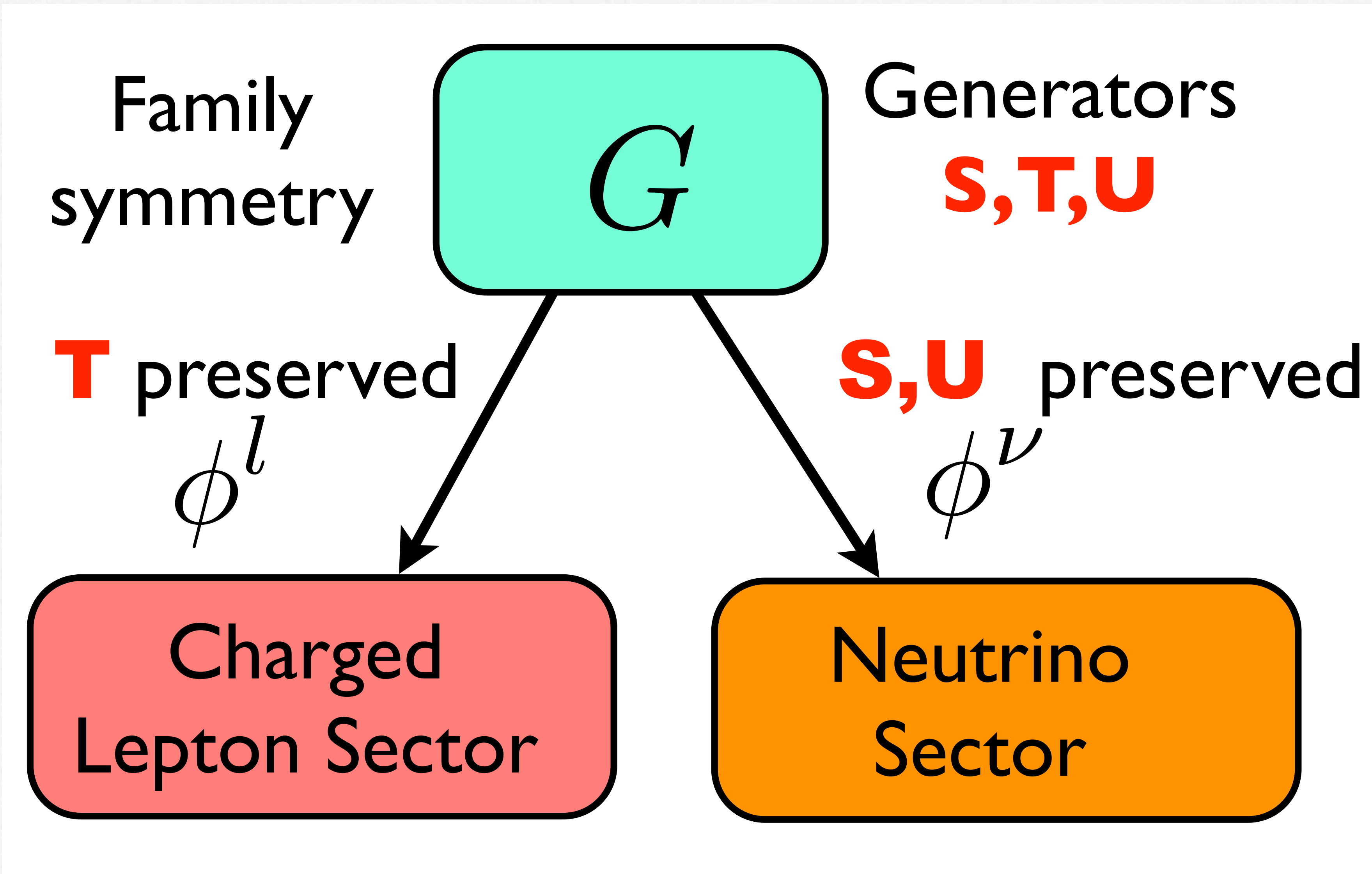
Symmetry



Family Symmetry



Direct Models



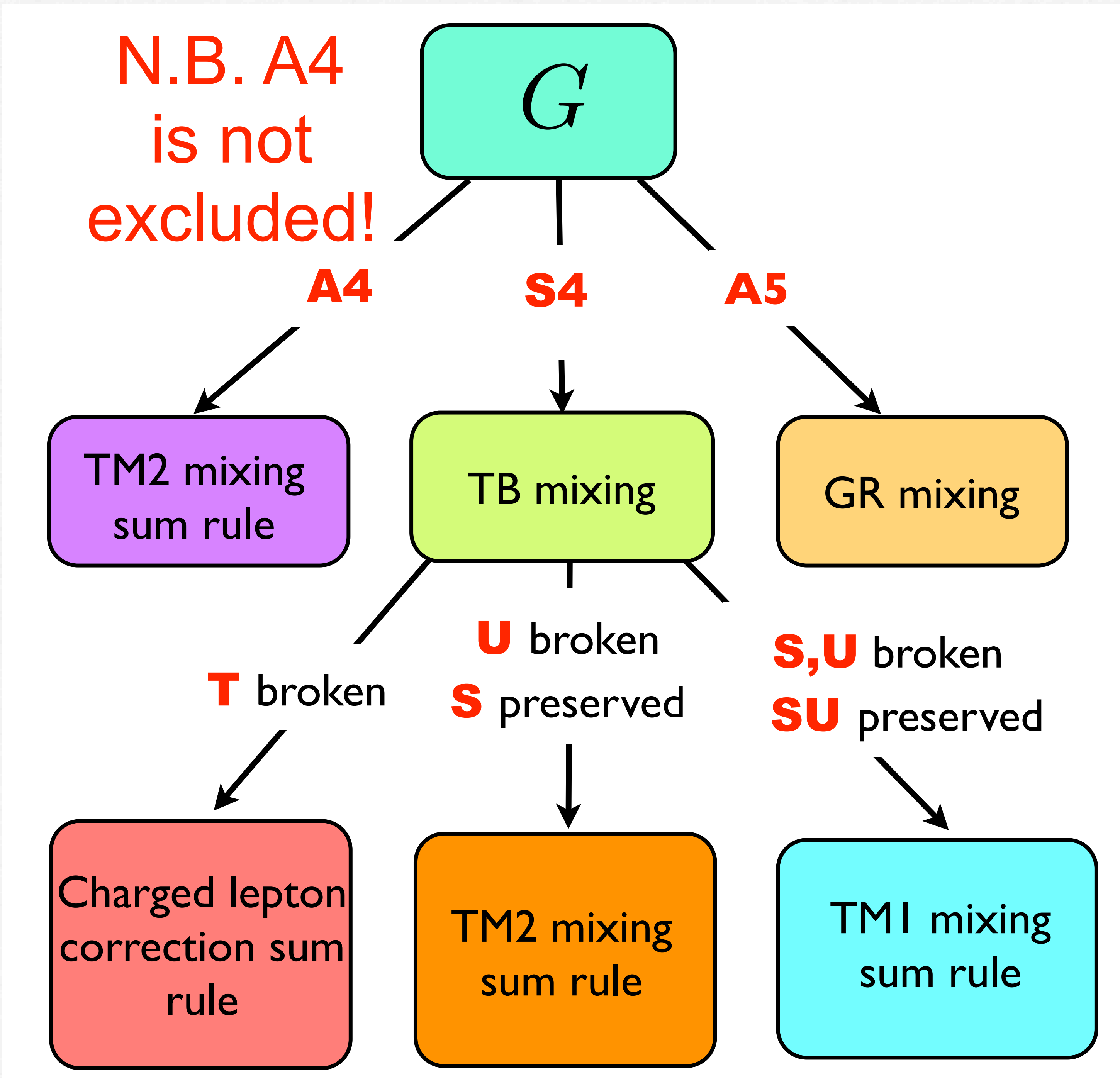
TBM from S_4

But TBM is excluded

