Hadron Structure from Lattice QCD

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Contents

Lattice regularization of QCD (LQCD)

QCD action

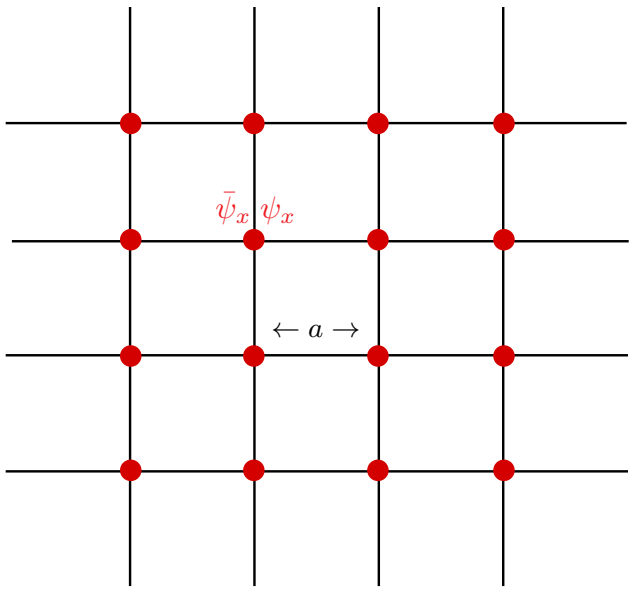
$$\mathcal{S}_{\mathrm{QCD}} = \int d^4x \left[-\frac{1}{2{g_0}^2} \operatorname{Tr} F^2[A] + \sum_f \bar{\psi}^f_x \left(\not{D}[A] - m_0^f \right) \psi^f_x \right]$$

Discretization of Euclidean space-time

- Introduce 4-dim lattice $L^3 \times T$

$$x = n\,a^4, \quad n \in \mathbf{Z}^4$$

- Quark fields ψ dwell on sites
- But: naive discretization of gluon field $A_\mu(x)$ not gauge-invariant at finite a



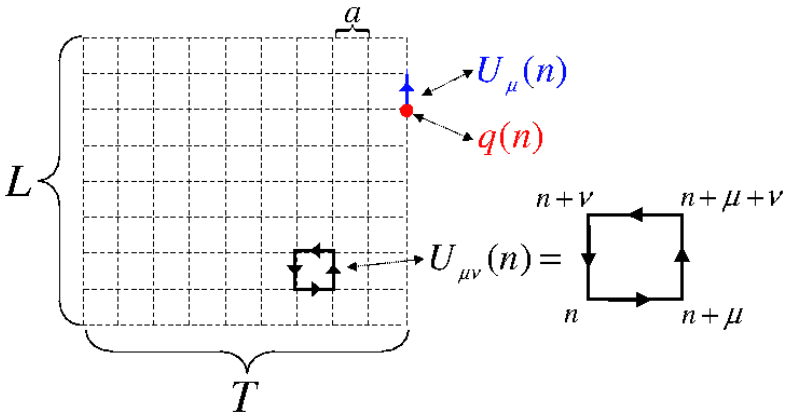
Lattice regularization of QCD (LQCD)

introduced by K. Wilson (1974-77)

Parallel transporter between x and $x+\hat{\mu}$

$$U_\mu(n) = \mathcal{P} e^{iag_0 \int_n^{n+\hat{\mu}} A_\mu(z) dz} \quad \in SU_c(3)$$

Using "links" $U_\mu(n)$ for discretization gives a gauge-invariant action for any a.



Example: unimproved Wilson action $\qquad (a \equiv 1)$

$$\mathcal{S}^W_{\mathrm{LQCD}} = \overbrace{\beta \sum_{n,\mu<\nu} \left(1 - \tfrac{1}{3}\mathfrak{Re}\,\mathrm{Tr}\,\Box_{n,\mu\nu}\right)}^{\text{gauge part}} + \overbrace{\sum_{n,m,f} \bar{\psi}^f_n D^W_{nm}[U,\kappa_f]\psi^f_m}^{\text{fermionic part}} \xrightarrow{a\to 0} S_{QCD}$$

- Wilson fermion matrix: $D^W_{nm}[U,\kappa_f] = \delta_{nm} - \kappa_f \sum_{\pm\mu} \delta_{m,n+\hat{\mu}}(1+\gamma_\mu)U_{n\mu}$
  (breaks chiral symmetry)
- QCD Parameters: $\beta \equiv 6/g_0^2, \qquad \kappa_f \equiv 1/(2m_0^f - 8)$

Other (improved) lattice actions possible, requirement: $\mathcal{S}_{\mathrm{LQCD}} \xrightarrow{a\to 0} S_{QCD}$

Lattice QCD calculation

Expectation value of an observable via path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod_{n\mu} dU_{n\mu} \, \underline{d\bar{\psi}_n \, d\psi_n} \, \mathcal{O}[U, \bar{\psi}, \psi] \; e^{-(S_g[U] + S_f[U, \bar{\psi}, \psi])}$$

- No gauge-fixing needed !
- Integrate over fermionic fields exactly which results in a modified action S_{eff} and $\mathcal{O}$ (non-local functionals of the links U)

$$S_{eff}[U; \beta, \kappa] = S_g[U; \beta] + \log \det D[U; \kappa]$$

Side remark: integration over Grassmann numbers gives

$$\underbrace{\det D}_{S \to S_{eff}} \cdot \underbrace{\sum_{k_1 \cdots k_n} \epsilon^{k_1 k_2 \cdot k_n}_{j_1 j_2 \cdots j_n} D^{-1}_{k_1 i_i} \cdots D^{-1}_{k_n i_n}}_{\mathcal{O} \to \hat{\mathcal{O}}} = \int \prod_n d\bar{\psi}_n d\psi_n \cdots \psi_{j_1} \bar{\psi}_{i_1} \cdots \psi_{n_1} \bar{\psi}_{i_n} \cdots e^{-\bar{\psi} D \psi}$$

- Leaves us with **Master integral** for expectation value

$$\langle \mathcal{O} \rangle_{\beta, \kappa, L} = \frac{1}{Z} \int \prod_{n\mu} dU_{n\mu} \; \hat{\mathcal{O}}[U] \; e^{-S_{eff}[U, \beta, \kappa]}$$

Lattice QCD calculation

Monte-Carlo calculation

Master integral is very-high dimensional integral

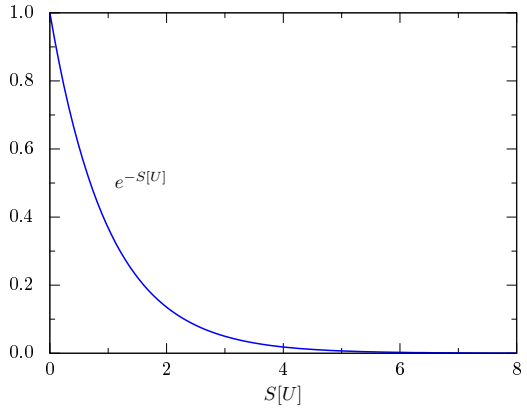
$$\langle \mathcal{O} \rangle_{\beta,\kappa,L} = \frac{1}{Z} \int \prod_{n\mu} dU_{n\mu}\ \hat{\mathcal{O}}[U]\ e^{-S_{eff}[U,\beta,\kappa]}$$

- Number of integration variables
  ($N_c = 3$, $N_d = 4$)

  $64^3 \times 128 \quad \times (N_c^2 - 1) \times N_d \quad = 1\,073\,741\,824$

  $16^3 \times 32 \quad \times (N_c^2 - 1) \times N_d \quad = 4\,194\,304$

- Calculate integrals stochastically



Monte-Carlo integration: Sample links U with probability density ("important sampling")

$$P[U^{(i)}] = \frac{1}{Z} e^{-S_{eff}[U^{(i)}]} \qquad \text{Markov chain:} \quad U^{(1)}, U^{(2)}, \ldots, U^{(N)} \quad \in SU(3)$$

Estimate for expectation value: $\underbrace{\overline{\mathcal{O}}}_{\text{"data"}} \equiv \frac{1}{N} \sum_{i=1}^{N} \hat{\mathcal{O}}[U^{(i)}] \xrightarrow{N\to\infty} \langle \mathcal{O} \rangle_{\beta,\kappa,L}$

Hadron mass from the lattice

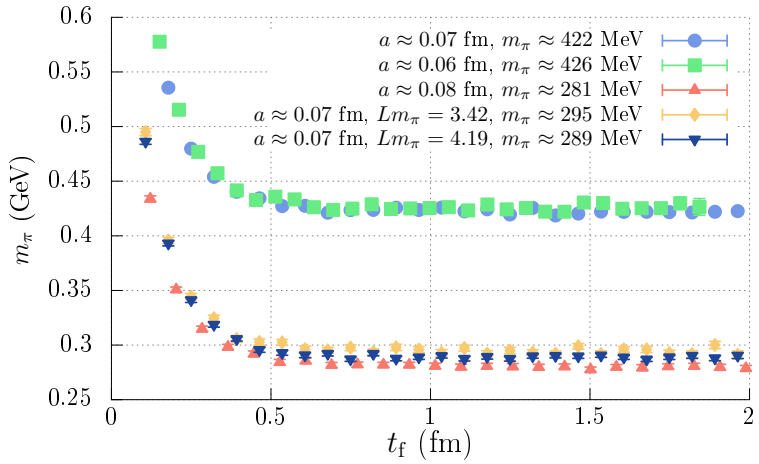
Hadronic 2-point function

$$C^h_{2pt}(\vec{p},t) = \frac{1}{\sqrt{V_S}} \sum_{\vec{x}} e^{i\vec{x}\cdot\vec{p}} \left\langle h(\vec{x},t)\, h(\vec{0},0) \right\rangle_U \overset{(\vec{p}=0)}{=} |Z_0|^2 e^{-m_0 t} + |Z_1|^2 e^{-m_1 t} + \ldots$$

- Large temporal extension to select m_0
- h has quantum number of hadron

  $h_{\pi^+}(x) = \bar{d}(x)\,\gamma_5\, u(x), \qquad x = (\vec{x}, t)$
- In practise: used smeared operators to improve overlap with ground state

pion mass



Monte-Carlo integration

$$\langle h_{\pi^+}(x)\bar{h}_{\pi^+}(y)\rangle = \frac{1}{N}\sum_{i=1}^{N} \underbrace{\mathrm{Tr}\left(\gamma_5 D^{-1}_{xy}[U^{(i)}]\gamma_5\, D^{-1}_{yx}[U^{(i)}]\right)}_{\text{expensive } \hat{\mathcal{O}}[U^{(i)}]} + O(1/\sqrt{N})$$

Hadron mass from the lattice

Hadronic 2-point function

$$C^h_{2pt}(\vec{p},t) = \frac{1}{\sqrt{V_S}} \sum_{\vec{x}} e^{i\vec{x}\cdot\vec{p}} \left\langle h(\vec{x},t)\, h(\vec{0},0) \right\rangle_U \overset{(\vec{p}=0)}{=} |Z_0|^2 e^{-m_0 t} + |Z_1|^2 e^{-m_1 t} + \ldots$$

