Hadron Structure from Lattice QCD

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\textsuperscript{b} SFB/TRR 55 “Hadron Physics from Lattice QCD”

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Contents

- Lattice QCD / hadron physics in a nutshell
- Hadronic quantities from lattice QCD
  - Lattice results of the RQCD collaboration near the physical point ($N_f = 2$)
    $g_A, g_S, g_T$, ..., $\langle x \rangle_{u-d}$, $\sigma_{\pi N}$, $\Delta \Sigma$, $\Delta s + \Delta \bar{s}$
- Summary
- Add-on: New input for functional approaches to hadron physics
  - Lattice results for Quark-Gluon and 3-Gluon vertex in Landau-gauge
Lattice regularization of QCD (LQCD)

QCD action

\[ S_{\text{QCD}} = \int d^4x \left[ -\frac{1}{2g_0^2} \text{Tr} F^2[A] + \sum_f \bar{\psi}_x^f \left( \mathcal{D}[A] - m_0^f \right) \psi_x^f \right] \]

Discretization of Euclidean space-time

- Introduce 4-dim lattice \( L^3 \times T \)
  \[ x = n a^4, \quad n \in \mathbb{Z}^4 \]
- Quark fields \( \psi \) dwell on sites
- But: naive discretization of gluon field \( A_\mu(x) \) not gauge-invariant at finite \( a \)
Lattice regularization of QCD (LQCD) introduced by K. Wilson (1974-77)

**Parallel transporter** between \( x \) and \( x + \hat{\mu} \)

\[
U_\mu(n) = P e^{iag_0 \int^n_{n+\hat{\mu}} A_\mu(z) dz} \in SU_c(3)
\]

Using "links" \( U_\mu(n) \) for discretization gives a gauge-invariant action for any \( a \).

**Example:** unimproved Wilson action \((a \equiv 1)\)

\[
S_{W}^{LQCD} = \beta \sum_{n,\mu<\nu} \left( 1 - \frac{1}{3} \Re \text{Tr} \Box_{n,\mu\nu} \right) + \sum_{n,m,f} \overline{\psi}_n D_{nm}^{W}[U,\kappa_f] \psi_m \xrightarrow{a \to 0} S_{QCD}
\]

- Wilson fermion matrix: \( D_{nm}^{W}[U,\kappa_f] = \delta_{nm} - \kappa_f \sum_{\pm \mu} \delta_{m,n+\hat{\mu}} (1 + \gamma_\mu) U_{n\mu} \) (breaks chiral symmetry)
- QCD Parameters: \( \beta \equiv 6/g_0^2, \quad \kappa_f \equiv 1/(2m_f^0 - 8) \)

Other (improved) lattice actions possible, requirement: \( S_{LQCD} \xrightarrow{a \to 0} S_{QCD} \)
Lattice QCD calculation

**Expectation value** of an observable via path integral

\[
\langle O \rangle = \frac{1}{Z} \int \prod_{n\mu} dU_{n\mu} \ d\bar{\psi}_n \ d\psi_n \ O[U, \bar{\psi}, \psi] \ e^{-(S_g[U]+S_f[U, \bar{\psi}, \psi])}
\]

- No gauge-fixing needed!
- Integrate over fermionic fields exactly which results in a modified action \( S_{\text{eff}} \) and \( O \) (non-local functionals of the links \( U \))

\[
S_{\text{eff}}[U; \beta, \kappa] = S_g[U; \beta] + \log \det D[U; \kappa]
\]

**Side remark:** integration over Grassmann numbers gives

\[
\det D \cdot \sum_{k_1 \cdots k_n} \epsilon_{j_1 j_2 \cdots j_n} D_{k_1 i_1}^{-1} \cdots D_{k_n i_n}^{-1} = \int \prod_n d\bar{\psi}_n d\psi_n \cdots \psi_1 \bar{\psi}_1 \cdots \psi_n \bar{\psi}_n \cdots e^{-\bar{\psi}D\psi}
\]

