Locating QCD’s critical end point

Christian S. Fischer

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Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP in press [1606.09602]
1. Introduction

2. Gluons, quarks and the CEP

3. Baryon effects on the CEP

\[ N - 1 = -1 + N_B + dq + dq \]
QCD phase transitions

2nd order

1st order

physical $m_{u,d,s}$

cross over

$m_s$

$m_{u,d}$
Locating QCD's critical end point

QCD phase transitions

\[ m_s \]

2\textsuperscript{nd} order

1\textsuperscript{st} order

cross over

physical \( m_{u,d,s} \)

\[ m_s^{\text{tri}} \]

1\textsuperscript{st} order

\[ m_{u,d} \]

chemical potential

QCD critical point

physical \( m_{u,d,s} \)

Is this happening ??

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Locating QCD’s critical end point
Lattice-QCD
- present: extrapolation
- future: exact methods?

DSE/FRG
- not exact, but allow for '10%-physics'

Is this happening??

QCD phase transitions

- 2nd order
- 1st order
- Physical m_{u,d,s}
- Cross over
- m_s
- m_{u,d}
- Chemical potential
- QCD critical point
- Physical m_{u,d,s}
- 2nd order
- 1st order

Locating QCD’s critical end point
Search for the CEP

- **Taylor expansion ($N_f=2$):**
  Datta, Gavai and Gupta, NPA 904-905 (2013) 883c
  Gavai, Gupta, PRD 71 (2005) 114014

- **Reweighting ($N_f=2+1$):**

- **Analytic continuation ($N_f=3$):**
  de Forcrand, Philipsen, JHEP 0811 (2008) 012;
  NPB 642 (2002) 290
Chiral transition line from analytic continuation

Lattice method:
- Calc. boundary at imaginary $\mu$ and extrapolate to real $\mu$
- Control systematics

Results:
- Larger curvature than previous results (but: different definitions and error budget)

Bellwied, Borsanyi, Fodor, Günther, Katz, Ratti and Szabo, PLB B 751 (2015) 559
1. Introduction

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\[ -1 = \frac{d}{dN} + \frac{d}{dN} + \frac{dq}{N} \]
QCD order parameters from propagators

Deconfinement:
- dressed Polyakov loop
- Polyakov loop potential

Chiral order parameter:
\[
\langle \bar{\Psi} \Psi \rangle = Z_2 N_c Tr D \frac{1}{T} \sum \int \frac{d^3 p}{(2\pi)^3} S(\vec{p}, \omega)
\]

\[
\Sigma = - \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi} \langle \bar{\Psi} \Psi \rangle \varphi
\]

Synatschke, Wipf, Wozar, PRD 75, 114003 (2007)
Bilgici, Bruckmann, Gattringer, Hagen, PRD 77 094007 (2008)
CF, PRL 103 052003 (2009)

\[
L = \frac{1}{N_c} Tr e^{i \beta A_0}
\]

\[
\frac{\delta (\Gamma - S)}{\delta A_0} = \frac{1}{2} - \frac{1}{6}
\]

Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011)
Fister, Pawlowski, PRD 88 045010 (2013)
CF, Fister, Luecker, Pawlowski, PLB 732 (2013)
Locating QCD’s critical end point

- **N_f=2+1-QCD with DSEs**

- Quenched: without quark-loop
- N_f=2: isospin symmetry
- N_f=2+1: solve coupled system of 2+3+3 equations
- Vertex: ansatz built along STI and known UV/IR behaviour

Quenched propagator from lattice

\[
\begin{align*}
-1 &\quad = \quad -1 \\
&\quad + \quad \text{up/down} \\
&\quad + \quad \text{strange}
\end{align*}
\]
Glue at finite temperature (T≠0)

T-dependent gluon propagator from quenched lattice simulations:

- **Crucial difference between magnetic and electric gluon**
- **Maximum of electric gluon near Tc**

Cucchieri, Maas, Mendes, PRD 75 (2007)
Cf, Maas, Mueller, EPJC 68 (2010)
Cucchieri, Mendes, PoS FACESQCD 007 (2010)
Aouane, Bornyakov, Ilgenfritz, Mitrjushkin, Muller-Preussker and Sternbeck, PRD 85 (2012) 034501
Silva, Oliveira, Bicudo, Cardoso, PRD 89 (2014) 074503