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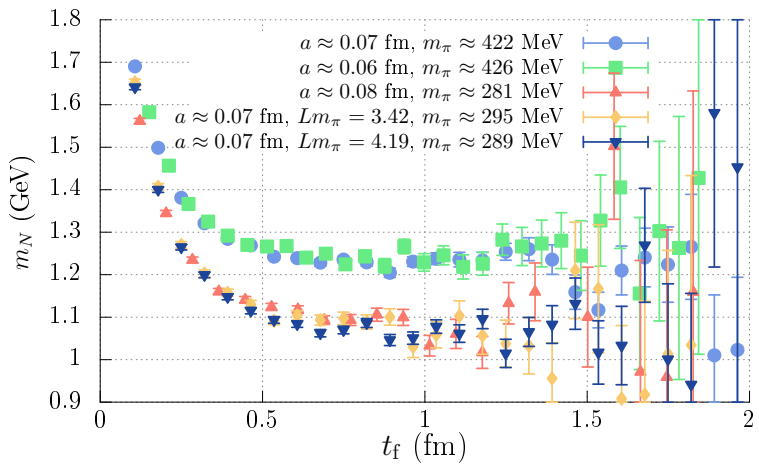
- In practise: used smeared operators to improve overlap with ground state

nucleon mass



Monte-Carlo integration

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Lattice QCD accellerator

Systematics

LQCD comes with the same parameters as QCD (coupling and quark masses) plus the lattice volume

$$\langle \mathcal{O} \rangle_{\beta,\kappa,L} = \frac{1}{Z} \int \prod_{n\mu} dU_{x\mu}\, \hat{\mathcal{O}}[U]\, e^{-S_{eff}[U,\beta,\kappa]}$$

Parameters: $\beta \equiv 6/g_0^2, \qquad \kappa_f \equiv 1/(2m_0^f - 8),$

Have to tune them to physical point, e.g.,

$$\frac{aM_2}{aM_1}(\beta,\kappa) \xrightarrow{\beta,L \text{ large}} ratio(\kappa) \xrightarrow{\kappa^f \to \kappa_c^f} \frac{M_2^{\text{phys}}}{M_1^{\text{phys}}}$$

Calculations close to physical point very expensive ($\rightarrow$ extrapolation)

Lattice spacing: $a = a(\beta)$

Monte-Carlo simulations in practise

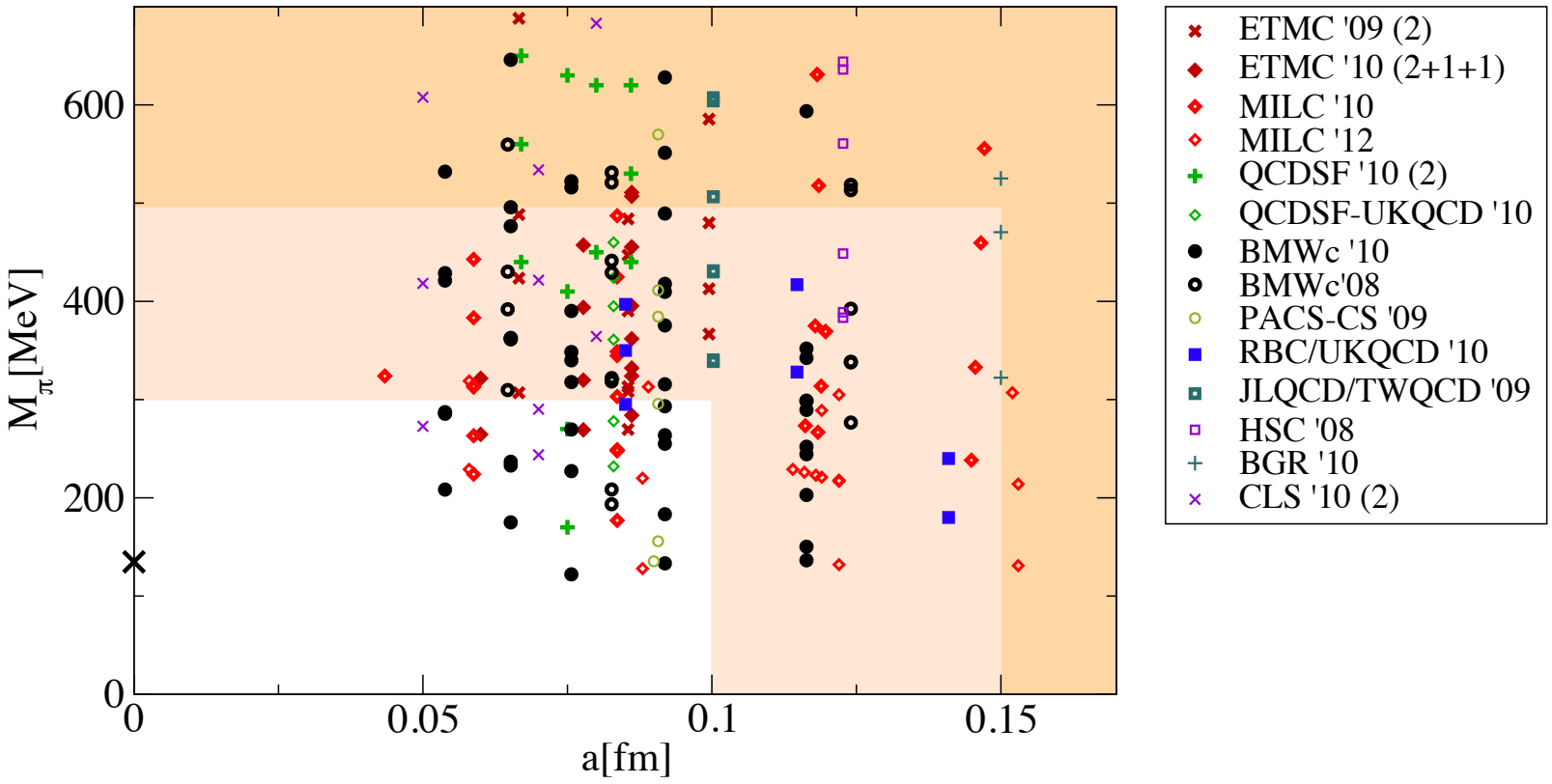
- $N_f = 2, N_f = 2{+}1, N_f = 2{+}1{+}1$
- Lattice spacings: $a \geq 0.04\,\text{fm}$, Volumes: $L \leq 6\,\text{fm}$

