Colored particle-in-cell simulations for heavy-ion collisions

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INSTITUTE FOR THEORETICAL PHYSICS

$\int dk \Pi$

FWF Der Wissenschaftsfonds.

TU WIEN

Introduction

Heavy-ion collision experiments investigate the properties of nuclear matter at high energies.

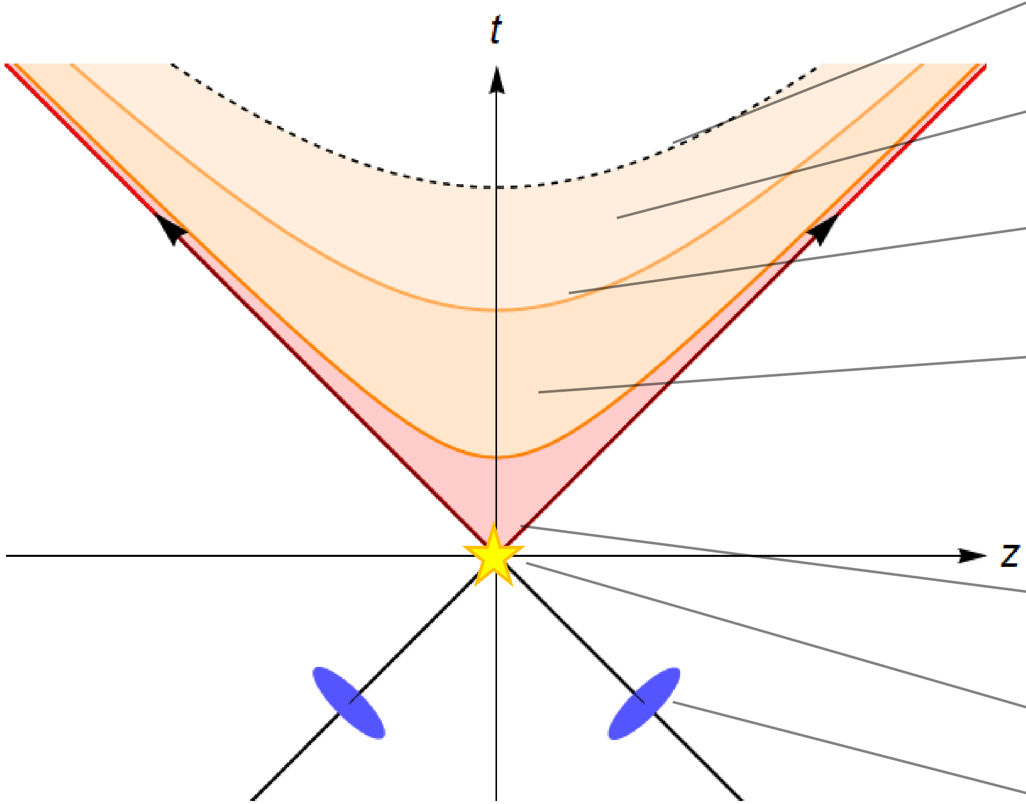
- Formation and evolution of the quark-gluon-plasma (QGP)?
- How does the QGP become isotropic and thermalized?
- What is the role of boost-invariance?

Various heavy-ion collision experiments:

- LHC (ALICE) @ CERN: Pb+Pb with **~5.5 TeV** per nucleon pair. $(\gamma \approx 2700)$
- RHIC @ BNL: Au+Au with **~200 GeV** per nucleon pair. $(\gamma \approx 100)$
- RHIC beam energy scan: **~7.7 – 62.4 GeV** $(\gamma \approx 4 – 30)$

Goal: Simulate heavy-ion collisions in the color glass condensate (CGC) framework with finite nucleus thickness. Possible with colored particle-in-cell (CPIC).

Stages of a heavy-ion collision



Freeze-out ($\tau \approx$ 15fm/c)**:**
interactions stop

Hadronic gas ($\tau \approx 10 - 15$fm/c)**:** hadrons
(kinetic transport theory)

Hadronization ($\tau \approx$ 10fm/c)**:**
confinement transition → hadron formation

QGP ($\tau \approx$ 1–10 fm/c)**:** quarks and gluons
(relativistic viscous hydrodynamics)
(almost) isotropic and in thermal equilibrium

Glasma ($\tau \approx$ 0–1 fm/c)**:** quasi-classical fields
(classical field equations)

Collision event

Initial state: Lorentz-contracted pancakes
(color glass condensate)

$$1\ \mathrm{fm}/c \approx 3.3 \cdot 10^{-24}\ s \approx 3.3\ ys$$

scope of this project

Color glass condensate

- The early stages of heavy-ion collisions can be described by **classical effective theory** in the color glass condensate (CGC) framework.
  [Gelis, Iancu, Jalilian-Marian, Venugopalan, Ann.Rev.Nucl.Part.Sci.60:463-489,2010]
- Hard quarks and gluons are approximated as classical color charges moving at the speed of light generating a classical gauge field.
- The gauge field describes the soft gluons in the nucleus.
- Static field configuration due to time dilation.
- Collision of two such classical fields creates the **Glasma**.
  [Gelis, Int.J.Mod.Phys. A28 (2013) 1330001]

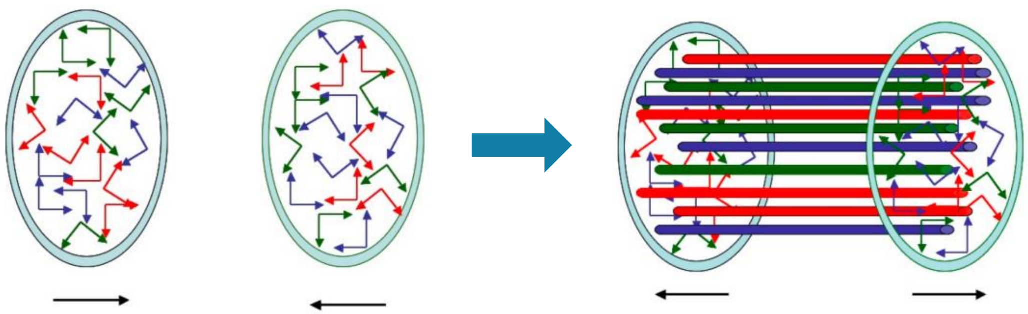
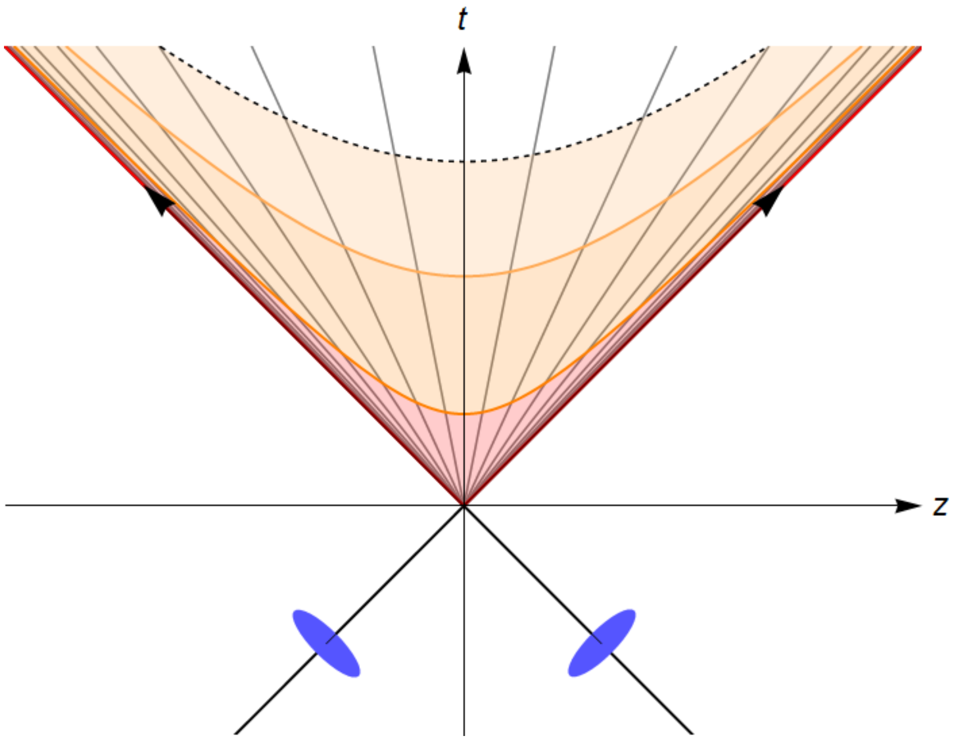


Figure from L. McLerran:
Proceedings of ISMD08, p.3-18 (2008)

Boost-invariant CGC collision



- CGC: Separation of hard and soft degrees of freedom, weak coupling
- Color currents of the nuclei restricted to the light cone and infinitely thin
- Analytical solutions exist for everything except the forward light cone
- Fields in the forward light cone are independent of rapidity η. Reduction from 3D+1 to 2D+1
- Need to solve 2D+1 source-free Yang-Mills equations in the forward light cone with Glasma initial conditions on the light cone

$$D_\mu F^{\mu\nu}(\tau, x_T) = 0$$

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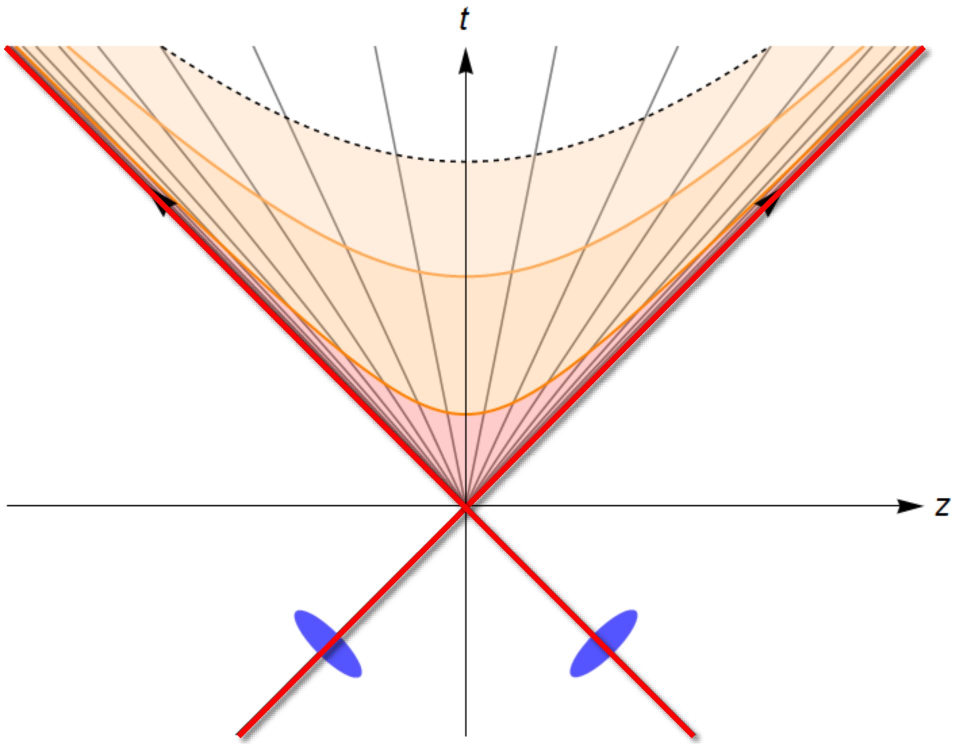

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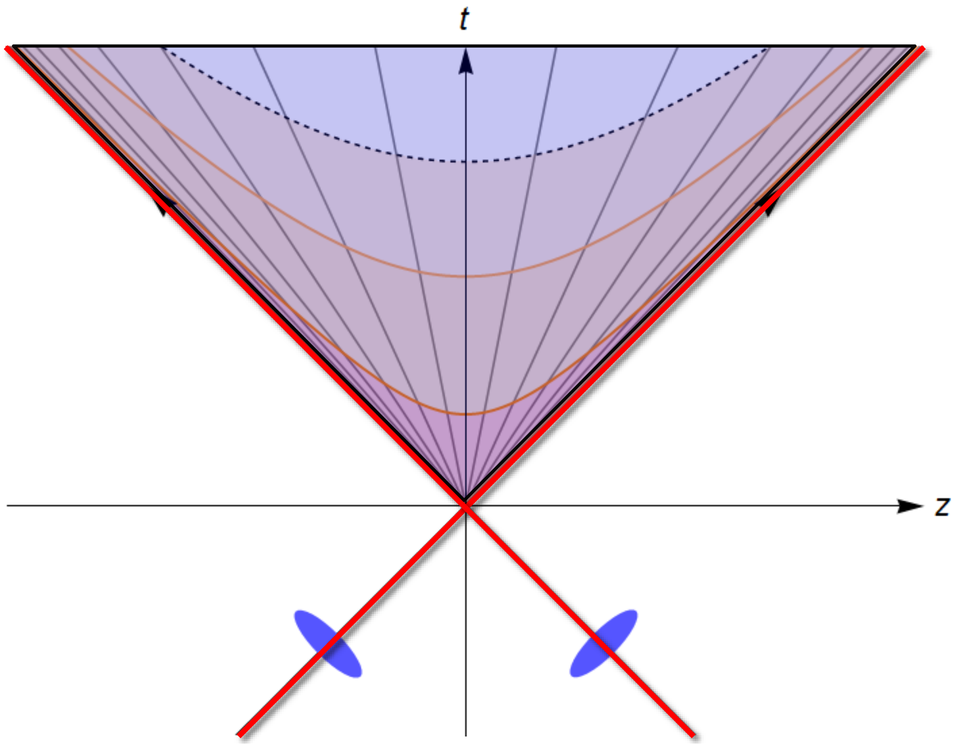

