Electromagnetic properties of pion and kaon in an effective quark theory

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Outline

- Quantum Chromodynamics
  fundamental theory of strong interaction, but difficult to solve directly.
- How can we describe hadrons from quarks and gluons ?
  1. Lattice QCD
     formulated as a lattice gauge theory.
  2. Effective quark models of QCD
     Nambu-Jona-Lasinio (NJL) model, Chiral Soliton model, Bag models, etc.
- Purpose of this study

  study electromagnetic properties of pion and kaon !
  Use the NJL model with proper time regularization. There is only one free model parameter, the dressed quark mass M.
- Comparison with the empirical data and QCD based predictions !

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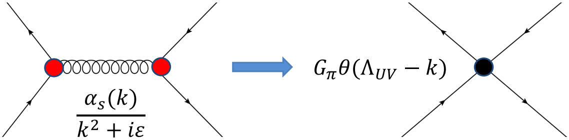
The NJL model

- The lagrangian of the three flavor NJL model with four-fermi interaction is given by

$$\mathcal{L}_{NJL} = \bar{\psi}(i\partial\!\!\!/ - m_q)\psi + G_\pi \sum_{a=0}^{8}\left[(\bar{\psi}\lambda_a\psi)^2 - (\bar{\psi}\lambda_a\gamma_5\psi)^2\right] - G_v \sum_{a=0}^{8}\left[(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2\right] \quad (1)$$

- In the NJL model, the effective gluon exchange is replaced by four fermi contact interaction:



Gap equation

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- In the mean field approximation, the current quark gets the effective mass M_q dynamically as shown below.

$$\begin{aligned} M_q &= m_q - 4G_\pi \langle \overline{q}q \rangle \\ &= m_q + 48iG_\pi \int \frac{d^4k}{(2\pi)^4} \frac{M_q}{k^2 - M_q^2 + i\epsilon} \end{aligned} \tag{2}$$

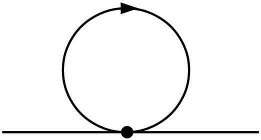

Figure : The quark self-energy in the mean filed approximation.

Description of mesons 1

- Mesons are described as relativistic bound states of a quark and an antiquark. Solving the Bethe - Salpeter equation for the quark - antiquark bound states in the pseudo - scalar (k $= \pi$, K) channels using the Random Phase Approximation, we get the T-matrices

$$T_k = \gamma_5 \lambda_\alpha \frac{-2iG_\pi}{1 + 2G_\pi \Pi_k(p^2)} \gamma_5 \lambda_\alpha \quad (3)$$

- The pion and kaon masses are defined by the poles of these T-matrices.

$$1 + 2G_\pi \Pi_k(p^2 = m_k^2) = 0 \quad (4)$$

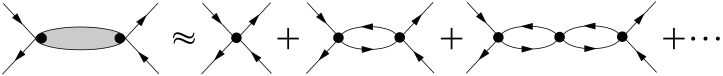

Figure : The quark and the antiquark T-matrix in the Random Phase Approximation.

Description of mesons 2

- Expanding $\Pi_k(p^2)$ near the pole as

$$\Pi_k(p^2) \sim \Pi_k(p^2 = m_k^2) + (p^2 - m_k^2)\Pi'_k(p^2)|_{p^2=m_k^2} \tag{5}$$

- T-matrix becomes

$$T_k \sim \gamma_5 \lambda_\alpha \frac{i g_k^2}{p^2 - m_k^2} \gamma_5 \lambda_\alpha, \quad g_k^{-1} = \sqrt{-\Pi'(p^2)|_{p^2=m_k^2}} \tag{6}$$

- Quark - meson interaction (of Yukawa type) is now derived from the original four fermi interaction ! We utilize this result for calculating the pion and kaon properties.

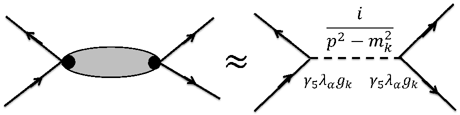

Figure : The quark - antiquark T-matrix is expressed by Yukawa type interaction mediated by pseudo - scalar mesons.

