Viscous chemical equilibration and cavitation at LHC energies

V. Sreekanth

Centre for High Energy Physics, IISc Bangalore

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Chemical equilibration and cavitation in the early stage of ultrarelativistic heavy-ion collisions

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arxiv1610.xxxx

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Quark and gluon distribution functions in a viscous quark-gluon plasma medium and dilepton production via $q\bar{q}-$ annihilation
Vinod Chandra and V. Sreekanth
Phys.Rev. D92 (2015) 9, 094027

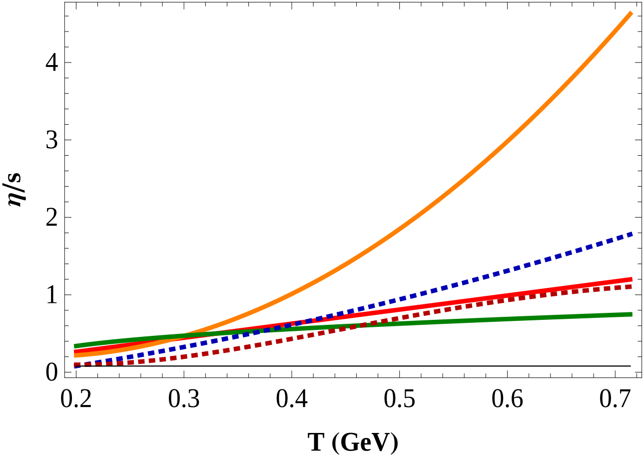
Outline

- Introduction
- Shear viscosity of QGP: η/s
- Second order dissipative hydro
- Shear viscosity induced cavitations
- Chemical equilibration in early times
- Combined effect
- Summary

- It is believed that perfect fluid is created at RHIC experiments
- QGP at RHIC energies: lowest value of $\eta/s \sim 1/4\pi$
- Shear viscosity at the high energies: many temperature dependent forms are used by different groups
- most of these calculations are done using hydrodyamical approaches, assuming instantantaneous chemical equilibration of matter together with thermal equilibration
- In heavy ion collisions number changing processes involving gluons and quark- anti-quarks govern the chemical equilibration of QGP

Temperature dependent Shear viscosity at LHC energies

- Viscous hydrodynamics for LHC energies
- Different prescriptions for *temperature dependent* η/s [Nakamura *et.al* 2005, S. Matiello *et.al* 2005 etc.] to calculate flow properies [H. Niemi *et.al*,PRL,2011; U.Heinz *et.al* 2011, 2015]



Relativistic hydrodyamical equations are obtained using

$$T^{\mu\nu} = \varepsilon\, u^{\mu}\, u^{\nu}\; - P\, \Delta^{\mu\nu} + \Pi^{\mu\nu}$$

$$u^{\nu} \partial_{\mu} T^{\mu\nu} = 0 \text{ (NR limit: Continuity eq)}$$
$$\Delta_{\alpha\nu}\, \partial_{\mu} T^{\mu\nu} = 0 \text{ (NR limit: Euler eq)}$$

$$\begin{aligned} D\varepsilon + (\varepsilon + P)\,\theta - \Pi^{\mu\nu} \nabla_{(\mu}\, u_{\nu)} &= 0\,, \\ (\varepsilon + P)\, Du^{\alpha} - \nabla^{\alpha} P + \Delta_{\alpha\nu}\, \partial_{\mu} \Pi^{\mu\nu} &= 0\,. \end{aligned}$$

$(D \equiv u^{\mu}\partial_{\mu},\ \theta \equiv \partial_{\mu}\, u^{\mu},\ \nabla_{\alpha} = \Delta_{\mu\alpha}\partial^{\mu}$ and $A_{(\mu}\, B_{\nu)} = \frac{1}{2}[A_{\mu}\, B_{\nu} + A_{\nu}\, B_{\mu}])$

