On DIS in the Color Dipole Picture: Color Transparency and Saturation

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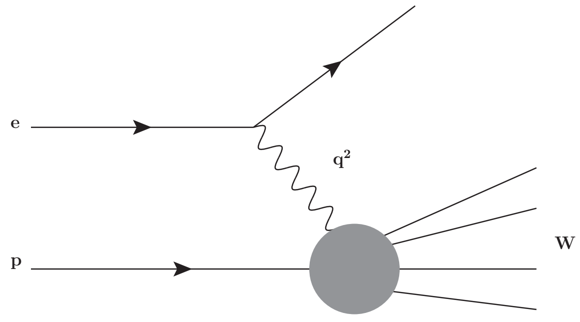
1. Introduction

Deep inelastic scattering (DIS), HERA 1992 to 2007:



DIS at low values of

$$x \equiv x_{bj} \simeq \frac{Q^2}{W^2}, \text{ where}$$
$$5 \cdot 10^{-4} \leq x \leq 10^{-1}$$
$$0 \leq Q^2 \leq 100 GeV^2$$

$$\begin{aligned} Q^2 &\equiv -q^2 > 0, \\ x_{bj} &= \frac{Q^2}{W^2 + Q^2 + M_p^2} \cong \frac{Q^2}{W^2}. \end{aligned}$$

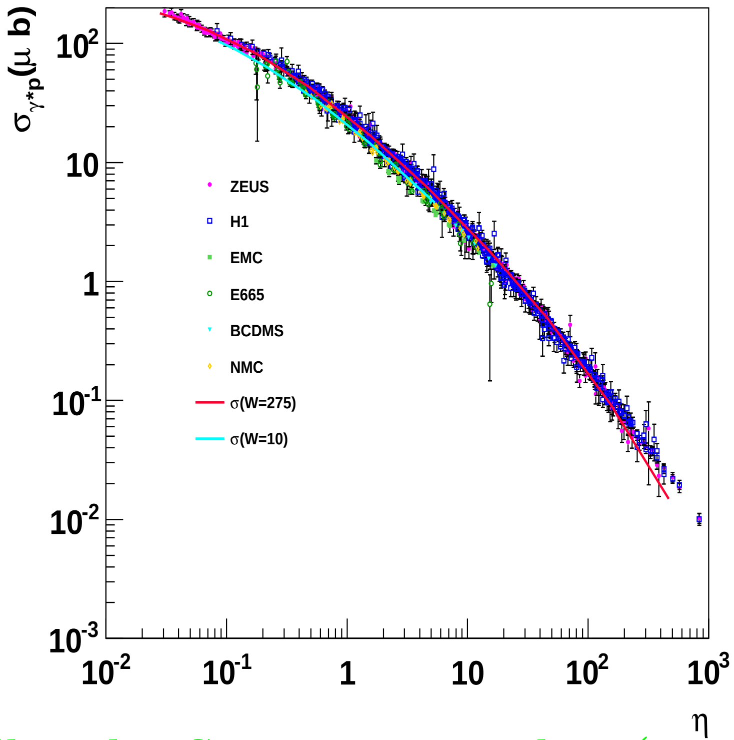
$$\begin{aligned} \sigma_{\gamma^* p}(W^2, Q^2) &= \sigma_{\gamma_L^* p}(W^2, Q^2) + \sigma_{\gamma_T^* p}(W^2, Q^2) \\ &\equiv \sigma_{\gamma_T^* p}(W^2, Q^2)(1 + R(W^2, Q^2)), \end{aligned}$$

$$\begin{aligned} F_2(x, Q^2) &\cong \frac{Q^2}{4\pi^2 \alpha} \sigma_{\gamma^* p}(W \cong \frac{Q^2}{x}, Q^2); \\ F_L &= \frac{R}{1+R} F_2. \end{aligned}$$

Low-x Scaling

Empirically : $\eta(W^2,Q^2) \equiv \frac{Q^2+m_0^2}{\Lambda_{sat}^2(W^2)}$,

$\Lambda_{sat}^2(W^2) \sim (W^2)^{C_2}$

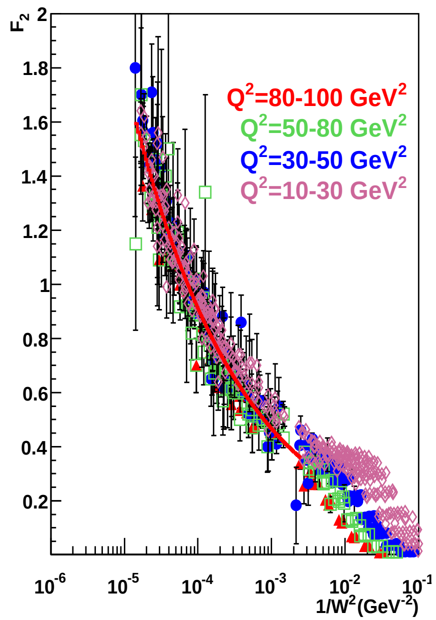


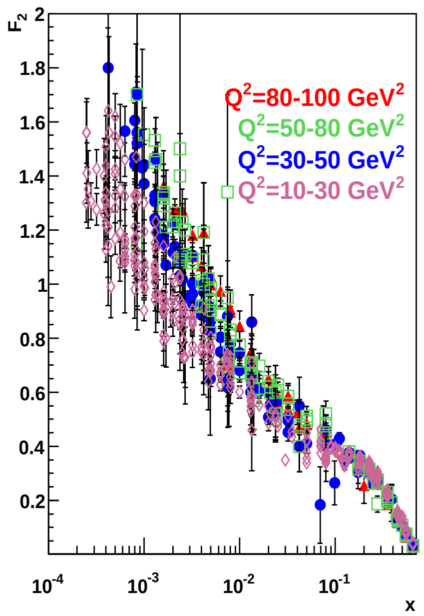
Schildknecht, Surrow, Tentyukov (2000)

$$\sigma_{\gamma^*p}(W^2,Q^2) = \sigma_{\gamma^*p}(\eta(W^2,Q^2))$$
$$\sim \sigma^{(\infty)} \begin{cases} ln\frac{1}{\eta(W^2,Q^2)} & , \quad \text{for } \eta(W^2,Q^2) \ll 1 \\ \frac{1}{\eta(W^2,Q^2)} & , \quad \text{for } \eta(W^2,Q^2) \gg 1 \end{cases}$$

The W-dependence

$$\begin{aligned} F_2(x,Q^2) &\cong \frac{Q^2}{4\pi^2\alpha}\left(\sigma_{\gamma_L^* p}(W^2,Q^2)+\sigma_{\gamma_T^* p}(W^2,Q^2)\right) \\ &= \frac{\sum_q Q_q^2}{4\pi^2}\int dz \int d\vec{l}_\perp^{\,2}\vec{l}_\perp^{\,2}\tilde{\sigma}(\vec{l}_\perp^{\,2}, z(1-z), W^2)(1+2\rho) \\ &= F_2(W^2) \text{ for } x<0.1. \end{aligned}$$





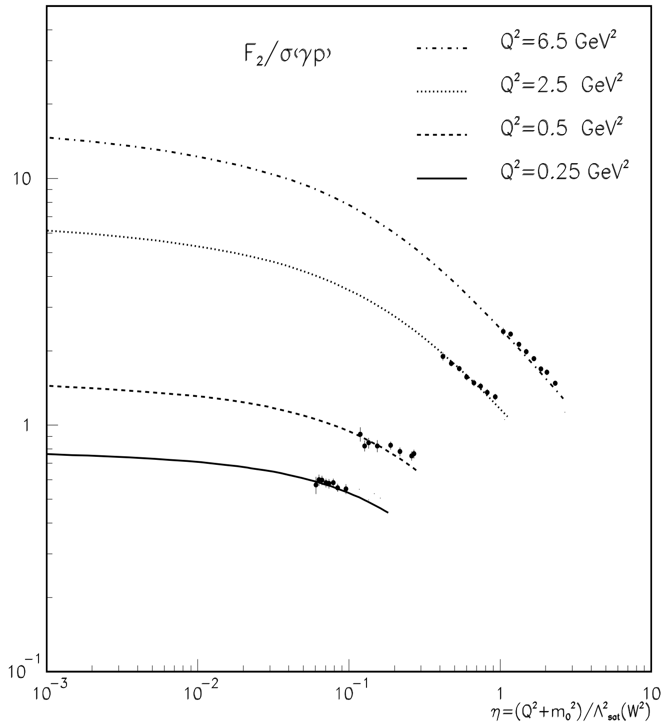
Prabhdeep Kaur (2010)

The limit of $\eta(W^2, Q^2) \to 0$, or $W^2 \to \infty$ at Q^2 fixed

$$\lim_{\substack{W^2\to\infty \\ Q^2 \text{ fixed}}} \frac{\sigma_{\gamma^* p}(\eta(W^2,Q^2))}{\sigma_{\gamma^* p}(\eta(W^2,Q^2=0))} = \lim_{\substack{W^2\to\infty \\ Q^2 \text{ fixed}}} \frac{\ln\left(\frac{\Lambda^2_{sat}(W^2)}{m_0^2}\frac{m_0^2}{(Q^2+m_0^2)}\right)}{\ln\frac{\Lambda^2_{sat}(W^2)}{m_0^2}} = 1 + \lim_{\substack{W^2\to\infty \\ Q^2 \text{ fixed}}} \frac{\ln\frac{m_0^2}{Q^2+m_0^2}}{\ln\frac{\Lambda^2_{sat}(W^2)}{m_0^2}} = 1.$$

$$\sigma_{\gamma^* p}\left(\eta(W^2,Q^2=0)\right) = \sigma_{\gamma p}(W^2)$$

D. Schildknecht, DIS 2001 (Bologna)



$$\lim_{\substack{W^2\to\infty \\ Q^2 \text{fixed}}} \frac{F_2(x\cong Q^2/W^2, Q^2)}{\sigma_{\gamma p}(W^2)} = \frac{Q^2}{4\pi^2\alpha}.$$

$Q^2[GeV^2]$	$W^2[GeV^2]$	$\frac{\sigma_{\gamma^* p}(\eta(W^2,Q^2))}{\sigma_{\gamma p}(W^2)}$
1.5	2.5×10^7	**0.5**
	1.26×10^{11}	**0.63**

