Colored particle-in-cell simulations for heavy-ion collisions

International School of Nuclear Physics, 38th Course
21.09.2016, Erice, Sicily

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Introduction

Heavy-ion collision experiments investigate the properties of nuclear matter at high energies.

- Formation and evolution of the quark-gluon-plasma (QGP)?
- How does the QGP become isotropic and thermalized?
- What is the role of boost-invariance?

Various heavy-ion collision experiments:

- LHC (ALICE) @ CERN: Pb+Pb with \(~5.5\) TeV per nucleon pair. (\(\gamma \approx 2700\))
- RHIC @ BNL: Au+Au with \(~200\) GeV per nucleon pair. (\(\gamma \approx 100\))
- RHIC beam energy scan: \(~7.7 – 62.4\) GeV (\(\gamma \approx 4 – 30\))

**Goal:** Simulate heavy-ion collisions in the color glass condensate (CGC) framework with finite nucleus thickness. Possible with colored particle-in-cell (CPIC).
Stages of a heavy-ion collision

- **Initial state:** Lorentz-contracted pancakes (color glass condensate)
- **Glasma** ($\tau \approx 0-1$ fm/c): quasi-classical fields (classical field equations)
- **QGP** ($\tau \approx 1-10$ fm/c): quarks and gluons (relativistic viscous hydrodynamics) (almost) isotropic and in thermal equilibrium
- **Hadronization** ($\tau \approx 10$ fm/c): confinement transition $\rightarrow$ hadron formation
- **Hadronic gas** ($\tau \approx 10-15$ fm/c): hadrons (kinetic transport theory)
- **Freeze-out** ($\tau \approx 15$ fm/c): interactions stop

$1 \text{ fm/c} \approx 3.3 \cdot 10^{-24} \text{ s} \approx 3.3 \text{ ys}$

**scope of this project**
Color glass condensate


- Hard quarks and gluons are approximated as classical color charges moving at the speed of light generating a classical gauge field.

- The gauge field describes the soft gluons in the nucleus.

- Static field configuration due to time dilation.

Boost-invariant CGC collision

- CGC: Separation of hard and soft degrees of freedom, weak coupling
- Color currents of the nuclei restricted to the light cone and infinitely thin
- Analytical solutions exist for everything except the forward light cone
- Fields in the forward light cone are independent of rapidity $\eta$. Reduction from 3D+1 to 2D+1
- Need to solve 2D+1 source-free Yang-Mills equations in the forward light cone with Glasma initial conditions on the light cone

\[ D_\mu F^{\mu\nu}(\tau, x_T) = 0 \]
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Finite nucleus thickness

- Extended color currents need to be taken into account.
- Fields depend on rapidity.
- Need to solve full 3D+1 Yang-Mills equation with currents.

$$D_\mu F^{\mu\nu}(t,z,x_T) = J^\nu$$
$$D_\mu J^\mu(t,z,x_T) = 0$$

Colored particle-in-cell (CPIC) provides a framework to numerically solve the field and current equations on a lattice.
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Colored particle-in-cell (CPIC) method

Collision of two nuclei in the lab frame:

- Color currents included, tied to the light-cone
- Sample color charge density with a number of (computational) particles.
- Parallel transport of charges
- NGP interpolation: Current $J_\mu$ on the grid generated by particle movement.
- $J_\mu$ as input for field equations of motion on the lattice.

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Nearest-grid-point method (NGP)
Initial conditions

- Temporal gauge \((A_0 = 0)\) suitable for numerical time evolution.
- Asymptotically pure gauge “trails” behind nuclei.
- Fixed boundary conditions on the longitudinal boundaries are required.
- Random charge densities \(\rho_{(1,2)}\) are sampled from McLerran-Venugopalan (MV) model.

\[\text{[McLerran, Venugopalan: PRDD49 (1994) 3352-3355]}\]

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\[ \langle \hat{\rho}^a(x_T)\hat{\rho}^b(x'_T) \rangle = g^2 \mu^2 \delta^{(2)}(x_T - x'_T) \delta^{ab} \]

\[ \rho(t, z, x_T) = f(z - t)\hat{\rho}(x_T) \]

UV & IR regulation

\[ m \approx 2 \text{ GeV} \]
\[ \Lambda_{UV} \approx 10 \text{ GeV} \]

MV parameter \(\mu \approx 0.5 \text{ GeV (Au)}\)

\[ Q_s \approx 2 \text{ GeV} \]
Simulation overview

1. Initialize random charges and fields of two colliding nuclei.

2. Simulation cycle:
   a. Move particles and apply parallel transport.
   b. Generate currents from particle movement.
   c. Evolve fields in time with currents as input.
   d. Compute observables \( (T_{\mu\nu}, \varepsilon, p_L, p_T, \ldots) \).