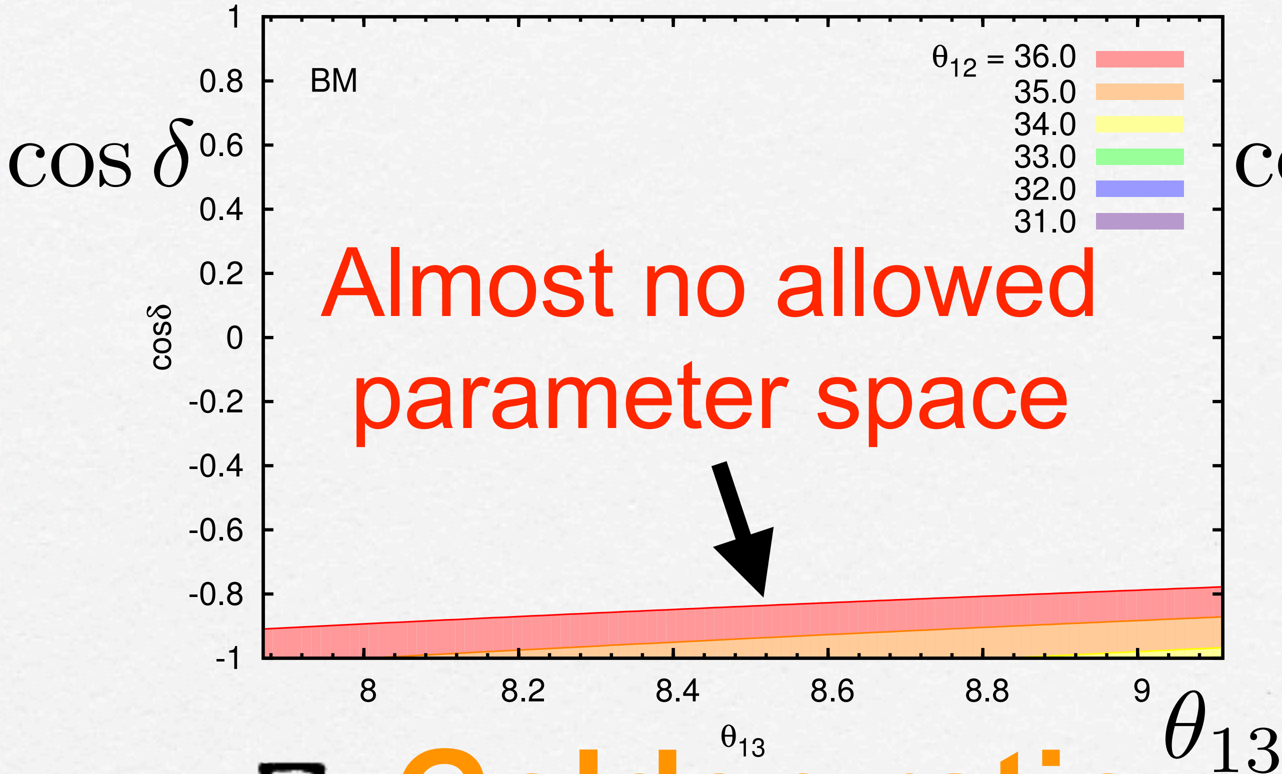
so: $\Delta(6n^2)$

But many people don't like such large groups

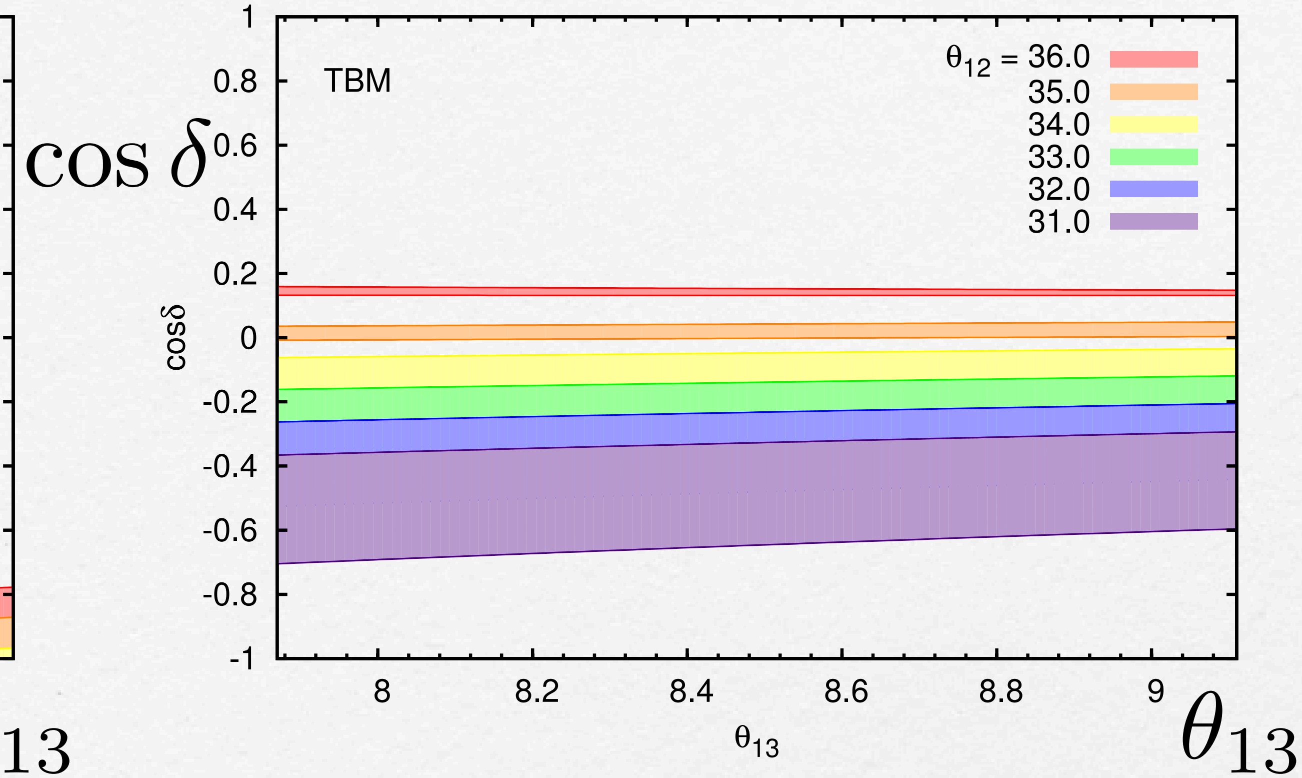
Semi-Direct Models (*with small groups*)