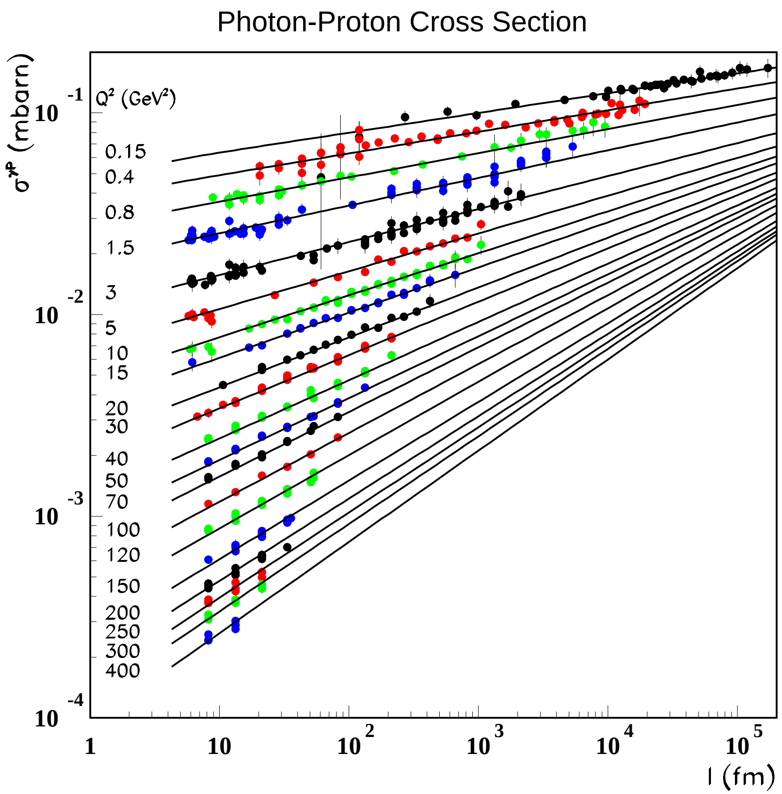
Observation by Caldwell

$$\sigma_{\gamma^* p}(W^2, Q^2) = \sigma_0(Q^2) \left(\frac{1}{2}\frac{W^2}{Q^2}\right)^{\lambda_{eff}(Q^2)} \equiv \sigma_0(Q^2) l^{\lambda_{eff}(Q^2)}$$

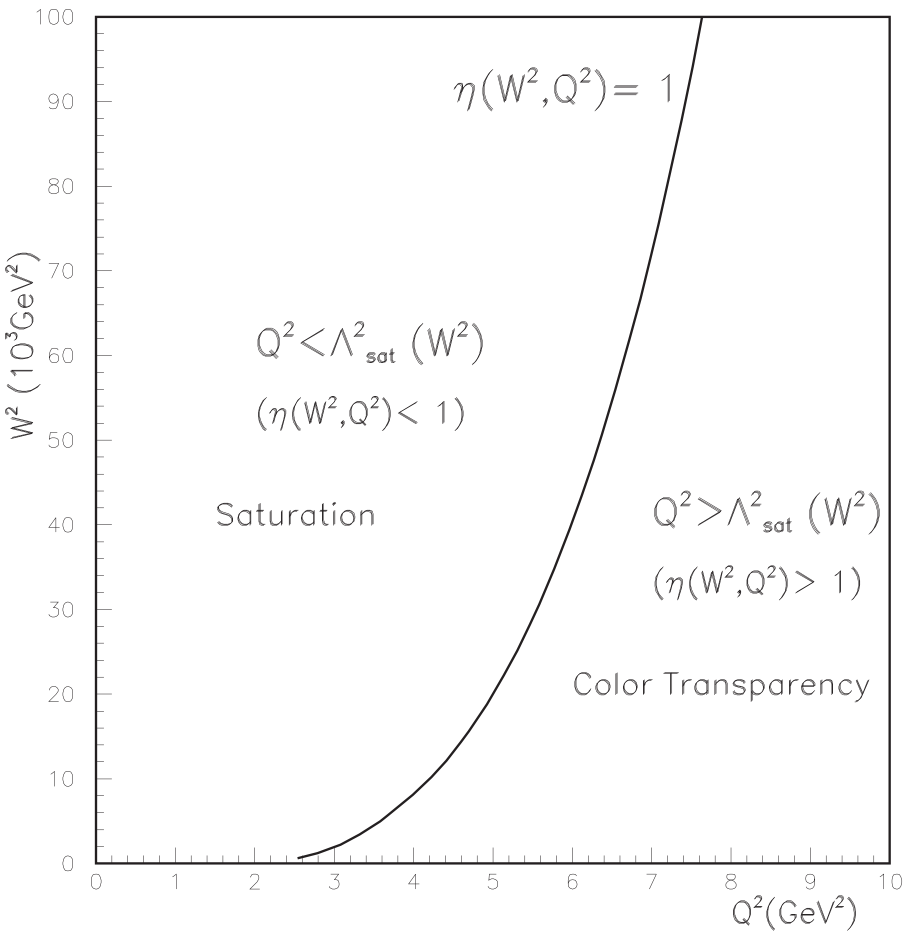
A. Caldwell (2008)

Q^2-independent limit at approximately

$$W^2 \simeq 10^9 Q^2.$$



The (Q^2, W^2) plane



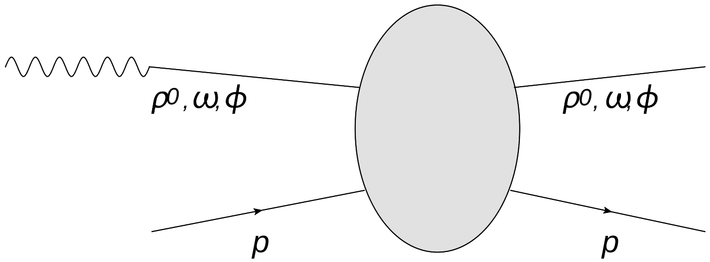
The experimentally observed behavior

follows from the Color Dipole Picture (CDP)

of deep-inelastic scattering for $x \stackrel{\sim}{<} 0.1$.

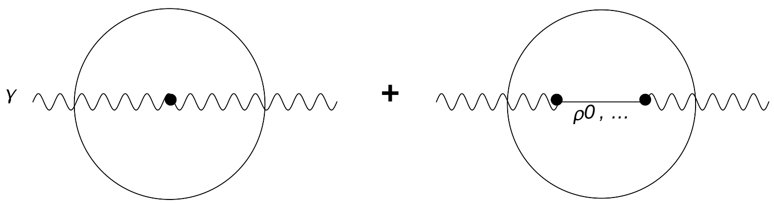
2. Photon-hadron interactions: Late 1960's, early 1970's.

1960's Vector Meson Dominance



J.J. Sakurai (1960, ...)

Shadowing in γA interactions



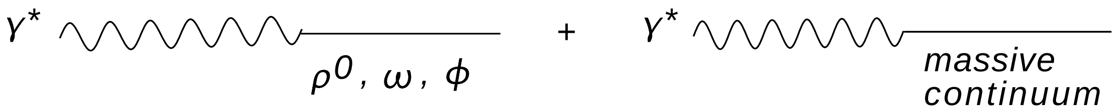
Leo Stodolsky (1967)

1969 DIS SLAC-MIT Collaboration

Bjorken scaling,

Feynman,parton model

(1972)