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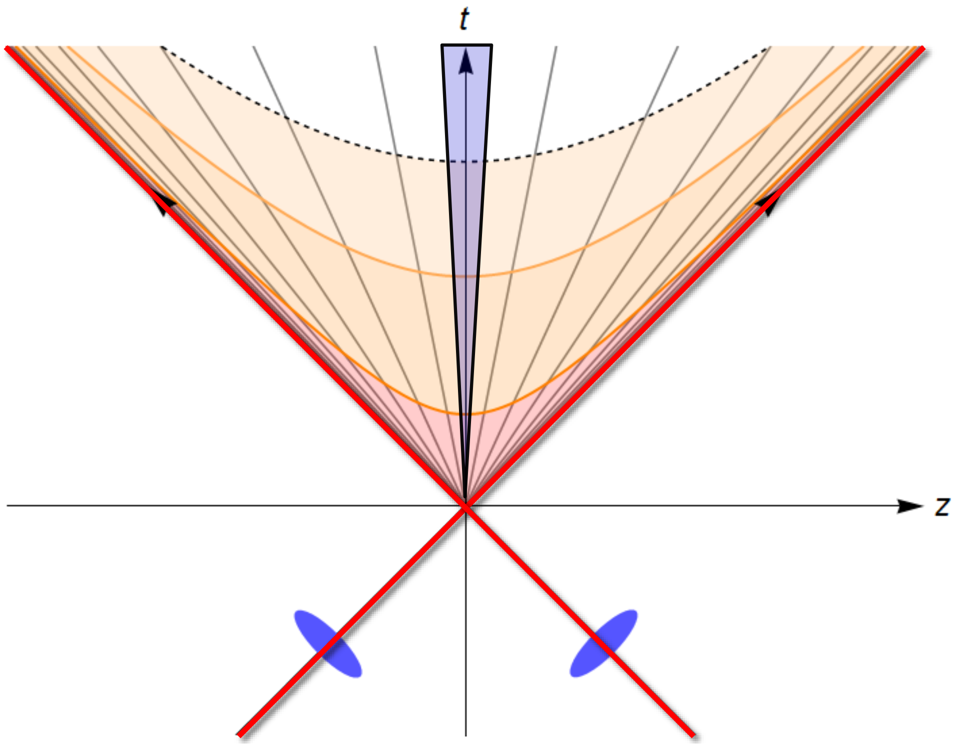

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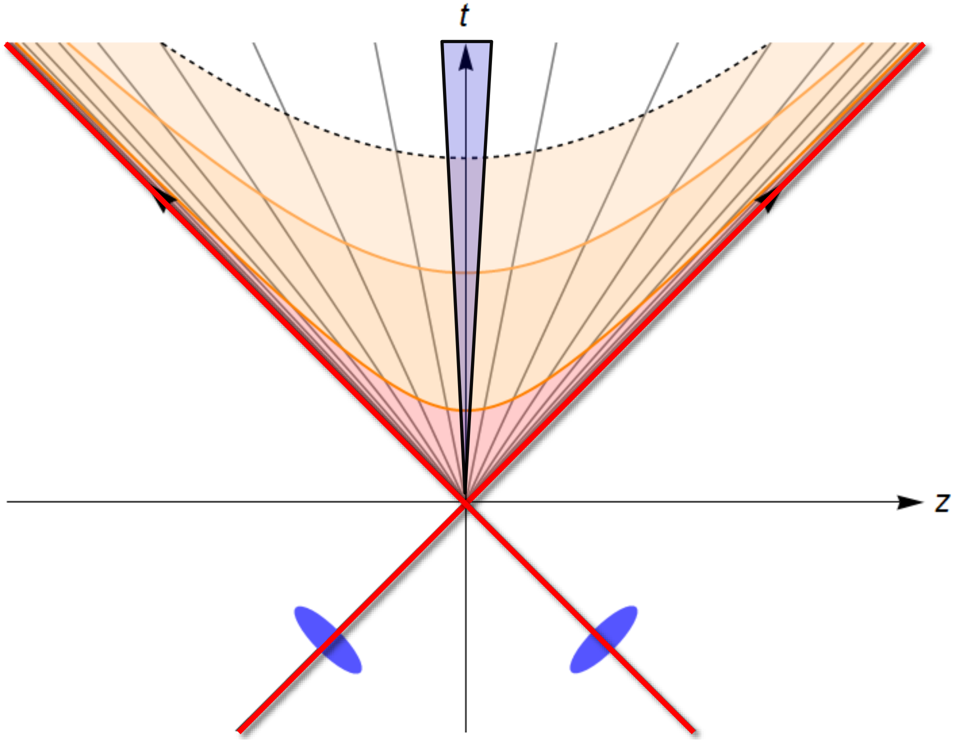

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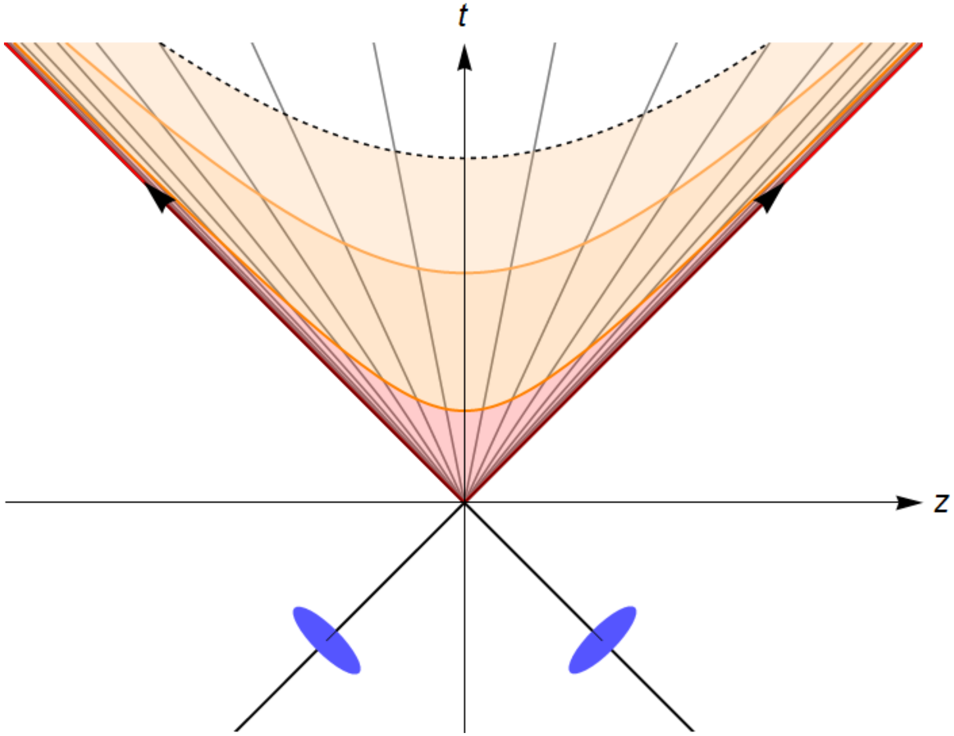
Finite nucleus thickness



- Extended color currents need to be taken into account.
- Fields depend on rapidity.
- Need to solve full 3D+1 Yang-Mills equation with currents.

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Colored particle-in-cell (CPIC) provides a framework to numerically solve the field and current equations on a lattice.

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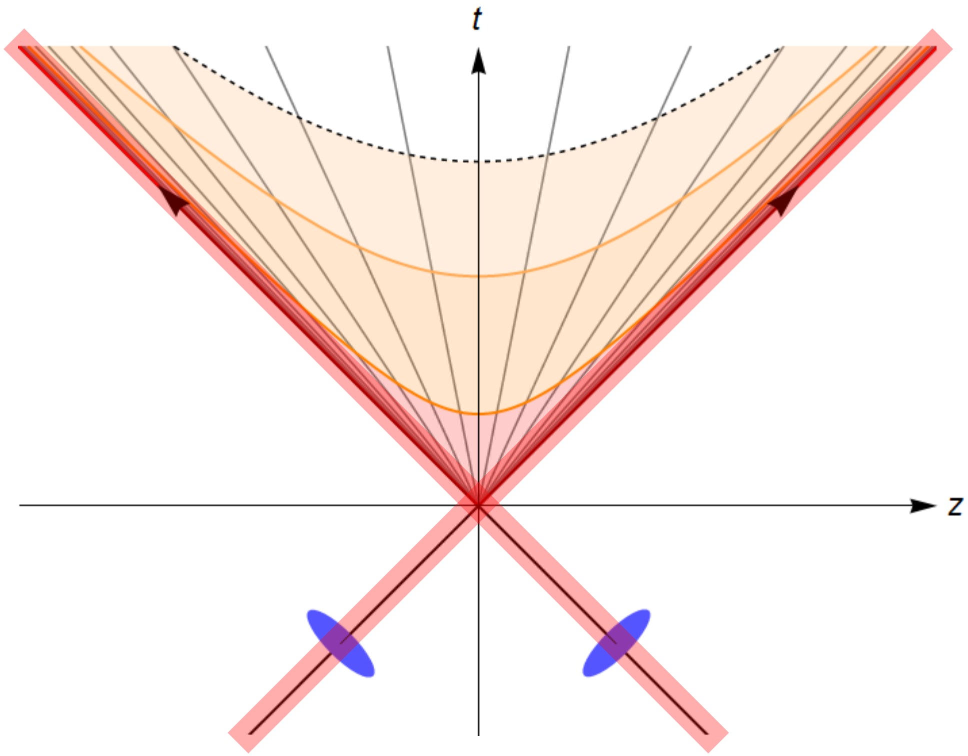

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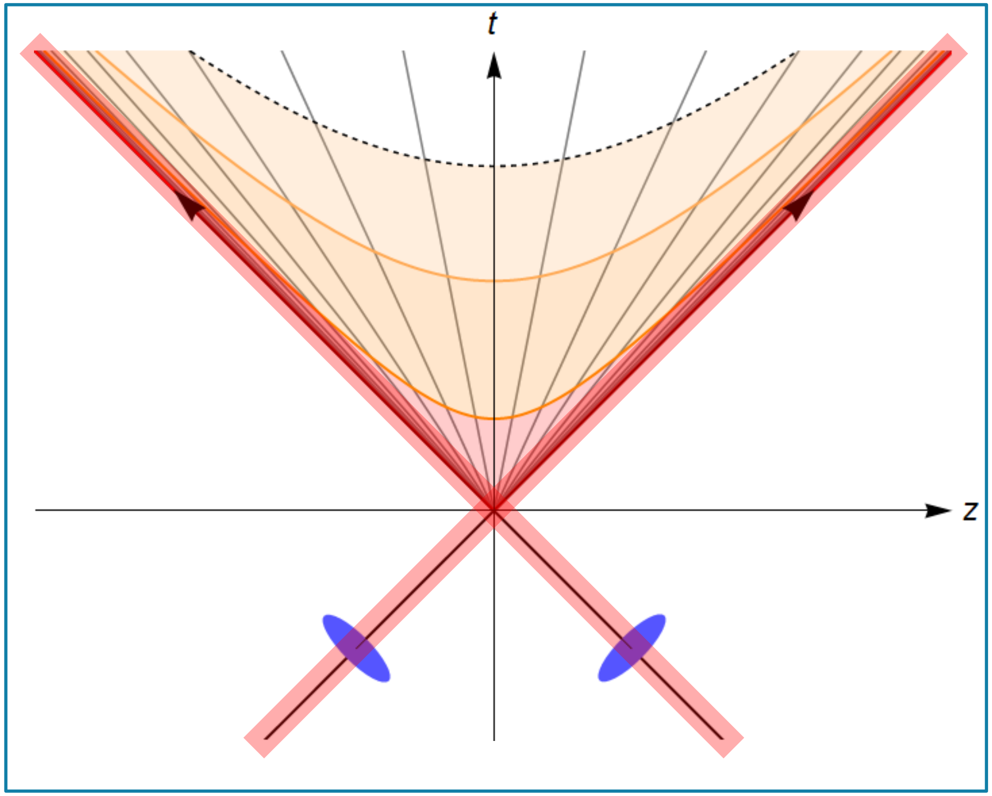