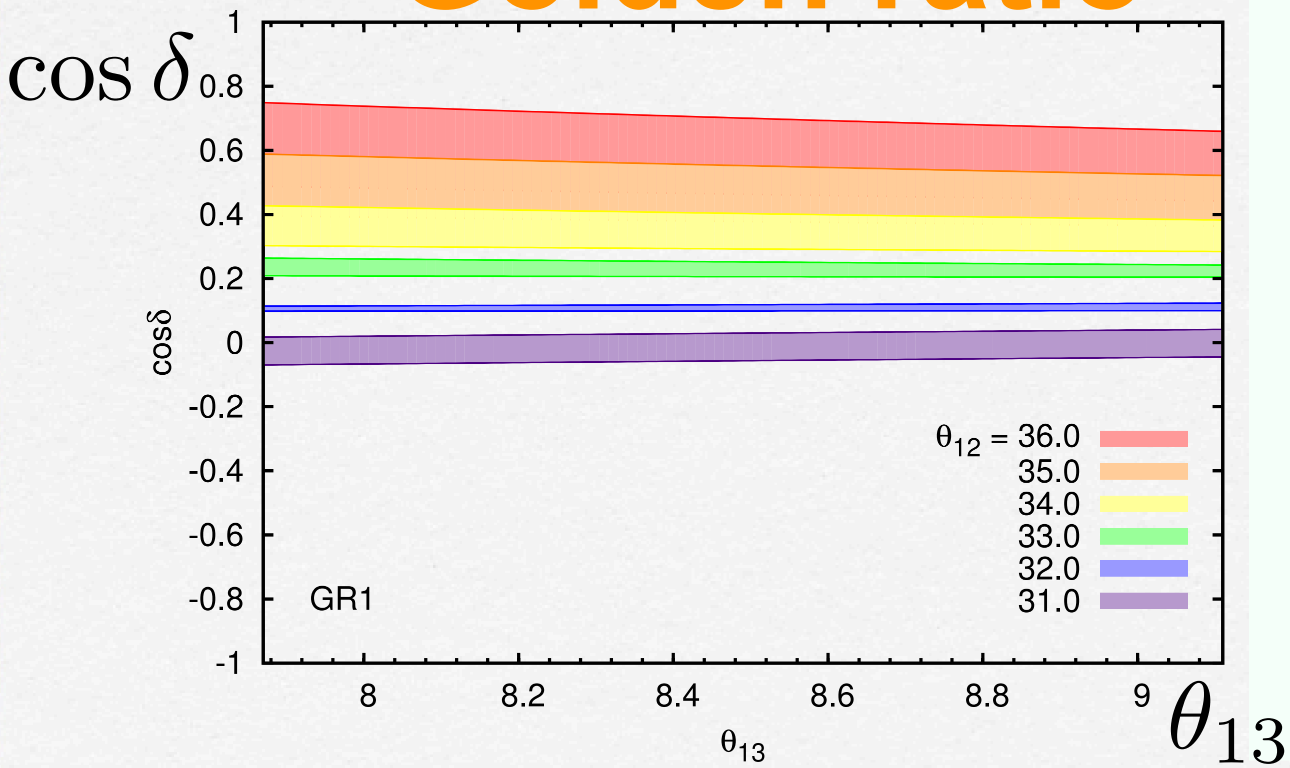
Bimaximal



Tri-bimaximal



Golden ratio



Solar Sum Rules

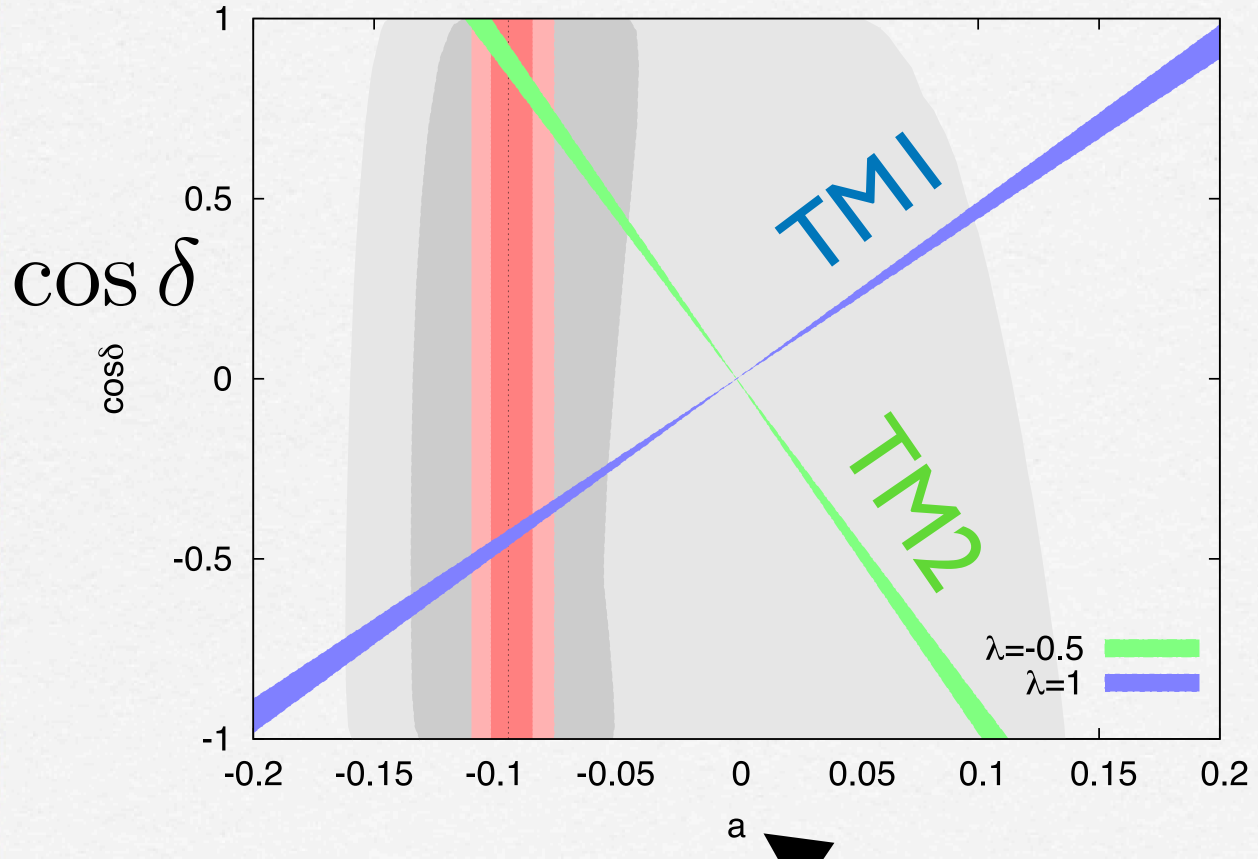
(from charged lepton corrections)