Volume 40B, number 1 PHYSICS LETTERS 12 June 1972

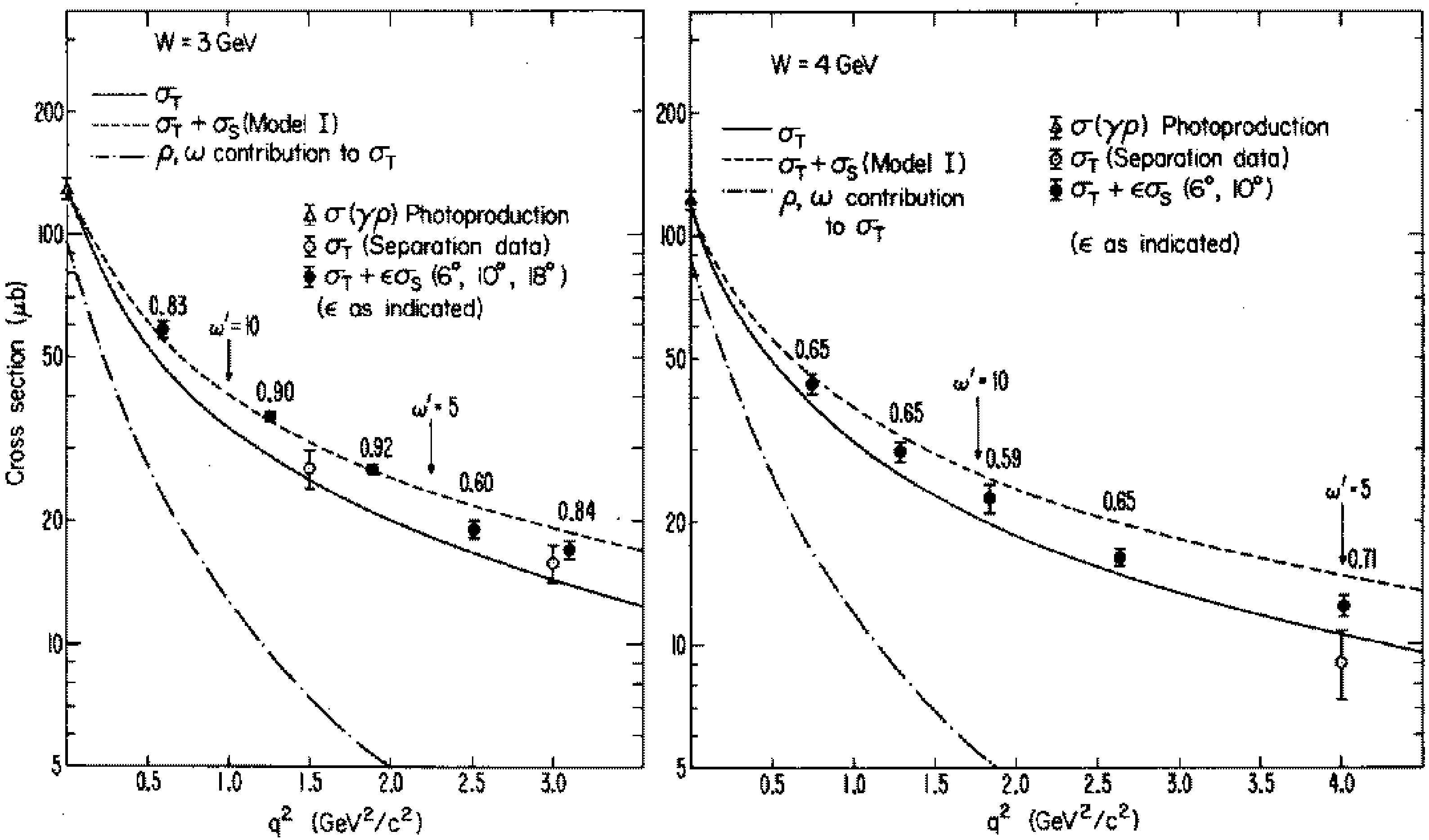
GENERALIZED VECTOR DOMINANCE AND INELASTIC ELECTRON-PROTON SCATTERING *

J. J. SAKURAI and D. SCHILDKNECHT **

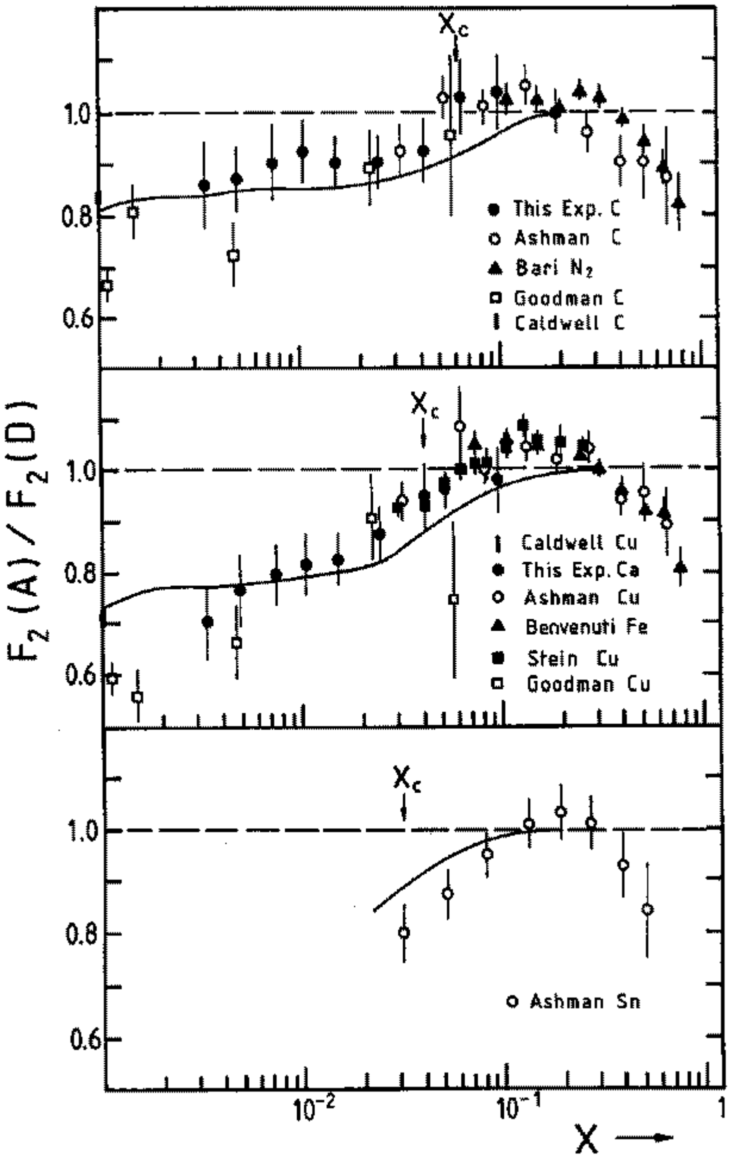
Department of Physics, University of California, Los Angeles, USA

Received 30 March 1972

We propose a model of inelastic electron-proton scattering which takes into account the coupling of the photon to higher-mass vector states. Both the virtual photon-proton cross section σ_T (predicted with essentially no adjustable parameters) and the q^2 dependence of R are in exceedingly good agreement with the SLAC-MIT data in the diffraction region.


W = 3 GeV
σT
σT + σS (Model I)
ρ, ω contribution to σT
σ(γp) Photoproduction
σT (Separation data)
σT + εσS (6°, 10°, 18°)
(ε as indicated)
0.83
ω′ = 10
0.90
ω′ = 5
0.92
0.60
0.84
Cross section (μb)
q² (GeV²/c²)
W = 4 GeV
σT
σT + σS (Model I)
ρ, ω contribution to σT
σ(γp) Photoproduction
σT (Separation data)
σT + εσS (6°, 10°)
(ε as indicated)
0.65
0.65
ω′ = 10
0.59
0.65
ω′ = 5
0.71
q² (GeV²/c²)

1989 Shadowing EMC Collaboration



D. Schildknecht (1973)

C. Bilchak and D. Schildknecht (1989)

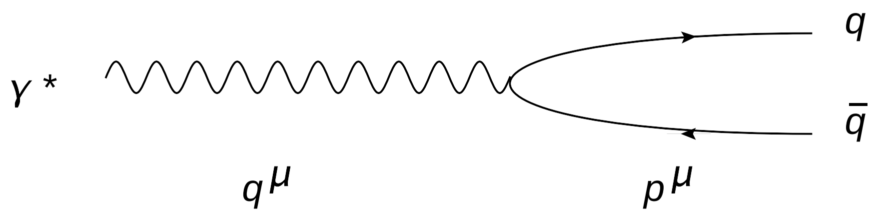
1994 HERA

DIS for $x_{bj} \ll 0.1$

High-mass diffractive production

("rap-gap"events).

Life time of hadronic fluctuations $\gamma^* \to \rho^0$, $\gamma^* \to q\bar{q}$



i) Four-momentum-conserving transition to virtual state, e.g. ρ^0, $q\bar{q}$ state

$$p^\mu = q^\mu,$$

$$p^2 = q^2 < 0, \qquad p^2 \neq M^2_{q\bar{q}},$$

Propagator: $\frac{1}{-q^2 + M^2{}_{q\bar{q}}} = \frac{1}{Q^2 + M^2_{q\bar{q}}}$.

ii) Equivalently: Three-momentum-conserving transition to on-shell $q\bar{q}$ state

$\vec{p} = \vec{q};$

$p^2 = M^2_{q\bar{q}}; \quad q^2 = (q^0)^2 - (\vec{q})^2 < 0; \quad Q^2 = -q^2;$

$$\Delta E = p^0 - q^0 = \frac{M^2_{q\bar{q}} + Q^2}{p^0 + q^0}$$

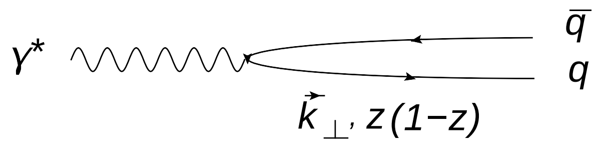
$$\cong \frac{M^2_{q\bar{q}} + Q^2}{2q^0}.$$

$$\tau = \frac{1}{\Delta E} = \frac{2M_p \nu}{Q^2 + M^2_{q\bar{q}}} \frac{1}{M_p} \gg \frac{1}{M_p}.$$

$(q\bar{q})p$ interaction cross section dependent on W (Q^2 and x dependence excluded).

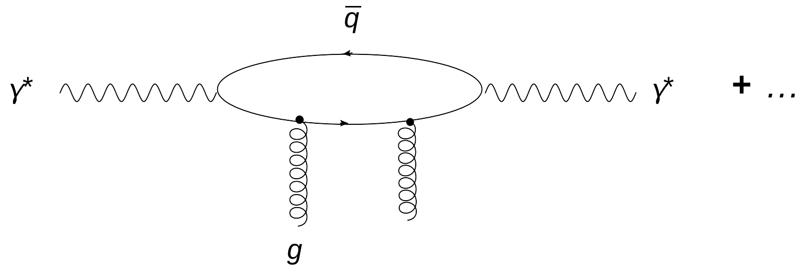
Modern picture of low-x DIS:

i) $q\bar{q}$ internal structure



Nikolaev, Zakharov (1991)

ii) $q\bar{q}$-dipole interaction



Low (1975)

Nussinov (1975)

Invariant mass of $q\bar{q}$ state

$$k^2 = k'^2 = m_q^2 = 0$$

$$M_{q\bar{q}}^2 = (k + k')^2 = (2k^0_{C.M.})^2$$
$$= 4\frac{\vec{k}_\perp{}^2}{\sin^2\vartheta_{C.M.}}$$

In terms of z:

$$k^3 = zq^3;$$

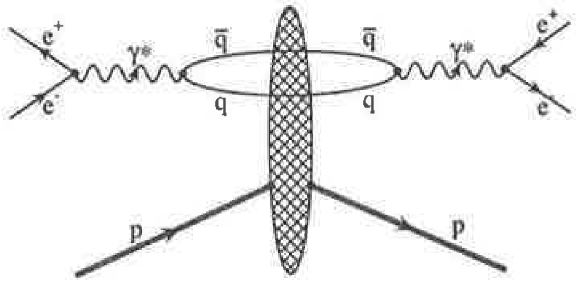
$$k'^3 = (1 - z)q^3;$$

$$M_{q\bar{q}}^2 = \frac{\vec{k}_\perp{}^2}{z(1-z)};$$

$$\sin^2\vartheta_{C.M.} = 4z(1 - z)$$

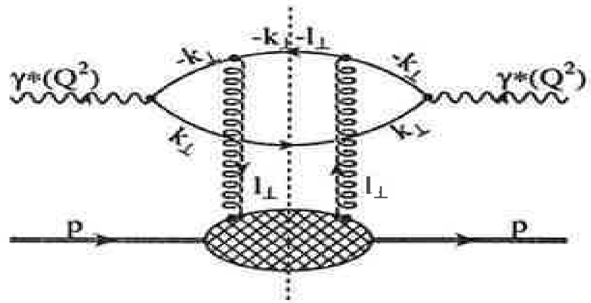
3. The Color Dipole Picture (CDP).

The longitudinal and the transverse photoabsorption cross section

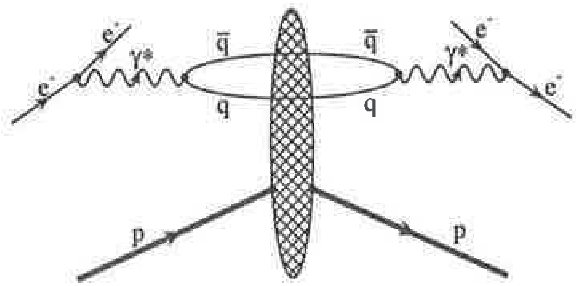


(a)

channel 1:

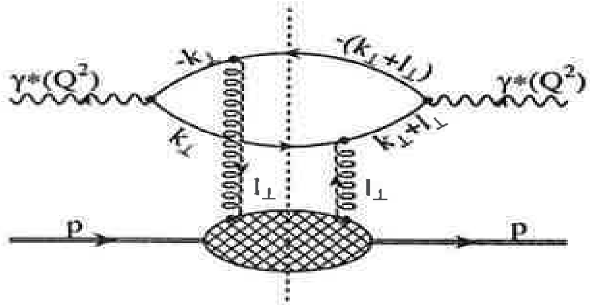


(a)

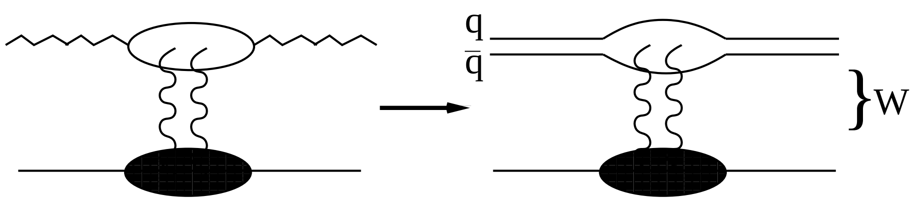


(b)

channel 2:



(b)



$\gamma^* p \rightarrow \gamma^* p \longrightarrow (q\bar{q})p \rightarrow (q\bar{q})p$

A) $\boldsymbol{\sigma_{\gamma^*_{L,T}}(W^2, Q^2) = \int dz \int d^2\vec{r}_\perp |\psi_{L,T}(\vec{r}_\perp, z(1-z), Q^2)|^2 \;\; \sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2)}$

Remarks:

i) $\boldsymbol{|\psi_{L,T}(\vec{r}_\perp, z(1-z), Q^2)|}$**:** **Probability for** $\gamma^*_{L,T} \to q\bar{q}$ **fluctuation (QED)**

Note: $\vec{r}^{\,2}_\perp \sim \frac{1}{Q^2}$

ii) $\boldsymbol{\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2)}$**:** $\boldsymbol{(q\bar{q})p}$ **cross section dependent on** $\boldsymbol{W^2}$ **(not on** $\boldsymbol{x \equiv \frac{Q^2}{W^2}}$**)**

B) Gauge-invariant two-gluon coupling:

$$\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2) = \int d^2\vec{l}_\perp \tilde{\sigma}(\vec{l}_\perp^{\,2}, z(1-z), W^2)\left(1 - e^{-i\,\vec{l}_\perp \cdot \vec{r}_\perp}\right)$$

Nikolaev, Zakharov (1991)

Cvetic, Schildknecht, Shoshi(2000)

Assume $\vec{l}_\perp^2 \leq \vec{l}_{\perp \mathrm{Max}}^2(W^2)$.