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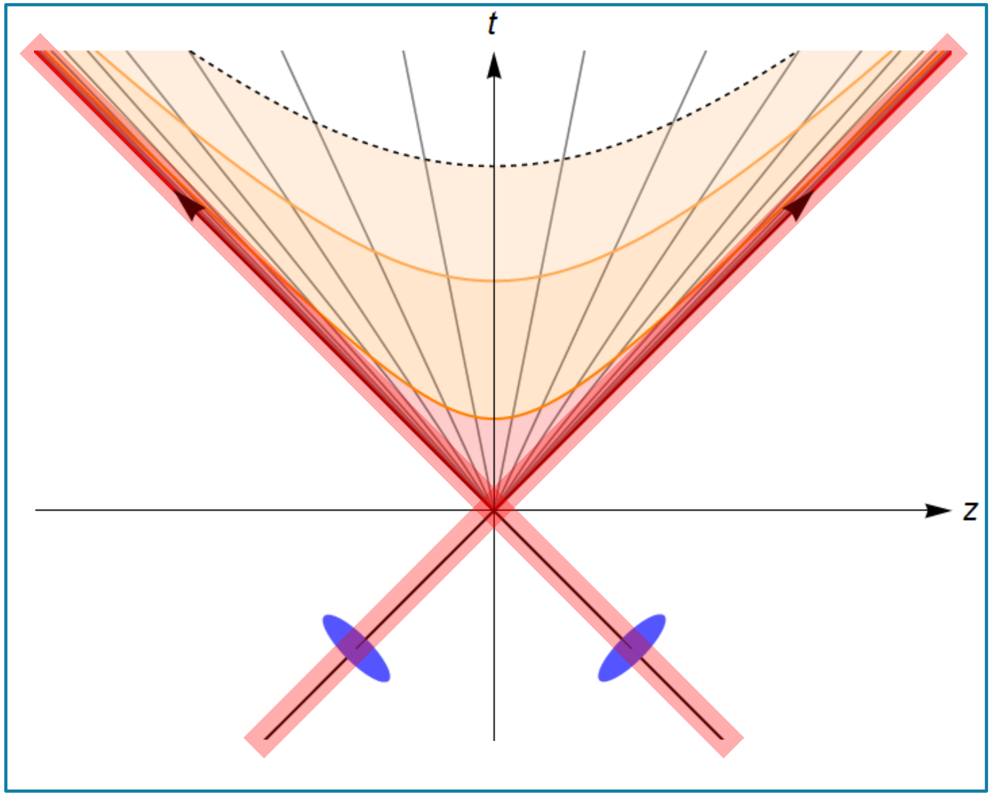

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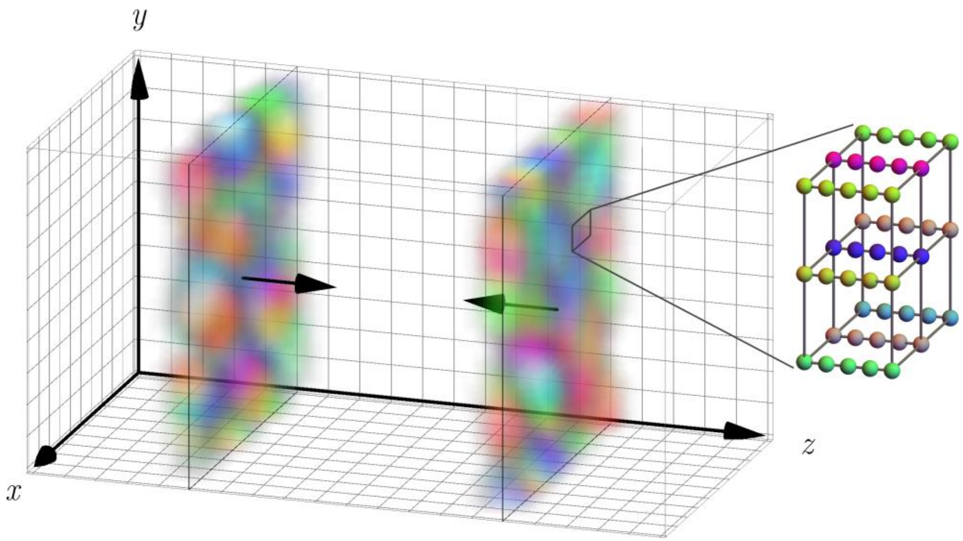
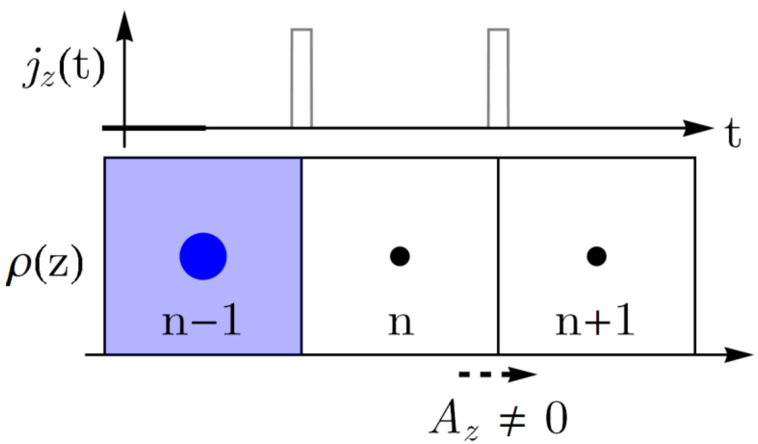


Nearest-grid-point method (NGP)



Collision of two nuclei in the lab frame:

- Color currents included, tied to the light-cone
- Sample color charge density with a number of (computational) particles.
- Parallel transport of charges
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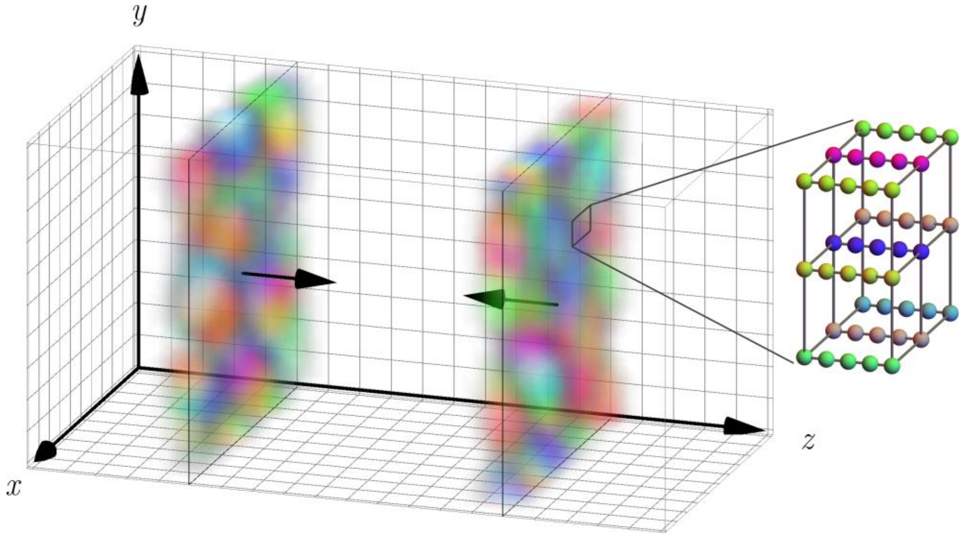

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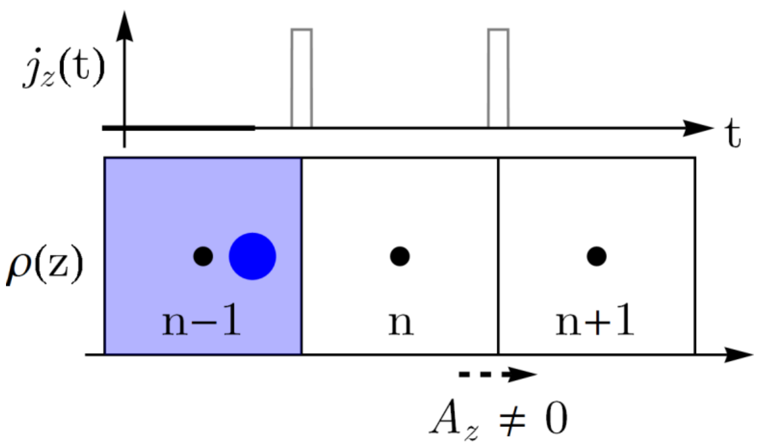

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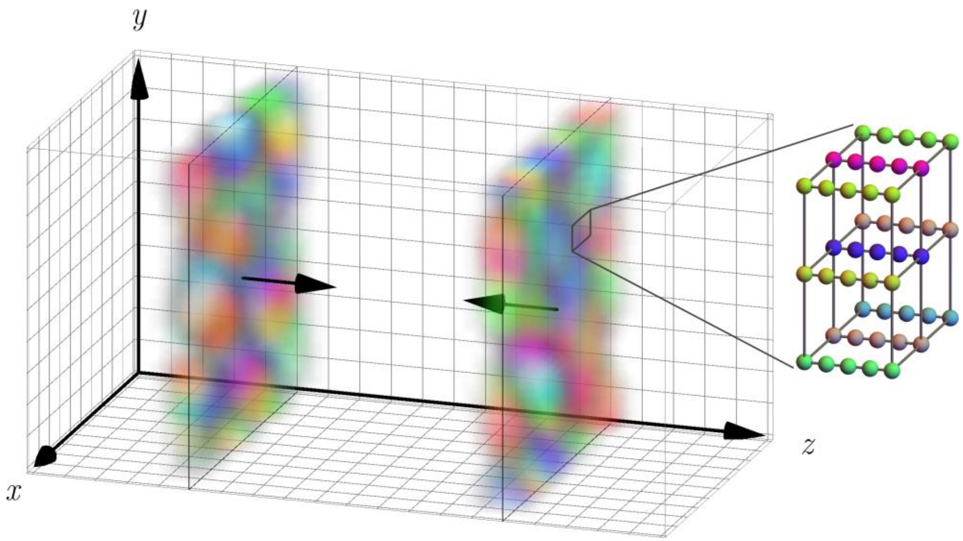

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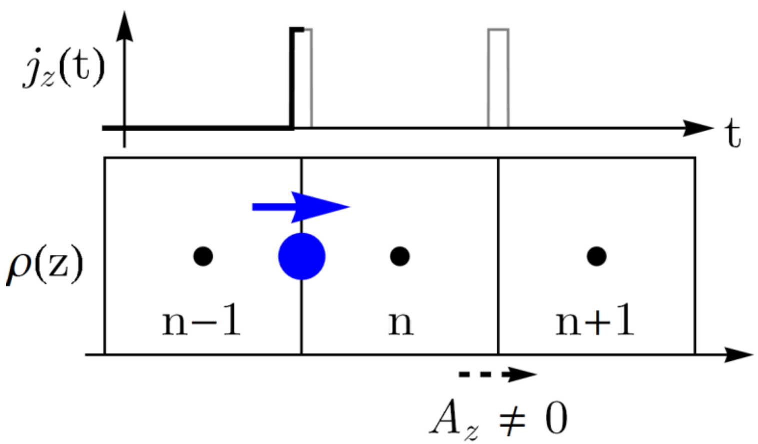

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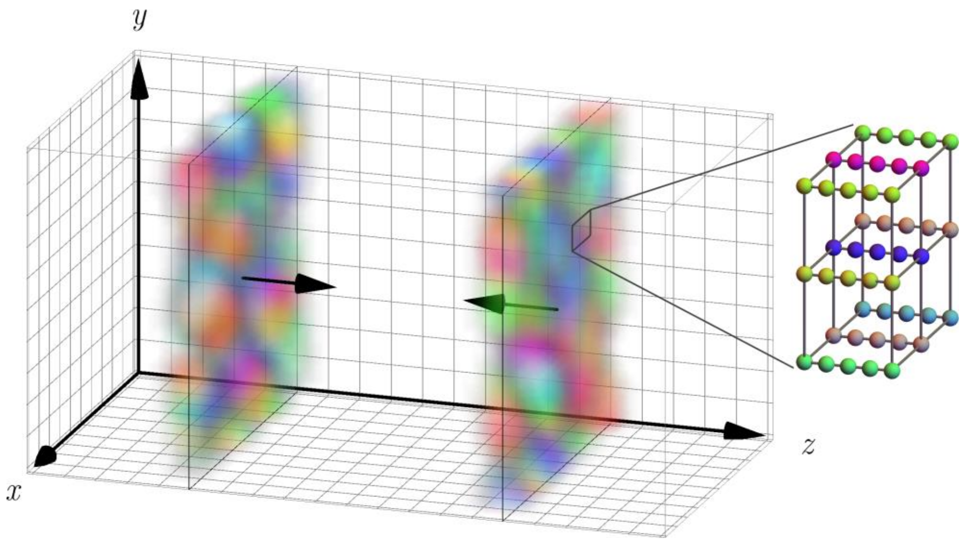