- The structure of viscous tensor can be determined with help of the definition of the entropy current s^{μ} and demanding the validity of second law of thermodynamics:

$$\partial_{\mu} s^{\mu} \geq 0 \;\; (s = \tfrac{\varepsilon + P}{T})$$

- Second order hydrodynamics (Isreal-Stewart) is obtained by using

$$s^\mu = su^\mu - \frac{\beta_0}{2T} u^\mu \Pi^2 - \frac{\beta_2}{2T} u^\mu \pi_{\alpha\beta}\pi^{\alpha\beta} + \mathcal{O}(\Pi^3)$$

- Now $\partial_\mu s^\mu \geq 0$ gives *dynamical evolution equations* for $\pi_{\mu\nu}$ and Π

$$\pi_{\alpha\beta} = \eta \left(\nabla_{<\alpha} u_{\beta>} - \pi_{\alpha\beta} TD\left(\frac{\beta_2}{T}\right) - 2\beta_2 D\pi_{\alpha\beta} - \beta_2 \pi_{\alpha\beta}\partial_\mu u^\mu \right) ,$$

$$\Pi = \zeta \left(\nabla_\alpha u^\alpha - \frac{1}{2}\Pi \; TD\left(\frac{\beta_0}{T}\right) - \beta_0 D\Pi - \frac{1}{2}\beta_0 \Pi \partial_\mu u^\mu \right) ,$$

  The coefficients β_0 and β_2 are related with the relaxation time by

$$\tau_\Pi = \zeta\,\beta_0\,, \tau_\pi = 2\eta\,\beta_2.$$

- Unlike first order (Navier-Stokes) this description is *causal* and no *instabilities*

Bjorken's prescription

Bjorken's prescription to describe the one dimensional boost invariant expanding flow:-

- convenient parametrization of the coordinates using the proper time $\tau = \sqrt{t^2 - z^2}$ and space-time rapidity $y = \frac{1}{2} ln[\frac{t+z}{t-z}]$;

$$t = \tau \cosh y \text{ and } z = \tau \sinh y$$

- in the local rest frame of the fireball $u^\mu = (\cosh y, 0, 0, \sinh y)$, form of $T^{\mu\nu} = \mathrm{diag.}(\varepsilon, P_\perp, P_\perp, P_z)$
- effective pressure in the transverse and longitudinal directions

$$\begin{aligned} P_\perp &= P + \frac{1}{2}\Phi \\ P_z &= P - \Phi \end{aligned}$$

- Φ is the non-equilibrium contributions to the equilibrium pressure P coming from shear $(\pi^{ij} = \mathrm{diag}(\Phi/2, \Phi/2, -\Phi)$

$$\frac{\partial \varepsilon}{\partial \tau} = -\frac{1}{\tau}(\varepsilon + P - \Phi)\,,$$

$$\frac{\partial \Phi}{\partial \tau} = -\frac{\Phi}{\tau_\pi} + \frac{2}{3}\frac{1}{\beta_2 \tau} - \frac{\Phi}{2}\left[\frac{1}{\tau} + \frac{T}{\beta_2}\partial_\tau\left(\frac{\beta_2}{T}\right)\right]$$

where $\Phi = \pi^{00} - \pi^{zz}$.

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where $\Phi = \pi^{00} - \pi^{zz}$.

- We consider the EoS of a relativistic gas of massless quarks and gluons:

$$\varepsilon = 3P = (a_2 + 2b_2)\, T^4;$$
$$a_2 = \frac{8\pi^2}{15}, b_2 = \frac{7\pi^2 N_f}{40}.$$

Equations governing longitudinal expansion

$$
\begin{aligned}
\frac{\partial \varepsilon}{\partial \tau} &= -\frac{1}{\tau}(\varepsilon + P - \Phi), \\
\frac{\partial \Phi}{\partial \tau} &= -\frac{\Phi}{\tau_\pi} + \frac{2}{3}\frac{1}{\beta_2 \tau} - \frac{\Phi}{2}\left[\frac{1}{\tau} + \frac{T}{\beta_2}\partial_\tau\left(\frac{\beta_2}{T}\right)\right]
\end{aligned}
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where $\Phi = \pi^{00} - \pi^{zz}$.

