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Two-photon exchange corrections in elastic lepton-proton scattering at small momentum transfer

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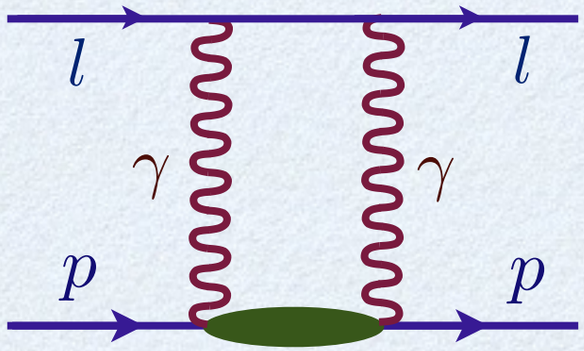
Outline

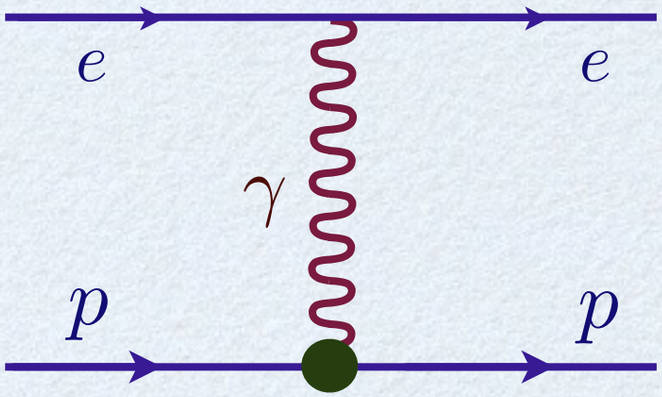
- Motivation
- Elastic lepton-proton scattering
- Forward limit of TPE. Corrections to HFS
- Elastic contribution to TPE corrections. DR framework
- Low-momentum transfer expansion
- Inelastic TPE contribution



Form factors in OPE approximation

OPE amplitude

$$T = \frac{e^2}{Q^2}(\bar{u}(k',h')\gamma_\mu u(k,h))(\bar{u}(p',\lambda')\Gamma^\mu(Q^2)u(p,\lambda))$$



momentum transfer $Q^2 = -(k-k')^2$

kinematic variables $\tau = \frac{Q^2}{4M^2}, \qquad \epsilon = \frac{\nu^2 - \tau(1+\tau)}{\nu^2 + \tau(1+\tau)}$

photon-proton vertex

$$\Gamma^\mu(Q^2) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2(Q^2)$$

Sachs form factors

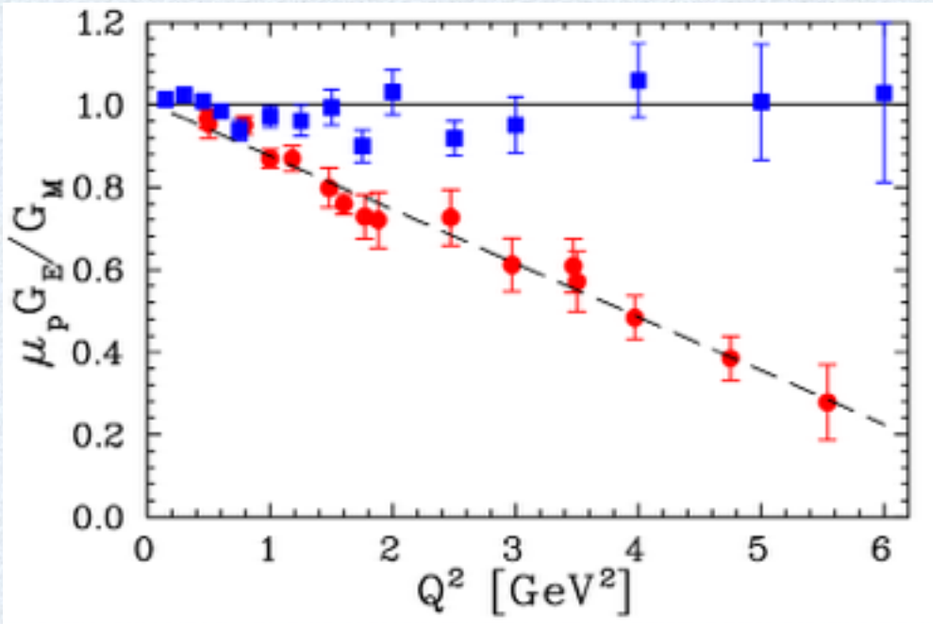
$$G_E = F_1 - \tau F_2, \qquad G_M = F_1 + F_2$$