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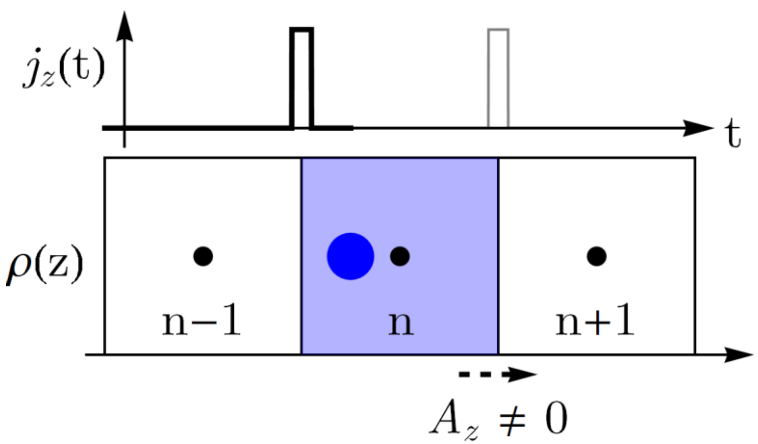

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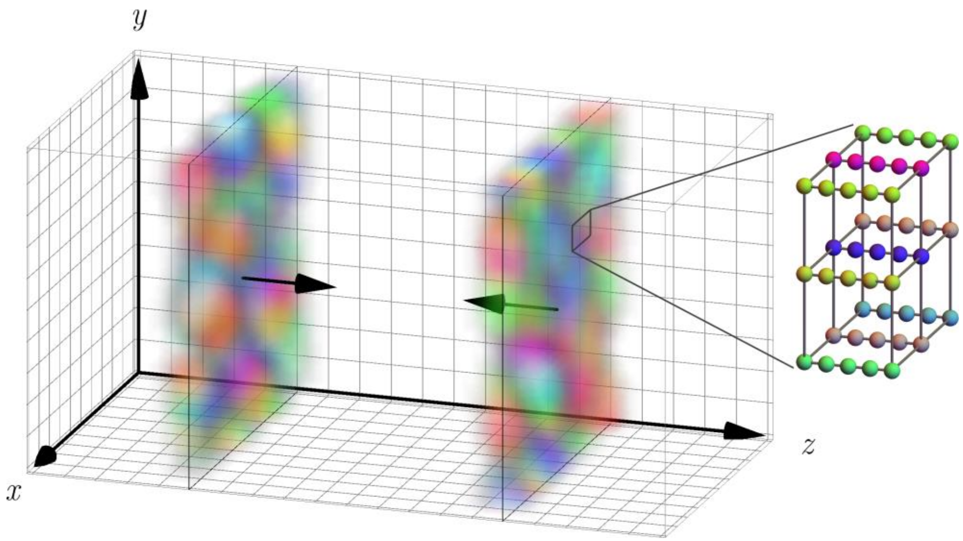

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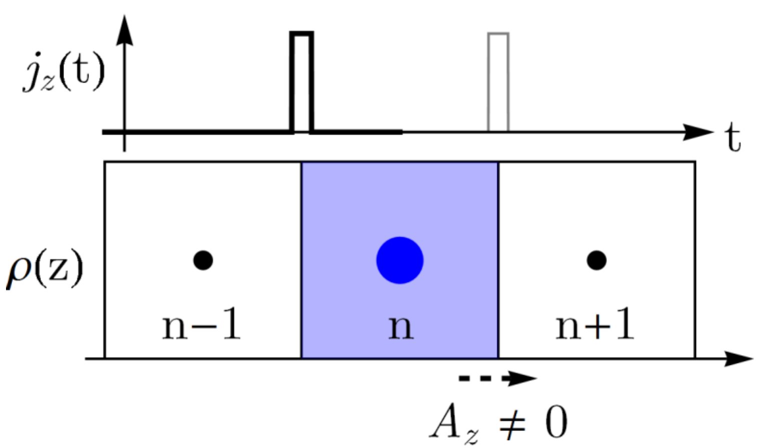

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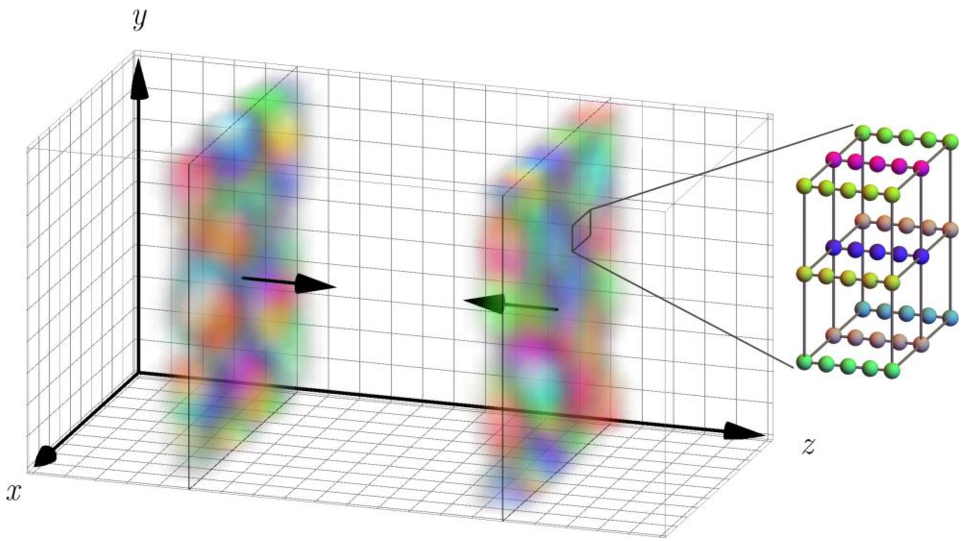

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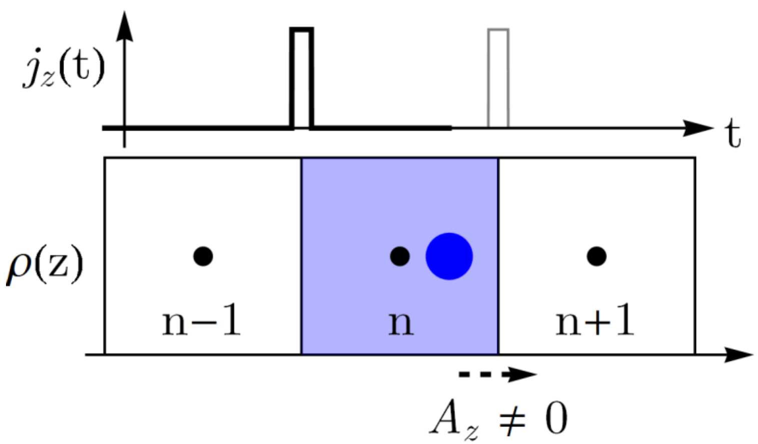

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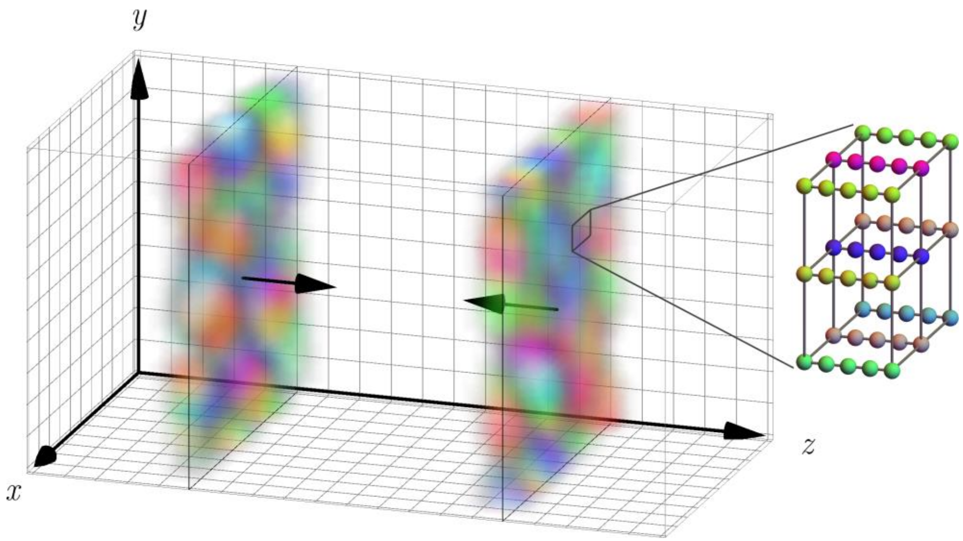

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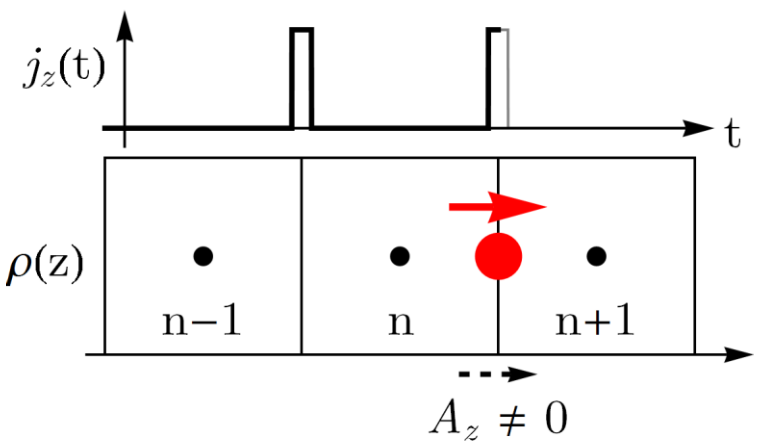

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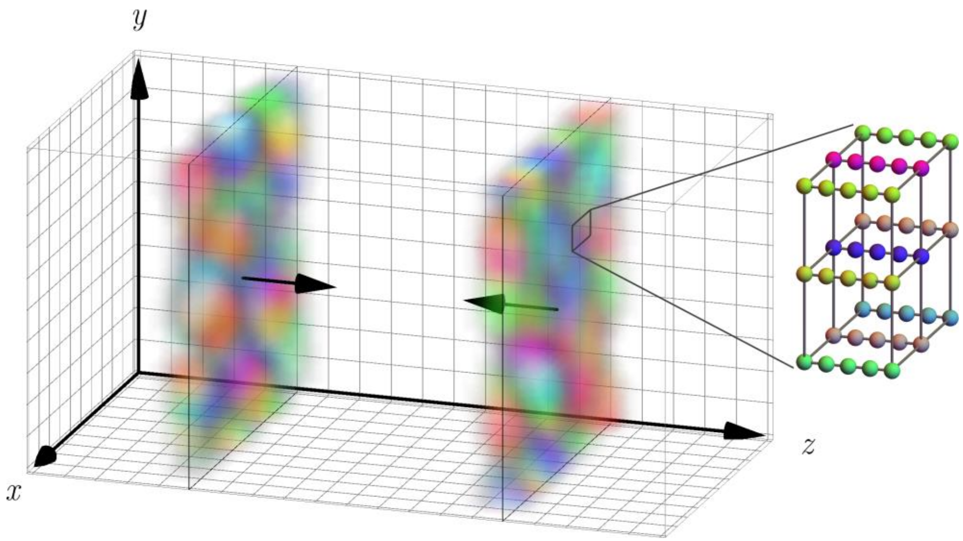

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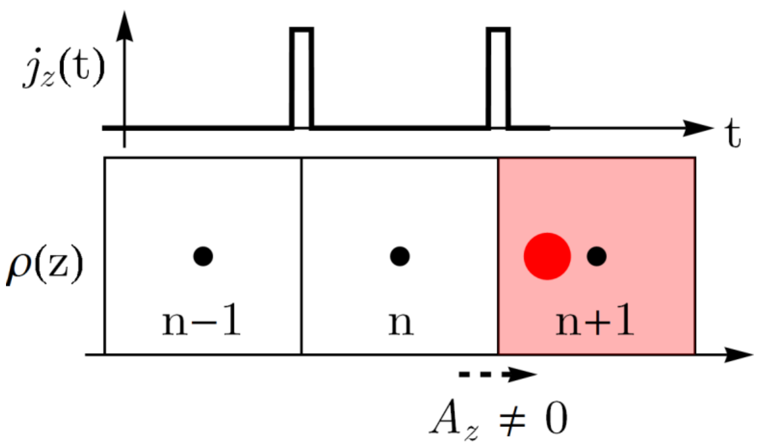