- Leaves us with **Master integral** for expectation value

\[
\langle O \rangle_{\beta, \kappa, L} = \frac{1}{Z} \int \prod_{n\mu} dU_{n\mu} \ \hat{O}[U] \ e^{-S_{\text{eff}}[U, \beta, \kappa]}
\]
Master integral is very-high dimensional integral

\[
\langle O \rangle_{\beta, \kappa, L} = \frac{1}{Z} \int \prod_{n\mu} dU_{n\mu} \hat{O}[U] e^{-S_{\text{eff}}[U, \beta, \kappa]}
\]

- Number of integration variables
  \((N_c = 3, N_d = 4)\)
  
  \[
  64^3 \times 128 \times (N_c^2 - 1) \times N_d = 1073741824
  \]
  
  \[
  16^3 \times 32 \times (N_c^2 - 1) \times N_d = 4194304
  \]

- Calculate integrals stochastically

Monte-Carlo integration: Sample links \(U\) with probability density
("important sampling")

\[
P[U^{(i)}] = \frac{1}{Z} e^{-S_{\text{eff}}[U^{(i)}]}
\]

Markov chain: \(U^{(1)}, U^{(2)}, \ldots, U^{(N)} \in SU(3)\)

Estimate for expectation value:

\[
\overline{\langle O \rangle} \equiv \frac{1}{N} \sum_{i=1}^{N} \hat{O}[U^{(i)}] \xrightarrow{N \to \infty} \langle O \rangle_{\beta, \kappa, L}
\]
Hadron mass from the lattice

Hadronic 2-point function

\[
C_{2pt}^h(\vec{p}, t) = \frac{1}{\sqrt{V_S}} \sum_{\vec{x}} e^{i \vec{x} \cdot \vec{p}} \left\langle h(\vec{x}, t) h(\vec{0}, 0) \right\rangle_U
\]

\[
(\vec{p}=0) = |Z_0|^2 e^{-m_0 t} + |Z_1|^2 e^{-m_1 t} + \ldots
\]

\[\begin{align*}
\text{Monte-Carlo integration} & \quad \langle h_{\pi^+}(x) \bar{h}_{\pi^+}(y) \rangle = \frac{1}{N} \sum_{i=1}^{N} \text{Tr} \left( \gamma_5 D_{xy}^{-1}[U^{(i)}] \gamma_5 D_{yx}^{-1}[U^{(i)}] \right) + O(1/\sqrt{N})
\end{align*}\]

- Large temporal extension to select \( m_0 \)
- \( h \) has quantum number of hadron
  \[ h_{\pi^+}(x) = \bar{d}(x) \gamma_5 u(x), \quad x = (\vec{x}, t) \]
- In practise: used smeared operators to improve overlap with ground state

\[ \begin{array}{ll}
a \approx 0.07 \text{ fm}, m_\pi \approx 422 \text{ MeV} & \text{a} \approx 0.06 \text{ fm}, m_\pi \approx 426 \text{ MeV} \\
\text{a} \approx 0.08 \text{ fm}, m_\pi \approx 281 \text{ MeV} & \text{a} \approx 0.07 \text{ fm}, Lm_\pi = 3.42, m_\pi \approx 295 \text{ MeV} \\
\text{a} \approx 0.07 \text{ fm}, Lm_\pi = 4.19, m_\pi \approx 289 \text{ MeV} \\
\end{array} \]
Hadron mass from the lattice

**Hadronic 2-point function**

\[ C_{2pt}^h(\vec{p}, t) = \frac{1}{\sqrt{V_S}} \sum_{\vec{x}} e^{i\vec{x} \cdot \vec{p}} \left\langle h(\vec{x}, t) h(\vec{0}, 0) \right\rangle_U \]

\[ = \left| Z_0 \right|^2 e^{-m_0 t} + \left| Z_1 \right|^2 e^{-m_1 t} + \ldots \]