Ballett, SK, Luhn, Pascoli, Schmidt

1410.7573

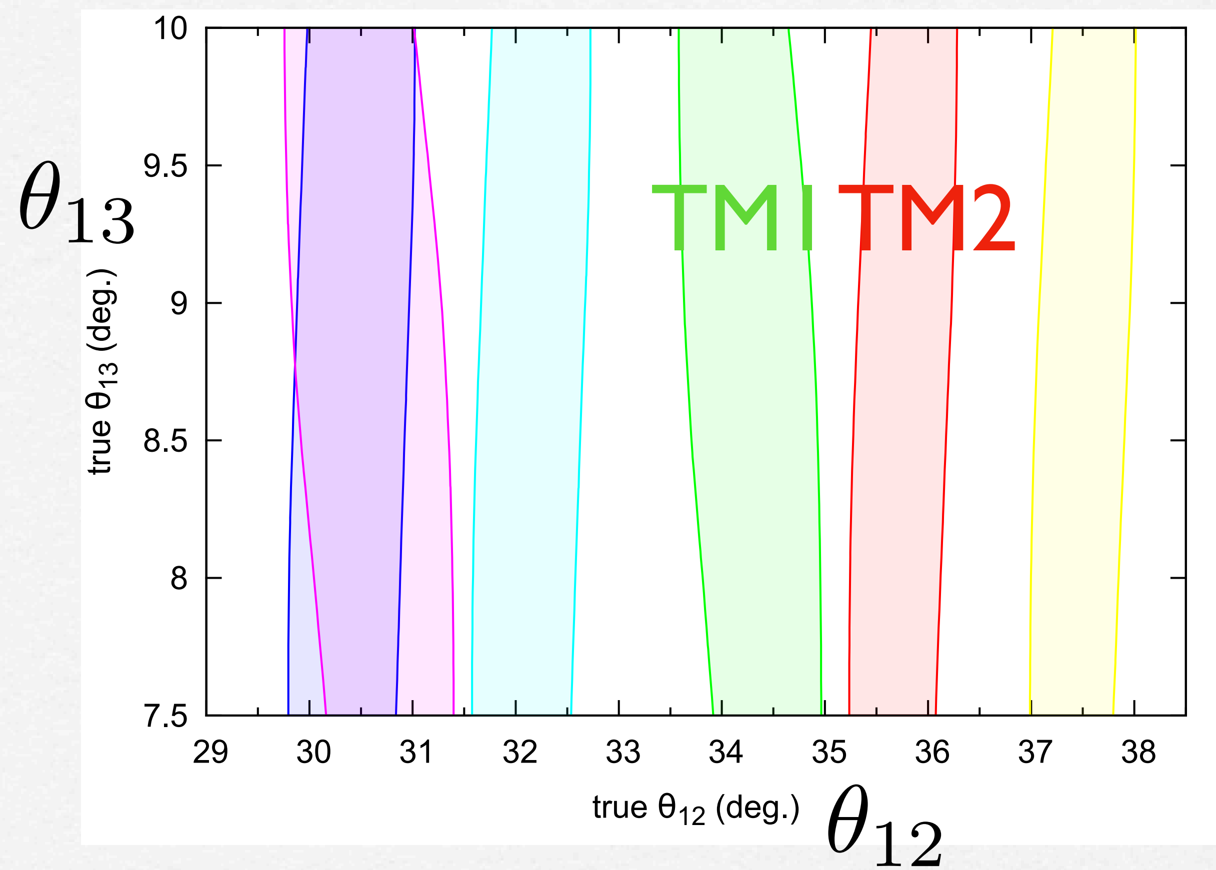
Atmospheric Sum Rules

1308.4314



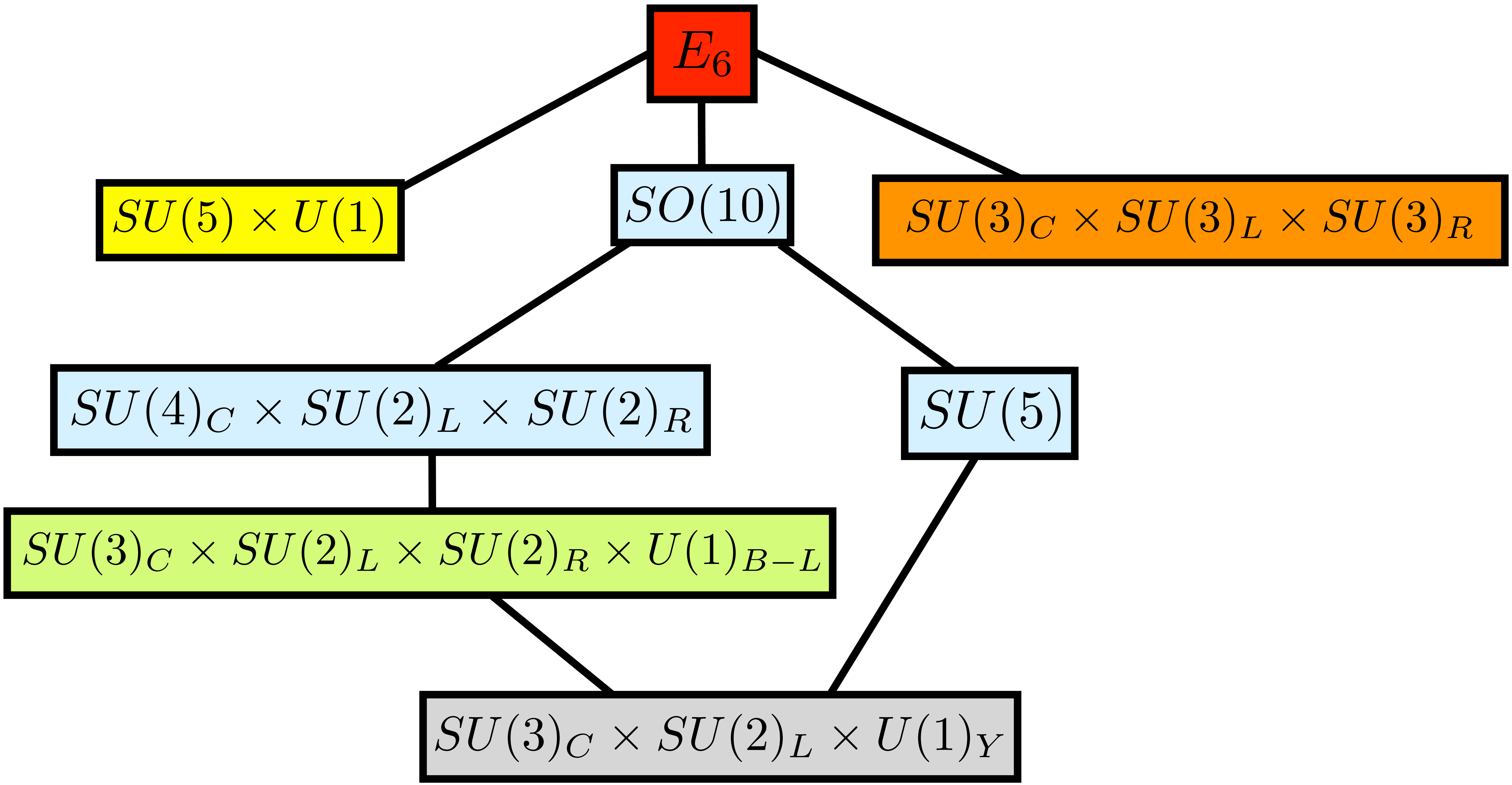
$$\sin\theta_{23} = \frac{1+a}{\sqrt{2}}$$