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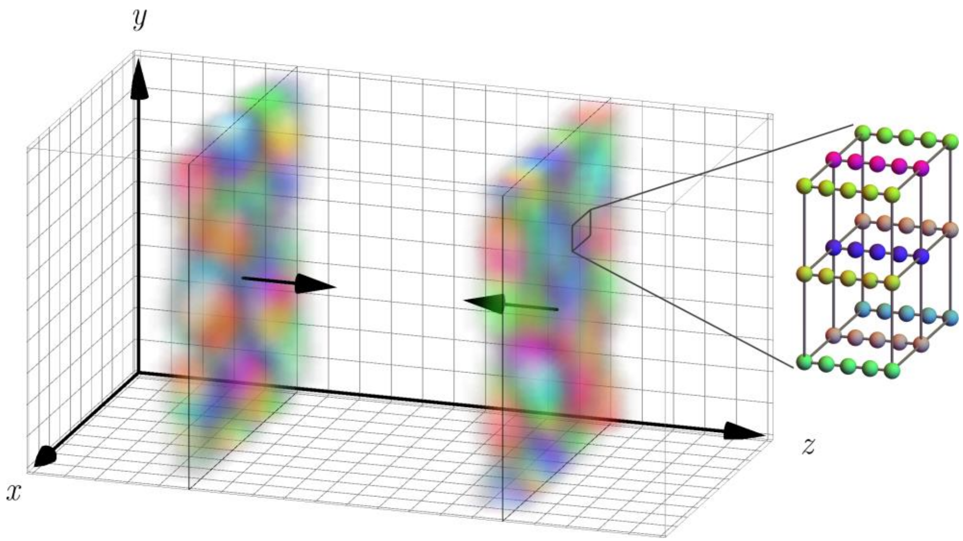

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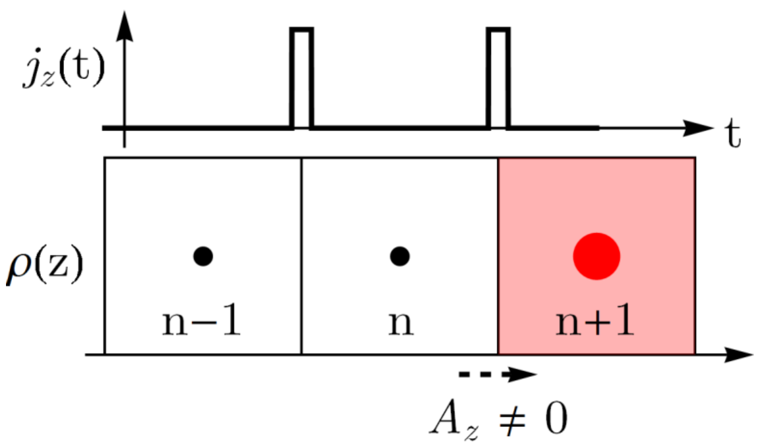

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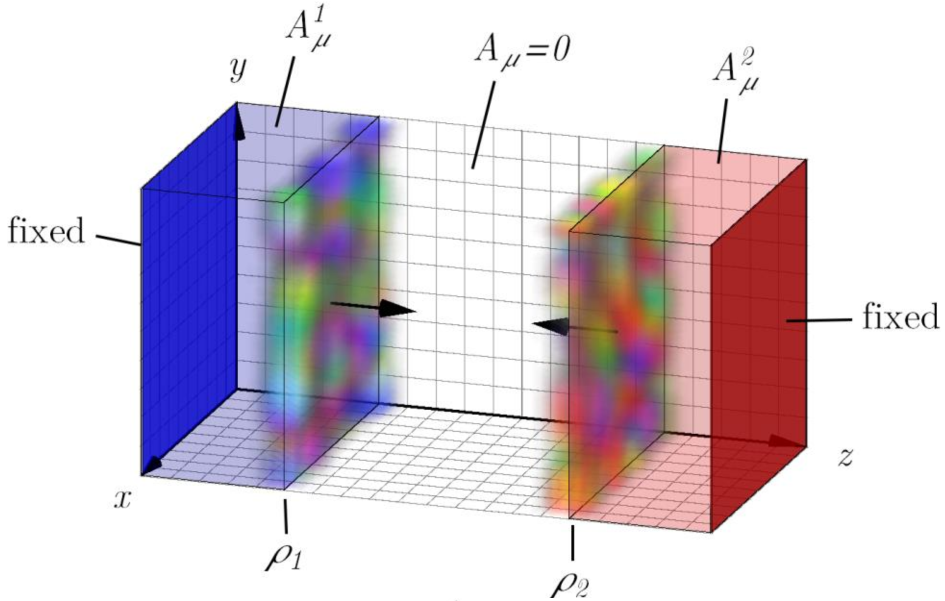

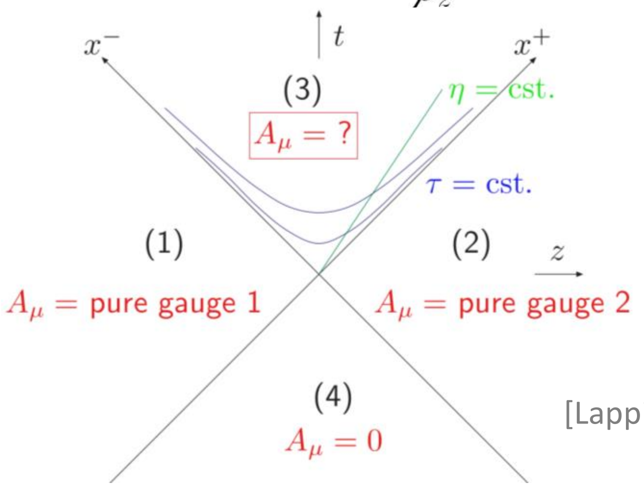

[Lappi, Phys.Lett. B643 (2006) 11-16]

- Temporal gauge ($A_0 = 0$) suitable for numerical time evolution.
- Asymptotically pure gauge "trails" behind nuclei.
- Fixed boundary conditions on the longitudinal boundaries are required.
- Random charge densities $\rho_{(1,2)}$ are sampled from **McLerran-Venugopalan (MV)** model.

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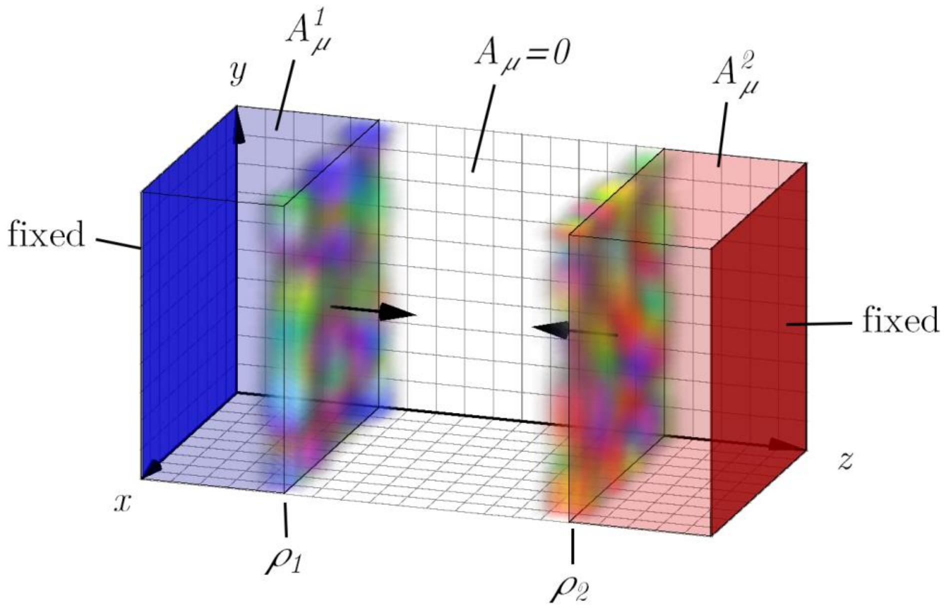

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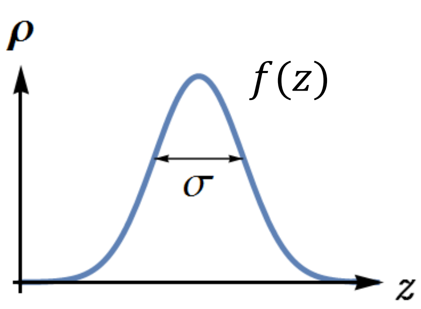


finite longitudinal thickness

$$\boxed{\begin{gathered}\langle \hat{\rho}^a(\boldsymbol{x}_T)\hat{\rho}^b(\boldsymbol{x'}_T)\rangle = g^2\mu^2\delta^{(2)}(\boldsymbol{x}_T - \boldsymbol{x'}_T)\delta^{ab} \\ \rho\,(t, z, \boldsymbol{x}_T) = f(z-t)\hat{\rho}(\boldsymbol{x}_T)\end{gathered}}$$

MV parameter $\mu \approx 0.5$ GeV (Au)

$Q_s \approx 2$ GeV

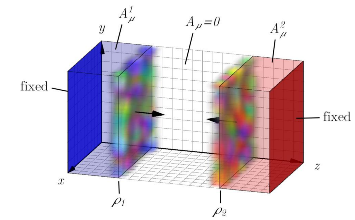
UV & IR regulation

$m \approx 2$ GeV

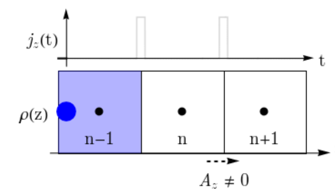
$\Lambda_{UV} \approx 10$ GeV

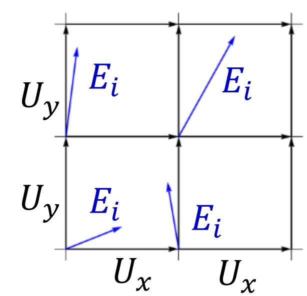
Simulation overview

1. Initialize random charges and fields of two colliding nuclei.



2. **Simulation cycle**:
   a. Move particles and apply parallel transport.
   b. Generate currents from particle movement.
   c. Evolve fields in time with currents as input.
   d. Compute observables ($T_{\mu\nu}$, ε, p_L, p_T, …).