Rosenbluth separation

$$\frac{d\sigma^{unpol}}{d\Omega} \sim (\epsilon G_E^2(Q^2) + \tau G_M^2(Q^2))$$

Polarization transfer

$$\frac{P_T}{P_L} \sim \frac{G_E(Q^2)}{G_M(Q^2)}$$



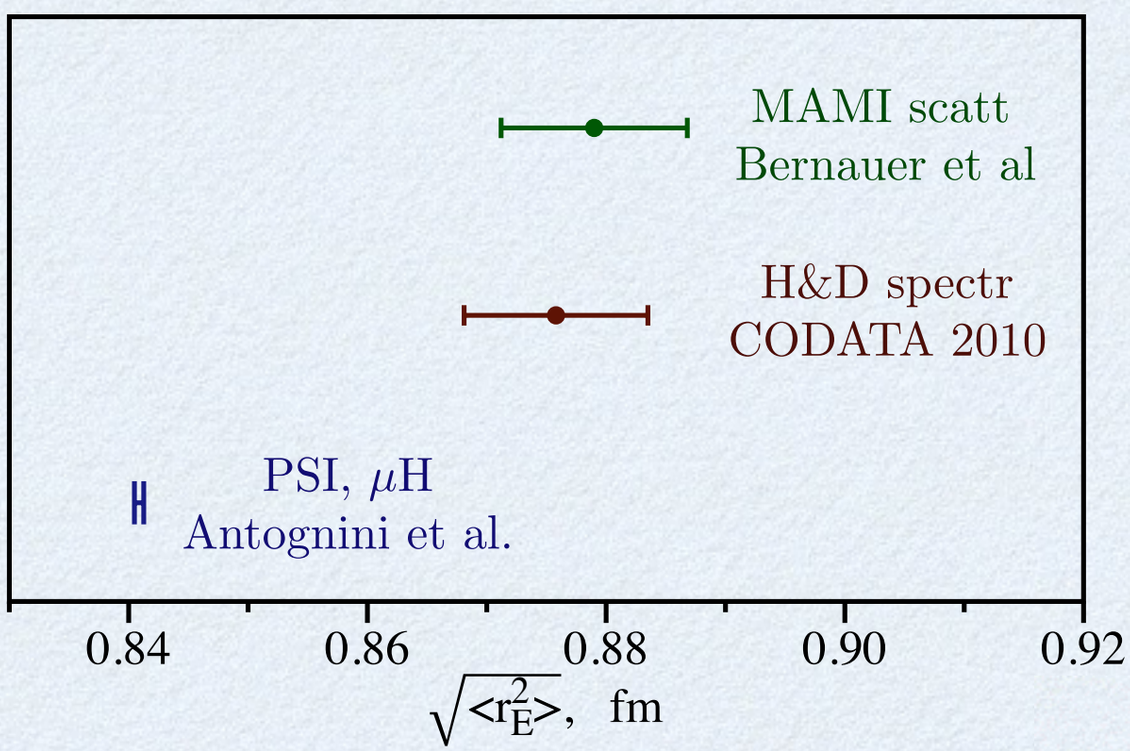
J. Arrington (2003)

A possible explanation - two-photon exchange

Proton radius puzzle

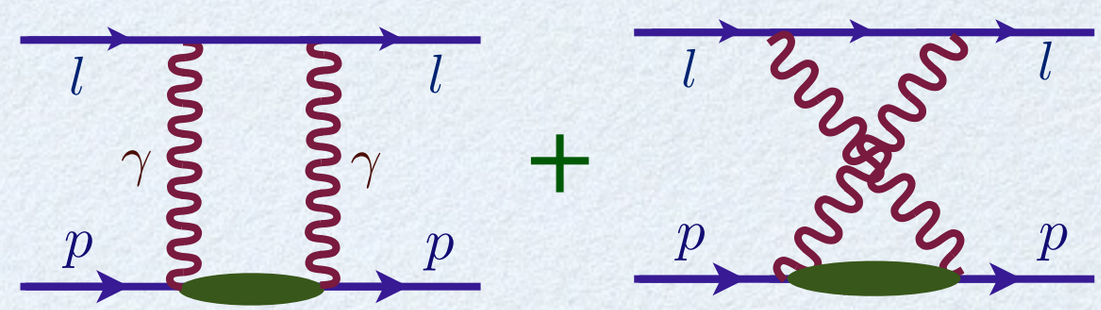
electric charge radius $< r_E^2 >= -\frac{1}{6}\frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2}$

	e hydrogen	μ hydrogen
Lamb shift	$r_E = 0.8758 \pm 0.0077 fm$	$r_E = 0.8409 \pm 0.0004 fm$
ep-elastic scattering	$r_E = 0.879 \pm 0.008 fm$	4-7 σ difference !