1406.0308



5 sigma allowed regions after JUNO

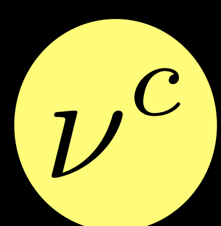
Grand Unified Theories

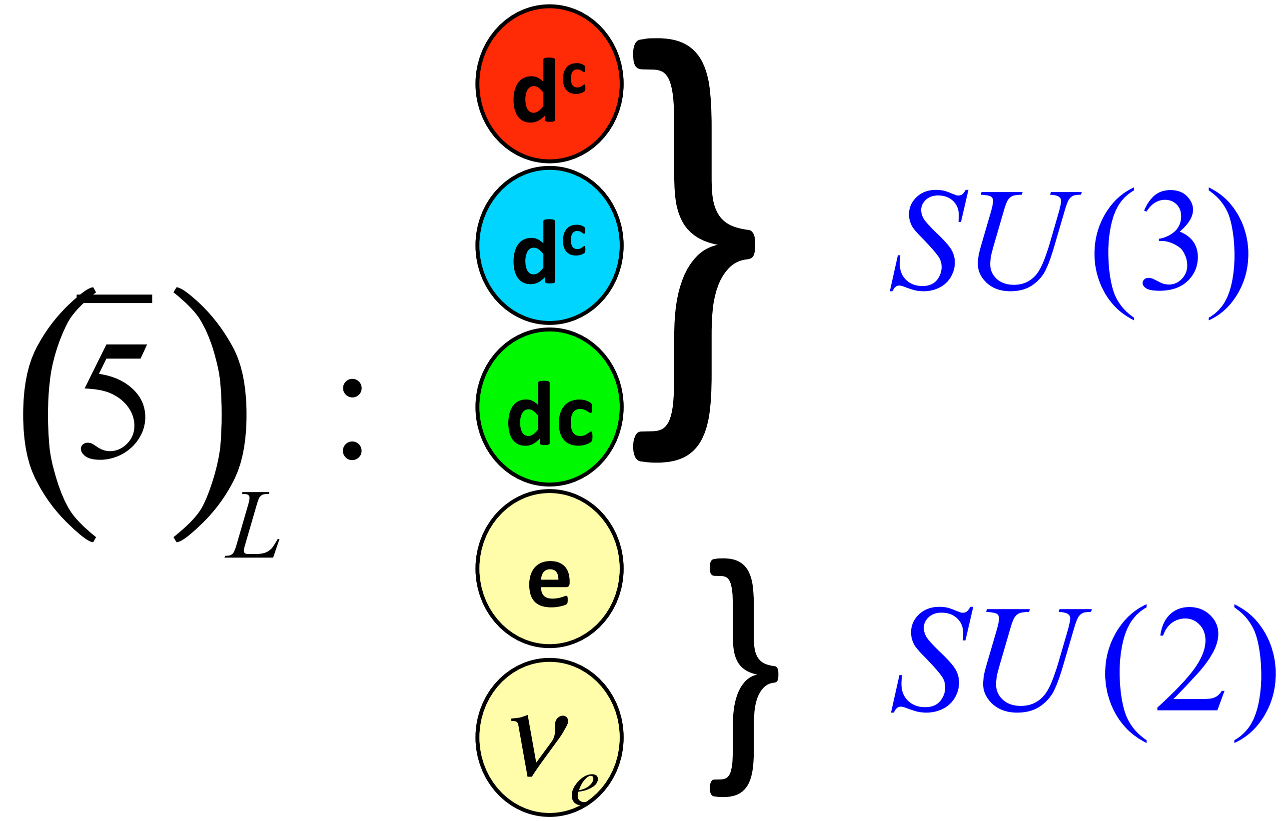


SU(5) GUT

Georgi, Glashow

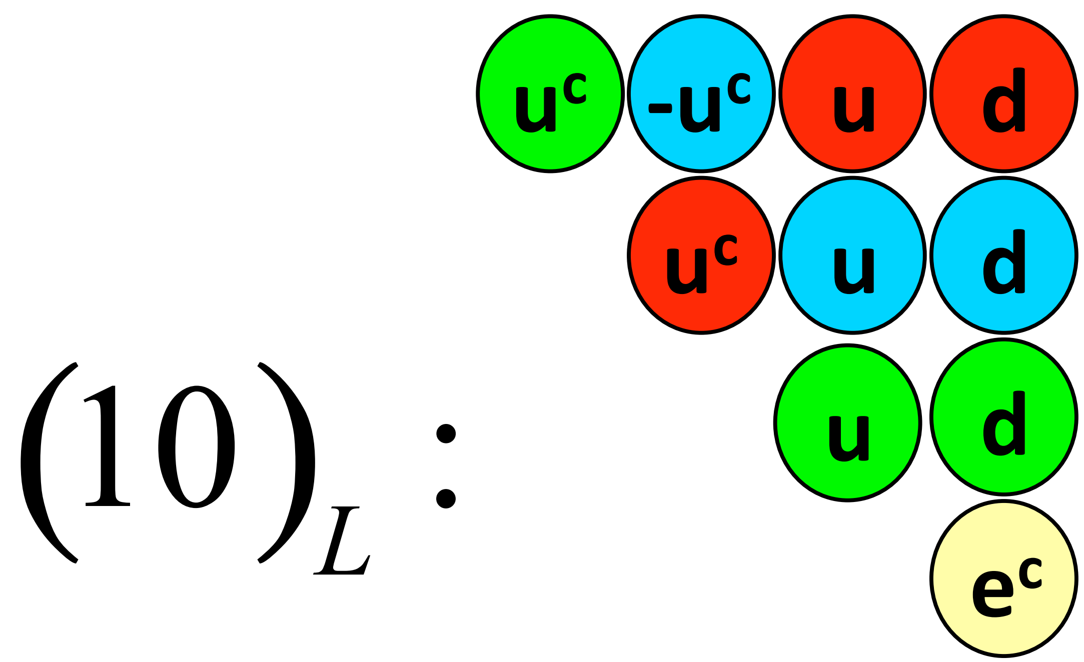
Right-handed neutrino is singlet





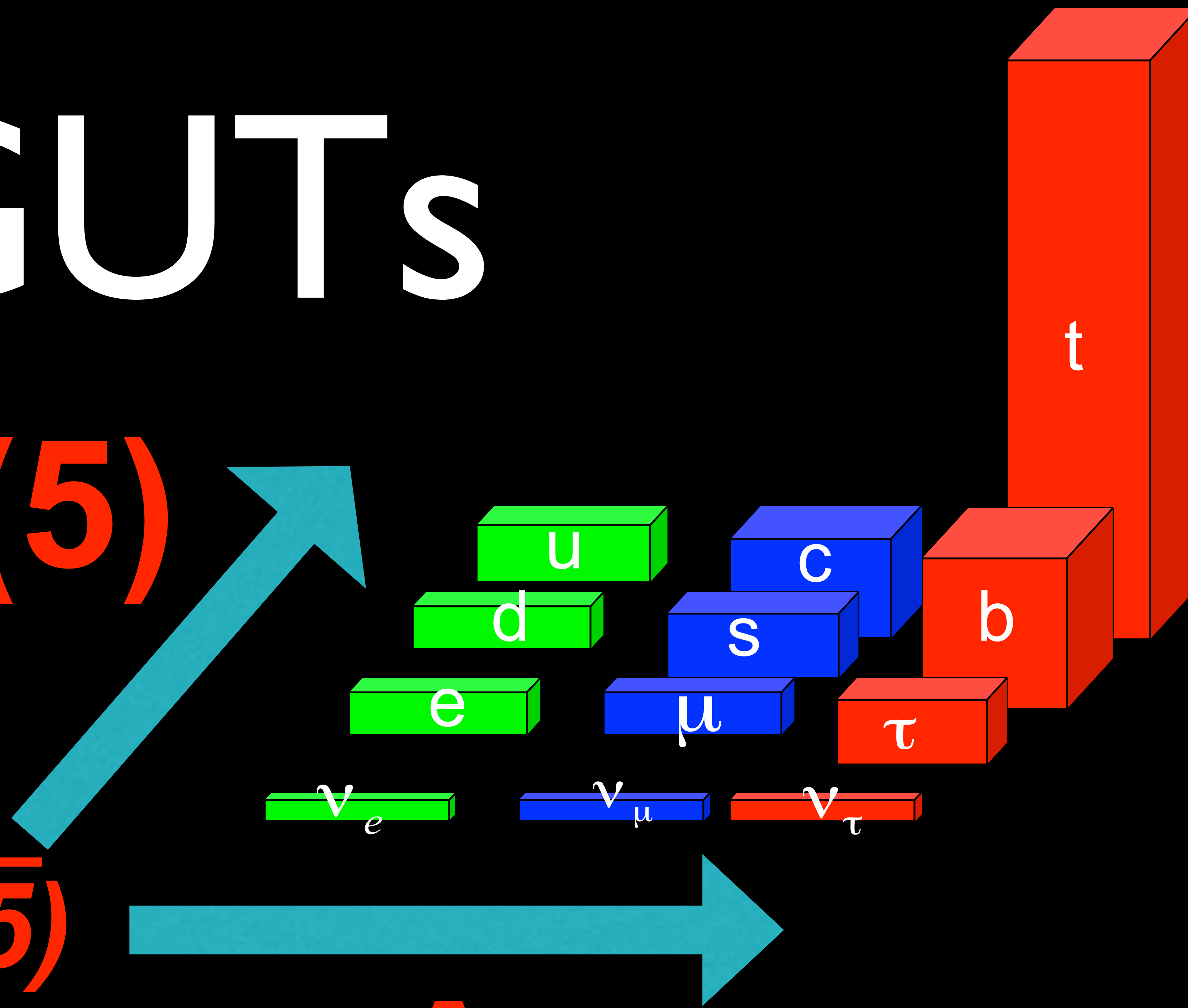
$$\overline{\mathbf{5}} = d^c(\overline{\mathbf{3}}, \mathbf{1}, 1/3) \oplus L(\mathbf{1}, \overline{\mathbf{2}}, -1/2),$$

$$\mathbf{0} = u^c(\overline{\mathbf{3}}, \mathbf{1}, -2/3) \oplus Q(\mathbf{3}, \mathbf{2}, 1/6) \oplus e^c(\mathbf{1}, \mathbf{1}, 1),$$