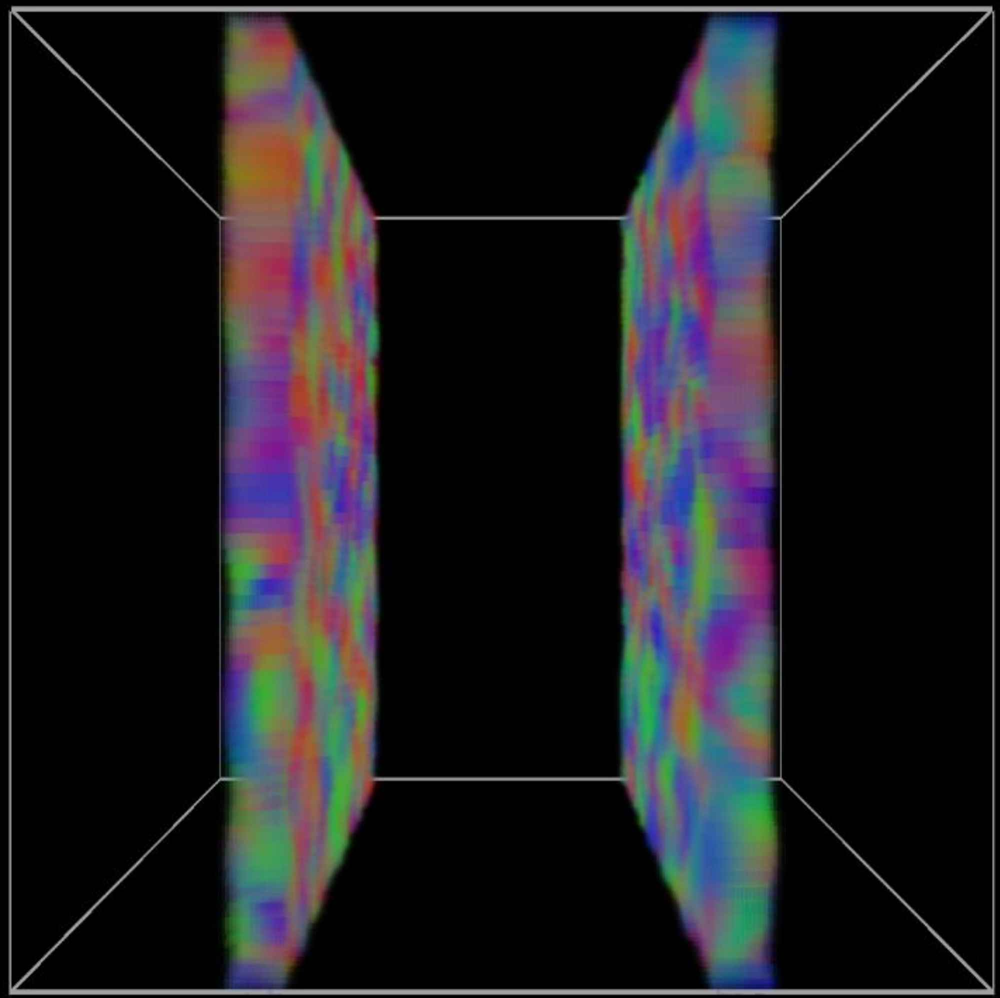


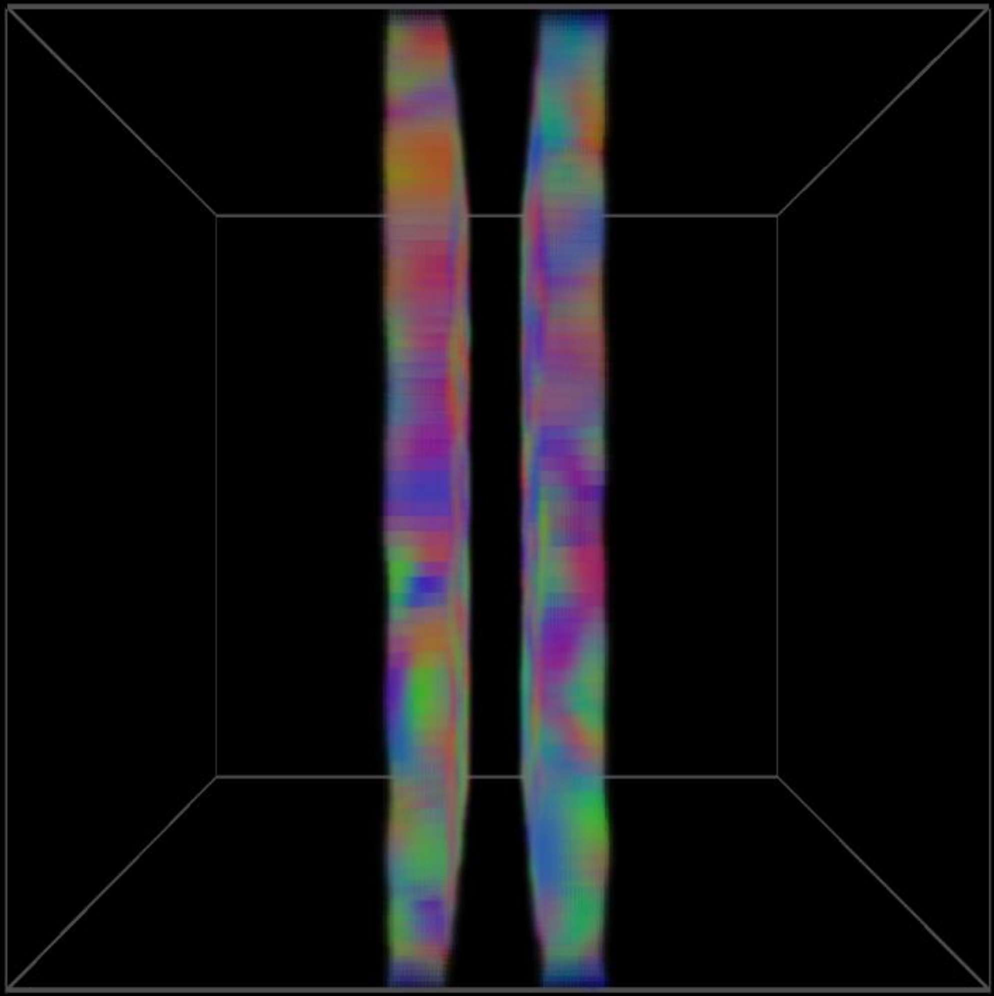
3. Average over many random events.

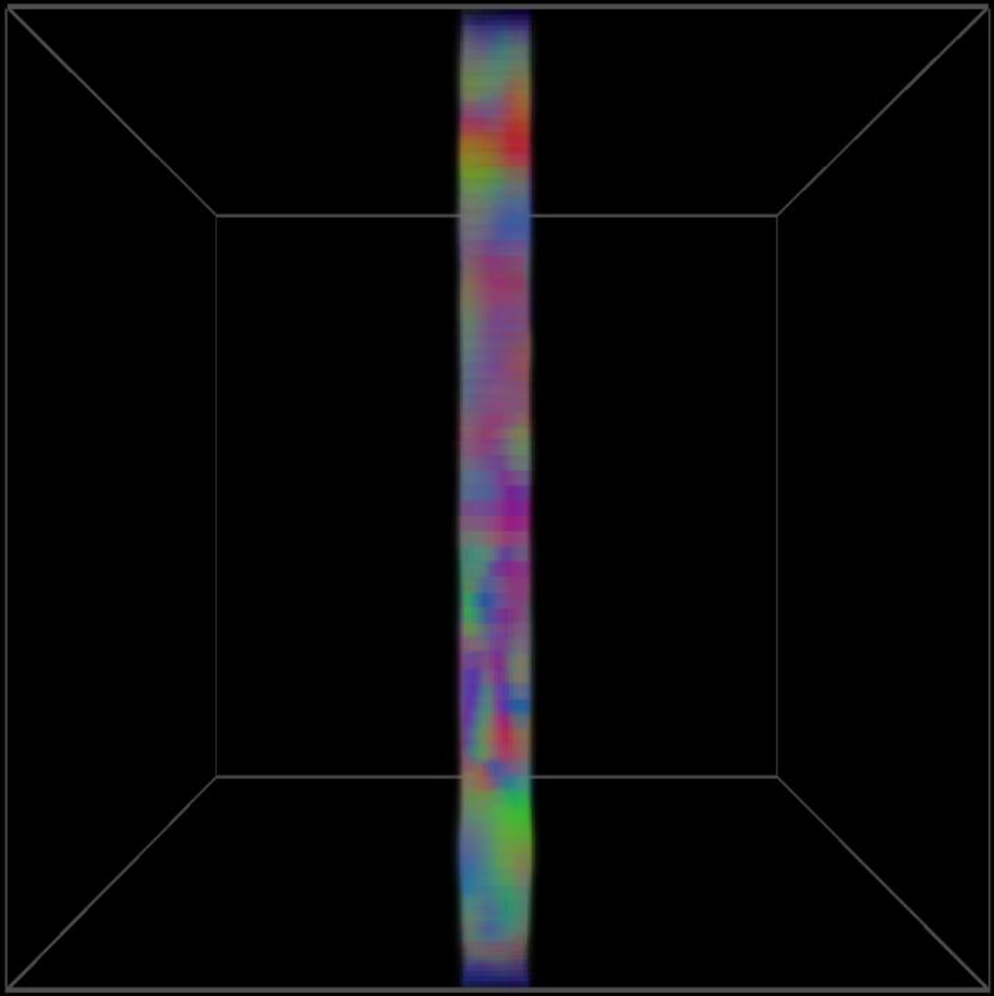
$\langle T_{\mu\nu}\rangle, \langle\varepsilon\rangle, \langle p_L\rangle, \langle p_T\rangle, \dots$

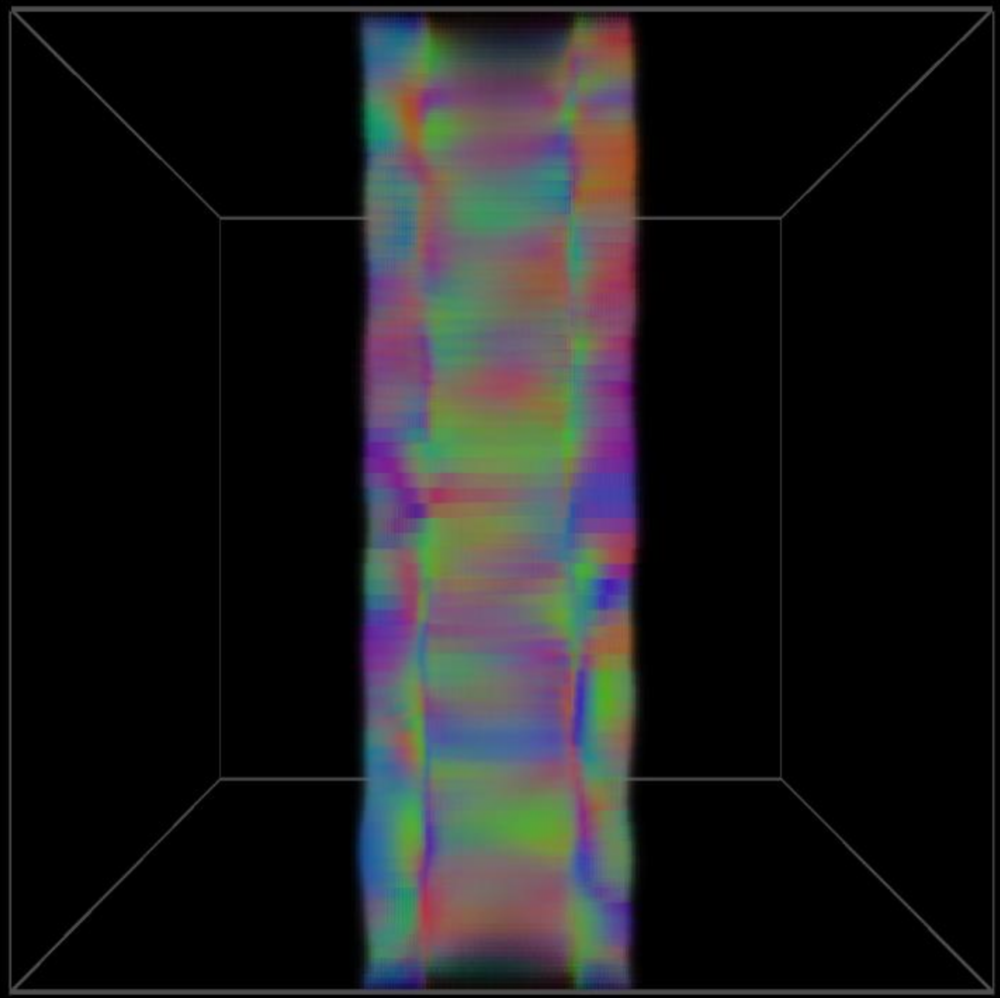
Numerical results

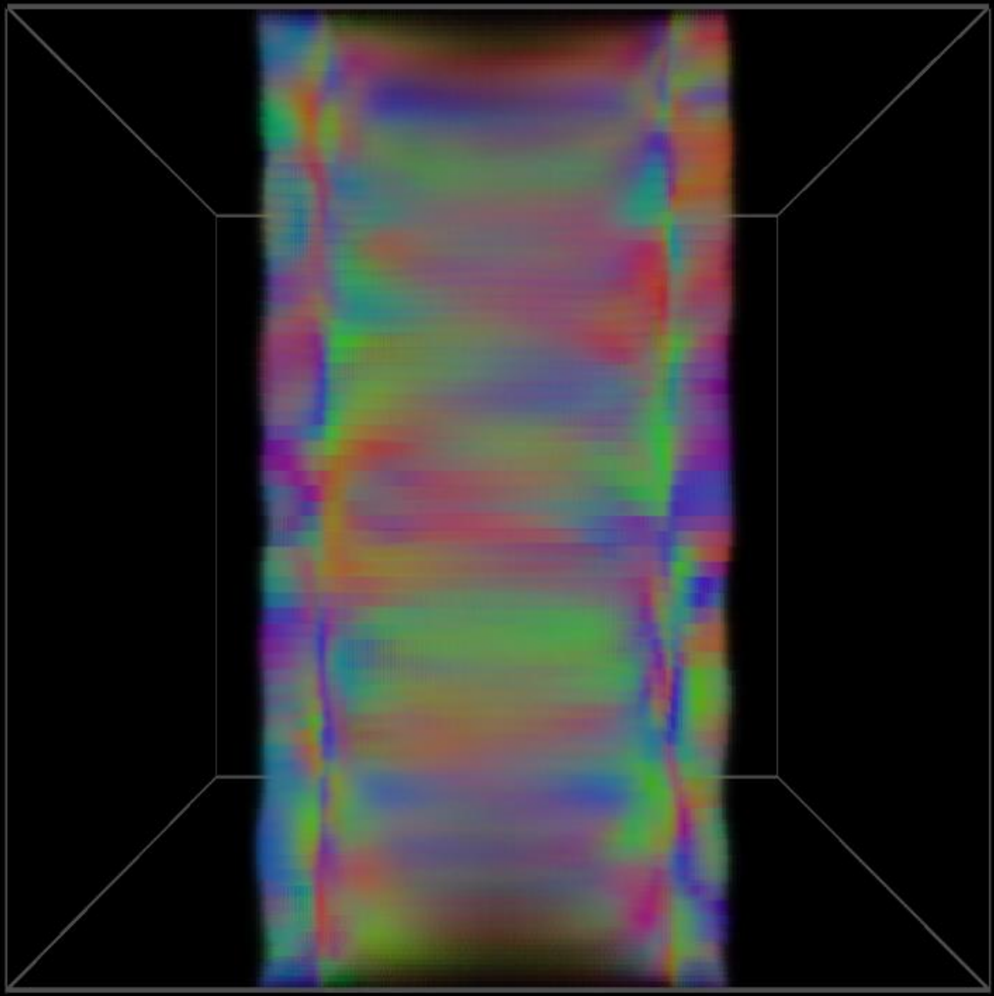
Au-Au collision in the MV model, SU(2)