TPE hadronic correction is dominant

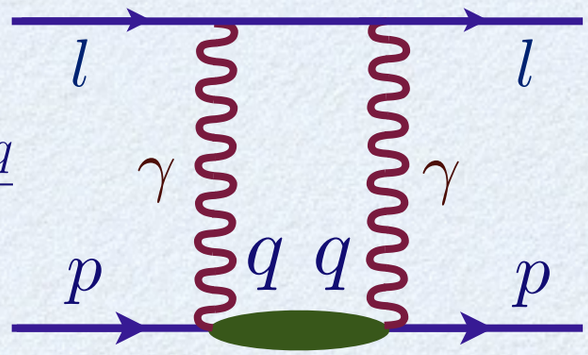
uncertainty in scattering experiments



$$\sigma^{exp} \equiv \sigma_{1\gamma}(1 + \delta_{soft} + \delta_{2\gamma})$$

magnetic radius depends on TPE

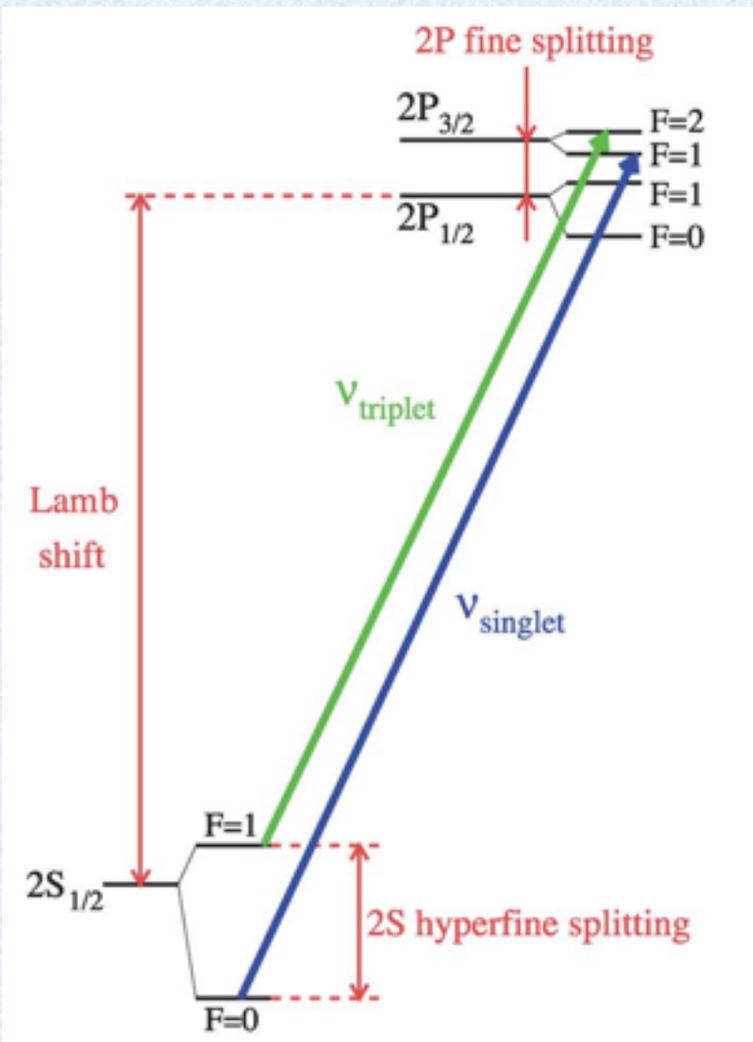
TPE correction to hydrogen spectroscopy



$$\nu_\gamma = \frac{p \cdot q}{M}$$

Shift of S-levels energy $\Delta E_{nS} = \frac{\mathcal{M}|\psi_n(0)|^2}{4Mm}$

TPE blob - forward Compton scattering



A. Altonini et al. (2013)

Lamb shift through unpolarized structure functions $F_1,\ F_2$

$$\Delta E_{n,S} \sim \int \mathrm{d}\nu_\gamma \mathrm{d}Q^2 \left\{T_1(0,Q^2),\ F_1(\nu_\gamma,Q^2), F_2(\nu_\gamma,Q^2)\right\}$$

C. Carlson and M. Vanderhaeghen (2011)

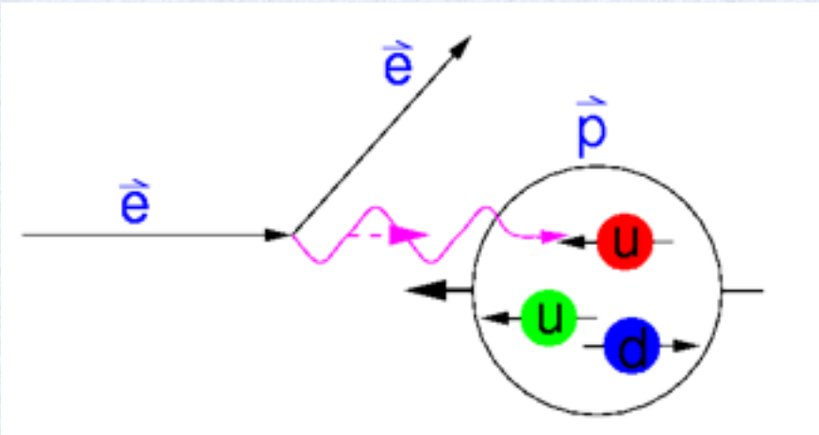
Correction of order 10% of the radius puzzle

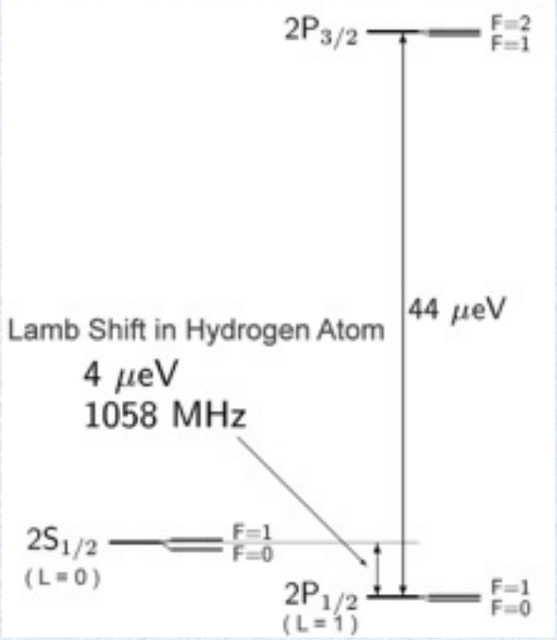
HFS correction through spin structure functions $g_1,\ g_2$

C. Carlson, V. Nazaryan, K. Griffioen (2011)

$$\Delta E_{n,S}^{HFS} \sim \int \mathrm{d}\nu_\gamma \mathrm{d}Q^2 \{g_1(\nu_\gamma,Q^2), g_2(\nu_\gamma,Q^2)\}$$

Dispersion relation framework





$f(z)$

analyticity

experimental cross sections

energy levels correction

optical theorem

$$\Re\mathcal{F}(\nu) = \frac{2\nu}{\pi}\mathcal{P}\int_{\nu_{min}}^{\infty}\frac{\Im\mathcal{F}(\nu'+i0)}{\nu'^2-\nu^2}d\nu'$$

DR

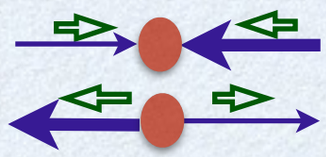
amplitudes: imaginary parts

amplitudes: real parts

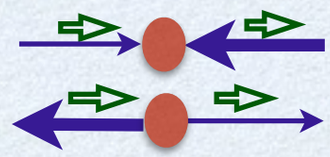
Forward scattering amplitudes

Angular momentum conservation → 3 amplitudes

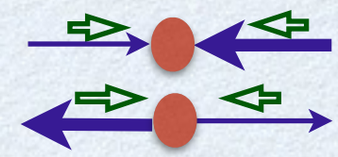
W. Grein and P. Kroll (1978)



$T_1 = T_{++,++}$



$T_3 = T_{+-,+-}$



$T_5 = T_{--,++}$

Crossing symmetric variable $\nu = ME$ is related to lepton energy

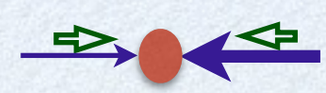
Amplitudes has definite crossing properties with respect E → -E

$$f_+ = \frac{T_1 + T_3}{2} \qquad g = \frac{T_5}{2} \qquad f_- = \frac{T_1 - T_3}{2}$$