3. Average over many random events.

\[ \langle T_{\mu\nu} \rangle, \langle \varepsilon \rangle, \langle p_L \rangle, \langle p_T \rangle, \ldots \]
Numerical results

Au-Au collision in the MV model, SU(2)
Comparison to boost-invariant results

- Check validity of simulation results with finite nucleus thickness by comparing to analytical boost-invariant results.
- Compare boost-invariant Glasma initial conditions to simulated fields and vary thickness parameter $\sigma$.

Energy density component $\text{tr} E_L^2(x_T)$ in the transverse plane at $\eta = 0$. 

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Comparison to boost-invariant results

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- Compute correlation between analytic and numerical results as a function of $\sigma$.
  - Thick nuclei: low correlation
  - Thin nuclei: high correlation
  - Numerical instabilities prohibit very thin nuclei.

  (but it’s just a question of lattice sizes)
Pressure anisotropy (1)

- Compute longitudinal and transverse pressure $p_L(z)$ and $p_T(z)$ as a function of the longitudinal coordinate $z$.

→ Pronounced pressure anisotropy

$t_0 = -1 \text{ fm/c}$
(before collision)

$t_1 = +2 \text{ fm/c}$
(after collision)

$t_2 = +5 \text{ fm/c}$
(late times)
Pressure anisotropy (2)

- **Isotropization**: initial pressure anisotropy should vanish after ~ 0.1 fm/c to a few fm/c.

"Thick" nuclei ($\gamma \sim 20 \sim 40$)

- Analyze pressure to energy density ratio in the central region at $\eta = 0$.
- Thick nuclei: pronounced pressure anisotropy (free-steaming).
- Slight movement towards isotropization visible, but it is too slow.
- Negative longitudinal pressures?
Pressure anisotropy (2)

- **Isotropization**: initial pressure anisotropy should vanish after $\sim 0.1$ fm/c to a few fm/c.

- Analyze pressure to energy density ratio in the central region at $\eta = 0$.
- Thin nuclei: negative longitudinal pressures
- Observables always influenced by presence of the nuclei at early times.
Initial conditions are still missing random longitudinal structure. Longitudinal randomness...

- leads to higher energy density in the Glasma. [Fukushima, PRD 77 (2008) 074005]
- could further break boost-invariance.

Possible consequence: faster isotropization times? → future work!

Current implementation

Boosted

"at rest"

Longitudinal randomness

"at rest"

Boosted
**Longitudinal structure (2)**

**First check:** Light-like Wilson line expectation value $\langle \text{tr}(V) \rangle$ of a single nucleus is sensitive to longitudinal structure.

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**Embedded 2D MV-model:**

$$\langle \hat{\rho}^a(x_T)\hat{\rho}^b(x'_T) \rangle = g^2\mu^2\delta^{(2)}(x_T - x'_T)\delta^{ab}$$

$$\rho(t, z, x_T) = f(z - t)\hat{\rho}(x_T)$$

**3D MV-model:**

(with random longitudinal structure)

$$\langle \rho^a(t, x)\rho^b(t', x') \rangle = g^2\mu^2f(z)\delta^{(3)}(x - x')\delta^{ab}$$

$f(z)$ ... longitudinal profile function

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**Improvement method for boost-invariant simulations**

[Fukushima, PRD 77 (2008) 074005]
Conclusions and summary

• Simulating CGC collisions in 3D+1 with finite nucleus thickness in the laboratory frame using CPIC is viable.
• Boost-invariant results reproduced in the limit of thin nuclei.
• We observe a pronounced pressure anisotropy after the collision.
• Observed isotropization too slow.

Future:
• Study effects of initial conditions with random longitudinal structure on isotropization
• Corrections to initial Glasma energy density due to finite thickness

Phys.Rev. D94 (2016) no.1, 014020
open source: https://github.com/openpixi
Thank you for your attention!