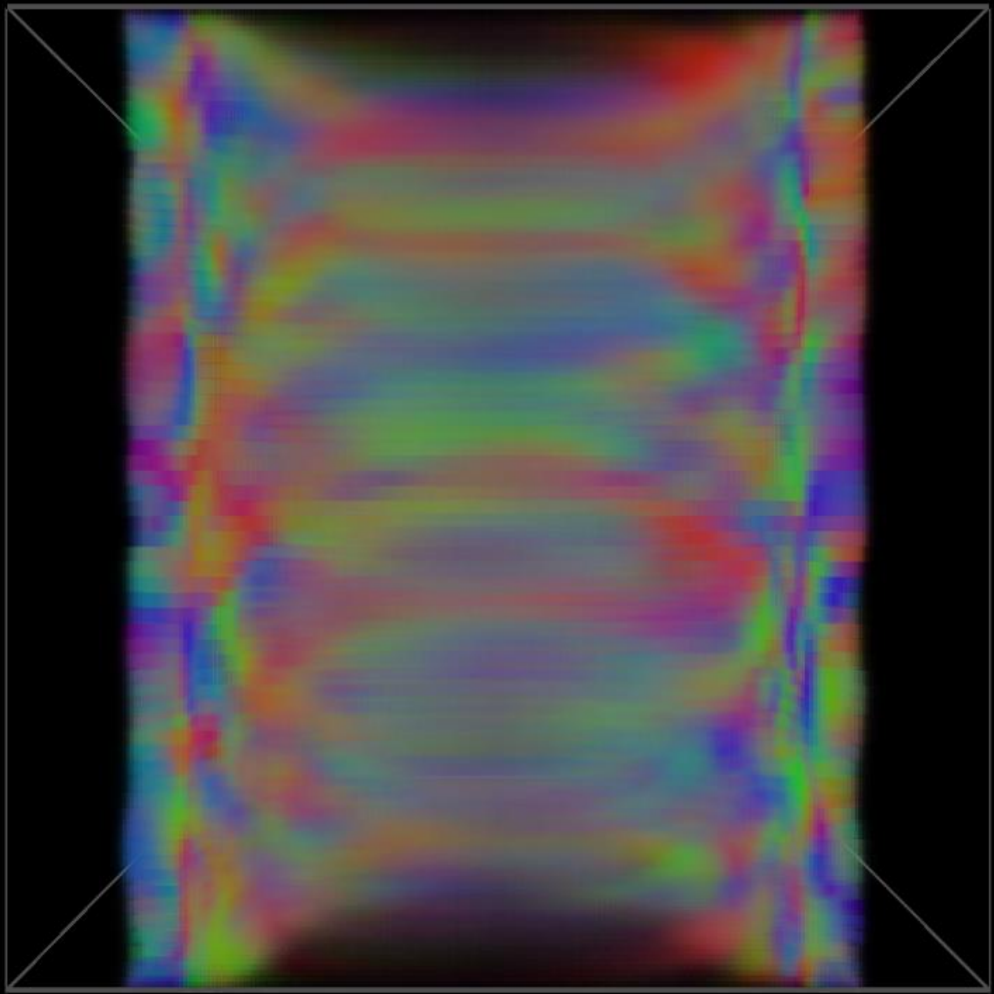








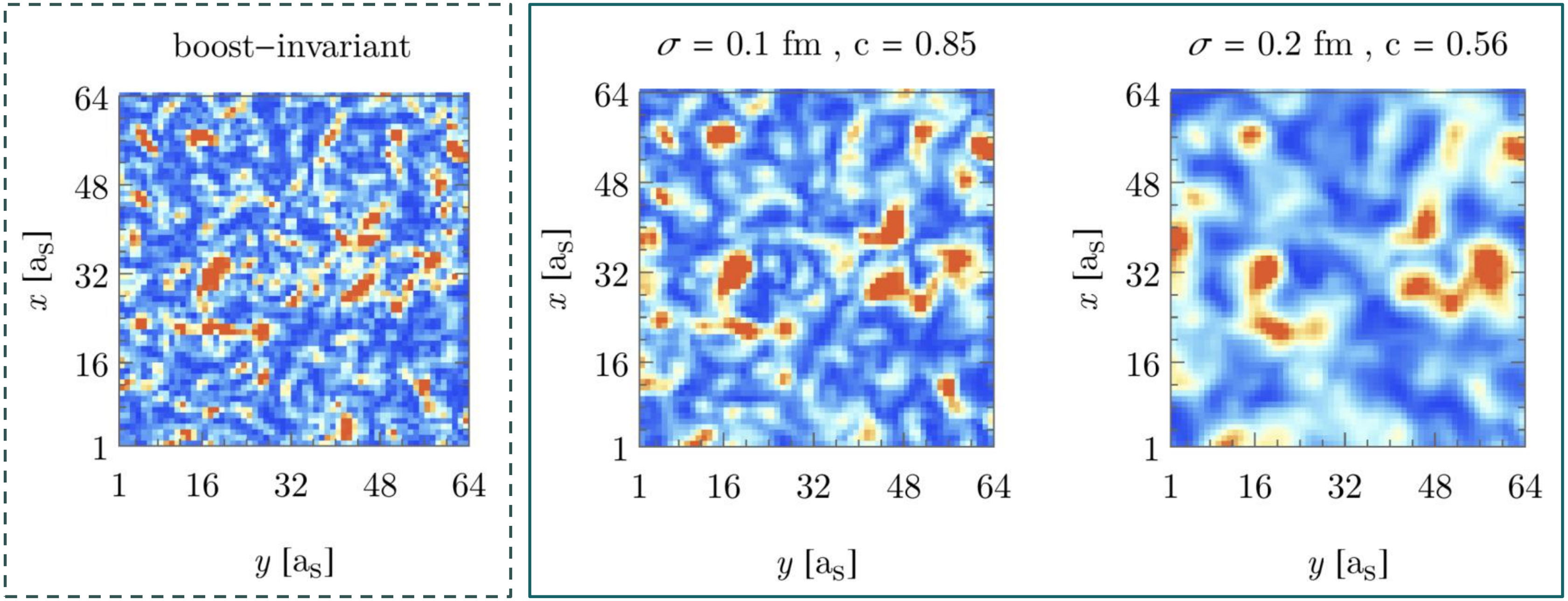




Comparison to boost-invariant results

- Check validity of simulation results with finite nucleus thickness by comparing to analytical boost-invariant results.
- Compare boost-invariant Glasma initial conditions to simulated fields and vary thickness parameter σ.

Energy density component $\mathrm{tr}E_L^2(x_T)$ in the transverse plane at $\eta = 0$.



analytic result

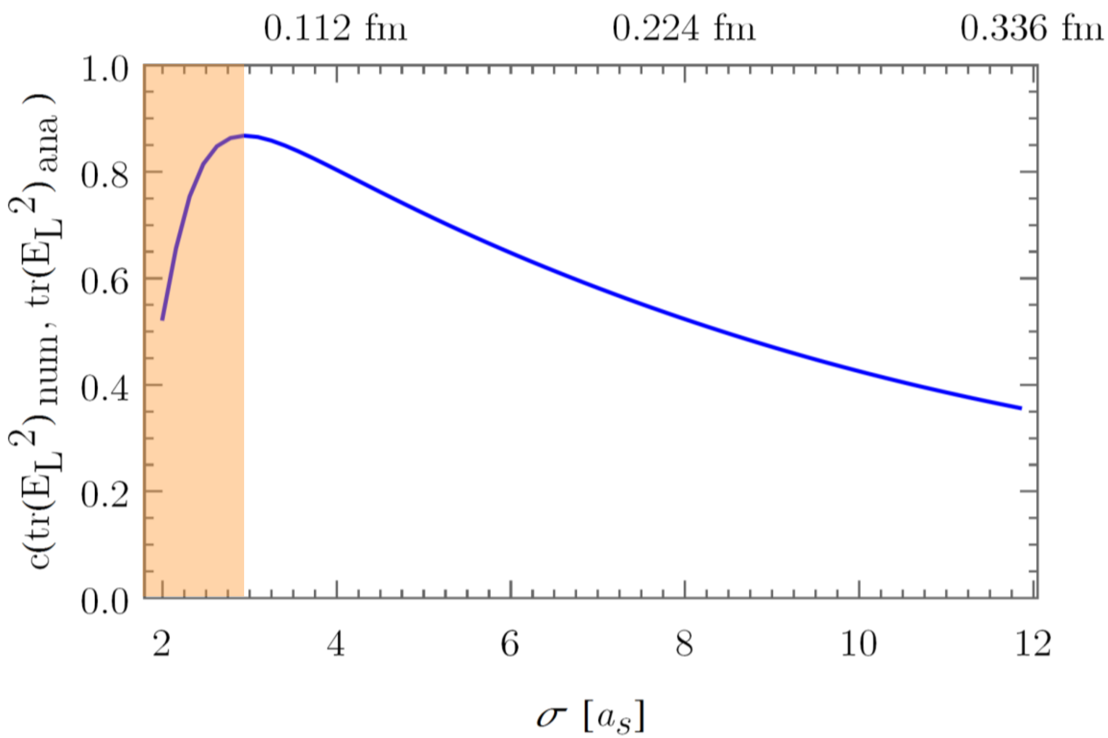
simulation result

Comparison to boost-invariant results

- Check validity of simulation results with finite nucleus thickness by comparing to analytical boost-invariant results.
- Compare boost-invariant Glasma initial conditions to simulated fields and vary thickness parameter σ.

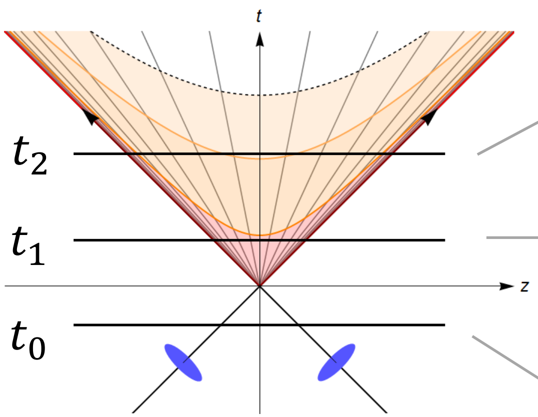
- Compute correlation between analytic and numerical results as a function of σ.
- Thick nuclei: low correlation
- Thin nuclei: high correlation
- Numerical instabilities prohibit very thin nuclei.

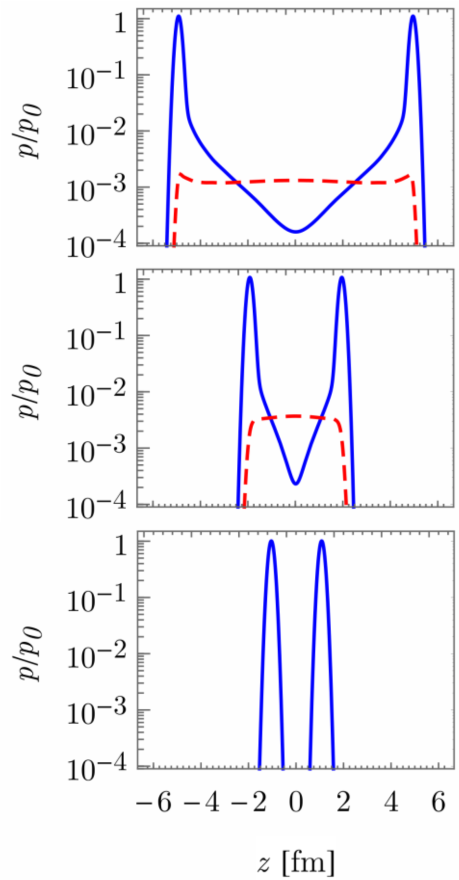
(but it's just a question of lattice sizes)



Pressure anisotropy (1)

- Compute longitudinal and transverse pressure $p_L(z)$ and $p_T(z)$ as a function of the longitudinal coordinate z.