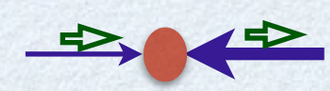
Optical theorem determines imaginary parts

$$\Im f_+(E) \sim \sigma_{++}(E) + \sigma_{+-}(E)$$
$$\Im f_-(E) \sim \sigma_{++}(E) - \sigma_{+-}(E)$$
$$\Im g(E) \sim \sigma_{\perp}(E) - \sigma_{||}(E)$$

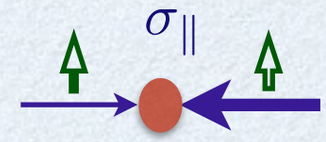
σ_{++}



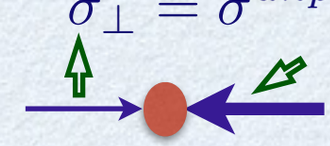
σ_{+-}



$\sigma_{||}$



$\sigma_{\perp} = \sigma^{unpol}$



Dispersion relations. TPE HFS correction

$$\Re f_+^{2\gamma}(E) - \Re f_+^{2\gamma}(E_0) = \frac{4M(E^2 - E_0^2)}{\pi} \int_m^\infty \frac{E'\sqrt{E'^2 - m^2}\cdot\sigma^{1\gamma}_{unpol}(E')}{(E'^2 - E^2)(E'^2 - E_0^2)} \mathrm{d}E' \quad ✗$$

$$\Re f_-^{2\gamma}(E) = \frac{2ME}{\pi} \int_m^\infty \frac{\sqrt{E'^2 - m^2}(\sigma^{1\gamma}_{++}(E') - \sigma^{1\gamma}_{+-}(E'))}{E'^2 - E^2} \mathrm{d}E' \quad ✓$$

$$\Re g^{2\gamma}(E) = \frac{4M}{\pi} \int_m^\infty \frac{E'\sqrt{E'^2 - m^2}(\sigma^{1\gamma}_{\perp}(E') - \sigma^{1\gamma}_{||}(E'))}{E'^2 - E^2} \mathrm{d}E' \quad ✓$$

verified to one-loop level in QED

O.Tomalak and V. Pascalutsa (in preparation)

Similar to light by light scattering except f_+

V. Pascalutsa and M. Vanderhaeghen (2010)

Interaction Hamiltonian

$$H = -f_+ - 4g\vec{s}\cdot\vec{S} - 4(f_- + g)(\vec{s}\cdot\hat{k})(\vec{S}\cdot\hat{p})$$

~~Lamb shift $f_+^{2\gamma}(m) \leftarrow F_1, F_2$~~

HFS correction $f_-^{2\gamma}(m),\ g^{2\gamma}(m) \leftarrow g_1, g_2$

S-levels HFS correction

$$\Delta E_S = E_F(1 + \Delta^S)$$

$$\mu_P e^2 \Delta^S = -g(m) + \frac{1}{2} f_-(m)$$

Zemach correction is reproduced

$$\Delta = \frac{8\alpha m M}{\pi(M+m)} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left(\frac{G_M(Q^2)G_E(Q^2))}{\mu_P} - 1 \right)$$

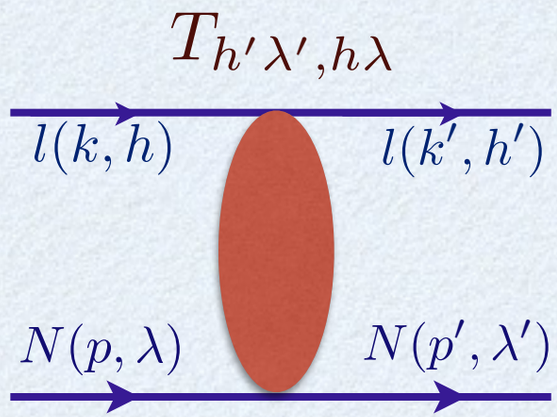
A.C. Zemach (1956)

Recoil correction only with BC sum rule

$$\int_{\nu_{thr}}^\infty g_2(\nu_\gamma, Q^2) \frac{M\mathrm{d}\nu_\gamma}{\nu_\gamma^2} = \frac{1}{4} F_2(Q^2) G_M(Q^2)$$

$$g^{2\gamma}(m) + f_-^{2\gamma}(m) = 0$$

Structure amplitudes



$Q^2 = -(k-k')^2$ momentum transfer

$s = (p+k)^2$, $u = (k-p')^2$, $\nu = \frac{s-u}{4}$ crossing symmetric variable

ϵ photon polarization parameter

Discrete symmetries → 6 structure amplitudes

Goldberger et al. (1957)

Electron scattering is described by 3 structure amplitudes

$$T^{non-flip} = \frac{e^2}{Q^2}\bar{l}(k',h')\gamma_\mu l(k,h).\bar{N}(p',\lambda')[\mathcal{G}_M(\nu,t)\gamma^\mu - \mathcal{F}_2(\nu,t)\frac{P^\mu}{M} + \mathcal{F}_3(\nu,t)\frac{\hat{K}P^\mu}{M^2}]N(p,\lambda)$$

P.A.M. Guichon and M. Vanderhaeghen (2003)

Muon scattering require lepton helicity-flip amplitudes

$m_l \neq 0$ →

$$T^{flip} = \frac{e^2}{Q^2}\frac{m_l}{M}\bar{l}(k',h')l(k,h).\bar{N}(p',\lambda')[\mathcal{F}_4(\nu,t) + \mathcal{F}_5(\nu,t)\frac{\hat{K}}{M}]N(p,\lambda) + \frac{e^2}{Q^2}\frac{m_l}{M}\mathcal{F}_6(\nu,t)\bar{l}(k',h')\gamma_5 l(k,h).\bar{N}(p',\lambda')\gamma_5 N(p,\lambda)$$