For fixed $|\vec{r}_\perp|$:

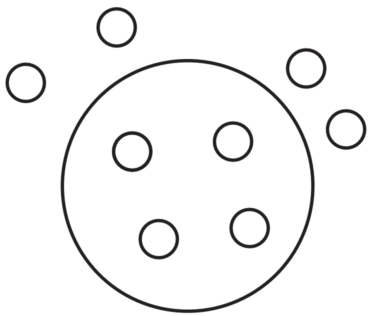
a) $\vec{l}_{\perp \mathrm{Max}}^2(W^2)\vec{r}_\perp^2 \ll 1$

$\sigma_{(q\bar{q})p} \sim \vec{r}_\perp^2 \longrightarrow$ "color transparency", $\sigma_{\gamma^* p} \sim \frac{1}{\eta(W^2,Q^2)} \sim \frac{\Lambda_{\mathrm{sat}}^2(W^2)}{Q^2}$.

b) $\vec{l}_{\perp \mathrm{Max}}^2(W^2)\vec{r}_\perp^2 \gg 1$

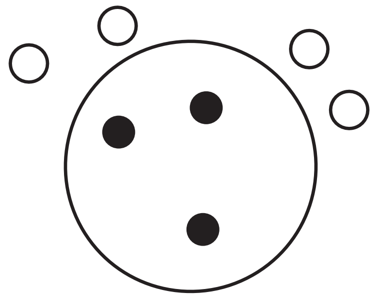
$\sigma_{(q\bar{q})p} \sim \sigma^{(\infty)}(W^2) \longrightarrow$ "saturation"$\sigma_{\gamma^* p} \sim \ln \frac{1}{\eta(W^2,Q^2)}$;

Color gauge invariant $q\bar{q}$ (dipole) interaction with gluon field in the nucleon implies low-x scaling.



Color Transparency

$$\eta(W^2, Q^2) \simeq \frac{Q^2}{\Lambda^2_{\rm sat}(W^2)} \gg 1$$



Saturation

hadron-like cross section

$$\eta(W^2, Q^2) \lesssim 1$$

The longitudinal-to-transverse ratio

$(q\bar{q})^{J=1}_{L,T}$ states : $\gamma^*_{L,T} \longrightarrow (q\bar{q})^{J=1}_{L,T}$

$$\sigma_{\gamma^*_{L,T}p}(W^2,Q^2) = \alpha \sum_q Q_q^2 \frac{1}{Q^2}\frac{1}{6} \begin{cases} \int d\vec{l}_\perp^{\prime 2} \vec{l}_\perp^{\prime 2} \bar{\sigma}_{(q\bar{q})^{J=1}_L p}(\vec{l}_\perp^{\prime 2}, W^2), \\ 2\int d\vec{l}_\perp^{\prime 2} \vec{l}_\perp^{\prime 2} \bar{\sigma}_{(q\bar{q})^{J=1}_T p}(\vec{l}_\perp^{\prime 2}, W^2). \end{cases} \qquad (\text{for } \eta \gg 1)$$

$$\vec{l}^2 = z(1-z)\vec{l}_\perp^2$$

$$\rho_W = \frac{\int d\vec{l}_\perp^{\prime 2} \vec{l}_\perp^{\prime 2} \bar{\sigma}_{(q\bar{q})^{J=1}_T p}(\vec{l}_\perp^{\prime 2}, W^2)}{\int d\vec{l}_\perp^{\prime 2} \vec{l}_\perp^{\prime 2} \bar{\sigma}_{(q\bar{q})^{J=1}_L p}(\vec{l}_\perp^{\prime 2}, W^2)}. \equiv \rho$$

$$R = \frac{1}{2\rho}.$$

Magnitude of ρ

Average transverse momentum of $q(\bar{q})$:

$$\langle \vec{l}_\perp^{\,2} \rangle_{L,T}^{\vec{l}_\perp^{\prime 2} = const} = \vec{l}_\perp^{\prime 2} \begin{cases} 6 \int dz z^2 (1-z)^2 = \frac{4}{20} \vec{l}_\perp^{\prime 2}, & (L) \\ \frac{3}{2} \int dz \; z(1-z)(1-2z(1-z)) = \frac{3}{20} \vec{l}_\perp^{\prime 2}, & (T) \end{cases}.$$

Assume that ρ is determined by average transverse size of $L(T)$. Uncertainty principle:

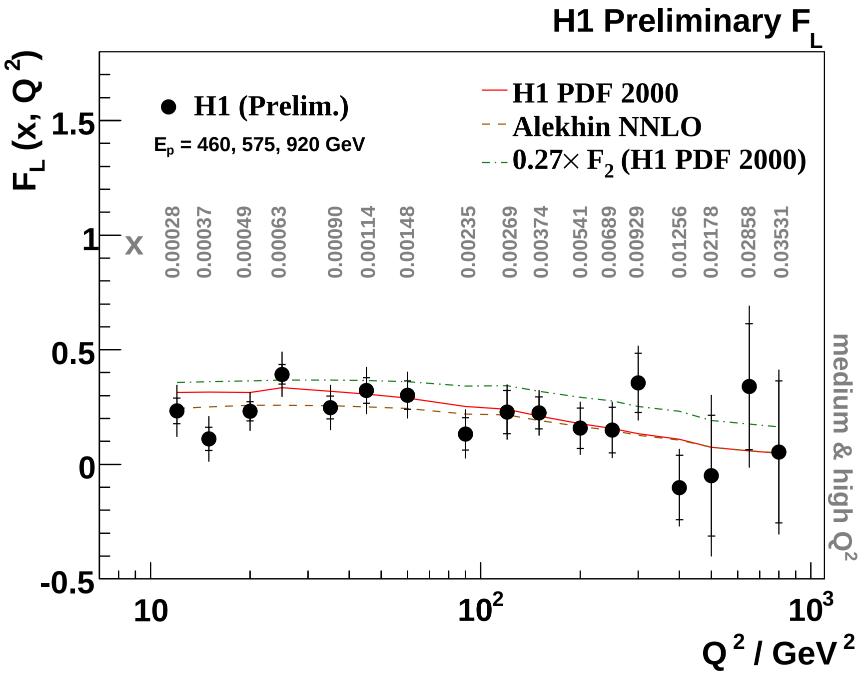
$$\rho = \frac{\langle r_\perp^2 \rangle_T}{\langle \vec{r}_\perp^{\,2} \rangle_L} = \frac{\langle \vec{l}_\perp^{\,2} \rangle_L}{\langle \vec{l}_\perp^{\,2} \rangle_T} = \frac{4}{3}.$$

Kuroda, Schildknecht (2008)

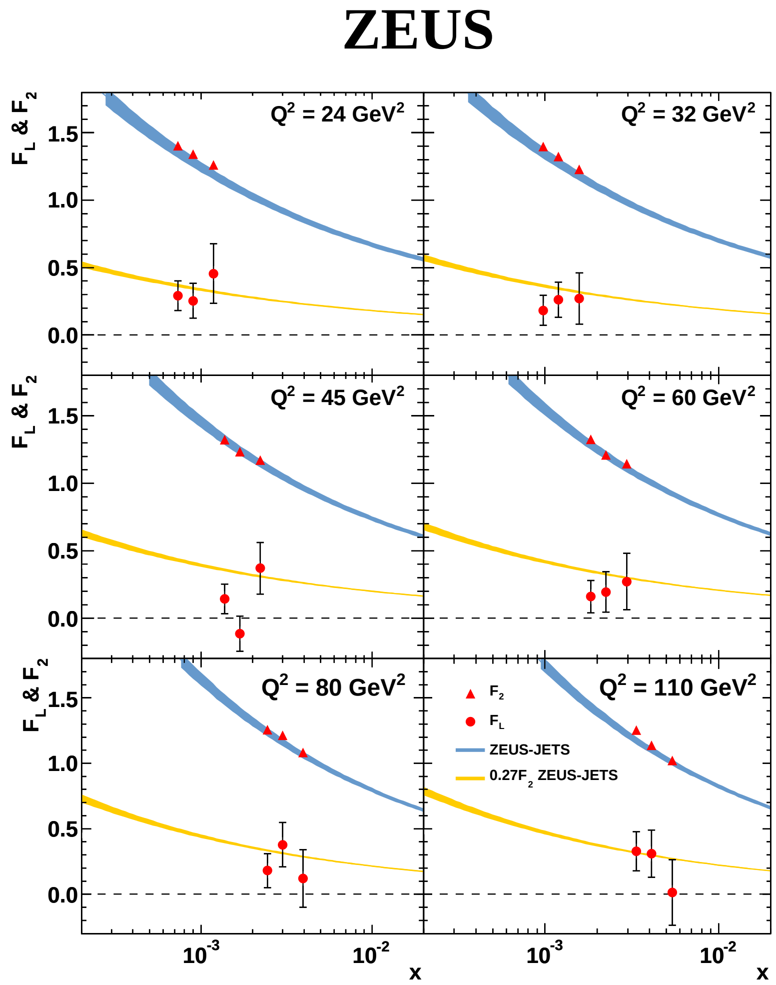
$$R = \frac{1}{2\rho} = \begin{cases} 0.5 & \text{for } \rho = 1, \\ \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8} = 0.375 & \text{uncertainty principle} \\ \frac{1}{4}, & \text{for } \rho = 2. \end{cases}$$