FRG: Fister, Pawlowski, arXiv:1112.5440
Approximation for Quark-Gluon interaction

- **T,μ,m-dependent vertex:**

\[
\Gamma_\nu(q, k, p) = \bar{Z}_3 \left( \delta_{4\nu} \gamma_4 \frac{C(k) + C(p)}{2} + \delta_{j\nu} \gamma_j \frac{A(k) + A(p)}{2} \right) \times \\
\times \left( \frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left( \frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right)
\]

- **Abelian WTI**
- **perturbation theory**

**Infrared ansatz:**
- d2 fixed to match gluon input
- d1 fixed via quark condensate (see later)
- correct UV (quant.) and IR-behavior (qual.)

explicit solutions at T=0: Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD PRD 93 (2016) 034026
Deconfinement transition in agreement with lattice QCD
Correct tricritical scaling

Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

CF, Luecker, Pawlowski, PRD 91 (2015) 1
Deconfinement transition in agreement with lattice QCD
Correct tricritical scaling

Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042
CF, Luecker, Pawlowski, PRD 91 (2015) 1
$N_f=2+1, \mu=0, \text{physical point}$

Lattice: Borsanyi et al. [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022
\( N_f=2+1, \mu=0, \text{physical point} \)

\[ l_s(T)/l_s(0) \]

Lattice: Borsanyi et al. [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022
**N_f=2+1, \( \mu=0 \), physical point**

- **Lattice QCD**
- **Quark Condensate**
- **dressed Polyakov Loop**

\[ \Delta_{l,s}(T)/\Delta_{l,s}(0) \]

\[ Z_L \]

- **Lattice**: Borsanyi et al. [Wuppertal-Budapest], JHEP 1009(2010) 073
- **DSE**: CF, Luecker, PLB 718 (2013) 1036
- **DSE**: CF, Luecker, Welzbacher, PRD 90 (2014) 034022

**quantitative agreement:** DSE prediction verified by lattice
Nf=2+1: Condensate and dressed Polyakov Loop

Quark condensate

Polyakov-Loop

\[ L = \frac{1}{N_c} \text{tr} \ e^{i g \int A_0} \]

CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273
$N_f=2+1$: phase diagram

- DSE: chiral crossover
- DSE: critical end point
- DSE: chiral first order
- DSE: deconfinement crossover

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Locating QCD’s critical end point
$N_f=2+1$: phase diagram

- DSE: chiral crossover
- DSE: critical end point
- DSE: chiral first order
- DSE: deconfinement crossover

CEP at large $\mu$


combined evidence of FRG and DSE: no CEP at $\mu_B/T<2$
$N_f=2+1$: phase diagram

Extrapolated curvature from lattice


CEP at large $\mu$

Combined evidence of FRG and DSE: no CEP at $\mu_B/T<2$
Nf=2+1+1: effects of charm

- Physical up/down, strange and charm quark masses
- Transition controlled by chiral dynamics
- *no lattice or model results available yet*
$\text{Nf}=2+1+1$: effects of charm

- Physical up/down, strange and charm quark masses
- Transition controlled by chiral dynamics
- No lattice or model results available yet

Effects very small!
Location of CEP in freeze-out landscape

Caveats:
- inhomogeneous phases
- effects of baryons?
- finite size
- ...