Challenge: Control over systematic error when extrapolating to physical point

Landscape of recent lattice simulations

Lattice QCD simulations approach (start approaching) the physical point

Lattice spacing vs. m_π

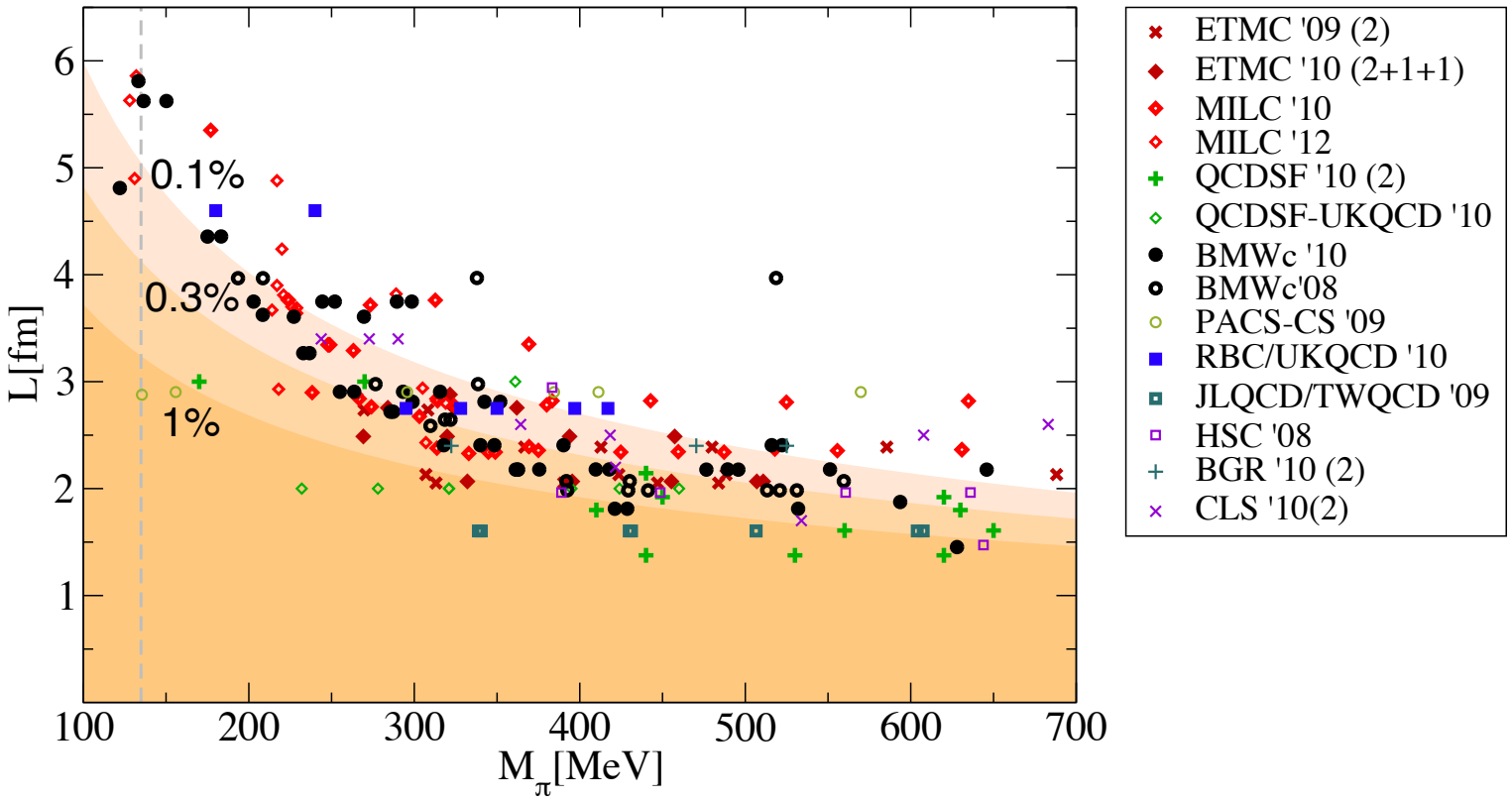


Figures from Hoelbling, arXiv:1410.3403

Landscape of recent lattice simulations

Lattice QCD simulations approach (start approaching) the physical point

Volume vs. m_π

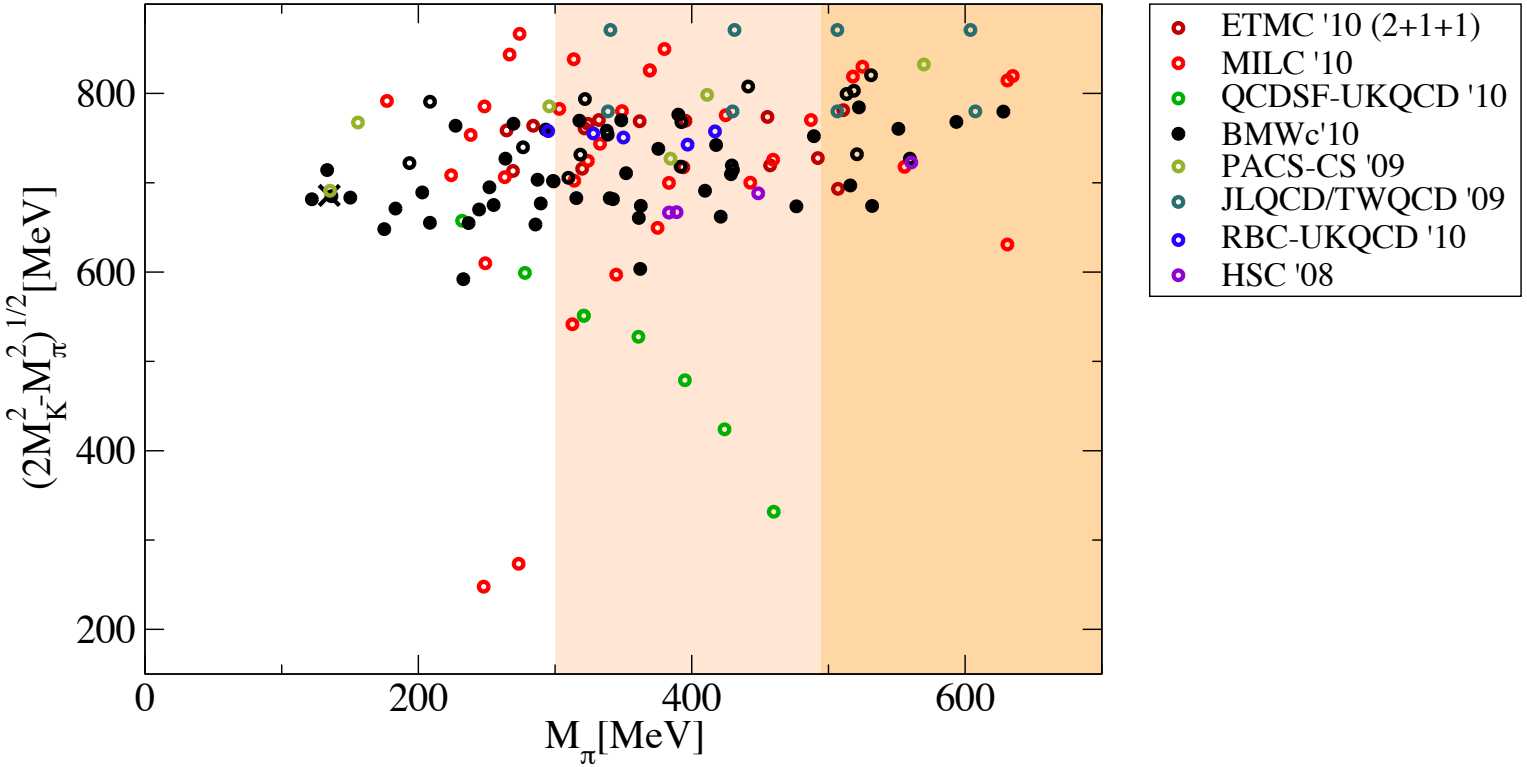


Figures from Hoelbling, arXiv:1410.3403

Landscape of recent lattice simulations

Lattice QCD simulations approach (start approaching) the physical point

Physical point vs. m_π



Figures from Hoelbling, arXiv:1410.3403

Hadron masses from the lattice

Lattice QCD works!

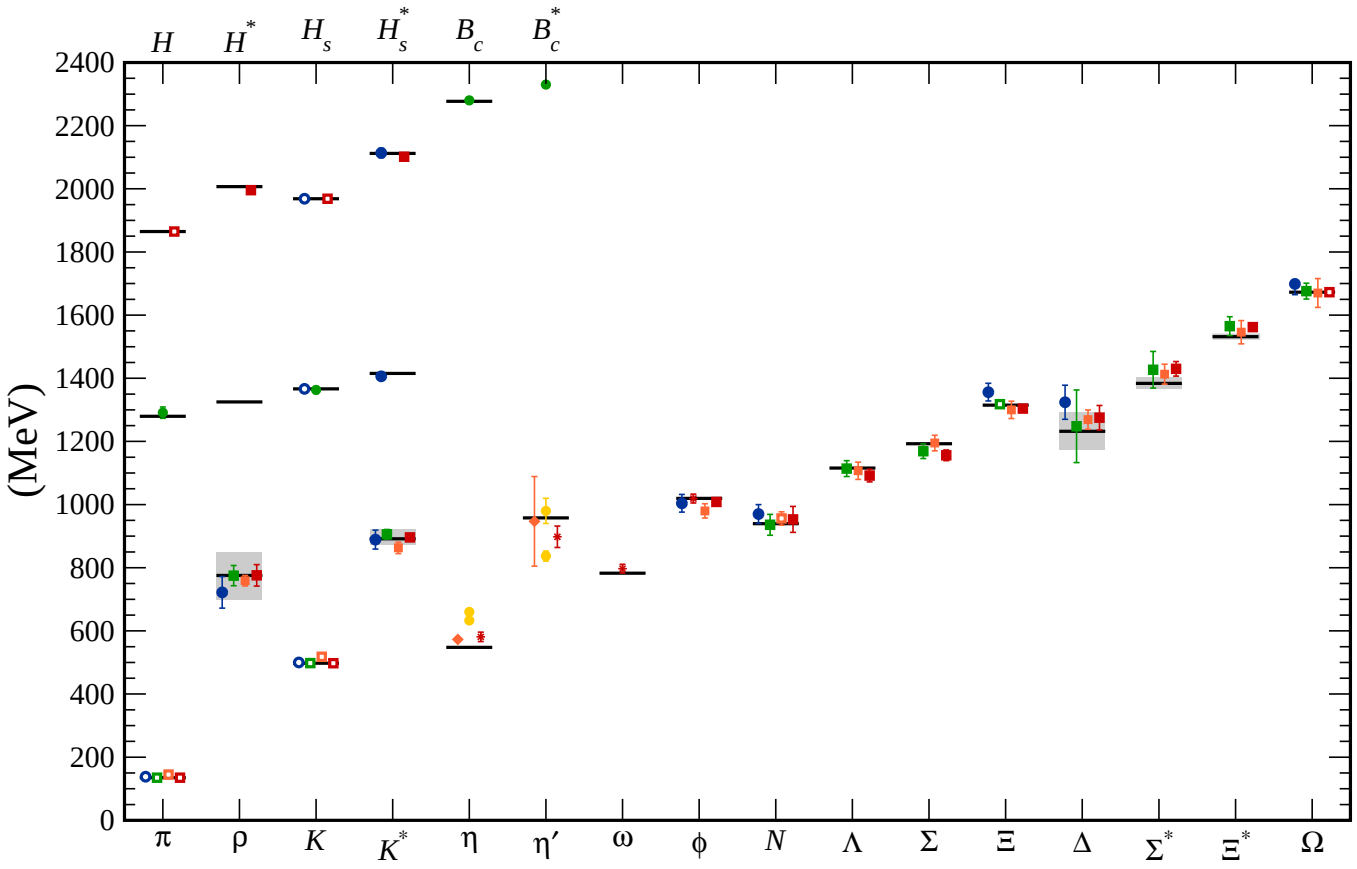


Figure : Hadron masses from lattice QCD: experimental vs. lattice QCD results (MILC, PACS-CS, BMW, QCDSF und RBC&UKQCD). From [Kronfeld arXiv:1203.1204].

Physics program of the RQCD collaboration

Hadron structure from lattice QCD

- Moments of parton distribution functions
  (Nucleon: arXiv:1207.1110, 1408.6850, pion in preparation)
- Nucleon isovector couplings (arXiv:1412.7336), sigma terms (arXiv:1206.7034, update in preparation), form factors and generalised form factors (in preparation)
- Moments of light-cone distribution amplitudes
  (Nucleon: arXiv:1403.4189, Pion: arXiv:1503.03656)
- Charmed baryon spectroscopy (arXiv:1503.08440)
- Strangeness Contribution to the Proton Spin (arXiv:1112.3354)
- ...

Up to now: $N_f = 2$ calculations from $m_\pi = 430 \ldots 150\,\mathrm{MeV}$ (this talk)

- safer extrapolation to physical point
- main uncertainties: volume, lattice spacing artefacts

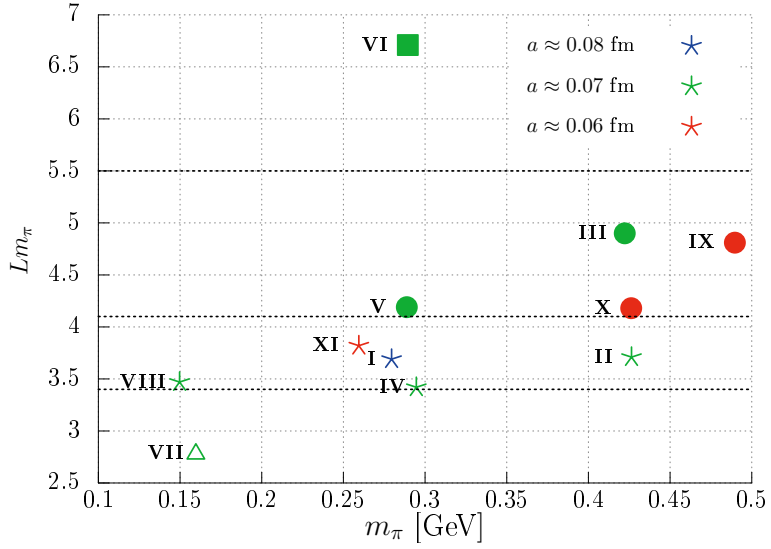
RQCD-Kollaboration: $N_f = 2$ ensembles

Action

- Wilson gauge
- $N_f = 2$ Wilson-clover fermions

Physical regime

- Lattice spacings:
  $a = 0.06 - 0.08\,\text{fm}$
  (2 spacings $m_\pi \approx 290$ MeV
  3 spacings $m_\pi \approx 425$ MeV).
- m_π down to 150 MeV.
- Lm_π up to 6.7,



Symbol	■	● ●	✳ ✳ ✳	△
Lm_π	≈ 6.7	> 4.1	> 3.4	≈ 2.8

Reduction of excited-state contaminations

- 300–600 Gauss-smearing iterations on top of APE smearing.
- Multi-exponential fits

Hadronic two- and three-point functions on the lattice

Hadronic matrix elements $\langle N|\mathcal{O}|N\rangle$ are obtained via:

1. two-point correlation functions (as above)
2. three-point correlation functions

$$C_{3pt}^{\mathcal{O}}(\tau, t, \vec{P}, \vec{P}') = \left\langle \tilde{h}(t, \vec{P}')\, \tilde{\mathcal{O}}(\tau, \Delta P)\, \tilde{h}^{\dagger}(0, \vec{P}) \right\rangle$$

- $\tilde{h}$ and $\tilde{\mathcal{O}}$ denote the Fourier transform of the hadronic and the insertion operator

$$\tilde{h}(t, \vec{P}) = \sum_{\vec{x}} e^{i\vec{P}\cdot\vec{x}} h(t, \vec{x}), \qquad \tilde{\mathcal{O}}(\tau, \Delta P) = \sum_{\vec{x}} e^{i\Delta P\cdot\vec{x}} \mathcal{O}(\vec{x}, \tau)$$

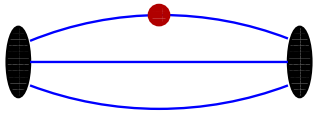
- Insertion operator: $\mathcal{O} = \{\gamma_5, \gamma_\mu, \gamma_\nu D_\mu, \ldots\}$ chosen wrt. desired matrix element, e.g.