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\end{aligned}
$$

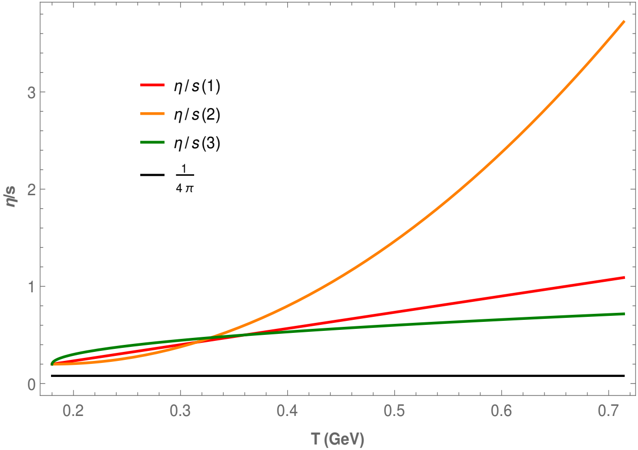
- η/s information enters via τ_π

Cavitation

- From the definition of longitudinal pressure $P_z = P - \Pi - \Phi$ it is clear that if either Π or Φ is large enough it can drive P_z to negative values.
- $Pz = 0$ defines the condition for the onset of *cavitation*
- In the case of relativistic fluids such as the QGP studied in heavy-ion collisions, cavitation would imply a phase transition from a deconfined plasma phase of quarks and gluons to a confined hadron-gas phase.
- The resulting medium would be highly inhomogeneous with (possibly short-lived) hadron gas bubbles expanding and collapsing
- Maybe more importantly, hadron gas dynamics would take over at temperatures above the QCD phase transition, which would have immediate consequences on the measured particle spectra.
- Bulk viscosity induced cavitation scenarios at RHIC energies [Mishustin et. al. (PRC 2008), K. Rajagopal et. al. (JHEP 2010), Sreekanth et al (JHEP 2010), Romatschke et al (JHEP 2014)]

Shear viscosity and cavitation at LHC energies

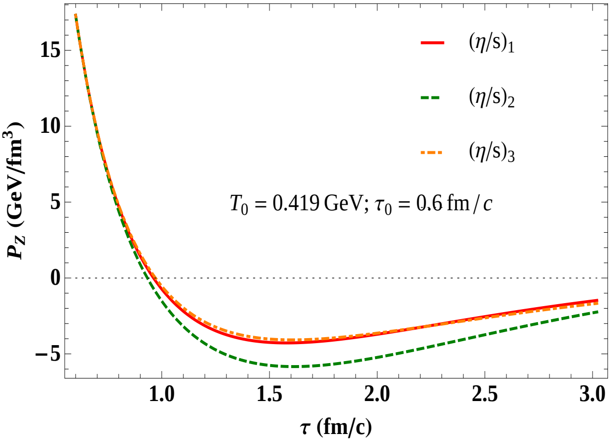
- High value of η/s [U.Heinz *et.al*, PRC, 2011] and cavitation



- $\eta/s_1 = 0.2 + 0.3 * \frac{T - T_c}{T_c}$; $\eta/s_2 = 0.2 + 0.4 * \left[\frac{T - T_c}{T_c}\right]^2$; $\eta/s_3 = 0.2 + 0.3 * \sqrt{\frac{T - T_c}{T_c}}$

Shear viscosity and cavitation at LHC energies

- High value of η/s and cavitation
- Cavitation sets in *very early* in all the cases ~ 1 fm/c



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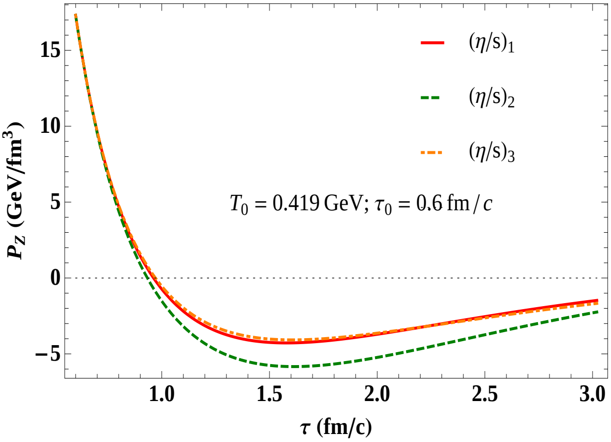