- Large temporal extension to select \( m_0 \)
- \( h \) has quantum number of hadron
- \( h_{\pi^+}(x) = \bar{d}(x) \gamma_5 u(x), \quad x = (\vec{x}, t) \)
- In practise: used smeared operators to improve overlap with ground state

**Monte-Carlo integration**

\[ \left\langle h_{\pi^+}(x) \bar{h}_{\pi^+}(y) \right\rangle = \frac{1}{N} \sum_{i=1}^{N} \text{Tr} \left( \gamma_5 D_{xy}^{-1}[U^{(i)}] \gamma_5 D_{yx}^{-1}[U^{(i)}] \right) + O(1/\sqrt{N}) \]

\[ \text{expensive } \hat{O}[U^{(i)}] \]

- **nucleon mass**

\[ a \approx 0.07 \text{ fm}, \; m_e \approx 422 \text{ MeV} \]
\[ a \approx 0.06 \text{ fm}, \; m_e \approx 426 \text{ MeV} \]
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Lattice QCD accelerator
Systematics

LQCD comes with the same parameters as QCD (coupling and quark masses) plus the lattice volume

\[
\langle \mathcal{O} \rangle_{\beta, \kappa, L} = \frac{1}{Z} \int \prod_{\mu} dU_{x, \mu} \hat{O}[U] e^{-S_{\text{eff}}[U, \beta, \kappa]}
\]

Parameters: \( \beta \equiv 6/g_0^2, \quad \kappa_f \equiv 1/(2m_f^0 - 8) \),

Have to tune them to physical point, e.g.,

\[
\frac{aM_2}{aM_1}(\beta, \kappa) \quad \overset{\beta, L \text{ large}}{\longrightarrow} \quad \text{ratio}(\kappa) \quad \overset{\kappa_f \rightarrow \kappa_f^c}{\longrightarrow} \quad \frac{M_{2}^{\text{phys}}}{M_{1}^{\text{phys}}}
\]

Calculations close to physical point very expensive (→ extrapolation)

Lattice spacing: \( a = a(\beta) \)

Monte-Carlo simulations in practise

- \( N_f = 2, \ N_f = 2+1, \ N_f = 2+1+1 \)
- Lattice spacings: \( a \geq 0.04 \text{ fm} \), Volumes: \( L \leq 6 \text{ fm} \)

Challenge: Control over systematic error when extrapolating to physical point
Landscape of recent lattice simulations
Lattice QCD simulations approach (start approaching) the physical point

Lattice spacing vs. $m_\pi$

Figures from Hoelbling, arXiv:1410.3403
Landscape of recent lattice simulations

Lattice QCD simulations approach (start approaching) the physical point

Volume vs. $m_\pi$

![Graph showing Volume vs. $m_\pi$](image)

Figures from Hoelbling, arXiv:1410.3403
Landscape of recent lattice simulations
Lattice QCD simulations approach (start approaching) the physical point

Physical point vs. $m_\pi$

Figures from Hoelbling, arXiv:1410.3403
Hadron masses from lattice QCD: experimental vs. lattice QCD results (MILC, PACS-CS, BMW, QCDSF und RBC&UKQCD). From [Kronfeld arXiv:1203.1204].
Hadron structure from lattice QCD

- Moments of parton distribution functions
- Nucleon isovector couplings (arXiv:1412.7336), sigma terms
  (arXiv:1206.7034, update in preparation), form factors and
  generalised form factors (in preparation)
- Moments of light-cone distribution amplitudes
- Charmed baryon spectroscopy (arXiv:1503.08440)
- Strangeness Contribution to the Proton Spin (arXiv:1112.3354)
- ...