$$F_L = \frac{R}{1+R} = \begin{cases} 0.33 \\ 0.27 \\ 0.20 \end{cases}$$

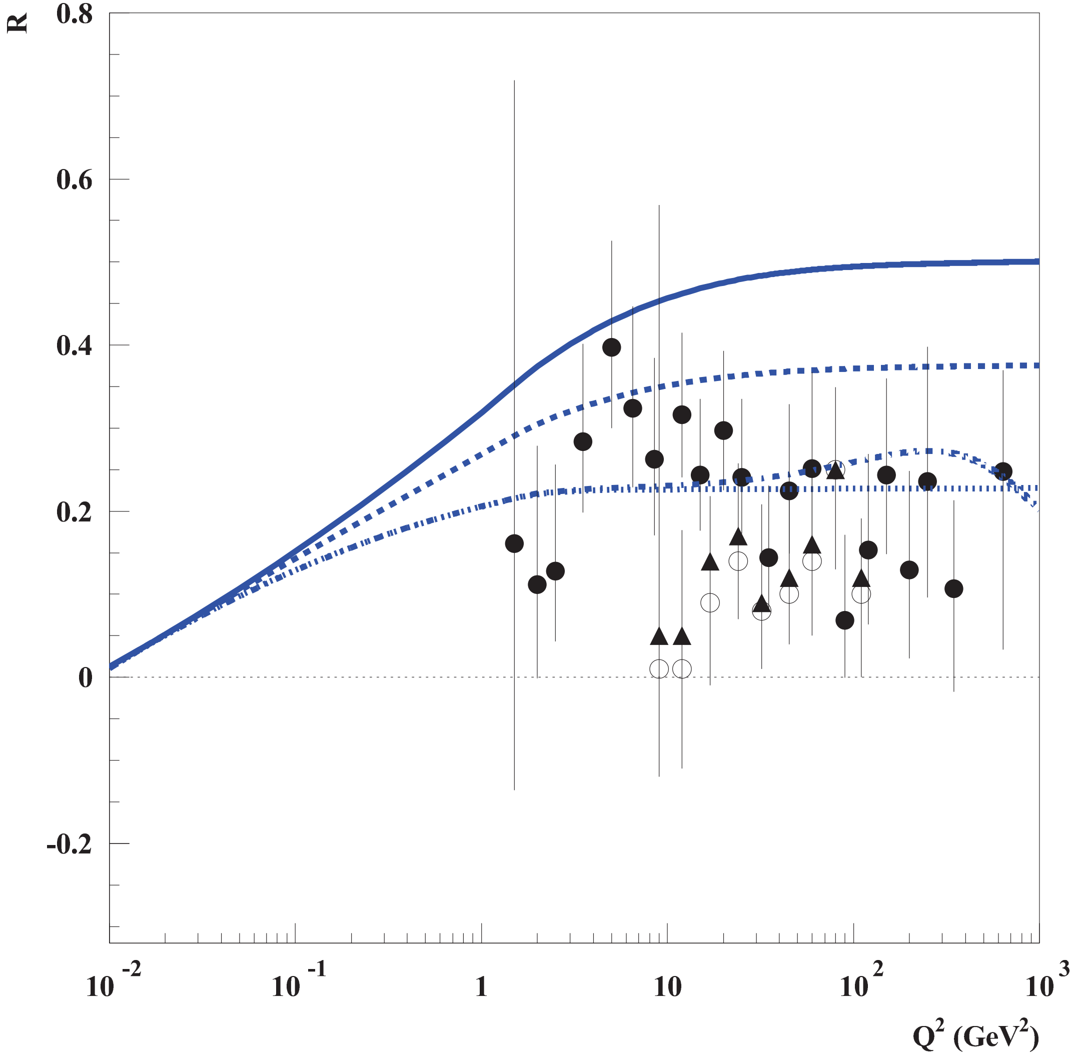
$$F_L = 0.27 F_2.$$



H1 Preliminary F_L
H1 (Prelim.)
E_p = 460, 575, 920 GeV
H1 PDF 2000
Alekhin NNLO
0.27× F_2 (H1 PDF 2000)
x
0.00028 0.00037 0.00049 0.00063 0.00090 0.00114 0.00148 0.00235 0.00269 0.00374 0.00541 0.00689 0.00929 0.01256 0.02178 0.02858 0.03531
medium & high Q²
F_L (x, Q²)
Q² / GeV²



ZEUS
Q² = 24 GeV²
Q² = 32 GeV²
Q² = 45 GeV²
Q² = 60 GeV²
Q² = 80 GeV²
Q² = 110 GeV²
F_L & F_2
F_2
F_L
ZEUS-JETS
0.27F_2 ZEUS-JETS
x



Experiment: $\boldsymbol{R(Q^2)|_{W \simeq 200\ \mathrm{GeV}}}$

H1 (2013)

ZEUS (2014)

Theory (CDP)

$\longleftarrow \quad \rho = 1$

$\longleftarrow \quad \rho = \frac{4}{3}$

$\longleftarrow \quad \rho = 2$

4. Ansatz for the Dipole Cross Section

Model-independently:

$$\sigma_{\gamma^* p} \sim \begin{cases} ln \frac{1}{\eta(W^2,Q^2)} & , \quad \eta(W^2,Q^2) \ll 1 \\ \frac{1}{\eta(W^2,Q^2)} & , \quad \eta(W^2,Q^2) \gg 1 \end{cases}$$

$$R = \begin{cases} 0 & \text{for } Q^2 = 0, \left(\eta = \frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}\right), \\ \frac{1}{2\rho} & \text{for } \eta(W^2,Q^2) \gg 1. \end{cases}$$

Interpolation between $\eta(W^2,Q^2) < 1$ and $\eta(W^2,Q^2) > 1$. by explicit ansatz for the dipole cross section.

Simple ansatz containing $\rho = 1$, $\left(R = \frac{1}{2\rho} = \frac{1}{2}\right)$:

Cvetic, Schildknecht, Surrow, Tentyukov (2001)

$$\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2) = \sigma^{(\infty)}(W^2)\left(1 - J_0\left(r_\perp\sqrt{z(1-z)}\Lambda_{sat}(W^2)\right)\right)$$

$$\begin{aligned}\sigma_{\gamma^* p}(W^2, Q^2) &= \sigma_{\gamma^* p}(\eta(W^2, Q^2)) + O\left(\frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}\right) = \\ &= \frac{\alpha R_{e^+e^-}}{3\pi}\sigma^{(\infty)}(W^2) I_0(\eta) + O\left(\frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}\right), \qquad R_{e^+e^-} = 3\sum_q Q_q^2.\end{aligned}$$

$$\begin{aligned}I_0(\eta(W^2, Q^2)) &= \frac{1}{\sqrt{1+4\eta(W^2,Q^2)}}\ln\frac{\sqrt{1+4\eta(W^2,Q^2)}+1}{\sqrt{1+4\eta(W^2,Q^2)}-1} \cong \\ &\cong \begin{cases}\ln\frac{1}{\eta(W^2,Q^2)} + O(\eta\ln\eta), & \text{for } \eta(W^2,Q^2) \to \frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}, \\ \frac{1}{2\eta(W^2,Q^2)} + O\left(\frac{1}{\eta^2}\right), & \text{for } \eta(W^2,Q^2) \to \infty,\end{cases}\end{aligned}$$

$\sigma^{(\infty)}(W^2)$ to be expressed in terms of $\sigma_{\gamma p}(W^2)$.

Refinements:

$$i)\rho = 1;$$

$$ii)\ m^2_{q\bar{q}} \leq m^2_1(W^2) = \xi \Lambda^2_{\rm sat}(W^2);$$

Kuroda, Schildknecht (2011)

Kuroda, Schildknecht, Surrow in preparation.

$$\sigma_{\gamma^* p}(W^2, Q^2) = \frac{\sigma_{\gamma p}(W^2)}{\lim\limits_{\eta \to \mu(W^2)} I_T^{(1)}\left(\frac{\eta}{\rho}, \frac{\mu}{\rho}\right)} \left(I_T^{(1)}\left(\frac{\eta}{\rho}, \frac{\mu}{\rho}\right) G_T(u) + I_L^{(1)}(\eta, \mu) G_L(u) \right)$$

$$G_{L,T}(u) = \frac{1}{2(1+u)^3} \begin{cases} 2u^3 + 6u^2, & (L), \\ 2u^3 + 3u^2 + 3u, & (T). \end{cases}$$

$$u = \frac{\xi}{\eta(W^2, Q^2)}; \qquad \mu(W^2) = \frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}.$$

$$
\boldsymbol{I_L^{(1)}(\eta,\mu) = \frac{\eta-\mu}{\eta}}
$$

$$
\boldsymbol{\times\left(1-\frac{\eta}{\sqrt{1+4(\eta-\mu)}}\ln\frac{\eta(1+\sqrt{1+4(\eta-\mu)})}{4\mu-1-3\eta+\sqrt{(1+4(\eta-\mu))((1+\eta)^2-4\mu)}}\right)},
$$

$$
\boldsymbol{I_T^{(1)}(\eta,\mu) = \frac{1}{2}\ln\frac{\eta-1+\sqrt{(1+\eta)^2-4\mu}}{2\eta} - \frac{\eta-\mu}{\eta} + \frac{1+2(\eta-\mu)}{2\sqrt{1+4(\eta-\mu)}}}
$$

$$
\boldsymbol{\times\ln\frac{\eta(1+\sqrt{1+4(\eta-\mu)})}{4\mu-1-3\eta+\sqrt{(1+4(\eta-\mu))((1+\eta)^2-4\mu)}}}.
$$

Comparison with experiment:

Kuroda, Schildknecht (2011)

- $\sigma_{\gamma p}(W^2)$ from Particle Data Group parameterization
- $\Lambda^2_{sat}(W^2) = C_1 \left(\frac{W^2}{W_0^2} + 1\right)^{C_2} \cong$ const $\left(\frac{W^2}{1GeV^2}\right)^{C_2}$