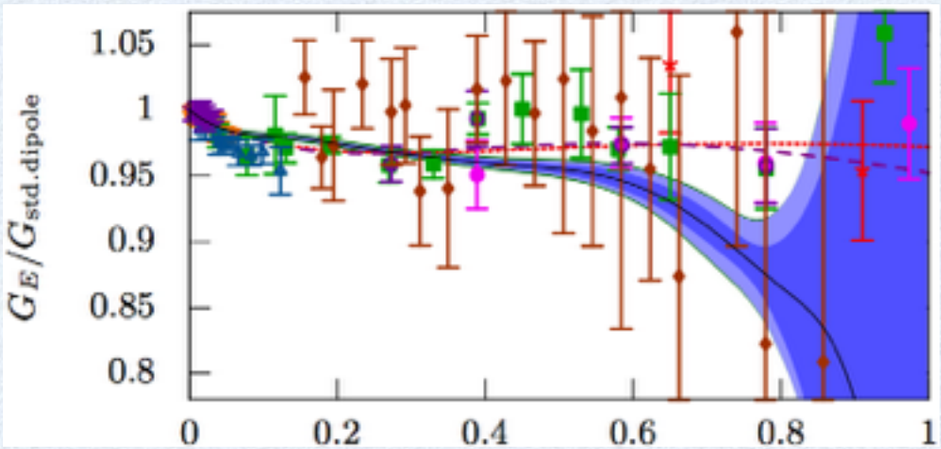
M. Gorchtein, P.A.M. Guichon and M. Vanderhaeghen (2004)

Leading TPE contribution to cross section - interference term with OPE

$$\delta_{2\gamma} \sim \Re\mathcal{G}_M, \Re\mathcal{F}_2, \Re\mathcal{F}_3, \Re\mathcal{F}_4, \Re\mathcal{F}_5$$

Fixed-t dispersion relation framework

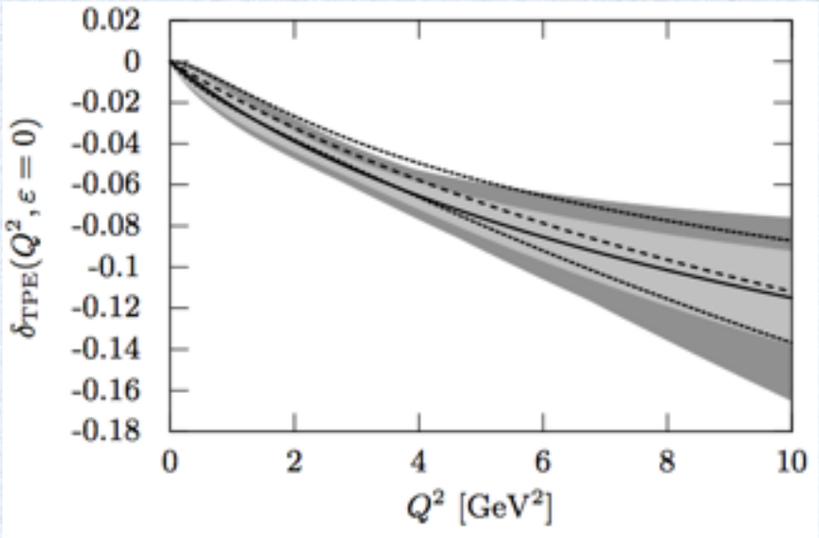
2γ corrections



on-shell one-photon amplitudes

$f(z)$

analyticity



exp. data/phenomenology

cross section correction

unitarity

$$\Re\mathcal{F}(\nu) = \frac{2\nu}{\pi}\mathcal{P}\int_{\nu_{min}}^{\infty}\frac{\Im\mathcal{F}(\nu'+i0)}{\nu'^2-\nu^2}d\nu'$$

amplitudes: imaginary parts

DR

amplitudes: real parts

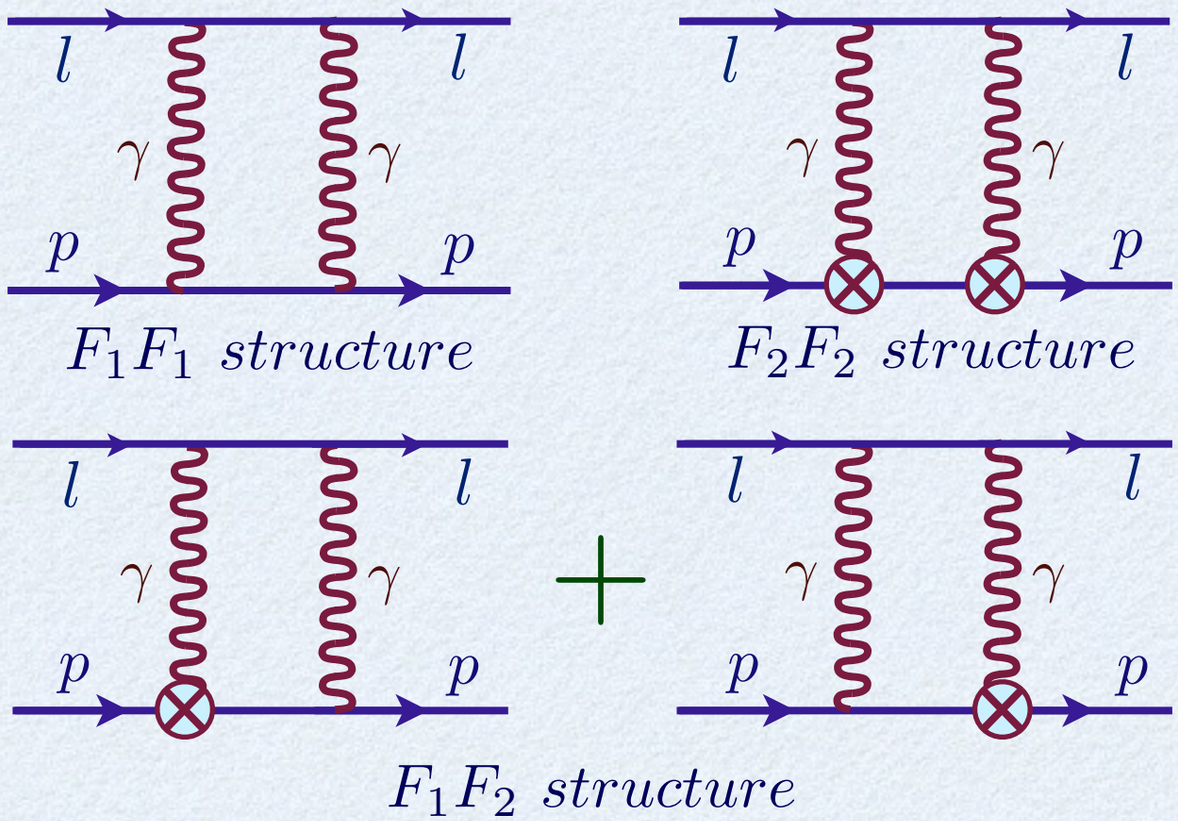
D. Borisyuk, A. Kobushkin (2008)