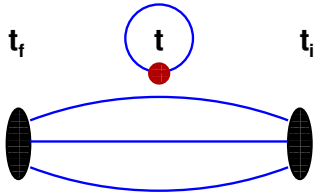
$$R(t, \tau) = \frac{C_{3pt}^{\mathcal{O}}(\tau, t)}{C_{2pt}(\tau)} = \frac{\langle N|\mathcal{O}|N\rangle}{2M_N} + \ldots$$

Illustration: Measurement of a three-point function on the lattice

Connected and disconnected contribution

Diagrams





On the Lattice

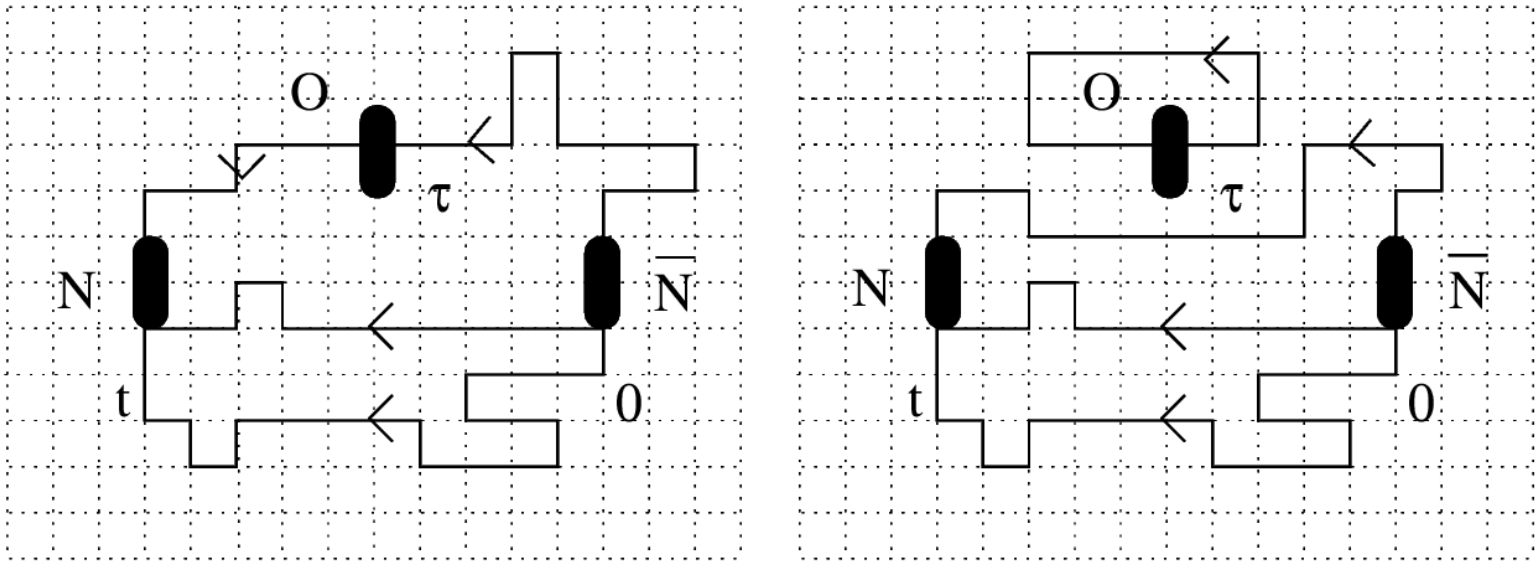


Figure : from R. Horsley (1999)

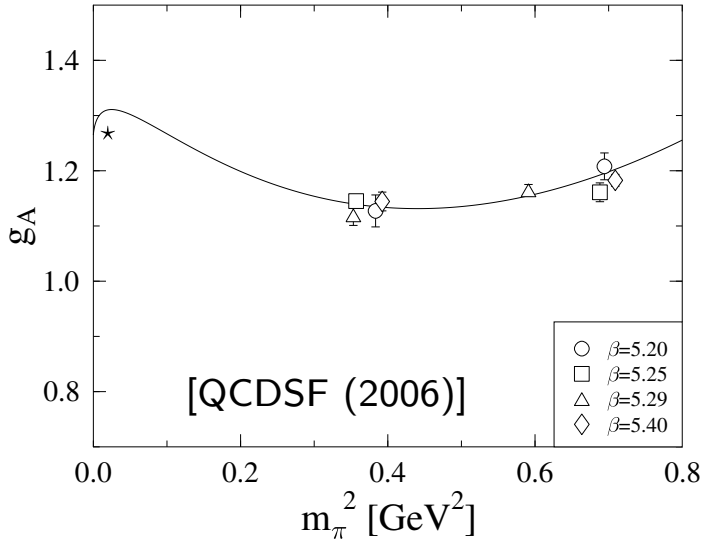
Nucleon axial charge g_A

- Associated with β decay (neutron $\rightarrow$ proton)
- Experimentally well known: $g_A = 1.2723(23) g_V$
  (isospin symmetric limit: $g_V = 1$)
- Theorie: $g_A = g_A(0)$ of the nucleon axial form factor:

$$\langle p|\bar{u}\gamma_\mu\gamma_5 d|n\rangle = \bar{u}_p(\mathbf{p}_\mathrm{f})\left[g_A(q^2)\gamma_\mu + \frac{\tilde{g}_P(q^2)}{2M_N}q_\mu\right]\gamma_5 u_n(\mathbf{p}_\mathrm{i})$$

Lattice QCD

- Good benchmark observable for a lattice calculation
- Challenge for many years (underestimate of exp. value)
- Conjecture: volume effect



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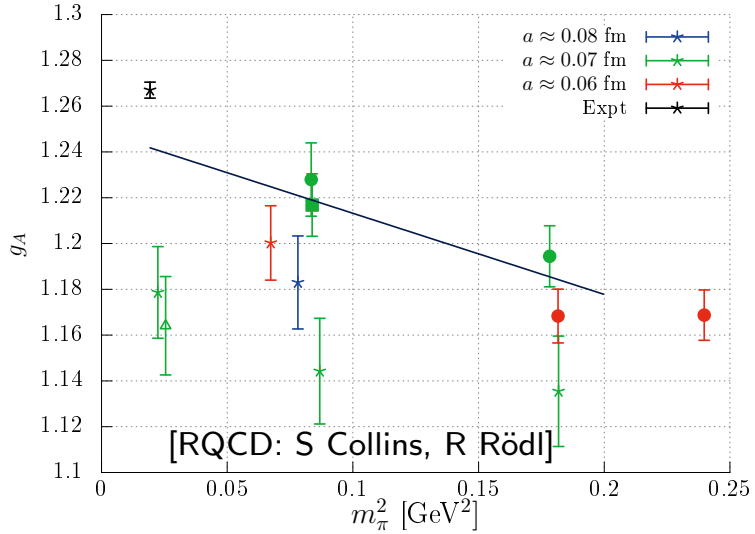
Lattice QCD

- New data from RQCD (arXiv:1412.7336)
- Clear volume effect for fixed lattice spacing
- No significant discret. errors for fixed volume



Symbol	■	● ●	* * *	△
Lm_π	≈ 6.7	> 4.1	> 3.4	≈ 2.8

Nucleon axial charge g_A: Finite volume effects

arXiv:1412.7336

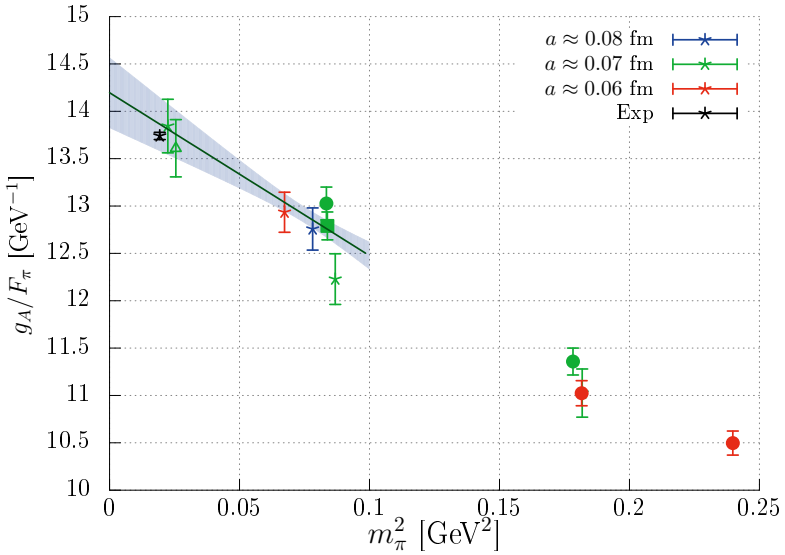
Prediction from χPT: Similar finite volume effects for g_A and F_π

Follow `QCDSF [1302.2233]` and plot/extrapolate ratio

Extrapolation to phys. point

Fit: $g_A/F_\pi = 13.88(29)\,\mathrm{GeV}^{-1}$

Exp: $g_A/F_\pi = 13.797(34)\,\mathrm{GeV}^{-1}$

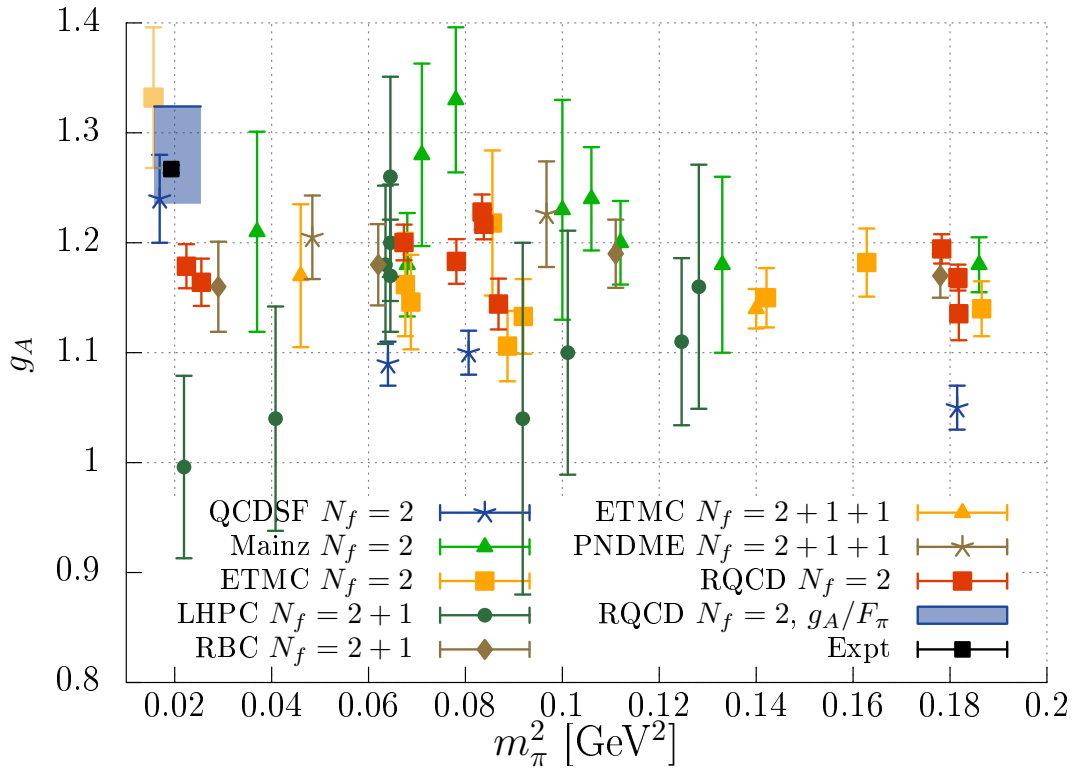


With $F_\pi(\mathrm{exp}) = 92.21$ MeV we obtain

$$g_A = 1.280(27)(35) \qquad \text{Exp:} \quad g_A = 1.2670(35)$$

Comparison of recent lattice results for g_A

arXiv:1412.7336



- World lattice data approaches physical regime ($m_\pi \approx 135\,\text{MeV}$)
- Reproduction of experimental value for g_A if excited-states and volume effects are under control

Nucleon (isovector) couplings

arXiv:1412.7336

Calculated also all other couplings

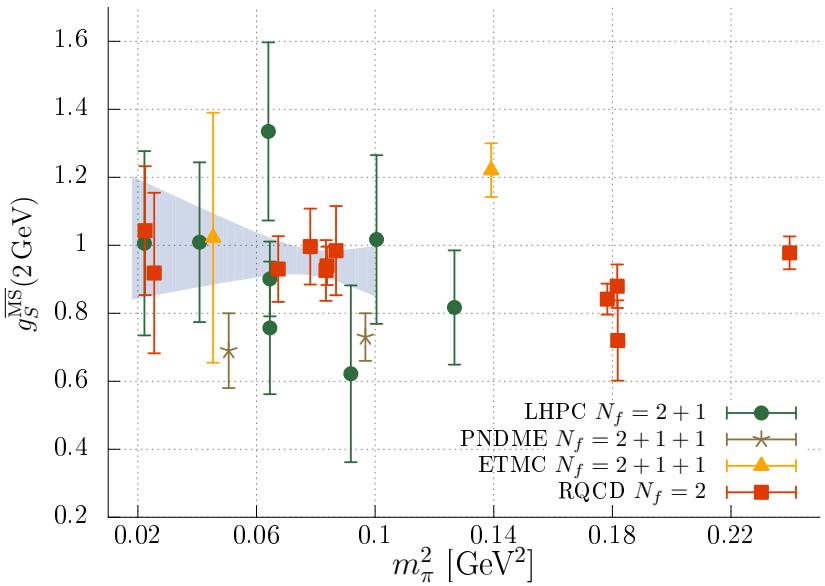
- Besides g_T, all couplings are directly obtained in the forward limit $q^2 = 0$
- g_T from the form factor data $g_T(q^2 \to 0)$