$t_2 = +5$ fm/c
(late times)

$t_1 = +2$ fm/c
(after collision)

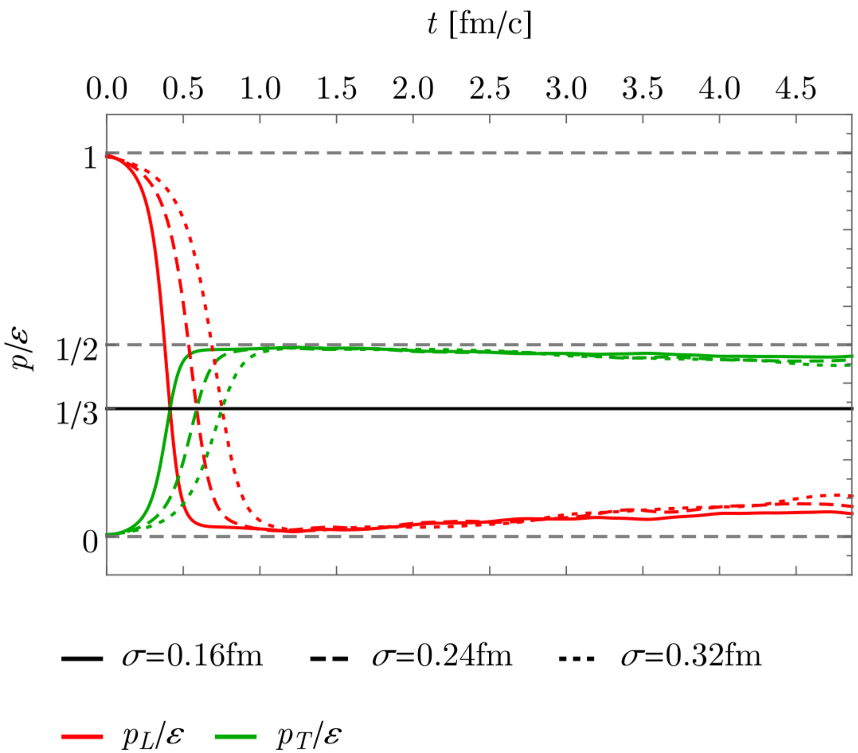
$t_0 = -1$ fm/c
(before collision)

→ Pronounced pressure anisotropy

Pressure anisotropy (2)

- **Isotropization**: initial pressure anisotropy should vanish after ~ 0.1 fm/c to a few fm/c.
- Boost-invariance breaking perturbations drive system towards isotropization. [Epelbaum, Gelis, PRL 111 (2013) 232301]. Finite thickness breaks boost-invariance.

"Thick" nuclei ($\gamma \sim 20-40$)

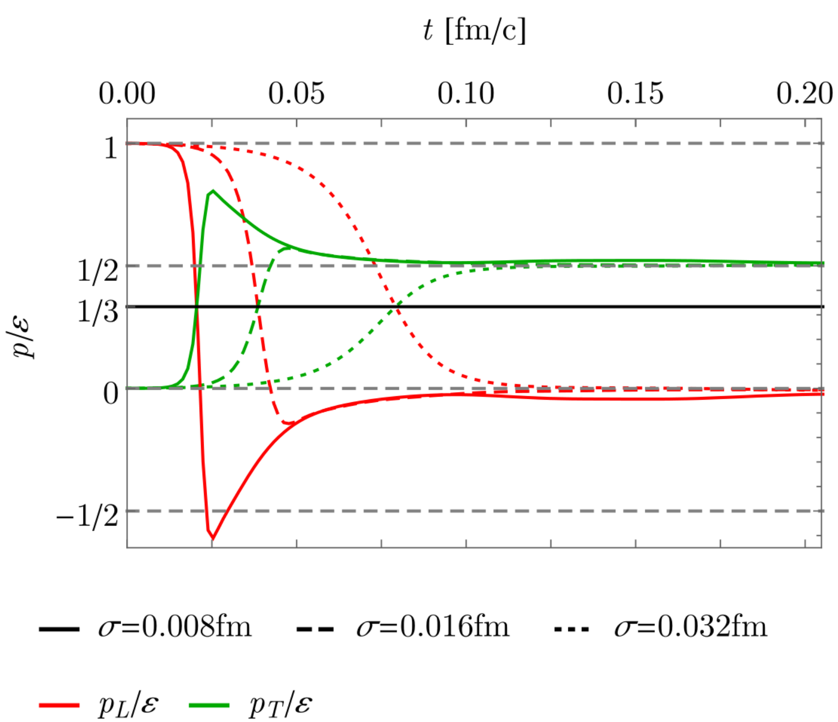


- Analyze pressure to energy density ratio in the central region at $\eta = 0$.
- Thick nuclei: pronounced pressure anisotropy (free-steaming).
- Slight movement towards isotropization visible, but it is too slow.
- Negative longitudinal pressures?

Pressure anisotropy (2)

- **Isotropization**: initial pressure anisotropy should vanish after ~ 0.1 fm/c to a few fm/c.
- Boost-invariance breaking perturbations drive system towards isotropization. [Epelbaum, Gelis, PRL 111 (2013) 232301]. Finite thickness breaks boost-invariance.

"Thin" nuclei ($\gamma \sim 200 - 1000$)



- Analyze pressure to energy density ratio in the central region at $\eta = 0$.
- Thin nuclei: negative longitudinal pressures
- Observables always influenced by presence of the nuclei at early times.

Longitudinal structure (1)

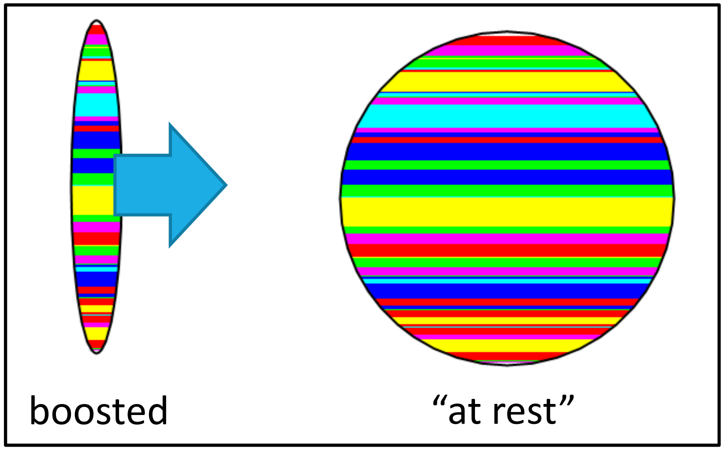
Initial conditions are still missing **random longitudinal structure**.

Longitudinal randomness…

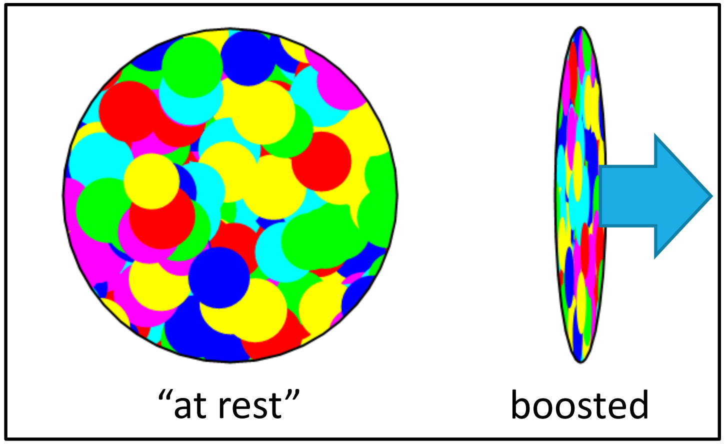
- leads to higher energy density in the Glasma.
  [Fukushima, PRD 77 (2008) 074005]
- could further break boost-invariance.

Possible consequence: **faster isotropization times? → future work!**

Current implementation



Longitudinal randomness



Longitudinal structure (2)

First check: Light-like Wilson line expectation value $\langle \mathrm{tr}(V) \rangle$ of a single nucleus is sensitive to longitudinal structure.

Embedded 2D MV-model:

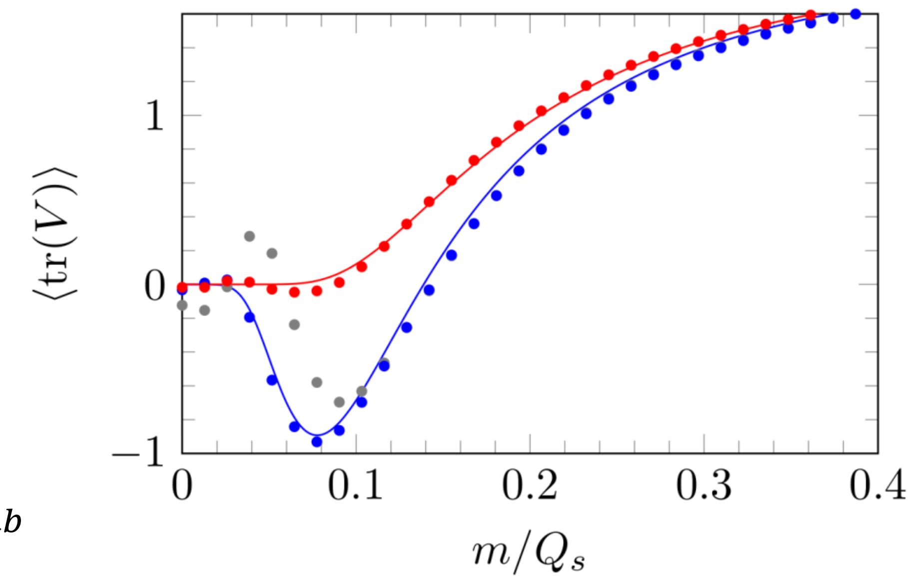
$$\left\langle \hat{\rho}^a(\boldsymbol{x}_T)\hat{\rho}^b(\boldsymbol{x'}_T)\right\rangle = g^2\mu^2\delta^{(2)}(\boldsymbol{x}_T - \boldsymbol{x'}_T)\delta^{ab}$$

$$\rho\,(t, z, \boldsymbol{x}_T) = f(z-t)\hat{\rho}(\boldsymbol{x}_T)$$

3D MV-model:
(with random longitudinal structure)

$$\left\langle \rho^a(t,\boldsymbol{x})\rho^b(t,\boldsymbol{x}')\right\rangle = g^2\mu^2 f(z)\delta^{(3)}(\boldsymbol{x}-\boldsymbol{x}')\delta^{ab}$$

$f(z)$ … longitudinal profile function



Lines: analytical result
Dots: numerical result

Blue: 2D MV-model
Red: 3D MV-model
Gray: intermediate

Improvement method for boost-invariant simulations
[Fukushima, PRD 77 (2008) 074005]

Conclusions and summary

- Simulating CGC collisions in 3D+1 with finite nucleus thickness in the laboratory frame using CPIC is viable.
- Boost-invariant results reproduced in the limit of thin nuclei.
- We observe a pronounced pressure anisotropy after the collision.
- Observed isotropization too slow.

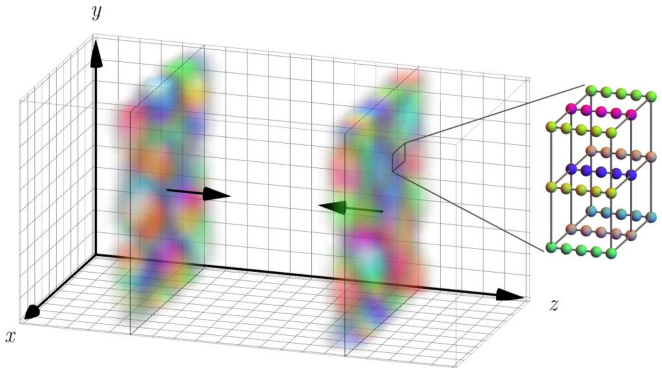
Future:

- Study effects of initial conditions with random longitudinal structure on isotropization
- Corrections to initial Glasma energy density due to finite thickness

Open Pixi

arXiv:1605.07184 [hep-ph]
Phys.Rev. D94 (2016) no.1, 014020
open source: https://github.com/openpixi

Thank you for your attention!



Open

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