Hadronic model

The one-photon exchange on-shell vertex

$$\Gamma^\mu(Q^2) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_2(Q^2)$$

P. G. Blunden, W. Melnitchouk, and J. A. Tjon (2003)



IR divergencies are subtracted

L.C. Maximon and J. A. Tjon (2000)

Point-like couplings → $F_1 = 1$ $\quad F_2 = \mu_p - 1$

Dipole FFs for G_M, G_E → $G_E = F_1 - \tau F_2$ $\quad G_M = F_1 + F_2$

$$(\tau = \frac{Q^2}{4M^2})$$

Hadronic model vs. dispersion relations

- Imaginary parts are the same
- Real parts are the same for

all F1F1 amplitudes

F1F2 amplitudes $\mathcal{G}_M \quad \mathcal{F}_2 \quad \mathcal{F}_3 \quad \mathcal{F}_5$

F2F2 amplitudes $\mathcal{F}_2 \quad \mathcal{G}_M + \frac{\nu}{M^2}\mathcal{F}_3 \quad \mathcal{F}_5$

Fixed-t subtracted dispersion relation works

F2F2 amplitudes $\mathcal{G}_M \quad \mathcal{F}_3 \quad \mathcal{F}_4 \quad \mathcal{F}_6$

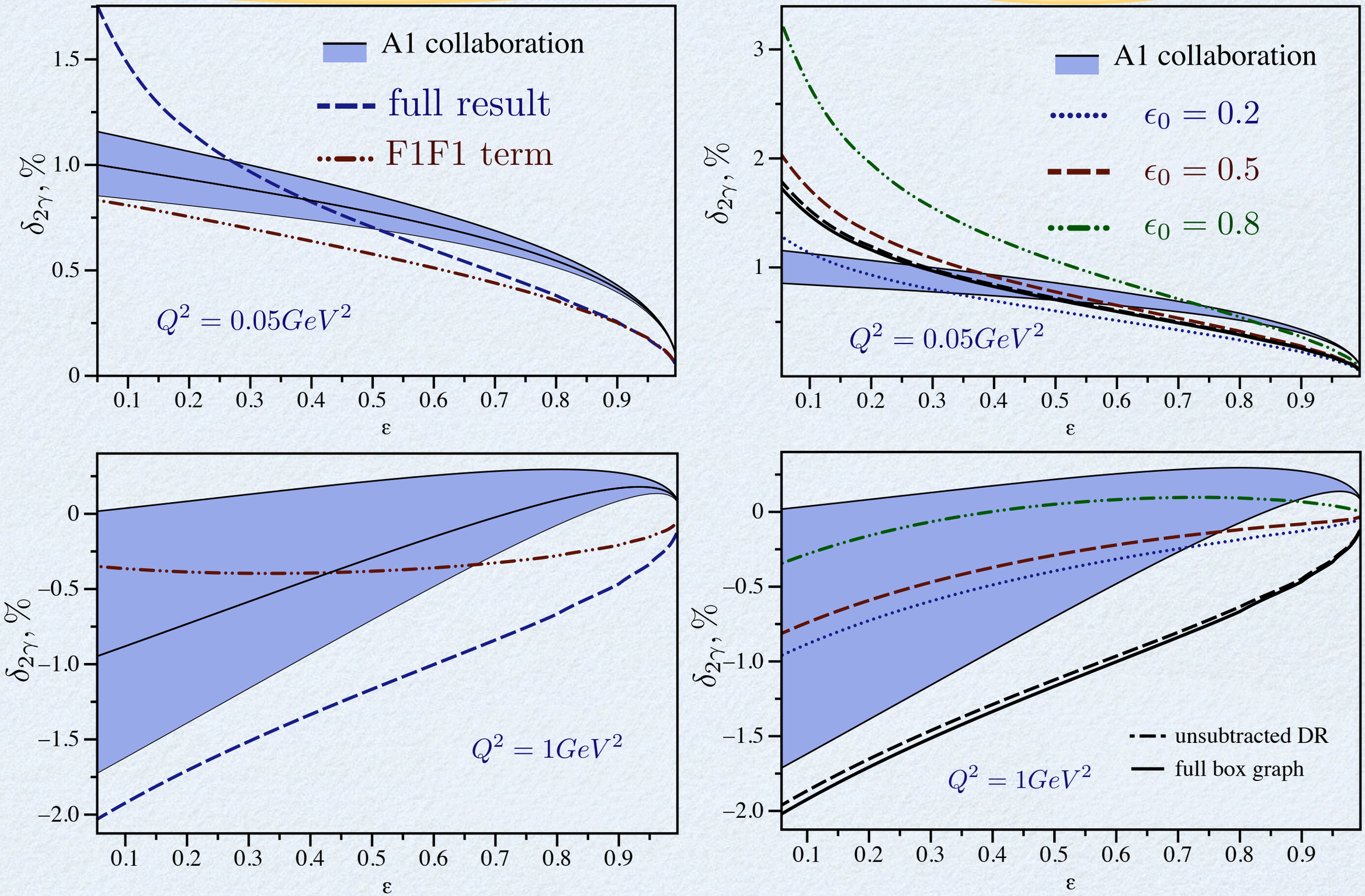
- Calculation based on DR for ep scattering
  - for amplitudes $\mathcal{G}_1,\ \mathcal{G}_2$ unsubtracted DR can be used
  - for amplitude $\mathcal{F}_3$ subtracted DR should be used
  - subtraction point $\Re\mathcal{F}_3^{F_2F_2}(\nu_0, Q^2)$ fixed from $\delta_{2\gamma}(\nu_0, Q^2)$ data

$$\mathcal{G}_1 = \mathcal{G}_M + \frac{\nu}{M^2}\mathcal{F}_3$$
$$\mathcal{G}_2 = \mathcal{G}_E + \frac{\nu}{M^2}\mathcal{F}_3$$