Figure taken from talk of T. Galatyuk, Erice 2016

Müller, Buballa and Wambach, PLB 727 (2013) 240

Nc=2: Brauner, Fukushima and Hidaka, PRD 80 (2009) 74035
Strodthoff, Schaefer and Smekal, PRD 85 (2012) 074007
1. Introduction

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\[ -1 = -1 + Nb + dq + dq \]
Low temperatures, large chemical potential: baryons are important degrees of freedom

How do baryons affect the quark condensate??
Baryon effects onto quark

- Dependence on T and \( \mu \) via -propagators -wave functions
- Exploratory calculation: use wave functions from T=\( \mu \)=0

Off-shell baryon in vertex

\[
-1 = -1 + \frac{\text{Off-shell baryon in vertex}}{\text{quark}}
\]
Vacuum: Baryons from BSEs

BSE for baryons (derived from equation of motion for G)

\[
\Gamma_i = T_{ij} \Gamma_j + T_{ik} \Gamma_k
\]

Faddeev equation (no three-body forces)

Diquark-quark
Vacuum: Baryons from BSEs

**BSE for baryons** (derived from equation of motion for G)

\[ \Gamma_i = T_i \Gamma_j + T_i \Gamma_k \]

**Faddeev equation** (no three-body forces)

\[ ^{-1} \phi = \Gamma_i \Gamma_j \Gamma_k \]

**Diquark-quark**

\[ \phi = \Gamma_i \Gamma_j \Gamma_k \]
Vacuum: Baryons from BSEs

BSE for baryons (derived from equation of motion for G)

Faddeev equation (no three-body forces)

Diquark-quark

\[ \Gamma_i \Gamma_j \Gamma_k = \Gamma_i \Gamma_j \Gamma_k \]

\[ \phi = \phi \]

\[ \frac{1}{k^2} = \frac{1}{k^2} + \]

\[ \alpha(k^2) = \pi \eta^\prime \left( \frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left( \frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2) \]

Input: Non-perturbative quark, quark-gluon interaction (RL)
## Vacuum: DSE/Faddeev landscape

<table>
<thead>
<tr>
<th>Quark-diquark</th>
<th>Contact interaction</th>
<th>QCD-based model</th>
<th>DSE (RL)</th>
<th>Three-quark</th>
<th>RL</th>
<th>bRL</th>
<th>bRL + 3q</th>
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- Roberts et al
- Oettel, Alkofer
- Roberts, Bloch
- Segovia et al.
- Eichmann, Alkofer
- Nicmorus, Krassnigg
- Eichmann, Alkofer
- Sanchis-Alepuz, CF
- Williams

Eichmann, N*-Workshop, Trento 2015
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Roper

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| N*(1535), . . .| ...                 | ...             | ...
| N → N*γ        | ...                 | ...             | ...

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- Eichmann, Alkofer Nicmorus, Krassnigg
- Eichmann, Alkofer Sanchis-Alepuz, CF
- Sanchis-Alepuz, CF Williams

Eichmann, N*-Workshop, Trento 2015
Vacuum: Light baryon spectrum

\[ \text{Diagram} = \text{Diagram} + \text{Diagram} + \text{Diagram} + \text{Diagram} \]
Three-body and diquark-quark approach agree qualitatively.

Spectrum in one-to-one agreement with experiment.

Correct level ordering (wo. coupled channel effects)!

Eichmann, CF, Sanchis-Alepuz, 1607.05748
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP in press [1606.09602]
Baryon effects on the CEP - results (N_f=2)

- Zero chemical potential: no effects after rescaling
- CEP: almost no effects

Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]
Baryon effects on the CEP - results ($N_f=2$)

Zero chemical potential: no effects after rescaling

CEP: almost no effects

But: strong $\mu$-dependence of baryon wave function may change situation…

Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]
QCD with finite chemical potential:

- back-reaction of quarks onto gluons important
- $N_f=2+1$ and $N_f=2+1+1$: CEP at $\mu_c/T_c > 3$
- charm quark does not influence CEP
- Baryon effects may or may not be significant for CEP…

Work in progress: - mesons and baryons at finite $T$ and $\mu$
- volume effects on CEP from DSEs
Need to take meson part of vertex explicitly into account

$T=0$: meson cloud corrections of order of 10-20 %

CF, Williams, PRD 78 (2008) 074006

$T=T_c$: meson corrections are dominant!

Critical scaling: $\langle \bar{\Psi}\Psi \rangle(t) \sim B(t) \sim t^{\nu/2}$

$\bar{f}_{\Pi,S}^2 \sim t^{\nu}$ \hspace{1cm} ($t = (T_c - T)/T_c$)

CF and Mueller, PRD 84 (2011) 054013