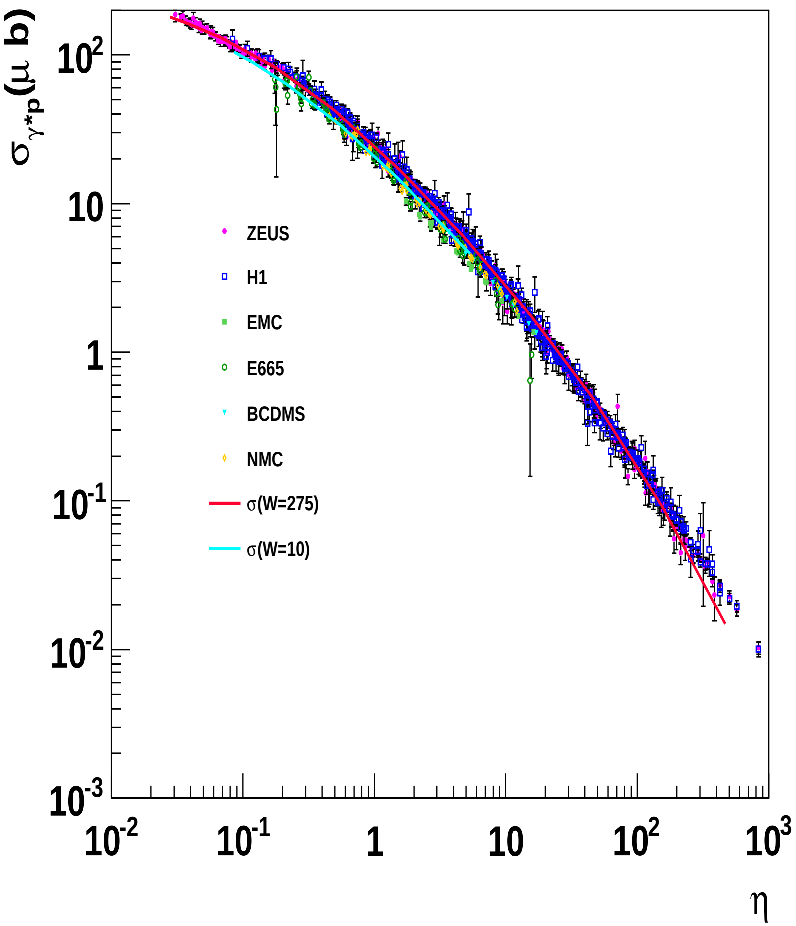
$$C_1 = 1.95 GeV^2$$

$$W_0^2 = 1081 GeV^2$$

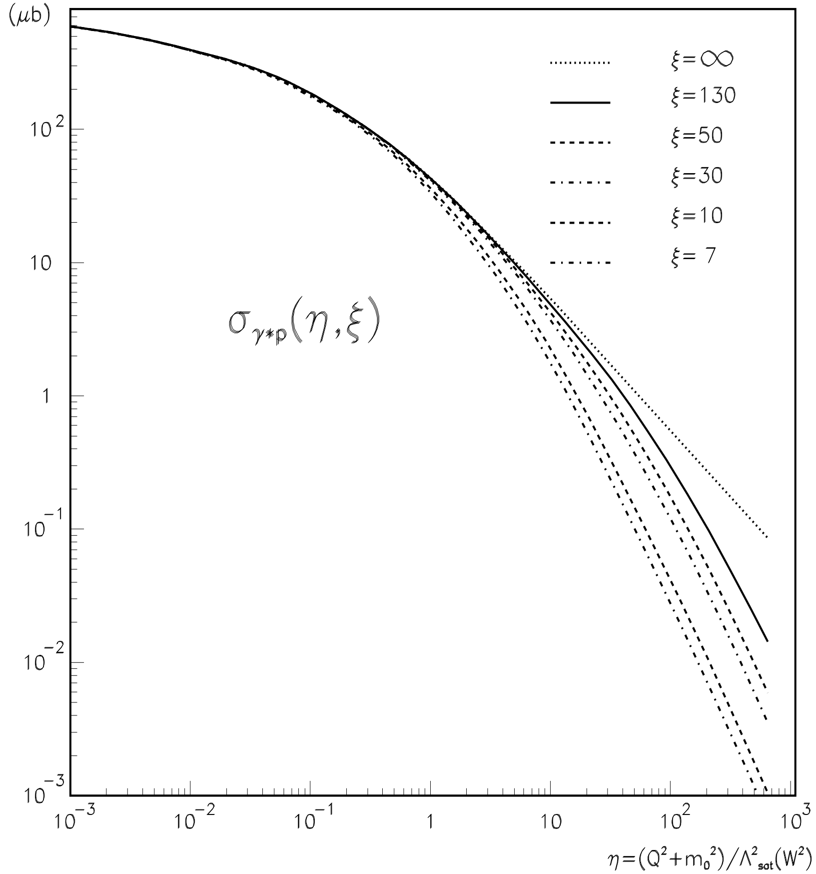
$$C_2 = 0.27(0.29)$$

$$m_0^2 = 0.15 GeV^2$$

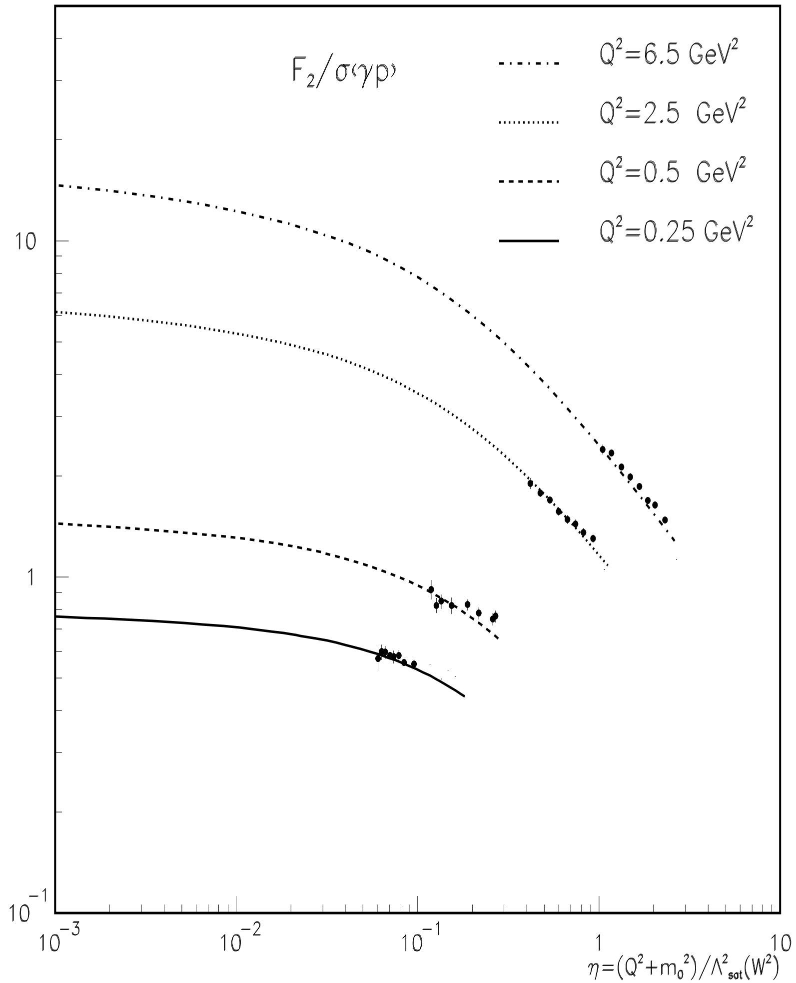
$$m_1^2(W^2) = \xi \Lambda^2_{sat}(W^2) = 130 \Lambda^2_{sat}(W^2)$$



σγ*p(μ b)
10²
10
1
10⁻¹
10⁻²
10⁻³
ZEUS
H1
EMC
E665
BCDMS
NMC
σ(W=275)
σ(W=10)
10⁻²
10⁻¹
1
10
10²
10³
η

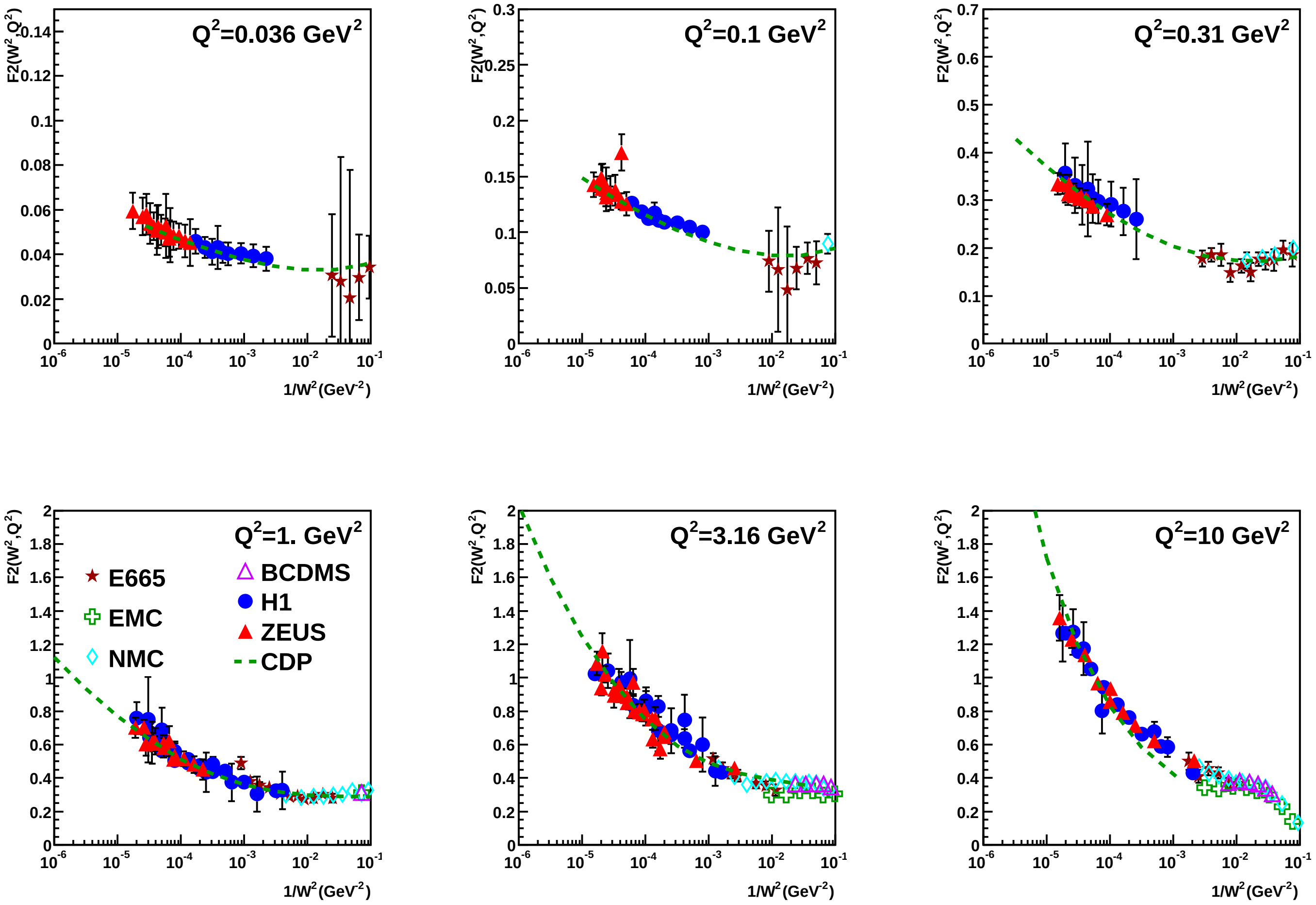


(μb)
ξ=∞
ξ=130
ξ=50
ξ=30
ξ=10
ξ= 7
σγ*p(η,ξ)
η=(Q²+m₀²)/Λ²sat(W²)