Up to now: $N_f = 2$ calculations from $m_\pi = 430 \ldots 150$ MeV (this talk)

- safer extrapolation to physical point
- main uncertainties: volume, lattice spacing artefacts
RQCD-Kollaboration: $N_f = 2$ ensembles

**Action**
- Wilson gauge
- $N_f = 2$ Wilson-clover fermions

**Physical regime**
- Lattice spacings:
  - $a = 0.06 - 0.08$ fm
  - (2 spacings $m_\pi \approx 290$ MeV
  - 3 spacings $m_\pi \approx 425$ MeV).
- $m_\pi$ down to 150 MeV.
- $Lm_\pi$ up to 6.7,

**Reduction of excited-state contaminations**
- 300–600 Gauss-smearing iterations on top of APE smearing.
- Multi-exponential fits
Hadronic two- and three-point functions on the lattice

Hadronic matrix elements $\langle N|O|N \rangle$ are obtained via:
1. two-point correlation functions (as above)
2. three-point correlation functions

$$C^O_{3pt}(\tau, t, \vec{P}, \vec{P}') = \left\langle \tilde{h}(t, \vec{P}') \tilde{O}(\tau, \Delta P) \tilde{h}^\dagger(0, \vec{P}) \right\rangle$$

- $\tilde{h}$ and $\tilde{O}$ denote the Fourier transform of the hadronic and the insertion operator

$$\tilde{h}(t, \vec{P}) = \sum_{\vec{x}} e^{i\vec{P}\cdot\vec{x}} h(t, \vec{x}), \quad \tilde{O}(\tau, \Delta P) = \sum_{\vec{x}} e^{i\Delta P\cdot\vec{x}} O(\vec{x}, \tau)$$

- Insertion operator: $O = \{\gamma_5, \gamma_\mu, \gamma_\nu D_\mu, \ldots\}$ chosen wrt. desired matrix element, e.g.

$$R(t, \tau) = \frac{C^O_{3pt}(\tau, t)}{C^O_{2pt}(\tau)} = \frac{\langle N|O|N \rangle}{2M_N} + \ldots$$
Illustration: Measurement of a three-point function on the lattice

Connected and disconnected contribution

Diagrams

On the Lattice

Figure: from R. Horsley (1999)
Nucleon axial charge $g_A$

- Associated with $\beta$ decay (neutron $\rightarrow$ proton)
- Experimentally well known: $g_A = 1.2723(23)g_V$
  (isospin symmetric limit: $g_V = 1$)
- Theorie: $g_A = g_A(0)$ of the nucleon axial form factor:

$$\langle p | \bar{u}\gamma_\mu\gamma_5 d | n \rangle = \bar{u}_p(p_f) \left[ g_A(q^2)\gamma_\mu + \tilde{g}_P(q^2)\frac{q_\mu}{2M_N} \right] \gamma_5 u_n(p_i)$$

Lattice QCD

- Good benchmark observable for a lattice calculation
- Challenge for many years (underestimate of exp. value)
- Conjecture: volume effect

![Graph showing $g_A$ vs. $m_\pi^2$](QCDSF (2006))
Nucleon axial charge $g_A$

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- Theorie: $g_A = g_A(0)$ of the nucleon axial form factor:

$$
\langle p|\bar{u}\gamma_\mu\gamma_5d|n\rangle = \bar{u}_p(p_f) \left[ g_A(q^2)\gamma_\mu + \frac{\tilde{g}_P(q^2)}{2M_N} q_\mu \right] \gamma_5 u_n(p_i)
$$

**Lattice QCD**

- New data from RQCD (arXiv:1412.7336)
- Clear volume effect for fixed lattice spacing
- No significant discret. errors for fixed volume
Prediction from $\chi$PT: Similar finite volume effects for $g_A$ and $F_\pi$

Follow QCDSF [1302.2233] and plot/extrapolate ratio

Extrapolation to phys. point

Fit: $g_A/F_\pi = 13.88(29) \text{ GeV}^{-1}$

Exp: $g_A/F_\pi = 13.797(34) \text{ GeV}^{-1}$

With $F_\pi(\text{exp}) = 92.21 \text{ MeV}$ we obtain

$$g_A = 1.280(27)(35) \quad \text{Exp: } g_A = 1.2670(35)$$
Comparison of recent lattice results for $g_A$

arXiv:1412.7336

- World lattice data approaches physical regime ($m_\pi \approx 135\,\text{MeV}$)
- Reproduction of experimental value for $g_A$ if excited-states and volume effects are under control
Nucleon (isovector) couplings