- Shear viscosity induced cavitation at LHC energies [Sreekanth et al (Phys Lett B 2011)]

Chemical non-equilibration in early times

- non-equilibrium effect is represented through *fugacities* - λ_i ($0 < \lambda_i \leq 1$) into distribution function,

$$f_i(p; \lambda_i, T) = \lambda_i \left(e^{\beta \cdot p} \pm \lambda_i \right)^{-1} \approx \lambda_i \left(e^{\beta \cdot p} \pm 1 \right)^{-1}.$$

('Boltzmann factorisation")

- The prominant reactions driving the equilibration are $gg \longleftrightarrow q\bar{q}$ and $gg \longleftrightarrow ggg$ and the correcponding *rate equations* are given by

$$\partial_\mu (n_g u^\mu) = n_g R_{2\to3} [1 - \lambda_g] - 2 n_g R_{g\to q} \left[1 - \frac{\lambda_q \lambda_{\bar{q}}}{\lambda_g^2} \right]$$

$$\partial_\mu (n_q u^\mu) = n_g R_{g\to q} \left[1 - \frac{\lambda_q \lambda_{\bar{q}}}{\lambda_g^2} \right] = \partial_\mu (n_{\bar{q}} u^\mu)$$

with $R_{2\to3} = \frac{1}{2} \langle \sigma_{2\to3} v \rangle n_g \simeq 2.1 \alpha_s^2 T (2\lambda_g - \lambda_g^2)^{1/2}$ and $R_{g\to q(\bar{q})} = \frac{1}{2} \langle \sigma_{g\to q} v \rangle n_g \simeq 0.24 N_f \alpha_s^2 \lambda_g T \, ln(5.5/\lambda_g)$. [Biro et. al PRC 93]

- At equilibrium, $\lambda_i = 1 \implies \partial_\mu (n_i u^\mu) = 0$

Longitudinal dynamics in presence of chemical equilibration

$$\begin{aligned}
\frac{\partial\varepsilon}{\partial\tau} &= -\frac{1}{\tau}(\varepsilon + P - \Phi)\,, \\
\frac{\partial\Phi}{\partial\tau} &= -\frac{\Phi}{\tau_\pi} + \frac{2}{3}\frac{1}{\beta_2\tau} - \frac{\Phi}{2}\left[\frac{1}{\tau} + \frac{T}{\beta_2}\partial_\tau\left(\frac{\beta_2}{T}\right)\right], \\
\frac{\partial n_i}{\partial\tau} + \frac{n_i}{\tau} &= R_i \;\; ; i = g, q
\end{aligned}$$

where $\Phi = \pi^{00} - \pi^{zz}$.

- We consider the EoS of a relativistic gas of massless quarks and gluons:

$$\begin{aligned}
\varepsilon &= 3P = (a_2\lambda_g + b_2[\lambda_q + \lambda_{\bar{q}}])\,T^4; a_2 = \frac{8\pi^2}{15}, b_2 = \frac{7\pi^2 N_f}{40}. \\
n &= (a_1\lambda_g + b_1[\lambda_q + \lambda_{\bar{q}}])\,T^3; a_1 = \frac{16\zeta(3)}{\pi^2}, b_1 = \frac{9\zeta(3)N_f}{2\pi^2}
\end{aligned}$$