Leptonic decay constants of pion and kaon

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- The pion and kaon leptonic decay constants are defined by the matrix element of the axial vector current between the meson state and the hadronic vacuum state.

$$\langle 0 | j_a^{\mu}(0) | k_b \rangle = i p^{\mu} f_k \quad (7)$$

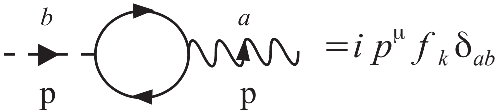

Figure : The dashed line represents a pseudo scalar meson and the wavy line an external axial vector field.

Regularization

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- The NJL model has two serious problems.
  1. Non-renormalizability
  2. Lack of the confinement
- The proper time regularization scheme is used, introducing the IR cut-off parameter Λ_{IR} to simulate the confinement.

$$\frac{1}{D^n} = \frac{1}{(n-1)!}\int_0^\infty d\tau\, \tau^{n-1} e^{-\tau D} \to \frac{1}{(n-1)!}\int_{\frac{1}{\Lambda_{UV}^2}}^{\frac{1}{\Lambda_{IR}^2}} d\tau\, \tau^{n-1} e^{-\tau D} \tag{8}$$

- IR cut-off Λ_{IR} eliminates the possibility of decaying hadrons into quarks.

  W. Bentz and A. W. Thomas, Nucl. Phys. A 696, 138 (2001)

Model parameters

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- Take the dressed u, d quark mass $M_u = M_d = M$ as the free parameter.

  The model parameters are M, M_s, G_π, G_v, Λ_{UV} and Λ_{IR}

The coupling constant G_π and the cut-off parameter Λ_{UV} are obtained so as to reproduce the pion mass m_π and the pion leptonic decay constant $f_\pi = 93.4$ MeV. We fixed $\Lambda_{IR} = 0.2$ GeV.

G_v and M_s are obtained by the ρ meson mass $m_\omega = m_\rho = 0.776$ GeV and the kaon mass $m_K = 0.494$GeV.

Then, the only free parameter is the dressed quark mass M.

Result 1

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- Model parameters and quark masses.

M	Λ_{UV}	G_π	G_v	M_s	m	m_s	m_s/m
0.20	1.24	2.36	2.08	0.467	0.0041	0.131	31.9
0.25	0.84	6.12	3.06	0.502	0.0086	0.227	26.5
0.30	0.71	10.6	4.52	0.540	0.0123	0.293	23.8
0.35	0.66	15.0	6.64	0.573	0.0150	0.331	22.1
0.40	0.64	19.3	9.60	0.609	0.0168	0.357	21.3
Latt.*	-	-	-	-	$0.0035^{+0.7}_{-0.2}$	0.95 ± 0.05	27.5 ± 1.0

Table : Masses and regularization parameters are in units of GeV, the Lagrangian couplings, G_π and G_v, are in units of GeV^{-2}.

(*) J. Beringer et al. [Particle Data Group](2012)

Result 2

- Kaon leptonic decay constant and quark condensates

M	f_K	$\langle\bar{\ell}\ell\rangle$	$\langle\bar{s}s\rangle$	$\langle\bar{s}s\rangle \,/\, \langle\bar{\ell}\ell\rangle$
0.20	0.128	$-(0.275)^3$	$-(0.329)^3$	1.71
0.25	0.110	$-(0.214)^3$	$-(0.224)^3$	1.15
0.30	0.100	$-(0.190)^3$	$-(0.180)^3$	0.85
0.35	0.094	$-(0.177)^3$	$-(0.159)^3$	0.72
0.40	0.091	$-(0.170)^3$	$-(0.148)^3$	0.70
Exp. & Latt*.	0.110 ± 0.7	$-(0.283 \pm 2)^3$	$-(0.290 \pm 15)^3$	1.08 ± 0.16

Table : The kaon leptonic decay constant is in units of GeV and the quark condensates are in units of GeV^3.