Calculated also all other couplings

- Besides $g_T$, all couplings are directly obtained in the forward limit $q^2 = 0$
- $g_T$ from the form factor data $g_T(q^2 \to 0)$

\[
\langle p|\bar{u}d|n\rangle = g_S(q^2)\bar{u}_p(p_f)u_n(p_i),
\]
\[
\langle p|\bar{u}\gamma_5 d|n\rangle = g_P(q^2)\bar{u}_p(p_f)\gamma_5 u_n(p_i),
\]
\[
\langle p|\bar{u}\gamma_{\mu} d|n\rangle = \bar{u}_p(p_f)\left[g_V(q^2)\gamma_{\mu} + \frac{\tilde{g}_T(q^2)}{2M_N}i\sigma_{\mu\nu}q^\nu\right]u_n(p_i),
\]
\[
\langle p|\bar{u}\gamma_\mu\gamma_5 d|n\rangle = \bar{u}_p(p_f)\left[g_A(q^2)\gamma_{\mu} + \frac{\tilde{g}_P(q^2)}{2M_N}q_\mu\right]\gamma_5 u_n(p_i),
\]
\[
\langle p|\bar{u}\sigma_{\mu\nu} d|n\rangle = g_T(q^2)\bar{u}_p(p_f)\sigma_{\mu\nu} u_n(p_i)
\]

- Induced tensor charge $\tilde{g}_T$ known $\to$ another benchmark observable
- Other charges more or less unknown, might be relevant for new physics processes or dark matter searches
- $g_T$ and $g_S$ accessible at present only through lattice simulation.
Lattice results for $g_S$ and $g_T$

We find $g_{\overline{MS}}(2\text{ GeV}) = 1.02(18)(30)$ and $g_{\overline{MS}}(2\text{ GeV}) = 1.005(17)(29)$ when approaching phys. point

- In agreement with other determinations
- Lattice spacing effects not excluded ($a = 0.06 \ldots 0.08$ fm)
Nucleon (isovector) couplings

arXiv:1412.7336: Final results extrapolated to physical point

Table: First errors contain statistics and systematics. The second errors are estimates of lattice spacing effects.

<table>
<thead>
<tr>
<th>Couplings</th>
<th>Our result</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_A$</td>
<td>1.280(44)(46)</td>
<td>1.2723(23)</td>
</tr>
<tr>
<td>$g_{S}^{\text{MS}}$ (2 GeV)</td>
<td>1.02(18)(30)</td>
<td>—</td>
</tr>
<tr>
<td>$g_{T}^{\text{MS}}$ (2 GeV)</td>
<td>1.005(17)(29)</td>
<td>—</td>
</tr>
<tr>
<td>$\tilde{g}_T$</td>
<td>3.00(08)(31)</td>
<td>3.7058901(5)</td>
</tr>
<tr>
<td>$g_P^*$</td>
<td>8.40(40)(159)</td>
<td>8.06(55)</td>
</tr>
</tbody>
</table>

- Couplings by definition valence quark quantities $\rightarrow$ no significant effects from including strange (or charm) sea quarks expected
- Chiral extrapolation to physical point under control
  (estimate at $m_\pi \approx 150$ MeV allows us to estimate uncertainties)
- Deviation of $\tilde{g}_T$ not yet clear
Moments of Parton Distributions (PDFs)

Nucleon parton distribution function
- Important for phenomenology
- Known for large range of $x = [0, 1]$ (long. mom. fraction)
- Good knowledge for $u$ and $d$
- For other quarks not as good

**Lattice QCD** allows only access to moments of PDFs

Lowest moment of nucleon PDF

$$\langle x \rangle_q = \int_0^1 dx \ x [q(x) + \bar{q}(x)]$$

Ever popular moment: $\langle x \rangle_{(u-d)}$
- no disconn. contributions needed
- discrepancy: lattice vs. exp.