Longitudinal dynamics in presence of chemical equilibration

$$
\begin{aligned}
\frac{\dot{T}}{T} + \frac{1}{3\tau} &= -\frac{1}{4}\frac{\dot{\lambda}_g + b_2/a_2\dot{\lambda}_q}{\lambda_g + b_2/a_2\lambda_q} + \frac{\Phi}{4\tau}\frac{1}{(a_2\lambda_g + b_2\lambda_q)\,T^4}\,, \\
\dot{\Phi} + \frac{\Phi}{\tau_\pi} &= \frac{8}{27\tau}\left[a_2\lambda_g + b_2\lambda_q\right]T^4 \\
&\quad -\frac{\Phi}{2}\left[\frac{1}{\tau} - 5\frac{\dot{T}}{T} - \frac{\dot{\lambda}_g + b_2/a_2\dot{\lambda}_q}{\lambda_g + b_2/a_2\lambda_q}\right]\,, \\
\frac{\dot{\lambda_g}}{\lambda_g} + 3\frac{\dot{T}}{T} + \frac{1}{\tau} &= R_3\left(1-\lambda_g\right) - 2R_2\left(1 - \frac{\lambda_q^2}{\lambda_g^2}\right)\,, \\
\frac{\dot{\lambda_q}}{\lambda_q} + 3\frac{\dot{T}}{T} + \frac{1}{\tau} &= R_2\frac{a_1}{b_1}\left(\frac{\lambda_g}{\lambda_q} - \frac{\lambda_q}{\lambda_g}\right)
\end{aligned}
$$

Numerical solving above set of equations, we get evolution profiles: $T(\tau)$, $\lambda_i(\tau)$ and $\Phi(\tau)$

- Initial conditions: T_0=0.570 GeV, $\tau_0 = 0.7$ fm/c, $\lambda_g^0 = 0.08$, $\lambda_q^0 = 0.02$ (HIJING - D. Dutta et al PRC 2009, T. Biro et al PRC 1993)
- $\tau_\pi = 3\frac{\eta/s}{T}$, $T_c = 180$ MeV and hydro equations of the code VISHNU [U Heinz]

Viscosity and chemical equilibration

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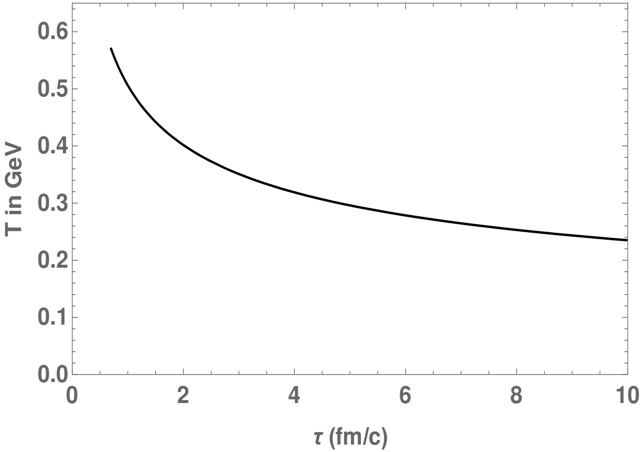
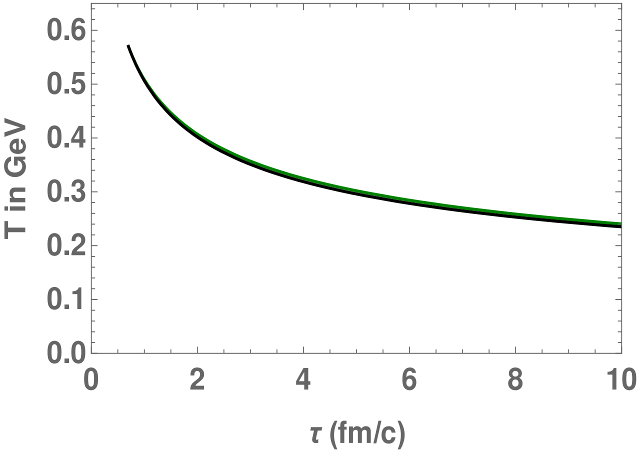
- $\tau_\pi = 3\frac{\eta/s}{T}$, $T_c = 180$ MeV and hydro equations of the code VISHNU [U Heinz]
- Take KSS limit: $\eta/s = 1/4\pi$