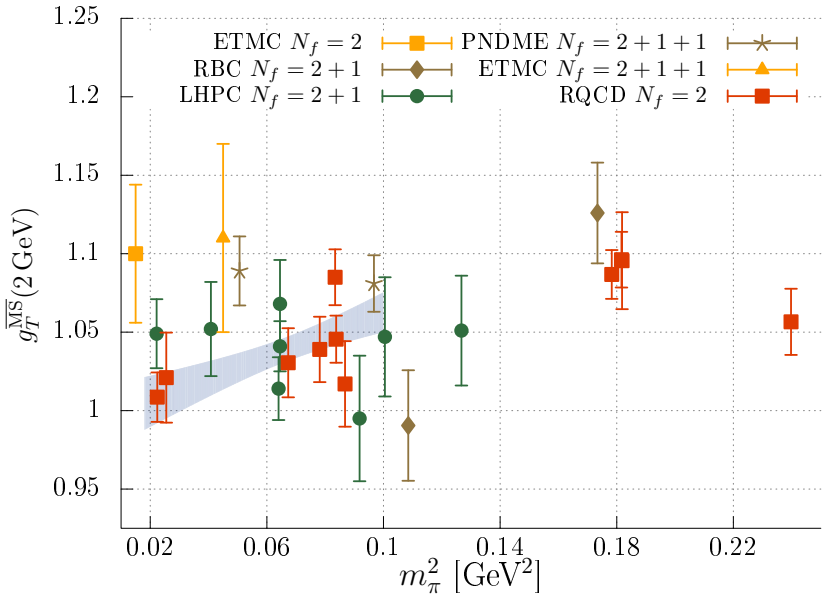
$$\begin{aligned}
\langle p|\bar{u}d|n\rangle &= g_S(q^2)\,\bar{u}_p(\mathbf{p}_\mathrm{f})u_n(\mathbf{p}_\mathrm{i})\,,\\
\langle p|\bar{u}\gamma_5 d|n\rangle &= g_P(q^2)\bar{u}_p(\mathbf{p}_\mathrm{f})\gamma_5 u_n(\mathbf{p}_\mathrm{i})\,,\\
\langle p|\bar{u}\gamma_\mu d|n\rangle &= \bar{u}_p(\mathbf{p}_\mathrm{f})\left[g_V(q^2)\gamma_\mu + \frac{\tilde{g}_T(q^2)}{2M_N}i\sigma_{\mu\nu}q^\nu\right]u_n(\mathbf{p}_\mathrm{i})\,,\\
\langle p|\bar{u}\gamma_\mu\gamma_5 d|n\rangle &= \bar{u}_p(\mathbf{p}_\mathrm{f})\left[g_A(q^2)\gamma_\mu + \frac{\tilde{g}_P(q^2)}{2M_N}q_\mu\right]\gamma_5 u_n(\mathbf{p}_\mathrm{i})\,,\\
\langle p|\bar{u}\sigma_{\mu\nu} d|n\rangle &= g_T(q^2)\bar{u}_p(\mathbf{p}_\mathrm{f})\sigma_{\mu\nu}u_n(\mathbf{p}_\mathrm{i})
\end{aligned}$$

- Induced tensor charge $\tilde{g}_T$ known $\to$ another benchmark observable
- Other charges more or less unknown, might be relevant for new physics processes or dark matter searches
- g_T and g_S accessible at present only through lattice simulation.

Lattice results for g_S and g_T

arXiv:1412.7336





- We find $g_S^{\overline{\rm MS}}(2\,{\rm GeV}) = 1.02(18)(30)$ and $g_T^{\overline{\rm MS}}(2\,{\rm GeV}) = 1.005(17)(29)$ when approaching phys. point
- In agreement with other determinations
- Lattice spacing effects not excluded ($a = 0.06 \ldots 0.08\,$fm)

Nucleon (isovector) couplings

arXiv:1412.7336: Final results extrapolated to physical point

Table : First errors contain statistics and systematics. The second errors are estimates of lattice spacing effects.

	Our result	Experiment
g_A	1.280(44)(46)	1.2723(23)
$g_S^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$	1.02(18)(30)	—
$g_T^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$	1.005(17)(29)	—
$\tilde{g}_T$	3.00(08)(31)	3.7058901(5)
g_P^*	8.40(40)(159)	8.06(55)

- Couplings by definition valence quark quantities $\rightarrow$ no significant effects from including strange (or charm) sea quarks expected
- Chiral extrapolation to physical point under control (estimate at $m_\pi \approx 150\,\mathrm{MeV}$ allows us to estimate uncertainties)
- Deviation of $\tilde{g}_T$ not yet clear

Moments of Parton Distributions (PDFs)

Nucleon parton distribution function

- Important for phenomenology
- Known for large range of $x = [0, 1]$ (long. mom. fraction)
- Good knowledge for u and d
- For other quarks not as good

Lattice QCD allows only access to moments of PDFs

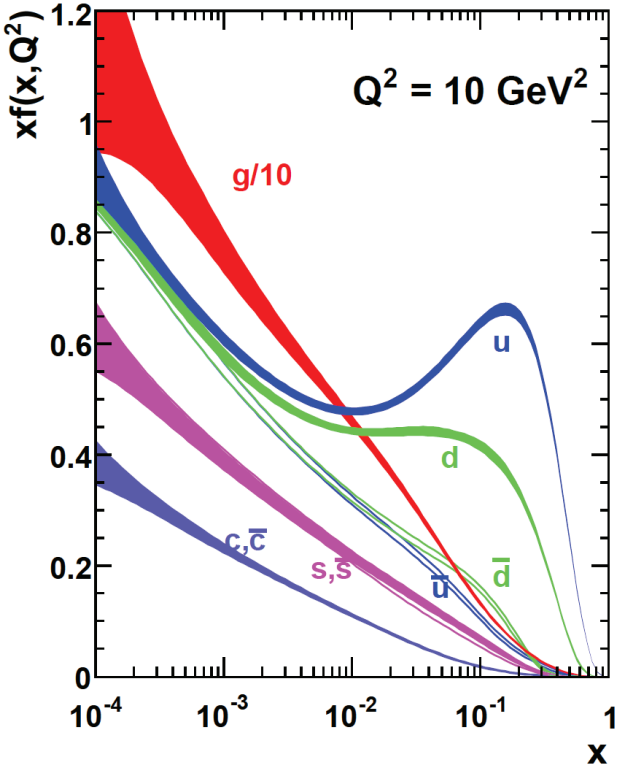
Lowest moment of nucleon PDF

$$\langle x \rangle_{(q)} = \int_0^1 dx\, x\, [q(x) + \bar{q}(x)]$$

Ever popular moment: $\langle x \rangle_{(u-d)}$

- no disconn. contributions needed
- discrepancy: lattice vs. exp.

Armstrong et al. (2012)



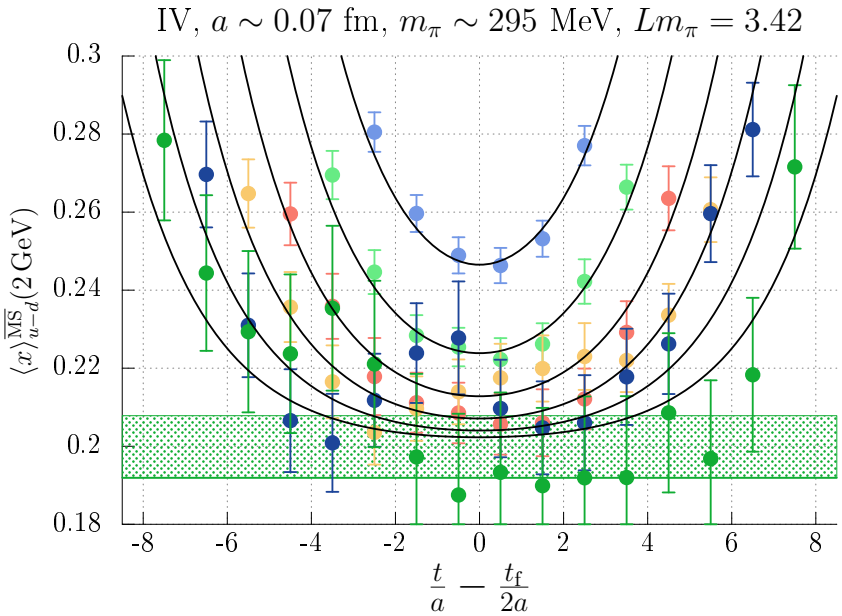
Reduction of systematic error for $\langle x \rangle_{u-d}^{\overline{MS}}$

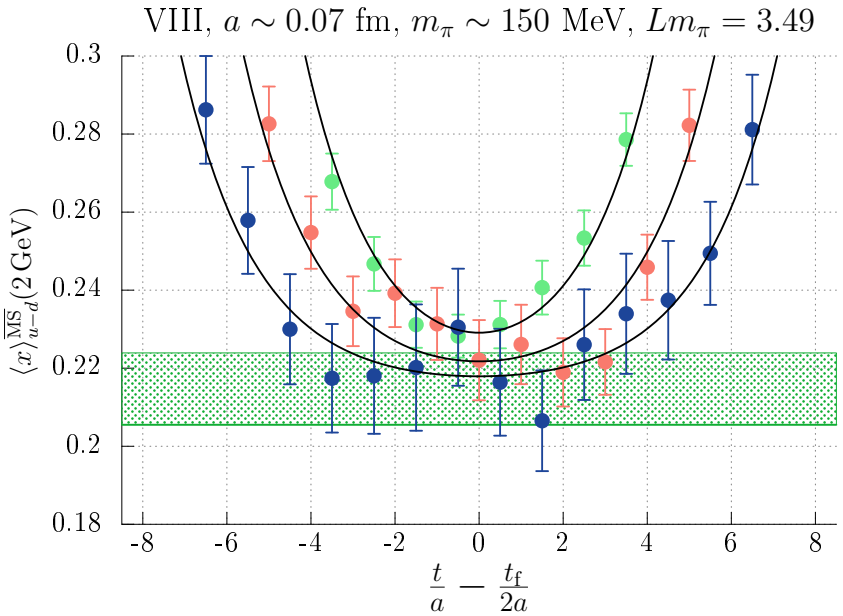
Fitting of leading excited states

- Access via 2- and 3-point functions (operator O with derivative)
- For moderate τ and $|\tau - t_f|$ also contributions from excited states

$$C_{2\mathrm{pt}}(t_\mathrm{f}) = |Z_0|^2 e^{-m_0 t_\mathrm{f}} + |Z_1|^2 e^{-m_1 t_\mathrm{f}} + \ldots$$