Armstrong et al. (2012)
Reduction of systematic error for $\langle x \rangle_{\overline{\text{MS}}}^{\text{MS}}_{u-d}$

Fitting of leading excited states

- Access via 2- and 3-point functions (operator $O$ with derivative)
- For moderate $\tau$ and $|\tau - t_f|$ also contributions from excited states

\[
C_{2\text{pt}}(t_f) = |Z_0|^2 e^{-m_0 t_f} + |Z_1|^2 e^{-m_1 t_f} + \ldots
\]

\[
C_{3\text{pt}}(t_f, t) = |Z_0|^2 \langle N_0 | O | N_0 \rangle e^{-m_0 t_f} + |Z_1|^2 \langle N_1 | O | N_1 \rangle e^{-m_1 t_f}
\]

\[
+ Z_1^* Z_0 \langle N_1 | O | N_0 \rangle e^{-m_0 t} e^{-m_1 (t_f - t)} + Z_0^* Z_1 \langle N_0 | O | N_1 \rangle e^{-m_1 t} e^{-m_0 (t_f - t)} + \ldots
\]

IV, $a \sim 0.07$ fm, $m_\pi \sim 295$ MeV, $Lm_\pi = 3.42$

VIII, $a \sim 0.07$ fm, $m_\pi \sim 150$ MeV, $Lm_\pi = 3.49$
Lowest moment of nucleon PDF $\langle x \rangle_{u-d}$

Status

- Deviation to phenom. value reduced (better treatment of excited-states)
- Discrepancy remains though
- Further issues?
Physics program of the RQCD collaboration

**Hadron structure from lattice QCD**

- Moments of parton distribution functions
- Nucleon isovector couplings (arXiv:1412.7336), sigma terms
  (arXiv:1206.7034, update in preparation), form factors and
  generalised form factors (in preparation) → \( J^q \)
- Moments of light-cone distribution amplitudes
- Charmed baryon spectroscopy (arXiv:1503.08440)
- Strangeness Contribution to the Proton Spin (arXiv:1112.3354)
- ...

**Up to now:** \( N_f = 2 \) calculations from \( m_\pi = 430 \ldots 150 \text{ MeV} \) (**this talk**)

- safer extrapolation to physical point
- main uncertainties: volume, lattice spacing artefacts
Pion-nucleon sigma term

Decomposition of nucleon mass

\[ M_N = \frac{\langle N|T_{44}|N \rangle}{\langle N|N \rangle} = \sum_q \sigma_q + \frac{4}{3\langle N|N \rangle} \langle N| \int d^3x \left( \frac{1}{8\pi\alpha} (E^2 - B^2) + \sum_q \bar{q} \gamma \cdot D q \right)|N \rangle \]

- \( T_{44} \ldots \) energy density component of energy-momentum tensor
- Nucleon mass is largely generated by gluon field energy (\( \approx 900 \) MeV) \((M_N = 938 \) MeV, \( m_u \approx 2 \) MeV, \( m_d \approx 4 \) MeV)
- Small fraction from sea and valence quarks

\( \sigma \)-terms

- parametrizes individual quark contribution \( f_q = \sigma_q / M_N \) to nucleon mass
  \[ \sigma_q = m_q \frac{\langle N|q\bar{q}|N \rangle}{\langle N|N \rangle} \]

- Pion-nucleon sigma term: \( \sigma_{\pi N} = \sigma_u + \sigma_d \)
- VEV \( \langle 0|\bar{q}q|0 \rangle \) is understood to be subtracted from \( \langle N|\bar{q}q|N \rangle \)
- Phenomenology does not give a clear answer on \( 45 - 65 \) MeV
- Lattice allows for direct calculation (involves disconnected contributions)
Most calculations via Feynman-Hellmann theorem

\[
\sigma_{\pi N} = m_\pi^2 \frac{\partial M_N(m_\pi)}{\partial m_\pi^2}
\]

New determinations are done directly and include contributions from disconnected quark loops (expensive and noisy)