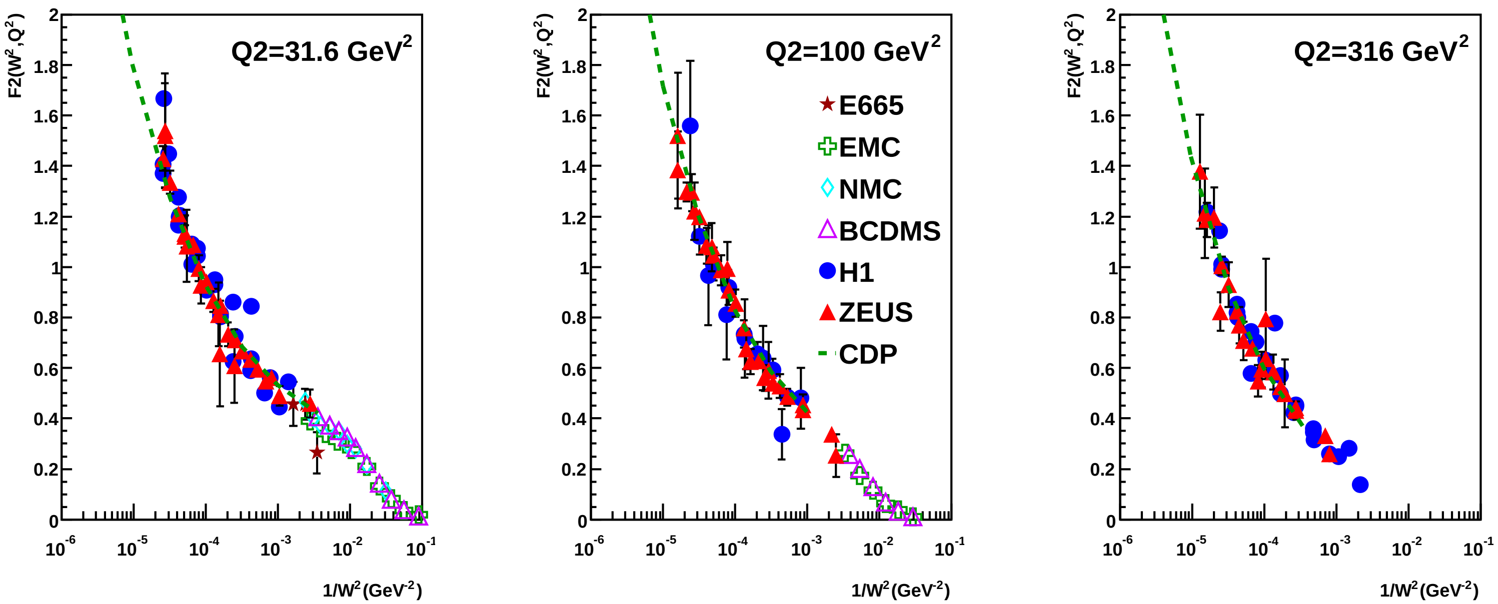


The approach to saturation.

Comparison with the experimental data for F_2 — Prabhdeep Kaur (2010)



Saturation limit: $\lim\limits_{\substack{W^2\to\infty \\ Q^2 \text{fixed}}} \frac{F_2(x\cong Q^2/W^2, Q^2)}{\sigma_{\gamma p}(W^2)} = \frac{Q^2}{4\pi^2\alpha}$



Q2=31.6 GeV^2
Q2=100 GeV^2
Q2=316 GeV^2
E665
EMC
NMC
BCDMS
H1
ZEUS
CDP
F2(W^2,Q^2)
1/W^2 (GeV^-2)

A Remark on : $F_2(W^2)$ in terms of gluon distribution:

$$F_2(W^2 = \frac{Q^2}{x}) = \frac{(2\rho+1)\sum Q_q^2}{3\pi}\xi_L^{C_2}\alpha_s(Q^2)G(x,Q^2) \qquad \eta(W^2,Q^2) \gg 1.$$

$$= \frac{(2\rho+1)\sum Q_q^2}{3\pi}\frac{1}{8\pi^2}\sigma_L^{(\infty)}\Lambda_{sat}^2(W^2). \qquad \text{color transparency}$$

(upon using $F_2 = f_2\left(\frac{W^2}{1GeV^2}\right)^{0.29} = \frac{(2\rho+1)\sum Q_q^2}{3\pi}\frac{1}{8\pi^2}\sigma_L^{(\infty)}\Lambda_{sat}^2(W^2)$.)

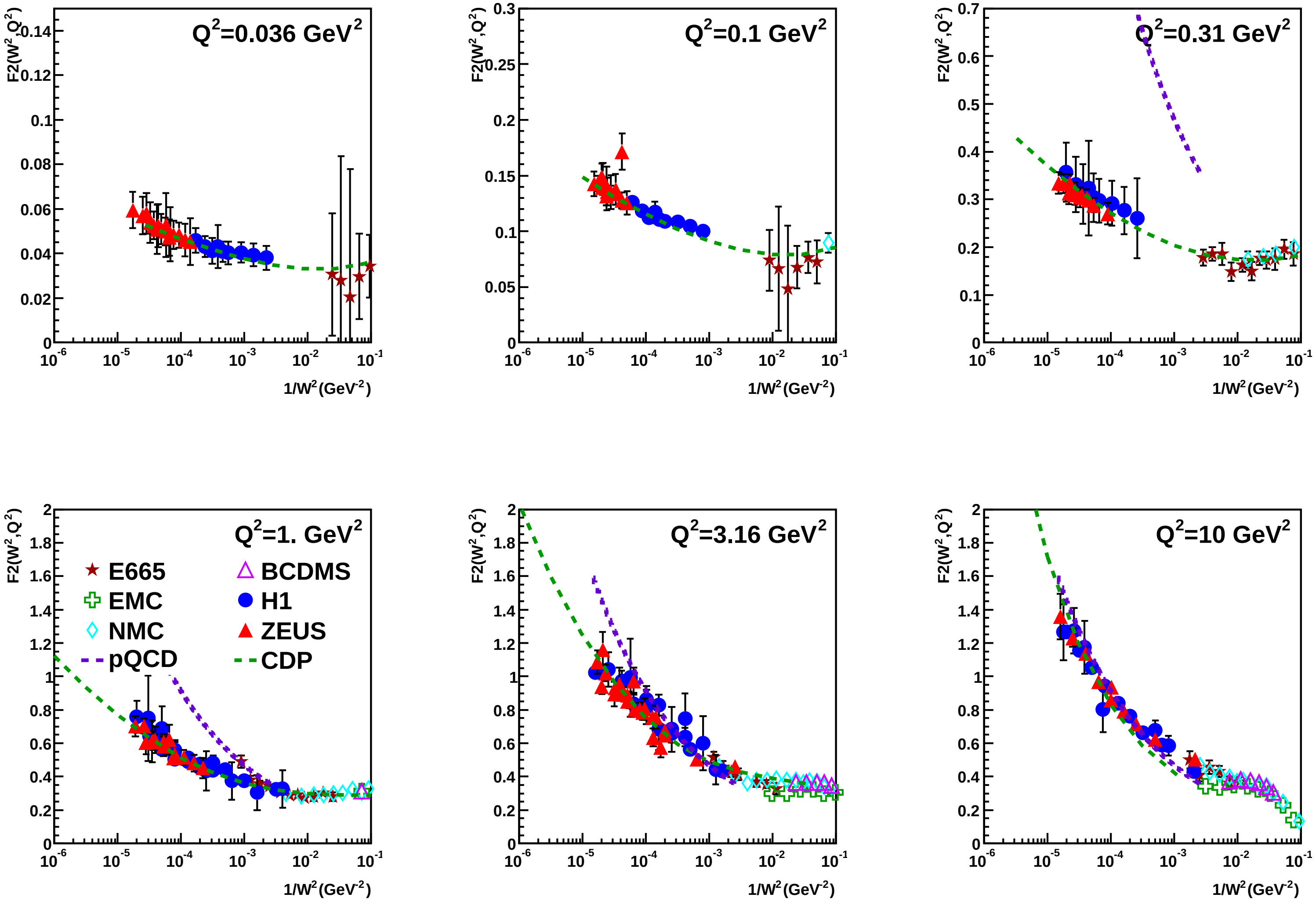
Saturation behavior:

$$F_2(W^2,Q^2) \sim Q^2\sigma_L^{(\infty)}\ln\frac{\Lambda_{\rm sat}^2(W^2)}{Q^2+m_0^2}$$

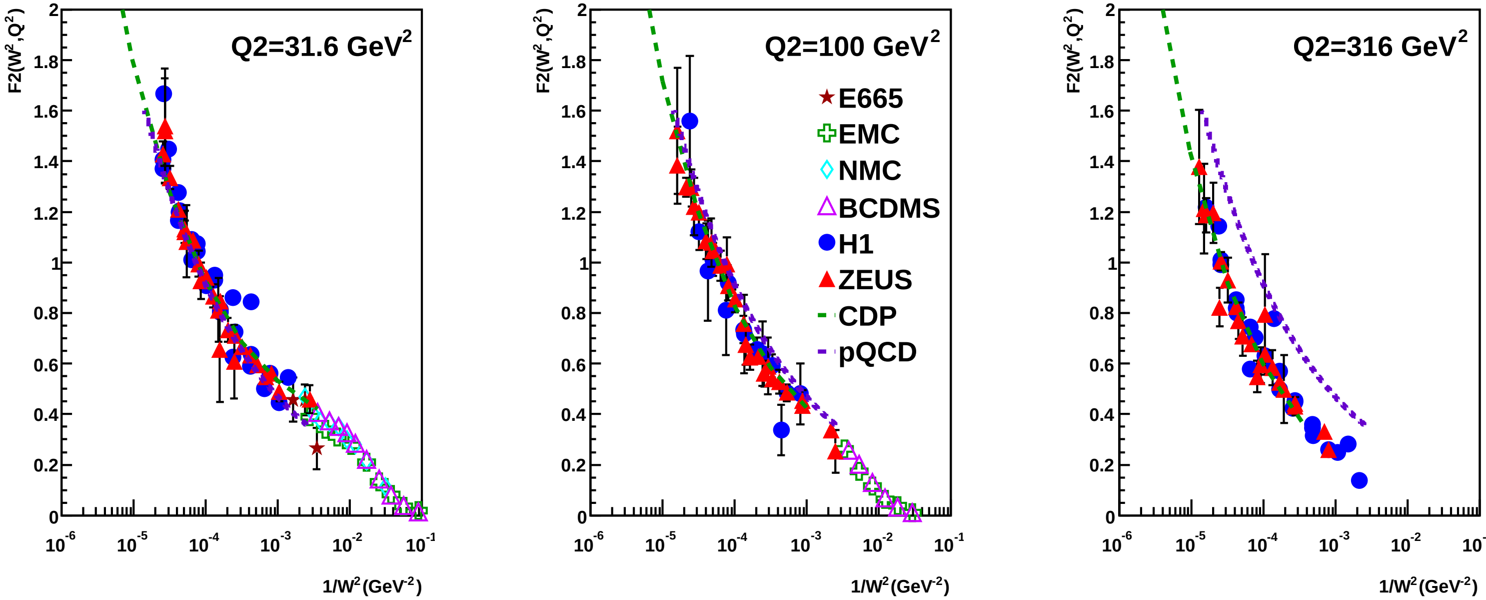
$$\sim Q^2\sigma_L^{(\infty)}\ln\left(\frac{\alpha_s(Q^2)G(x,Q^2)}{\sigma_L^{(\infty)}(Q^2+m_0^2)}\right), \qquad \eta(W^2,Q^2) \ll 1.$$

saturation

Logarithmic dependence on gluon distribution in saturation limit.