(*) J. Beringer et al. [Particle Data Group] (2012), C. McNeile *et al.* (2013)

- Most of previous NJL calculations used $M = 0.3 \sim 0.4$ GeV, but our above results favor for smaller M.
- How does the form factor behave for smaller M ?

Form factors of pion and kaon

- The form factor of meson is defined by the matrix element.

$$\int d^4z\, e^{-iqz} \left\langle p' \left| \overline{\psi}(z) \frac{1}{2} \left(\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8 \right) \gamma^\mu \psi(z) \right| p \right\rangle \equiv (2\pi)^4 \delta^4(p' - p - q) F(q^2) \tag{9}$$

- The corresponding Feynman diagrams are shown in the figure below.

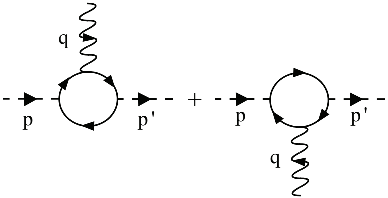

Figure : Feynman diagrams for the meson electromagnetic current.

Effects of pion cloud and vector mesons around the dressed quarks on the form factors

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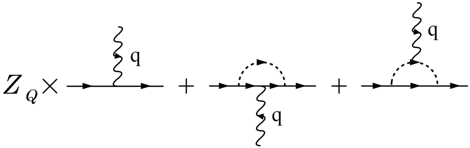
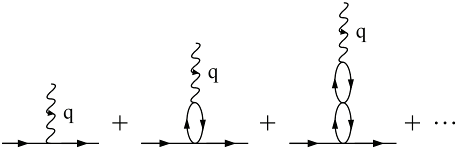
In this study, the pion and kaon form factors are determined at three levels of sophistication.

1. The dressed quarks are treated like point (bare) quarks.
2. Including the pion cloud around the dressed quarks. Note that the *s* quark does not couple to the pion.



3. Including the vector meson contributions (ρ, ω). Note that the *s* quark does not couple to the ρ and ω mesons.



Result 3

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- Pion form factor with $M = 0.25\,\text{GeV}$.

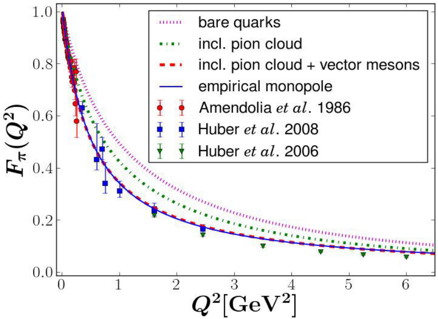

Figure : The purple line shows the result where the vertex is treated as point like; the green line includes the pion cloud effect; the red line is the full result including also vector mesons; the blue line is the empirical monopole function, given by $1/(1 + Q^2/0.517\ \text{GeV}^2)$.

Result 4

- Pion form factor with $M = 0.4\,\text{GeV}$.

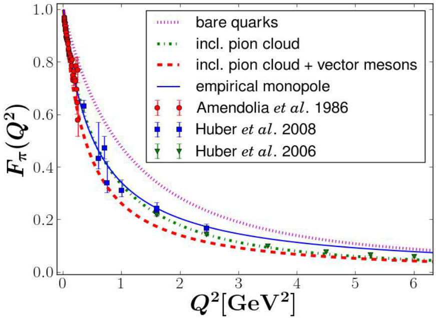

Figure : The purple line shows the result where the vertex is treated as point like; the green line includes the pion cloud effect; the red line is the full result including also vector mesons; the blue line is the empirical monopole function, given by $1/(1 + Q^2/0.517\ \text{GeV}^2)$.

Result 6

- Kaon form factor with $M = 0.25\,\text{GeV}$.