Central value changed over the years: 50...60 \rightarrow 30...40 MeV
Linear Fit with and without constraint $\sigma_{\pi N}(0) = 0$ gives $\sigma_{\pi N} \approx 35$ MeV

The non-vanishing light quark masses responsible for only 35 MeV of $M_N$

Not unexpected: $M_N \gg 0$ as $m_{ud} \to 0$

**Note:** quark contribution to pion mass $m_\pi/2$
Decomposition of pion mass and the Pion sigma term

\[ m_\pi \approx \frac{1}{2} m_\pi + \frac{3}{8} m_\pi + \frac{1}{8} m_\pi \]

with \( \sigma_\pi = m_{ud} \langle \pi | \bar{u}u + \bar{d}d | \pi \rangle = m_{ud} \frac{\partial m_\pi}{\partial m_{ud}} \)

The theoretical expectation \( \sigma_\pi \approx m_\pi / 2 \) is confirmed.
Side remark: Chiral extrapolation of the nucleon mass

- Lattice QCD simulations produce bare numbers
- Need to calculate physical quantity (mass, decay constant, ...)
- Need to extrapolate to physical point (systematic uncertainty)

- Extrapolation of $M_N$ data desired but yet too uncertain
- Sigma-term values (slope) constrain the chiral fit (basically only offset is left free)

→ $\sigma_{\pi N}$ values make chiral extrapolation of $M_N$ data much less ambiguous.
Lattice can contribute to many quantities that are hard to constrain by experiment: $\sigma_{\pi N}$, $g_s$, $g_T$, $\Delta q$, $\Delta \Sigma$, $J_q$, $\langle x \rangle_{\Delta q}$, ....

Benchmark observables help to improve lattice methods
- Hadron masses (2-point functions) come out nicely
- $g_A$ (3-point functions) seems to approach the physical value, once $Lm_\pi > 4$
- $\langle x \rangle_{u-d}$ comes out 20% bigger than expected

Up to now: $N_f = 2$ calculations from $m_\pi = 430 \ldots 150$ MeV with lattice spacings: $a = 0.06 \ldots 0.08$ fm (this talk)

Ultimately precision physics requires an extrapolation $a \to 0$.

**Upcoming:** $N_f = 2 + 1$ calculations targeting physical point and continuum limit (part of the CLS effort).

Disconnected quark line diagrams will more and more be included
- in the past often omitted and differences quoted: $g_A$, $\langle x \rangle_{u-d}$, ..., but no $\Delta s$, $\Delta \Sigma$, $J_q$, $\langle x \rangle_q$ etc.
- See, e.g., quark spin contribution to proton spin $\Delta \Sigma$
Proton Spin

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + \Delta G \]

Results for \( \overline{MS} \) scheme at \( \mu^2 = 7.4 \text{ GeV}^2 \)

\[ \Delta \Sigma = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} = 0.45(4)(9) \]

\[ \Delta s + \Delta \bar{s} = -0.020(10)(4) \]

Warning: no continuum limit, \( m_\pi \approx 290 \text{ MeV} \) \( \Rightarrow \) add 20\% systematic error.
Quark spin contributions to proton spin
Comparison of recent lattice calculations

Consistency between different determinations: small $\Delta s + \Delta \bar{s}$.
ETMC result shows statistical accuracy that is possible. Systematics!

[QCDSF: GB et al, 1112.3354; M Engelhardt, 1210.0025; ETMC: A Abdel-Rehim et al, 1310.6339; $\chi$QCD: Y Yang et al, unpublished.]
Leaving the traditional way lattice QCD is used for hadron physics

Lattice QCD

- gauge-invariant nonperturbative approach to QCD
- Systematically improvable
  (larger and finer lattices, tune bare parameters towards physical point)
- Allows access to many interesting quantities without a need to fix a gauge
Leaving the traditional way lattice QCD is used for hadron physics

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Lattice QCD allows more . . .