- $\eta/s = 0,\ \lambda_i = 1$



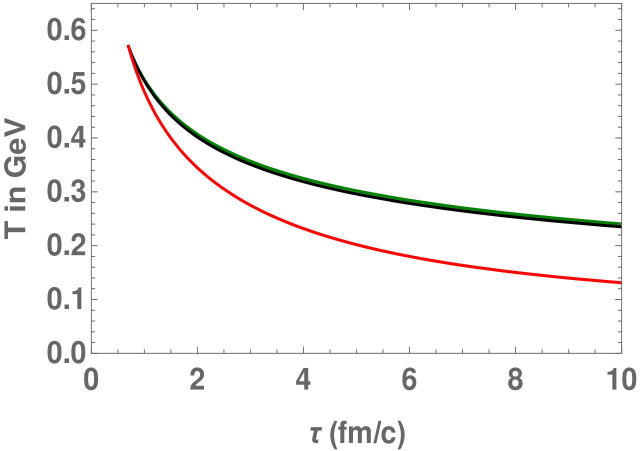
Viscosity and chemical equilibration

- $\eta/s = 1/4\pi$, $\lambda_i = 1$

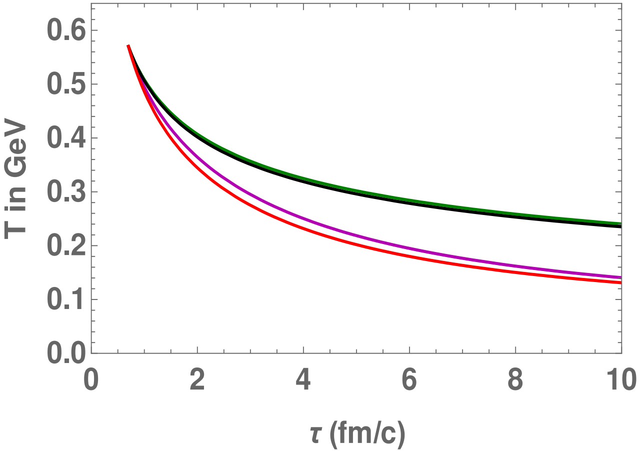


Viscosity and chemical equilibration

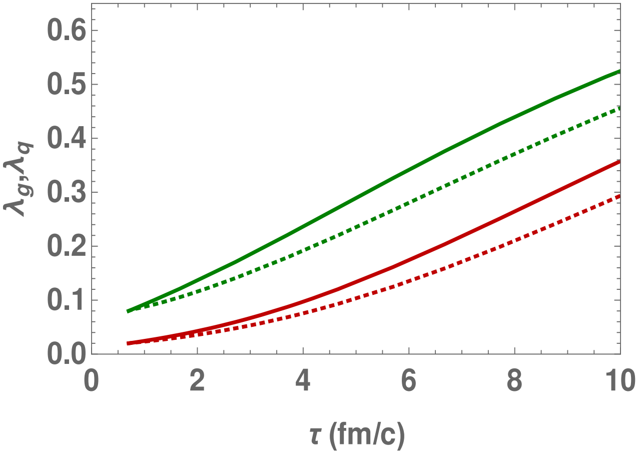
- $\eta/s = 0, \ \lambda_i < 1$



- $\eta/s = 1/4\pi$, $\lambda_i < 1$



Viscosity and chemical equilibration



Chemical equilibration and cavitation at LHC energies

Temperature dependent shear viscosity with *Chemical non-equilibrium* :

- η/s_1

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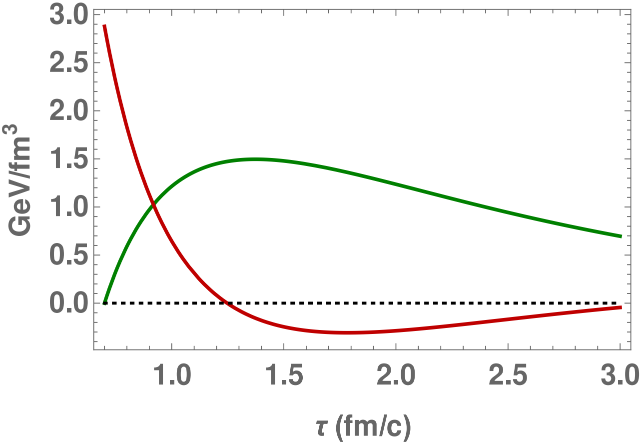
- η/s_1
- *Cavitation condition*: $Pz = [a_2\lambda_g(\tau) + 2b_2\lambda_q(\tau)]\ T(\tau)^4 - 3\Phi(\tau) = 0$