Flavoured GUTs

Björkeroth, de Anda,
de Medeiros Varzielas, SFK



SU(5)

$F \sim (3,\bar{5})$

$T_i \sim (1,10)$

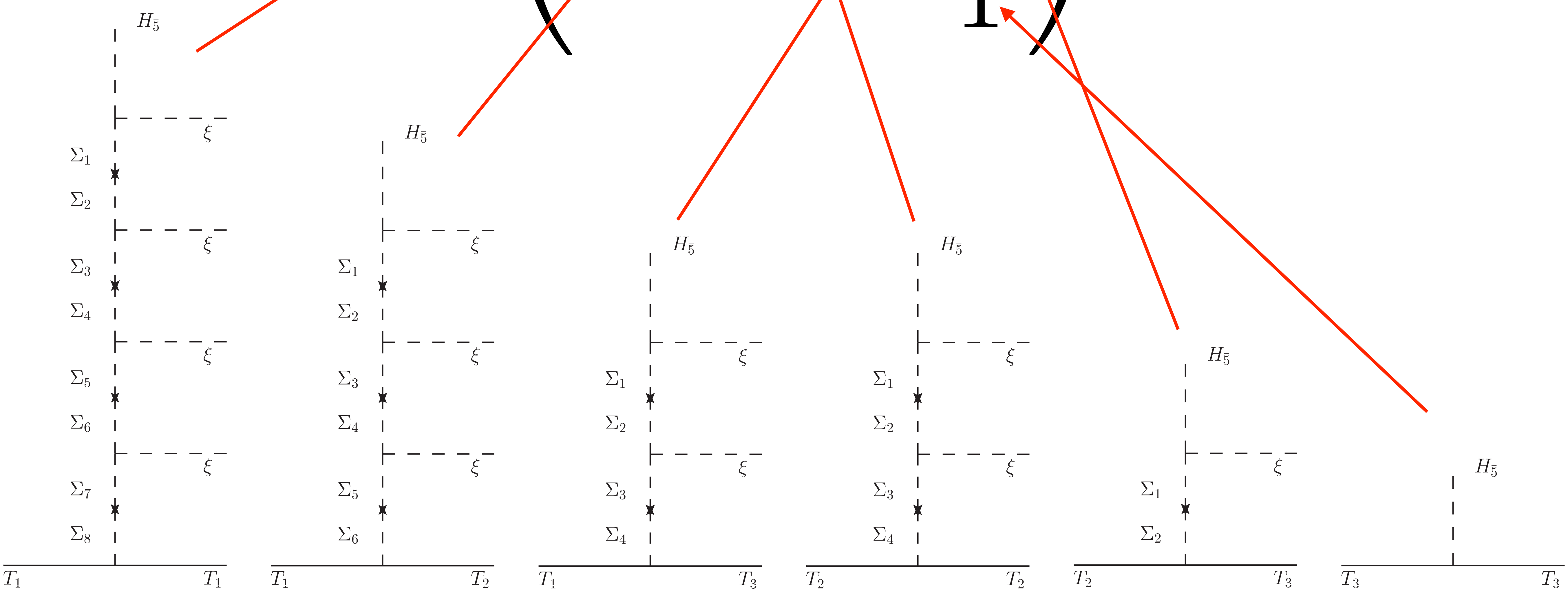
A_4

Up-type quarks

Froggatt-Nielsen

$$Y^u_{ij} \sim \begin{pmatrix} \tilde{\xi}^4 & \tilde{\xi}^3 & \tilde{\xi}^2 \\ & \tilde{\xi}^2 & \tilde{\xi} \\ & & 1 \end{pmatrix}$$

$$\tilde{\xi} = \langle \xi \rangle / M \sim 0.1$$

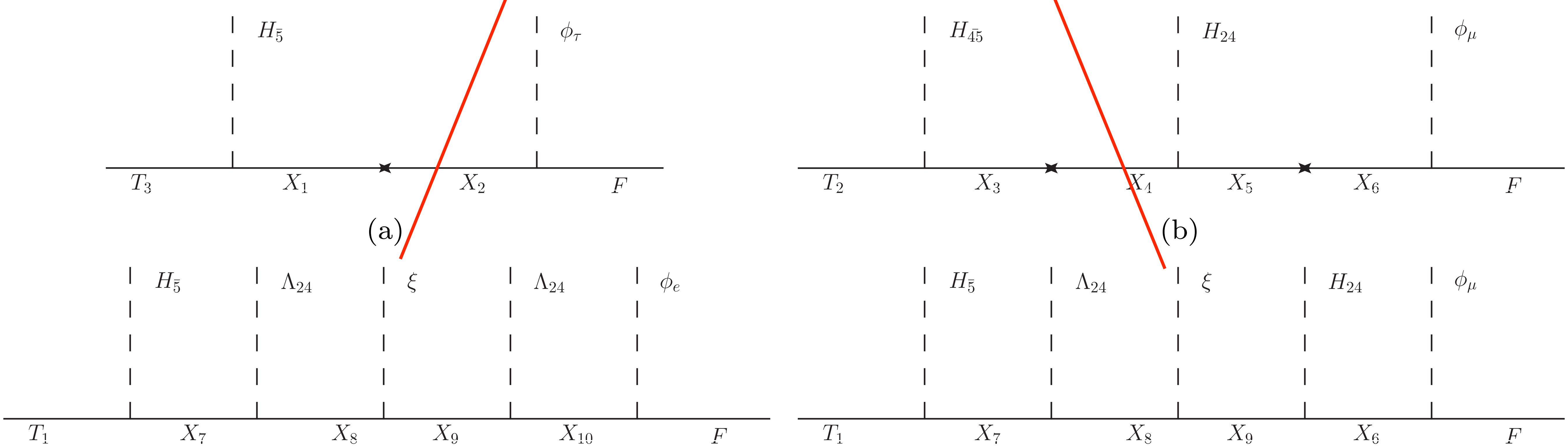


$$\langle \phi_e \rangle = v_e \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \langle \phi_\mu \rangle = v_\mu \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \langle \phi_\tau \rangle = v_\tau \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Down-type quarks

$$Y^d_{LR} \sim Y^e_{RL} \sim \begin{pmatrix} \frac{\langle \xi \rangle v_e}{v^2_{\Lambda_{24}}} & \frac{\langle \xi \rangle v_\mu}{v_{\Lambda_{24}} v_{H_{24}}} & 0 \\ 0 & \frac{v_{H_{24}} v_\mu}{M^2} & 0 \\ 0 & 0 & \frac{v_\tau}{M} \end{pmatrix}$$