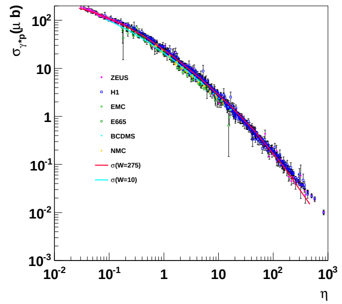


CDP and pQCD-improved parton model



CDP and pQCD-improved parton model

5. Conclusions

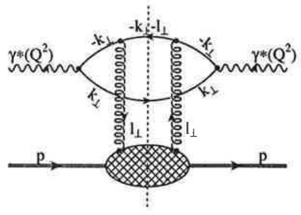


The empirically observed low-x ($x_{bj} \cong \frac{Q^2}{W^2} \leq 0.1$) scaling behavior,

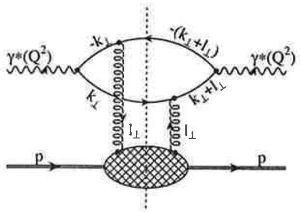
$$\sigma_{\gamma^* p}(W^2, Q^2) = \sigma_{\gamma^* p}\left(\eta(W^2, Q^2)\right),$$

where $\eta(W^2, Q^2) = \frac{Q^2 + m_0^2}{\Lambda^2_{\rm sat}(W^2)}$,

$$\Lambda^2_{\rm sat}(W^2) = C_1 \left(\frac{W^2}{1 \mathrm{GeV}^2}\right)^{C_2},$$

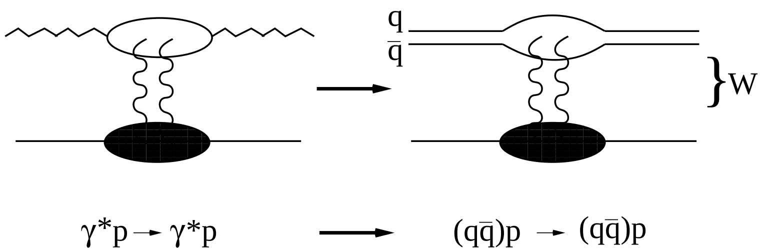


(a)

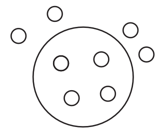


(b)

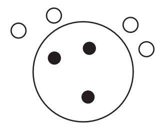
is a consequence of the color-gauge-invariant $q\bar{q}$-dipole interaction with the color field in the nucleon.



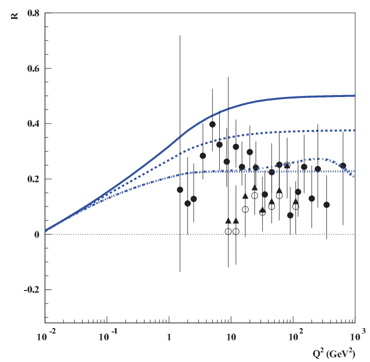
- For $\eta(W^2, Q^2) \gg 1$, color transparency, $\sigma_{q\bar{q})p} \sim \vec{r}_\perp^{\,2}$, implies $\sigma_{\gamma^* p} \sim \frac{1}{\eta}$.



- For $\eta(W^2, Q^2) \ll 1$, saturation, $\sigma_{q\bar{q})p} \sim \sigma^{(\infty)}(W^2)$, implies $\sigma_{\gamma^* p} \sim \sigma^{(\infty)}(W^2) \ln \frac{1}{\eta}$, i. e. hadronlike $\ln^2 W^2$ dependence at any Q^2 fixed.



- $R(W^2, Q^2) = \frac{\sigma_{\gamma_L^* p}(\eta(W^2,Q^2))}{\sigma_{\gamma_T^* p}(\eta(W^2,Q^2))} = \frac{1}{2\rho}$ for $\eta \gg 1$.



- Detailed model essentially based on a parameterization of

$$\Lambda^2_{\rm sat}(W^2) = C_1 \left(\frac{W^2}{1\mathrm{GeV}^2} \right)^{C_2}$$

shows agreement with all DIS data at low x, including $Q^2 = 0$ photoproduction.

Appendix

Equivalently, in terms of the variables:

$$\vec{r}_\perp{}' = \sqrt{z(1-z)}\,\vec{r}_\perp,$$

$$\vec{l}_\perp{}' = \frac{\vec{l}_\perp}{\sqrt{z(1-z)}},$$

Photon wave function (e.g. L):

$$K_0(r'_\perp Q) = \frac{1}{2\pi}\int d^2\vec{k}_\perp{}' \frac{1}{Q^2 + \vec{k}_\perp'^2} e^{-i\vec{r}_\perp{}'\cdot\vec{k}_\perp{}'}$$

$\gamma^* q\bar{q}$ **coupling :**

$$\sum_{\lambda=-\lambda=\pm 1} |j_L^{\lambda,\lambda'}|^2 = 4M_{q\bar{q}}^2 \left(d_{10}^1(z)\right)^2,$$

$$\sum_{\lambda=-\lambda'=\pm 1} |j_T^{\lambda,\lambda'}(+)|^2 = \sum_{\lambda=-\lambda=\pm 1} |j_T^{\lambda,\lambda'}(-)|^2 = 4M_{q\bar{q}}^2 \frac{1}{2}\left((d_{1-1}^1(z))^2 + (d_{11}^1(z))^2\right).$$

Upon introducing the cross section $\sigma_{(q\bar{q})^{J=1}_{L,T}p}(r'_\perp, W^2)$, for $(q\bar{q})^{J=1}_{L,T}p$ scattering

A) $\sigma_{\gamma^*_{L,T}p}(W^2, Q^2) = \frac{\alpha}{\pi}\sum_q Q_q^2 Q^2 \int dr'^2_\perp K^2_{0,1}(r'_\perp Q)\sigma_{(q\bar{q})^{J=1}_{L,T}p}(r'_\perp, W^2).$

Kuroda, Schildknecht (2011)

and

B)
$$\sigma_{(q\bar{q})^{J=1}_{L,T}p}(\vec{r}_\perp{}', W^2) = \int d^2\vec{l}_\perp{}' \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2)(1 - e^{-i\vec{l}_\perp{}'\cdot\vec{r}_\perp{}'})$$
$$= \pi \int d\vec{l}_\perp{}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2) \cdot \left(1 - \frac{\int d\vec{l}_\perp{}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2) J_0(l'_\perp r'_\perp)}{\int d\vec{l}_\perp{}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2)}\right)$$

For fixed dipole size, $r'_\perp$, dominant contribution to dipole cross section

$$\vec{l}_\perp{}'^2 \leq \vec{l}_{\perp \mathrm{Max}}'^2(W^2).$$

The Color Dipole Cross Section.

I) Color transparency

$$0 < l'_\perp r'_\perp < l'_{\perp\ Max}(W^2) r'_\perp \ll 1, \qquad J_0(l'_\perp r'_\perp) \cong 1 - \frac{1}{4}(l'_\perp r'_\perp)^2$$

$$\sigma_{(q\bar{q})^{J=1}_{L,T} p}(r'^2_\perp, W^2) =$$

$$= \frac{1}{4}\pi r'^2_\perp \int d\vec{l}'^2_\perp \vec{l}'^2_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_{L} p}(\vec{l}'^2_\perp, W^2) \begin{cases} 1, \\ \rho_W, \end{cases} \left(r'^2_\perp \ll \frac{1}{l'^2_{\perp\ Max}(W^2)} \right).$$

where $\int d\vec{l}'^2_\perp \vec{l}'^2_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_{T} p}(\vec{l}'^2_\perp, W^2) = \rho_W \int d\vec{l}'^2_\perp \vec{l}'^2_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_{L} p}(\vec{l}'^2_\perp, W^2)$.

$$\sigma_{(q\bar{q})^{J=1}_{L,T} p}(r'^2_\perp, W^2) = \frac{1}{4} r'^2_\perp \sigma^{(\infty)}_L (W^2) \Lambda^2_{sat}(W^2) \begin{cases} 1, \\ \rho_W, \end{cases} \left(r'^2_\perp \ll \frac{1}{l'^2_{\perp\ Max}(W^2)} \right).$$

where $\Lambda^2_{sat}(W^2) \equiv \frac{\int d\vec{l}'^2_\perp \vec{l}'^2_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_{L} p}(\vec{l}'^2_\perp, W^2)}{\int d\vec{l}'^2_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_{L} p}(\vec{l}'^2_\perp, W^2)}$

Strong cancellation between channel 1 and channel 2.

II) Saturation

$$l'_{\perp Max}(W^2) r'_{\perp} \gg 1,$$

huge integrations range in integral over $dl'^2_{\perp}$, many oscillations of $J_0(l'_{\perp} r'_{\perp})$, contribution from channel 2 vanishing

$$\sigma_{(q\bar{q})^{J=1}_{L,T} p}({r_{\perp}'}^2, W^2) \cong \pi \int d{\vec{l}_{\perp}}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T} p}({\vec{l}_{\perp}}'^2, W^2) \equiv \sigma^{(\infty)}_{L,T}(W^2),$$
$$\left(r'^2_{\perp} \gg \frac{1}{l'^2_{\perp\; Max}(W^2)} \right).$$

Unitarity: $\sigma^{(\infty)}_{L,T}(W^2)$ at most logarithmically dependent on W^2.

Thus: Property of dipole interaction:

$$\lim_{\substack{r'^2_{\perp}\text{fixed} \\ W^2 \to \infty}} \sigma_{(q\bar{q})^{J=1}_{L,T} p}(r'_{\perp}, W^2) = \lim_{\substack{r'^2_{\perp} \to \infty \\ W^2\text{fixed}}} \sigma_{(q\bar{q})^{J=1}_{L,T} p}(r'^2_{\perp}, W^2)$$

Photoabsorption Cross Section

Due to $K^2_{0,1}(r'_\perp Q) \sim \frac{\pi}{2r'_\perp Q} e^{-2r'_\perp Q}, \qquad (r'_\perp Q \gg 1)$, cross section determined by

$$r'^2_\perp < \frac{1}{Q^2}.$$

At fixed Q^2,

either $r'^2_\perp < \frac{1}{Q^2} < \frac{1}{\Lambda^2_{sat}(W^2)}$, color transparency: $Q^2 \gg \Lambda^2_{sat}(W^2)$

or $\frac{1}{\Lambda^2_{sat}(W^2)} < r'^2_\perp < \frac{1}{Q^2}$, saturation: $\Lambda^2_{sat}(W^2) \ll Q^2$.

$$\begin{aligned} \sigma_{\gamma^* p}(W^2, Q^2) &= \sigma_{\gamma^* p}(\eta(W^2, Q^2)) = \\ &= \frac{\alpha}{\pi} \sum_q Q_q^2 \begin{cases} \sigma_T^{(\infty)}(W^2) \ln \frac{1}{\eta(W^2,Q^2)}, & (\eta(W^2,Q^2) \ll 1) \quad \text{(sat.)}, \\ \frac{1}{6}(1+2\rho)\sigma_L^{(\infty)}(W^2) \frac{1}{\eta(W^2,Q^2)}, & (\eta(W^2,Q^2) \gg 1), \quad \text{(col.tr.)} \end{cases} \end{aligned}$$

$$\eta(W^2, Q^2) = \frac{Q^2 + m_0^2}{\Lambda^2_{sat}(W^2)}$$

Color-gauge-invariant $q\bar{q}$ (dipole) interaction with gluon field in the nucleon implies low-x scaling.