- Apply a gauge condition $\rightarrow$ access to nonperturbative and untruncated elementary $n$-point functions of QCD
  - 2-point functions: quark, gluon and ghost propagators
    Many lattice studies in the past helped to settle momentum dependence
  - 3-point functions: quark-gluon, 3-gluon, 4-gluon, ghost-gluon vertex

- Essential information for functional approaches to hadron physics
  (e.g. QCD Dyson-Schwinger and bound-state equations)
  - perturbative, semi-perturbative and non-perturbative regimes
  - e.g.: validity of Rainbow-Ladder truncations etc.
New lattice results for Quark-Gluon-Vertex in Landau gauge in collaboration with Kizilersu, Oliveira, Silva, Skullerud

**Quark-Gluon vertex**

$$G_{\mu}^{\bar{\psi} \psi^A} = \Gamma_{\lambda}^{\bar{\psi} \psi^A}(p, q) \cdot S(p) \cdot D_{\mu \lambda}(q) \cdot S(p + q)$$

- Non-perturbative structure mostly unknown, but crucial for many functional approaches to hadron physics (Dyson-Schwinger, Bethe-Salpether equations etc.)
- Up to now: only lattice data for quenched QCD

**Ball-Chiu-Parametrisation**

$$\Gamma_{\mu}^{\bar{\psi} \psi^A}(p, q) = \Gamma_{\mu}^{ST}(p, q) + \Gamma_{\mu}^{T}(p, q)$$

with

$$\Gamma_{\mu}^{ST}(p, q) = \sum_{i=1}^{4} \lambda_i (p^2, q^2) L_{i \mu}(p, q) \quad \text{satisfies Slavnov-Taylor identities}$$

$$\Gamma_{\mu}^{T}(p, q) = \sum_{i=1}^{8} \tau_i (p^2, q^2) T_{i \mu}(p, q) \quad \text{is transverse (} q_\mu \Gamma_{\mu}^{T} = 0 \text{)}.$$

The tensors $L_{\mu}$ and $T_{\mu}$ can be found in the literature: $L_{1 \mu}(p, q) = \gamma_\mu$, $L_{2 \mu}(p, q) = -\gamma_\mu (2p_\mu + q_\mu)$, \ldots, $T_{1 \mu}(p, q) = i[p_\mu q^2 - q_\mu (p \cdot q)]$, \ldots
New lattice results for Quark-Gluon-Vertex in Landau gauge

(Be careful, results are preliminary yet!)

**Example:** \( q = 0 \) (soft gluon) \( \rightarrow \tau_i(p, 0) = 0 \)

- Discretization effect not yet under control (hypercubic artefacts for \( p^2 > 3 \) due to lattice)
- Deviation from tree-level for \( p < 1 \) GeV
- For \( p > 1 \) GeV: Vertex \( \sim \) tree-level
  (might explain why Rainbow ladder truncation so successful)
Lattice results for 3-Gluon vertex (preliminary)
together with BSc. Balduf

\[ |p| = |q|, \theta = \text{angle}(p, q) \]

- Statistical noise grows with \(|p| = |q|\)
- Small quark-mass dependence (slight upward shift) for the available sets, but a clear quenching effect (vertical shift)
- No “zero-crossing” for \(p > 0.3\ \text{GeV}\) (\(p > 0.8\ \text{GeV} \) quenched)

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\(\Gamma / f^{-1}\) vs \(|p| / \text{GeV}\)
Lattice results for 3-Gluon vertex (preliminary) together with BSc. Balduf

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</table>

| | $|p| = |q|$, $\theta = \text{angle}(p, q)$ |
| | Statistical noise grows with $|p| = |q|$ |
| | Small quark-mass dependence (slight upward shift) for the available sets, but a clear quenching effect (vertical shift) |
| | No “zero-crossing” for $p > 0.3 \text{ GeV}$ ($p > 0.8 \text{ GeV}$ quenched) |

$\Gamma / F$ vs. $|p| / \text{GeV}$

Drei-Gluon-Vertex, $\theta = 90^\circ$
Thank you for your attention!