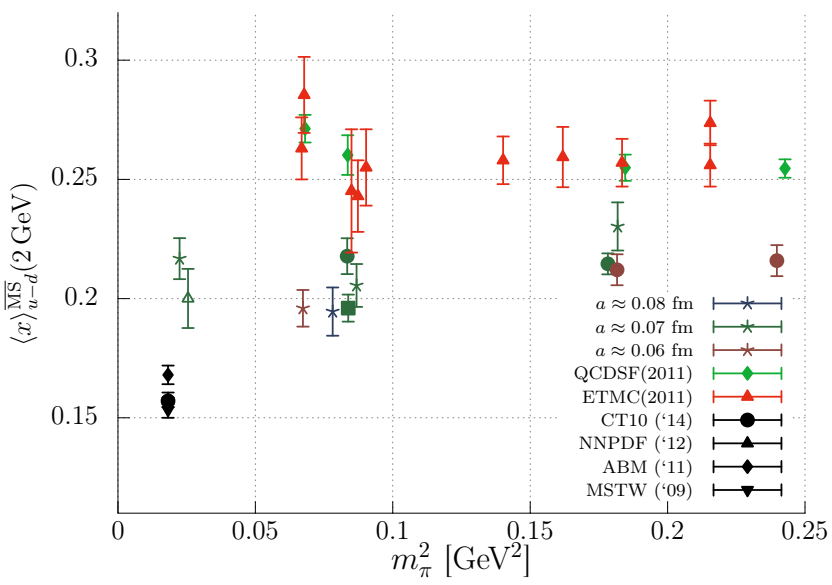
$$C_{3\mathrm{pt}}(t_\mathrm{f}, t) = |Z_0|^2 \langle N_0 | O | N_0 \rangle e^{-m_0 t_\mathrm{f}} + |Z_1|^2 \langle N_1 | O | N_1 \rangle e^{-m_1 t_\mathrm{f}} + Z_1^* Z_0 \langle N_1 | O | N_0 \rangle e^{-m_0 t} e^{-m_1 (t_\mathrm{f} - t)} + Z_0^* Z_1 \langle N_0 | O | N_1 \rangle e^{-m_1 t} e^{-m_0 (t_\mathrm{f} - t)} + \ldots$$

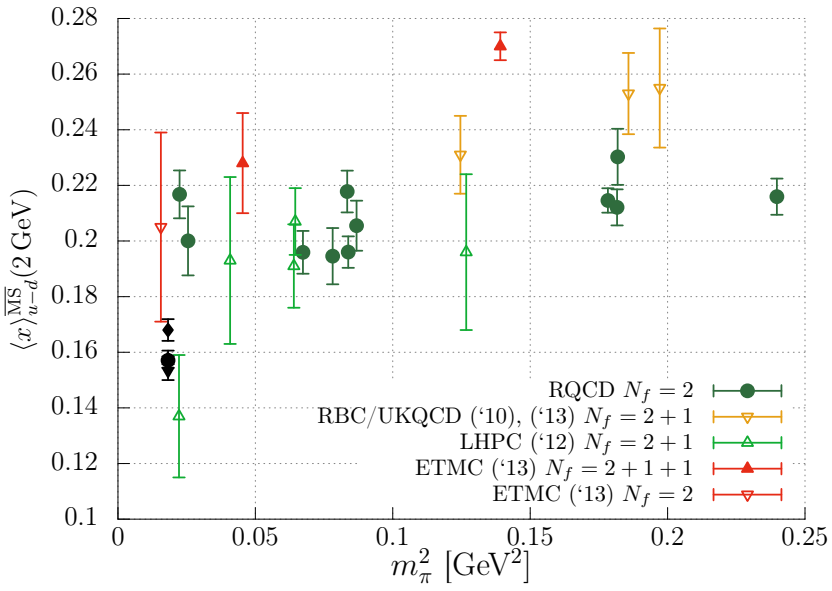




Lowest moment of nucleon PDF $\langle x \rangle_{u-d}$

Status





- Deviation to phenom. value reduced (better treatment of excited-states)
- Discrepancy remains though
- Further issues ?

Physics program of the RQCD collaboration

Hadron structure from lattice QCD

- Moments of parton distribution functions
  (Nucleon: arXiv:1207.1110, 1408.6850, pion in preparation)
- Nucleon isovector couplings (arXiv:1412.7336), sigma terms (arXiv:1206.7034, update in preparation), form factors and generalised form factors (in preparation) $\rightarrow J^q$
- Moments of light-cone distribution amplitudes
  (Nucleon: arXiv:1403.4189, Pion: arXiv:1503.03656)
- Charmed baryon spectroscopy (arXiv:1503.08440)
- Strangeness Contribution to the Proton Spin (arXiv:1112.3354)
- ...

Up to now: $N_f = 2$ calculations from $m_\pi = 430 \ldots 150\,\mathrm{MeV}$ (this talk)

- safer extrapolation to physical point
- main uncertainties: volume, lattice spacing artefacts

Pion-nucleon sigma term

Decomposition of nucleon mass

$$M_N = \frac{\langle N|T_{44}|N\rangle}{\langle N|N\rangle} = \underbrace{\sum_q \sigma_q}_{\text{small}} + \underbrace{\frac{4}{3\langle N|N\rangle}\left\langle N\left|\int d^3x\left(\frac{1}{8\pi\alpha}(\mathbf{E}^2-\mathbf{B}^2)+\sum_q \bar{q}\gamma\cdot\mathbf{D}q\right)\right|N\right\rangle}_{\text{dominant}}$$

- T_{44}... energy density component of energy-momentum tensor
- Nucleon mass is largely generated by gluon field energy (≈ 900 MeV) ($M_N = 938\,\text{MeV}$, $m_u \approx 2\,\text{MeV}$, $m_d \approx 4\,\text{MeV}$)
- Small fraction from sea and valence quarks

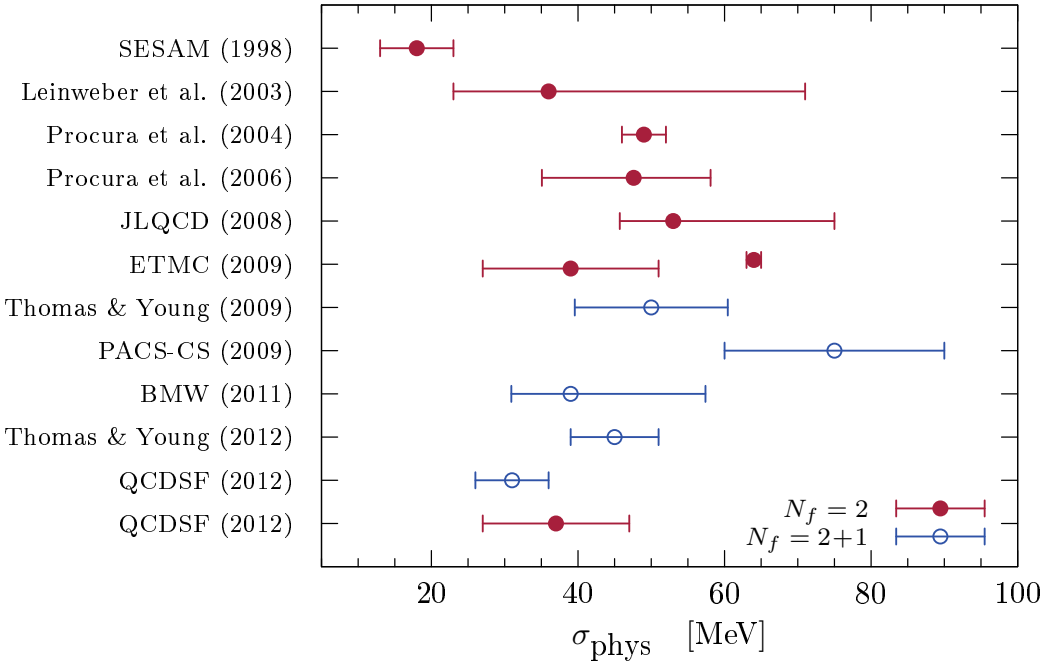
σ-terms

- parametrizes individual quark contribution $f_q = \sigma_q/M_N$ to nucleon mass

$$\sigma_q = m_q \frac{\langle N|q\bar{q}|N\rangle}{\langle N|N\rangle}$$

- $\boxed{\text{Pion-nucleon sigma term:} \quad \sigma_{\pi N} = \sigma_u + \sigma_d}$
- VEV $\langle 0|\bar{q}q|0\rangle$ is understood to be subtracted from $\langle N|\bar{q}q|N\rangle$
- Phenomenology does not give a clear answer on (45 – 65 MeV)
- Lattice allows for direct calculation (involves disconnected contributions)

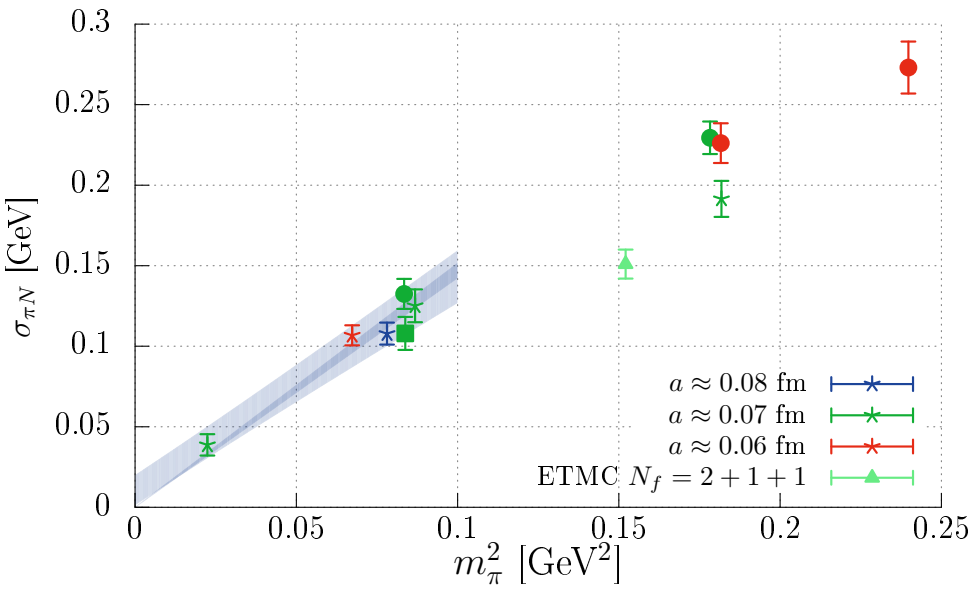
Pion-nucleon sigma term from the lattice



- Most calculations via Feynman-Hellmann theorem $\rightarrow \quad \sigma_{\pi N} = m_\pi^2 \dfrac{\partial M_N(m_\pi)}{\partial m_\pi^2}$
- New determinations are done directly and include contributions from disconnected quark loops (expensive and noisy)
- Central value changed over the years: $50 \ldots 60 \quad \rightarrow \quad 30 \ldots 40\,\text{MeV}$

Pion-nucleon sigma term from the lattice

New data from RQCD, to be published soon



$Lm_\pi \approx 6.7$: ■
$Lm_\pi > 4.1$: ● ●
$Lm_\pi > 3.4$: * * *

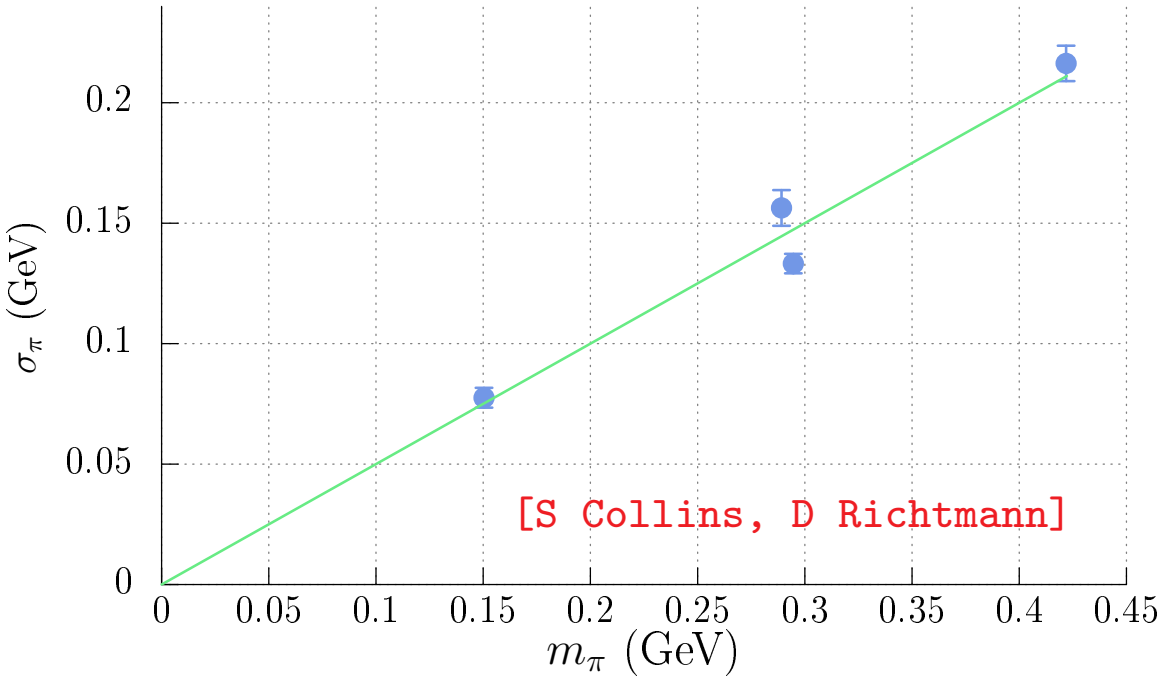
[S Collins]

- Linear Fit with and without constraint $\sigma_{\pi N}(0) = 0$ gives $\sigma_{\pi N} \approx 35\,\text{MeV}$
- The non-vanishing light quark masses responsible for only 35 MeV of M_N
- Not unexpected: $M_N \gg 0$ as $m_{ud} \to 0$

Note: quark contribution to pion mass $m_\pi/2$!