Chemical equilibration and cavitation at LHC energies

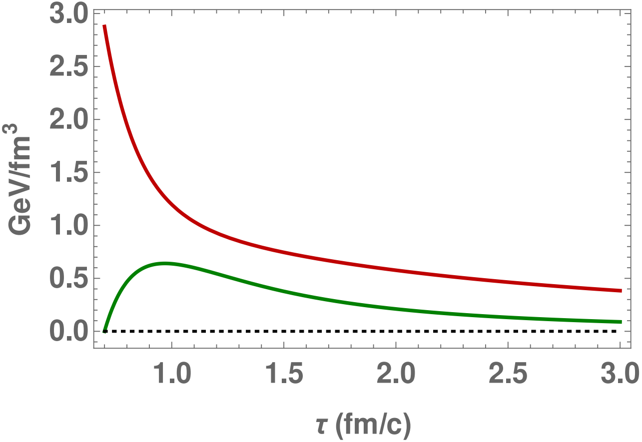
Temperature dependent shear viscosity with *Chemical non-equilibrium* :

- η/s_1
- *Cavitation condition*: $Pz = [a_2\lambda_g(\tau) + 2b_2\lambda_q(\tau)]\ T(\tau)^4 - 3\Phi(\tau) = 0$
- η/s driven cavitation survives and happens early times



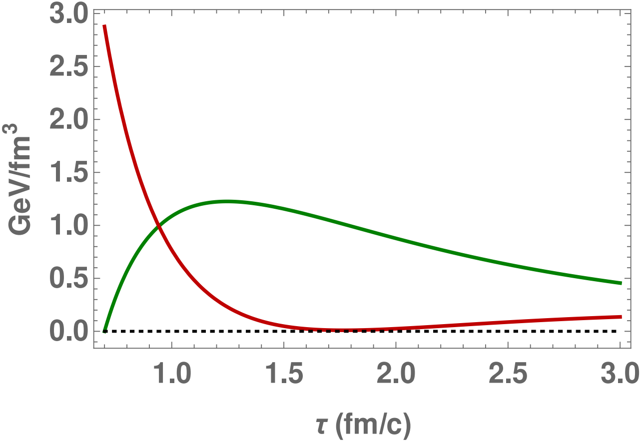
Supposing cavitations doesnt happen ⇒ bound on shear viscosity
(For bulk viscosity, equilibrated system - [P Romatschke et al JHEP 2015])

- $\eta/s = 1/4\pi$



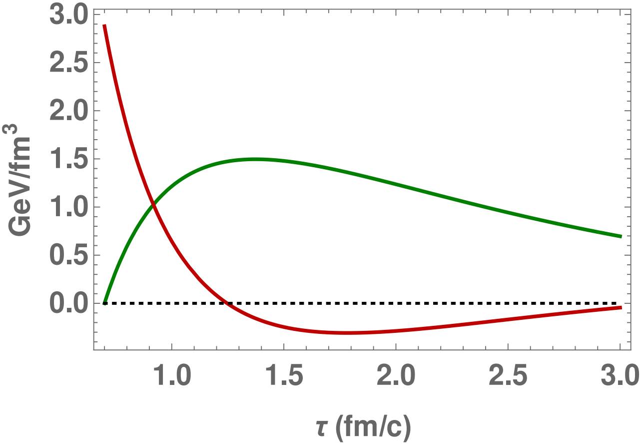
Supposing cavitations doesnt happen ⇒ bound on shear viscosity
(For bulk viscosity, equilibrated system - [P Romatschke et al JHEP 2015])

- $\eta/s = 6 * 1/4\pi$



Supposing cavitations doesnt happen ⇒ bound on shear viscosity
(For bulk viscosity, equilibrated system - [P Romatschke et al JHEP 2015])