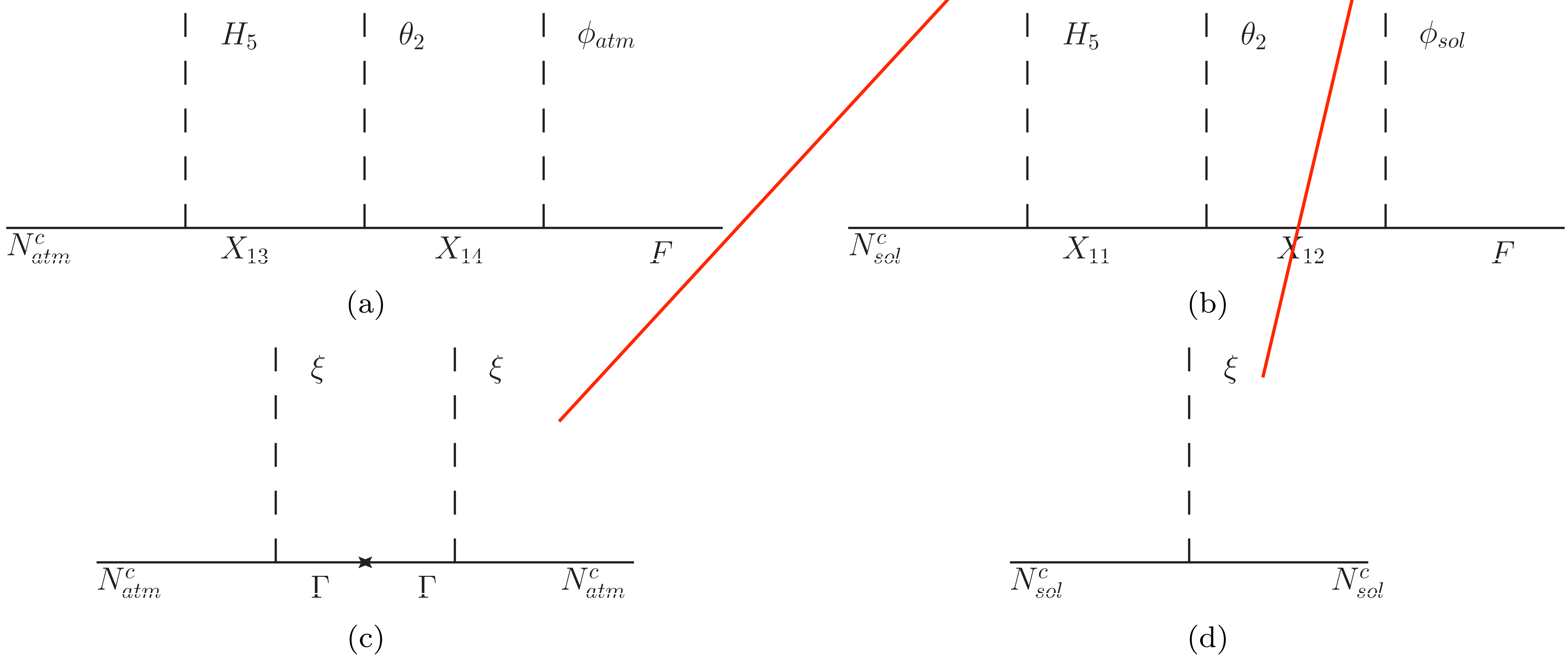
Charged Leptons



Neutrinos with Littlest Seesaw A

$$\langle \phi_{\rm atm} \rangle = v_{\rm atm} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \qquad \langle \phi_{\rm sol} \rangle = v_{\rm sol} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$$

$$m_D = \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix} \qquad M_R = \begin{pmatrix} M_{\rm atm} & 0 \\ 0 & M_{\rm sol} \end{pmatrix}$$



Summary of $A_4 \times SU(5)$

- Explains quark mass hierarchies, mixing angles and the CP phase.
- Reproduces Littlest Seesaw predictions (SUSY)
- Near maximal atmospheric mixing, normal hierarchy, $m_1=0$
- Z_9 flavour symmetry fixes the phase η to be 2pi/3
- Leptogenesis fixes $M_{atm} \sim 10^{10}$ GeV
- Renormalisable at GUT scale, SU(5) breaking potential, spontaneously broken CP.
- The MSSM is reproduced with R-parity from discrete Z_4^R .
- Doublet-triplet splitting via the Missing Partner mechanism.
- mu term is generated at the correct scale.
- Proton decay is sufficiently suppressed.
- Solves strong CP problem through the Nelson-Barr mechanism .

Many other possibilities

Mu-Chun

G_{GUT} / G_{FAM}	$SU(2)_L \times U(1)_Y$	$SU(5)$	PS	$SO(10)$
S_3	[29]			[150]
A_4	[36, 49, 51, 62, 151–154]	[155–158]	[66, 159, 160]	
T'		[161]		
S_4	[31, 49, 51, 154, 163]	[164, 165]	[162]	[166]
A_5	[51, 169]	[170]		
T_7	[171, 172]			
$\Delta(27)$	[173]			[174]
$\Delta(96)$	[175, 176]	[177]		[178]
D_N	[179]			
Q_N	[180]			
other	[181]	[182]	[183]	

Conclusions

- Origin of neutrino mass is unknown ($\therefore$ BSM)
- Roadmap of possibilities
- Attractive possibility is Type I seesaw
- Littlest seesaw predicts PMNS with RG corrections in SM large (SUSY small RG corrections, maximal θ_{23})
- Discrete Family Symmetry NOT EXCLUDED by θ_{13}
- Discrete Family Symmetry PREDICTS Sum Rules
- GUTs treats quarks and leptons on same footing
- Family symmetry x GUTs **unified theory of forces and flavour - in progress...**