Decomposition of pion mass and the Pion sigma term

$$m_\pi \approx \underbrace{\frac{1}{2} m_\pi}_{\sigma_\pi} + \underbrace{\frac{3}{8} m_\pi}_{\text{gluon int.}} + \underbrace{\frac{1}{8} m_\pi}_{\text{trace anomaly}} \qquad \text{with} \quad \sigma_\pi = m_{ud} \langle \pi | \bar{u}u + \bar{d}d | \pi \rangle = m_{ud} \frac{\partial m_\pi}{\partial m_{ud}}$$

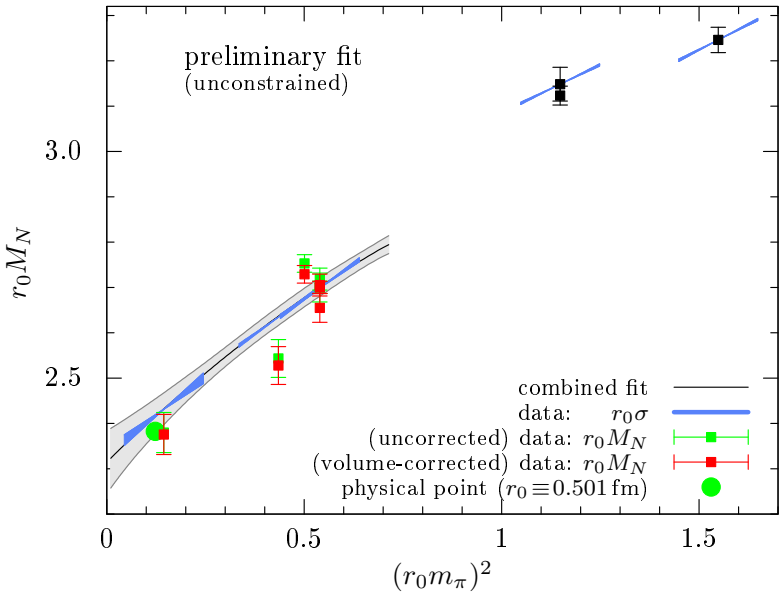


The theoretical expectation $\sigma_\pi \approx m_\pi/2$ is confirmed.

Side remark: Chiral extrapolation of the nucleon mass

- Lattice QCD simulations produce bare numbers
- Need to calculate physical quantity (mass, decay constant, . . .)
- Need to extrapolate to physical point (systematic uncertainty)

- Extrapolation of M_N data desired but yet too uncertain
- Sigma-term values (slope) constrain the chiral fit (basically only offset is left free)



$\rightarrow$ $\sigma_{\pi N}$ values make chiral extrapolation of M_N data much less ambiguous.

Summary

- Lattice can contribute to many quantities that are hard to constrain by experiment: $\sigma_{\pi N}$, g_S, g_T, Δq, $\Delta\Sigma$, J_q, $\langle x \rangle_{\Delta q}$, ...
- Benchmark observables help to improve lattice methods
  - Hadron masses (2-point functions) come out nicely
  - g_A (3-point functions) seems to approach the physical value, once $Lm_\pi > 4$
  - $\langle x \rangle_{u-d}$ comes out 20% bigger than expected
- Up to now: $N_f = 2$ calculations from $m_\pi = 430 \ldots 150\,\text{MeV}$ with lattice spacings: $a = 0.06 \ldots 0.08\,\text{fm}$ (this talk)
- Ultimately precision physics requires an extrapolation $a \to 0$.
- **Upcoming:** $N_f = 2 + 1$ calculations targetting physical point and continuum limit (part of the CLS effort).
- Disconnected quark line diagrams will more and more be included
  - in the past often omitted and differences quoted: g_A, $\langle x \rangle_{u-d}$, ..., but no Δs, $\Delta\Sigma$, J_q, $\langle x \rangle_q$ etc.
  - See, e.g., quark spin contribution to proton spin $\Delta\Sigma$

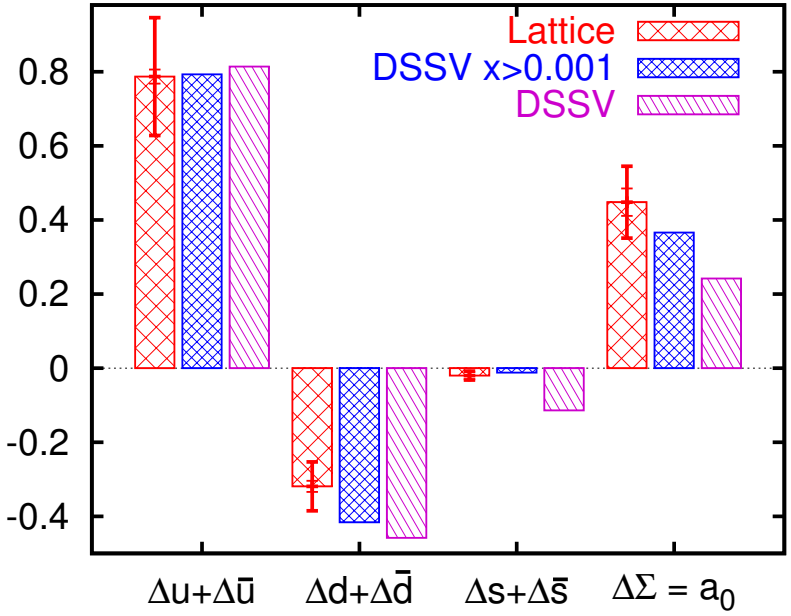
Quark spin contributions to proton spin

Bali et al., PRL108 (2012) 222001

- Proton Spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G$$

- Results for $\overline{\mathrm{MS}}$ scheme at $\mu^2 = 7.4\ \mathrm{GeV}^2$



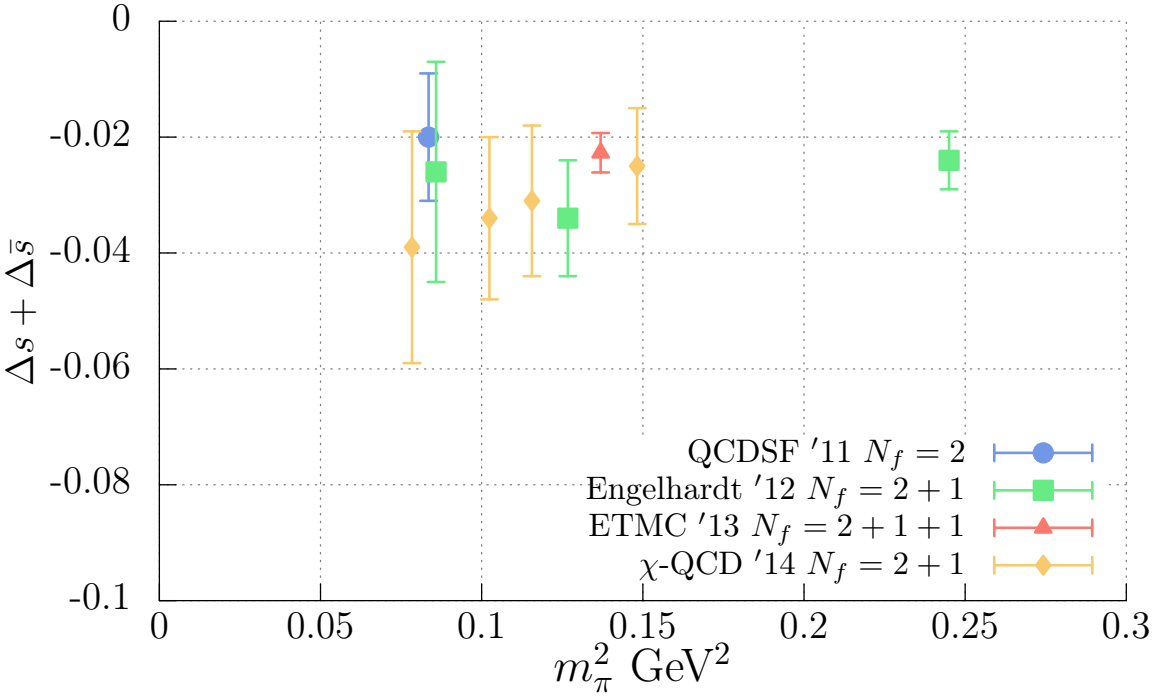
$$\Delta\Sigma = \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s} = 0.45(4)(9)$$

$$\Delta s + \Delta\bar{s} = -0.020(10)(4)$$

Warning: no continuum limit, $m_\pi \approx 290$ MeV $\Rightarrow$ add 20 % systematic error.

Quark spin contributions to proton spin

Comparison of recent lattice calculations



Consistency between different determinations: small $\Delta s + \Delta\bar{s}$.

ETMC result shows statistical accuracy that is possible. Systematics!

[QCDSF: GB et al, 1112.3354; M Engelhardt, 1210.0025; ETMC: A Abdel-Rehim et al, 1310.6339; χQCD: Y Yang et al, unpublished.]

Leaving the traditional way lattice QCD is used for hadron physics

Lattice QCD

- gauge-invariant nonperturbative approach to QCD
- Systematically improvable
  (larger and finer lattices, tune bare parameters towards physical point)
- Allows access to many interesting quantities without a need to fix a gauge

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- Allows access to many interesting quantities without a need to fix a gauge

Lattice QCD allows more ...

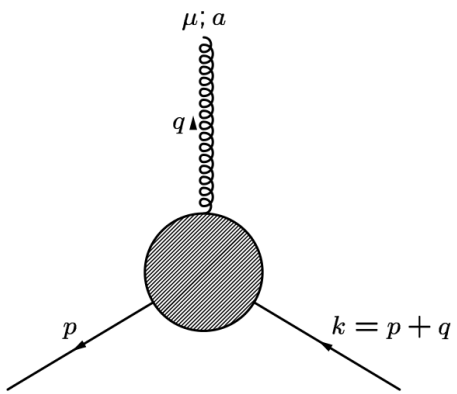
- Apply a gauge condition $\rightarrow$ access to nonperturbative and <u>untruncated</u> elementary n-point functions of QCD
  - <u>2-point functions:</u> quark, gluon and ghost propagators
    Many lattice studies in the past helped to settle momentum dependence
  - <u>3-point functions:</u> quark-gluon, 3-gluon, 4-gluon, ghost-gluon vertex
- Essential information for functional approaches to hadron physics (e.g. QCD Dyson-Schwinger and bound-state equations)
  - perturbative, semi-perturbative and non-perturbative regimes
  - e.g: validity of Rainbow-Ladder truncations etc.