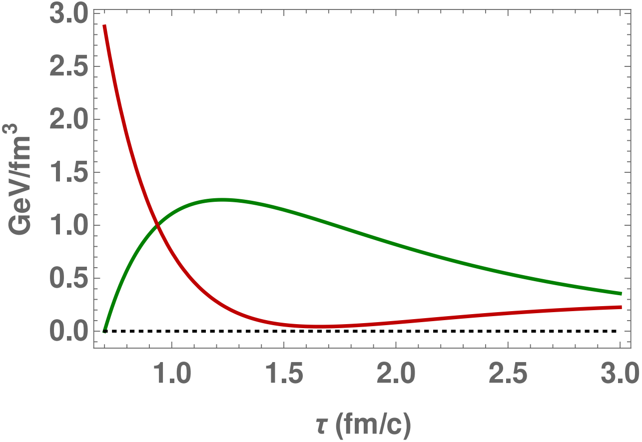
- Temperature dependent $\eta/s = \eta/s_1$



Bound on viscosity

Supposing cavitations doesnt happen $\Rightarrow$ bound on shear viscosity
(For bulk viscosity, equilibrated system - [P Romatschke et al JHEP 2015])

- Temperature dependent $\eta/s = \frac{\eta/s_1}{1.5}$

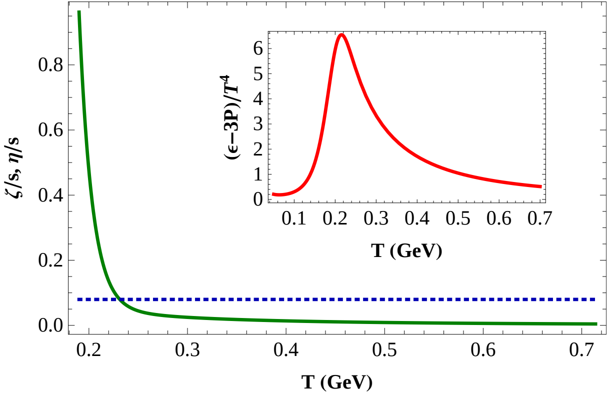


Summary and Conclusions

- We studied the effect of temperature dependent η/s on Chemical equilibration using second order hydrodynamics within Bjorken approximantion
- Study of shear viscosity driven cavitation scenarios under chemical equilibration
- Chemical non-equilibrium is not washing away cavitation scenarios
- Obtained bound on the values of $\eta/s(T)$ by assuming that cavitations doesnt occur
- 2+1 D and 3+1 D Hydro codes should check the longitudinal effective pressure
- 'Boltzmann factorisation' $\Longrightarrow$ relatively simple equations
- used relativistic massless equation of state : $\epsilon = 3P$
- the bulk-viscous contributions to the fluid flow profiles themselves have been neglected for simplicity We studied how this combined effect alters temperature profile the system
- Need to do a meticulous analysis of these effects within all allowed parameter ranges

THANK YOU

Special thanks to Marco Schramm, Christopher Jung, Shakeb Ahmed and others...



- $(\varepsilon - 3P)/T^4\,, \zeta/s$ (and $\eta/s = 1/4\pi$) as functions of temperature T. Around critical temperature ($T_c = .190$ GeV) $\zeta \gg \eta$ and departure of equation of state from ideal case is large.

Ideal Equation of State

In order to understand the effect of *non-ideal* EoS in hydrodynamical evolution and subsequent photon spectra we compare these results with that of an *ideal* EoS ($\varepsilon = 3P$).

- We consider the EoS of a relativistic gas of massless quarks and gluons. The pressure of such a system is given by

$$P = a\ T^4\,;\ a = \left(16 + \frac{21}{2} N_f\right)\ \frac{\pi^2}{90}$$

  where $N_f = 2$ in our calculations.

- Hydrodynamical evolution equations of such an EoS within ideal (without viscous effects) Bjorken flow can be solved analytically and the temperature dependence is given by

$$T = T_0\ \left(\frac{\tau_0}{\tau}\right)^{1/3},$$

  where τ_0 and T_0 are the initial time and temperature.

- effect of bulk viscosity can be neglected in the relativistic limit when the equation of state $3P = \varepsilon$ is obeyed