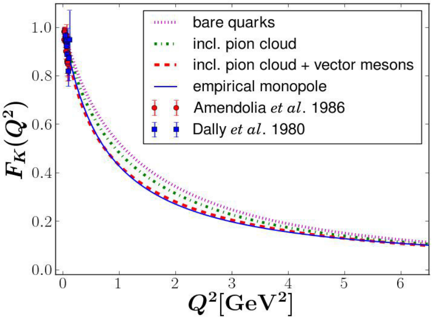

Figure : The purple line shows the result where the vertex is treated as point like; the green line includes the pion cloud effect; the red line is the full result including vector mesons; the blue line is the monopole function, given by $1/(1 + Q^2/0.744\ \text{GeV}^2)$.

Result 7

- Kaon form factor with $M = 0.4\,\text{GeV}$.

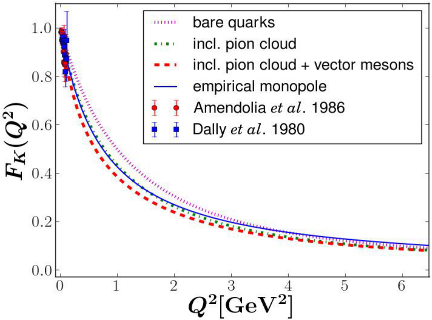

Figure : The purple line shows the result where the vertex is treated as point like; the green line includes the pion cloud effect; the red line is the full result including also vector mesons; the blue line is the monopole function, given by $1/(1 + Q^2/0.744\ \text{GeV}^2)$.

Result 8

pQCD predicts that $F_K(Q^2)/F_\pi(Q^2)$ should approach $f_K^2/f_\pi^2 \sim 1.4$ as $Q^2 \to \infty$ (G. R. Farrar and D. R. Jackson, (1979)).

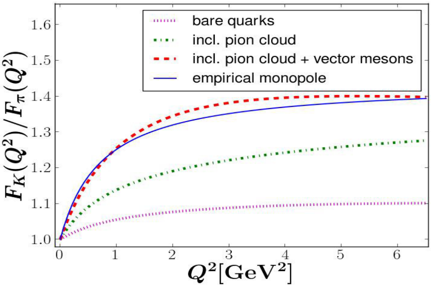

Figure : Results of the ratio of form factors $F_K(Q^2)/F_\pi(Q^2)$ for the case $M = 0.25$ GeV.

- Our results with $M = 0.25\,\mathrm{GeV}$ can also reproduce the pQCD prediction !

Result 9

- Charge radii of pion and kaon

M	$\langle r_\pi \rangle^{(b)}$	$\langle r_K \rangle^{(b)}$	$\langle r_\pi \rangle^{(b)} / \langle r_K \rangle^{(b)}$	$\langle r_\pi \rangle^{(\pi)}$	$\langle r_K \rangle^{(\pi)}$	$\langle r_\pi \rangle^{(\pi)} / \langle r_K \rangle^{(\pi)}$	$\langle r_\pi \rangle$	$\langle r_K \rangle$	$\langle r_\pi \rangle / \langle r_K \rangle$
0.20	0.455	0.430	1.06	0.527	0.481	1.09	0.645	0.571	1.13
0.25	0.489	0.465	1.05	0.589	0.530	1.11	0.690	0.608	1.14
0.30	0.497	0.474	1.05	0.627	0.553	1.13	0.724	0.630	1.15
0.35	0.488	0.468	1.04	0.649	0.562	1.15	0.750	0.643	1.17
0.40	0.472	0.453	1.04	0.663	0.563	1.18	0.773	0.653	1.18
Exp*.	-	-	-	-	-	-	0.672 ± 0.008	0.560 ± 0.31	1.2 ± 0.08

(*) J. Beringer et al. [Particle Data Group] (2012).

- The bare quarks case doesn't sensitive to the dressed quark mass M.
- The pion cloud effect is larger for the pion than for the kaon, and give a good description.
- The case of $M \sim 0.25$ GeV including the pion cloud and vector mesons can reproduce the empirical data !

Summary

In the NJL model, the case of dressed quark mass M $\sim$ 0.25 GeV gives a good description of

- Pion and kaon electromagnetic properties
- Current u,d and s quark masses
- The chiral condensate of u,d and s quarks
- The kaon leptonic decay constant

The pion cloud and vector mesons are very important to understand the electromagnetic properties of pion and kaon.