New lattice results for Quark-Gluon-Vertex in Landau gauge

in collaboration with Kizilersu, Oliveira, Silva, Skullerud

Quark-Gluon vertex

$$G_\mu^{\bar\psi\psi A} = \Gamma_\lambda^{\bar\psi\psi A}(p,q) \cdot S(p) \cdot D_{\mu\lambda}(q) \cdot S(p+q)$$



- Non-perturbative structure mostly unknown, but crucial for many functional approaches to hadron physics (Dyson-Schwinger, Bethe-Salpether equations etc.)
- Up to now: only lattice data for quenched QCD

Ball-Chiu-Parametrisation

$$\Gamma_\mu^{\bar\psi\psi A}(p,q) = \Gamma_\mu^{ST}(p,q) + \Gamma_\mu^{T}(p,q)$$

with $\Gamma_\mu^{ST}(p,q) = \sum_{i=1}^{4} \lambda_i(p^2,q^2) L_{i\mu}(p,q)$ satisfies Slavnov-Taylor identies

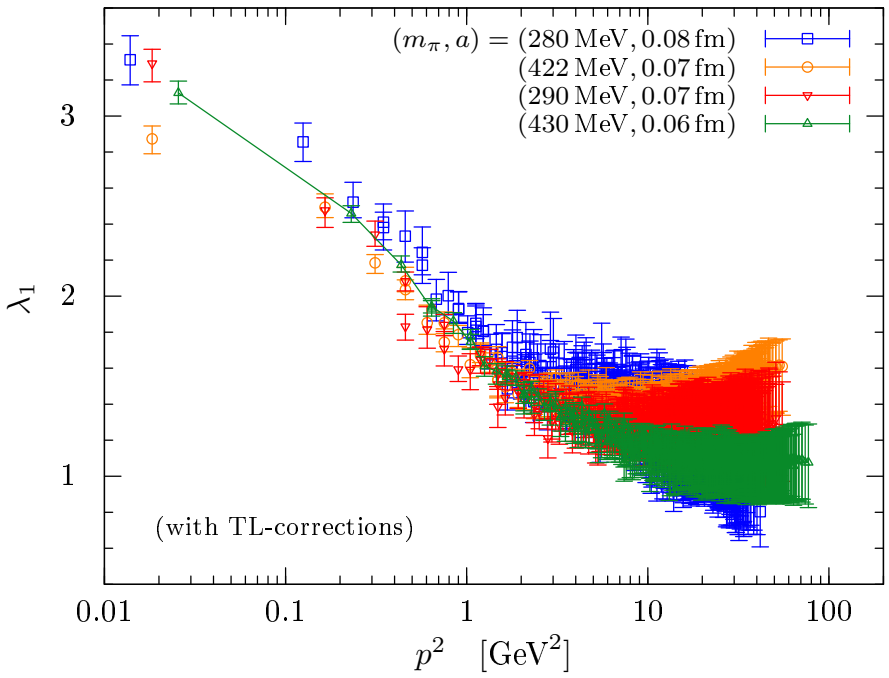
$\Gamma_\mu^{T}(p,q) = \sum_{i=1}^{8} \tau_i(p^2,q^2) T_{i\mu}(p,q)$ is transverse ($q_\mu \Gamma_\mu^T = 0$).

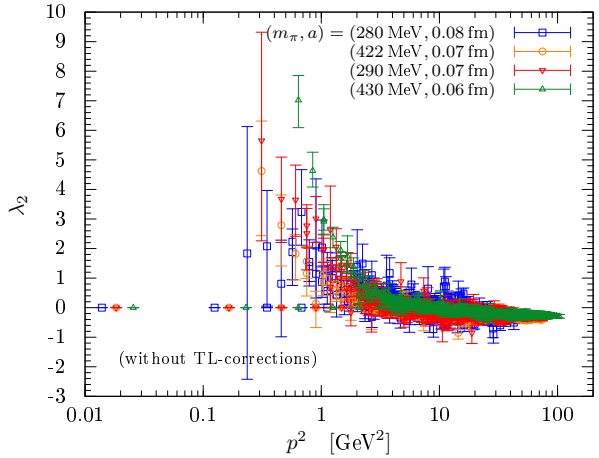
The tensors L_μ and T_μ can be found in the literature: $L_{1\mu}(p,q) = \gamma_\mu$, $L_{2\mu}(p,q) = -\gamma_\mu(2p_\mu + q_\mu)$, ..., $T_{1\mu}(p,q) = i[p_\mu q^2 - q_\mu(p\cdot q)]$, ...

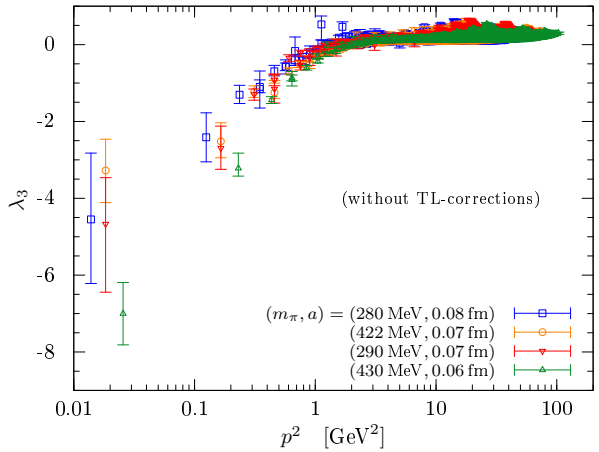
New lattice results for Quark-Gluon-Vertex in Landau gauge

(Be careful, results are preliminary yet!)

Example: $q = 0$ (soft gluon) $\rightarrow \tau_i(p, 0) = 0$



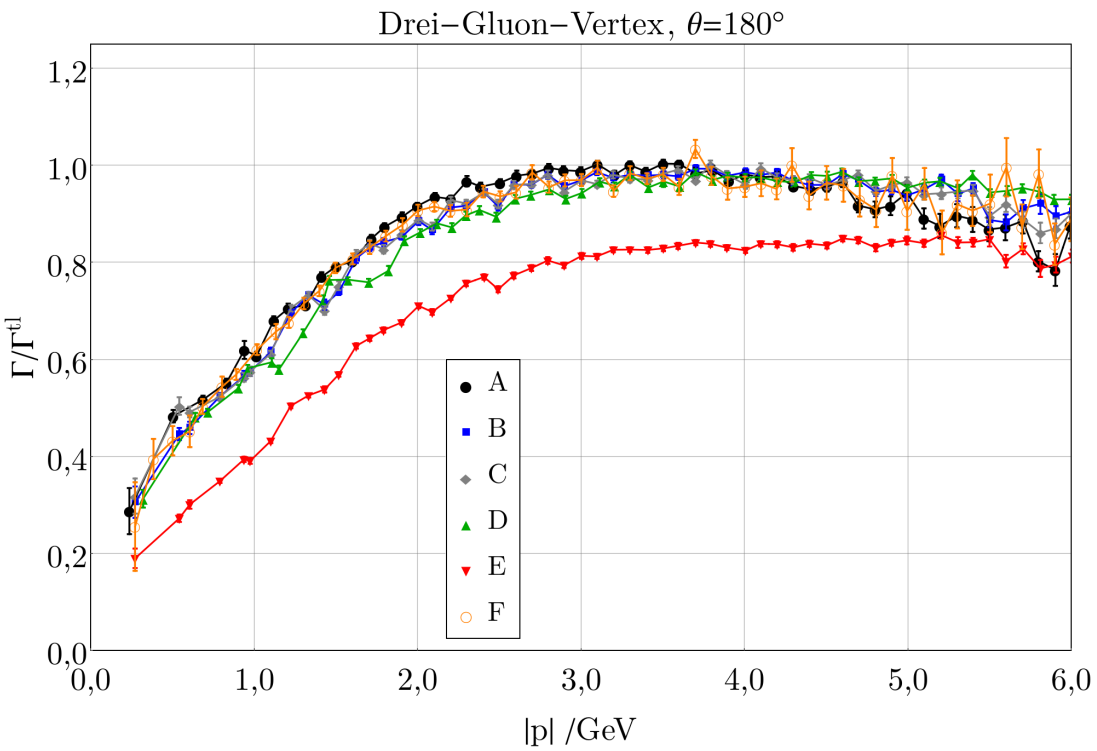




- Discretization effect not yet under control
  (hypercubic artefacts for $p^2 > 3$ due to lattice)
- Deviation from tree-level for $p < 1\,\text{GeV}$
- For $p > 1\,\text{GeV}$: Vertex $\sim$ tree-level
  (might explain why Rainbow ladder truncation so successful)

Lattice results for 3-Gluon vertex (preliminary)

together with BSc. Balduf



	a [fm]	m_π [GeV^2]
A	0.08	280
B	0.07	422
C	0.07	290
F	0.07	290
D	0.06	430
E	0.07	—

- $|p| = |q|$, $\theta = angle(p, q)$
- Statistical noise grows with $|p| = |q|$
- Small quark-mass dependence (slight upward shift) for the available sets, but a clear quenching effect (vertical shift)
- No "zero-crossing" for $p > 0.3\,\text{GeV}$ ($p > 0.8\,\text{GeV}$ quenched)

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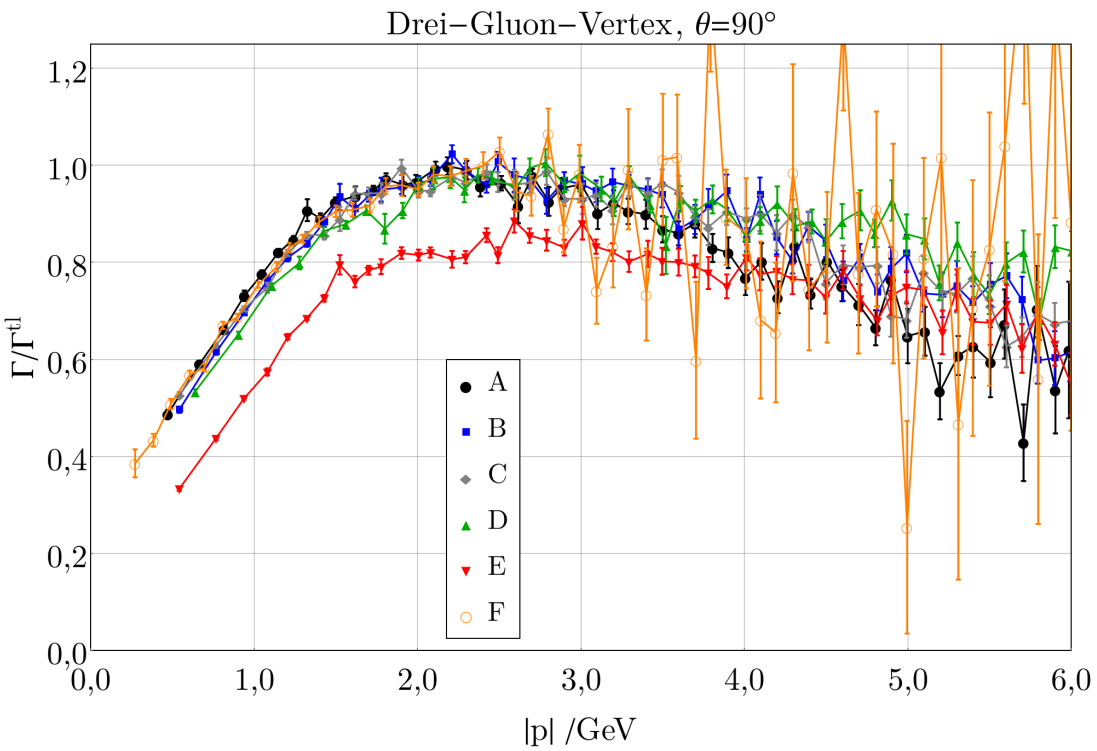

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Thank you for your attention!