2γ in e⁻p elastic scattering

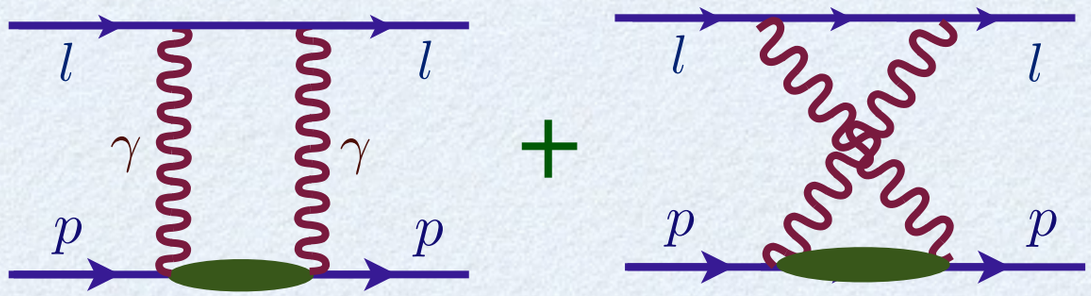
box diagram model calculation

subtracted DR



O. Tomalak and M. Vanderhaeghen (2015)

Near-forward inelastic TPE correction(e^-p)

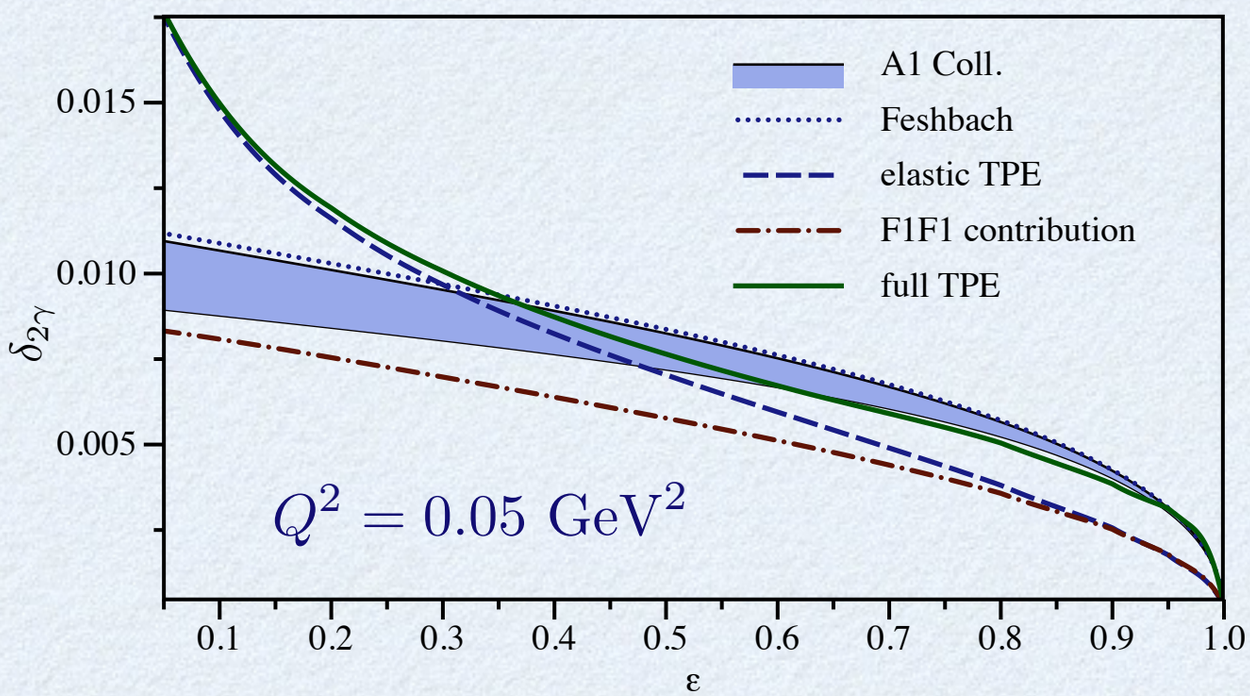


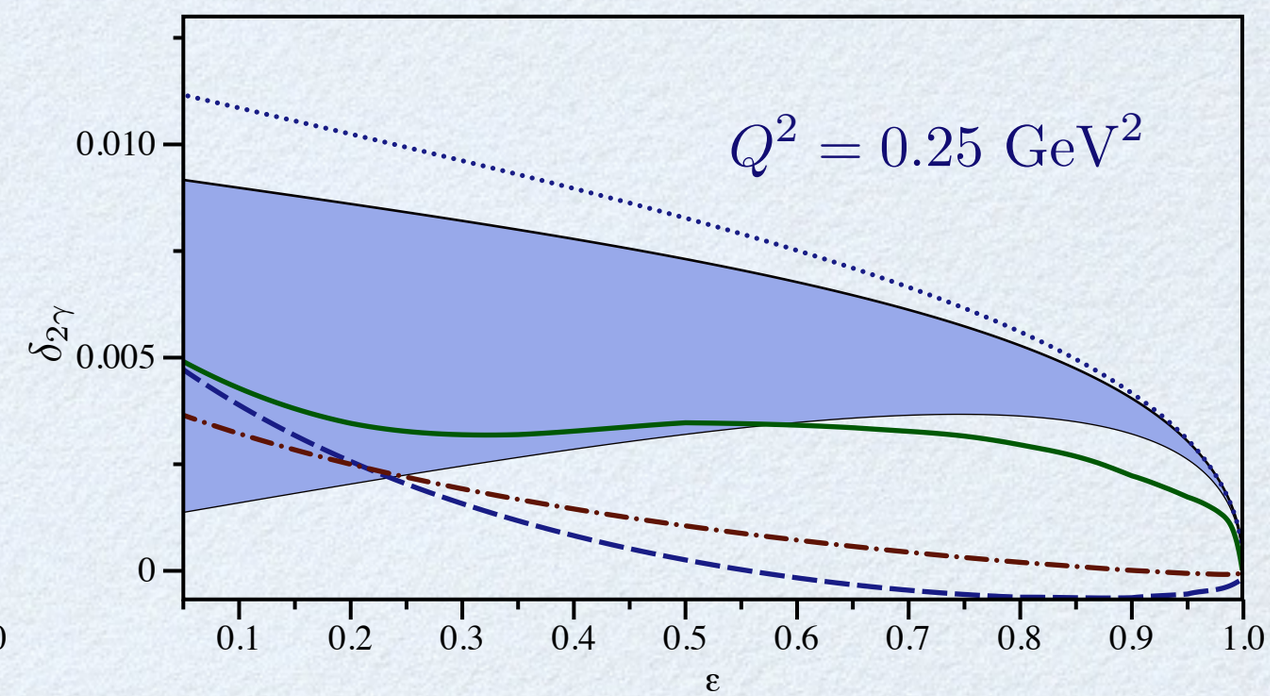
TPE blob - forward Compton scattering

perform the Wick rotation

$$\delta_{2\gamma} = \int \mathrm{d}\nu_\gamma \mathrm{d}\tilde{Q}^2 (w_1(\nu_\gamma, \tilde{Q}^2) \cdot F_1(\nu_\gamma, \tilde{Q}^2) + w_2(\nu_\gamma, \tilde{Q}^2) \cdot F_2(\nu_\gamma, \tilde{Q}^2))$$

- $Q^2 \ln Q^2$ term for ep scattering reproduced, no hadronic scale
- Uncertain Q^2 region of applicability





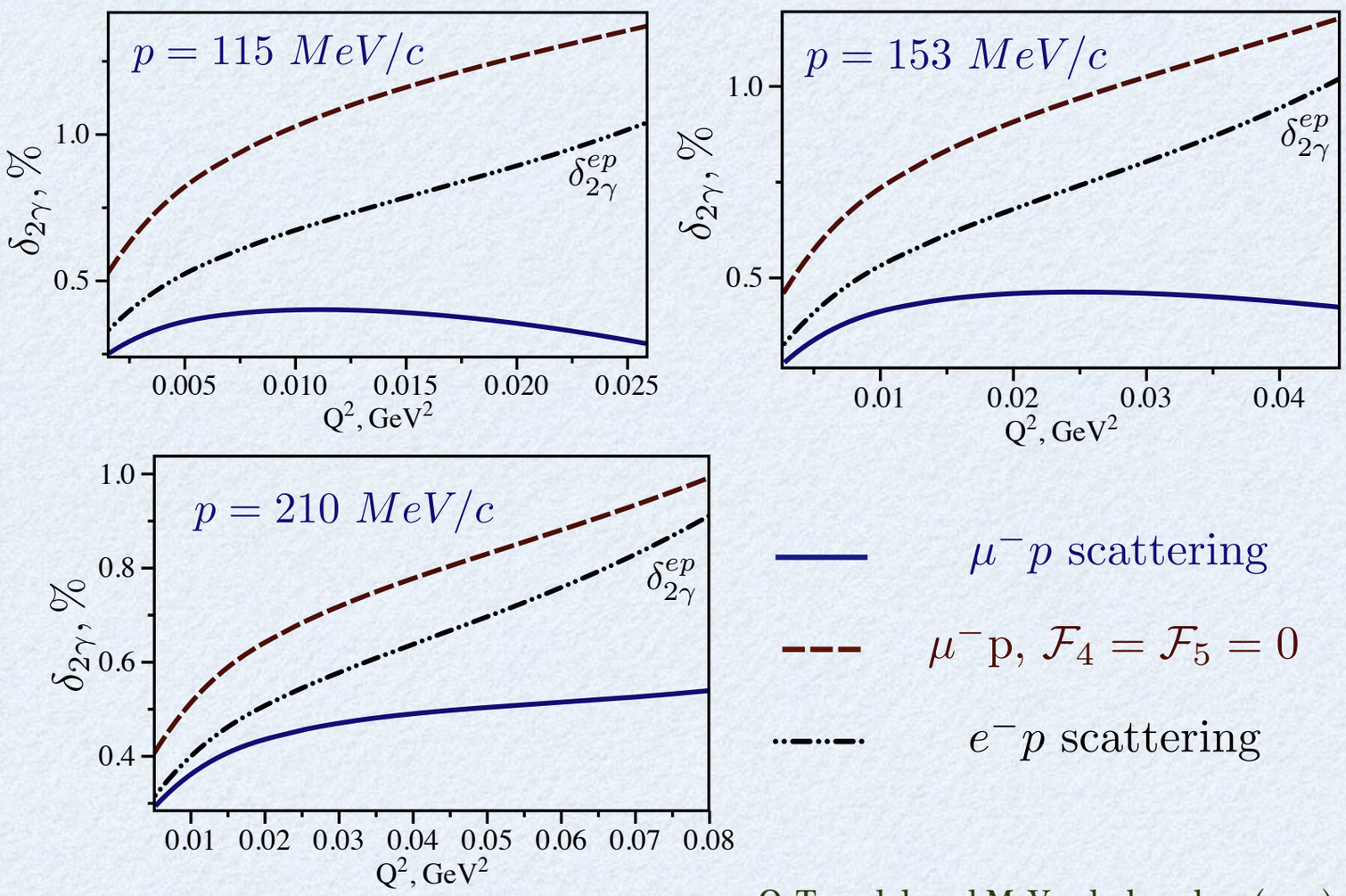
O. Tomalak and M. Vanderhaeghen (2015)

no significant influence on electric charge radius

μp experiment (MUSE) estimates

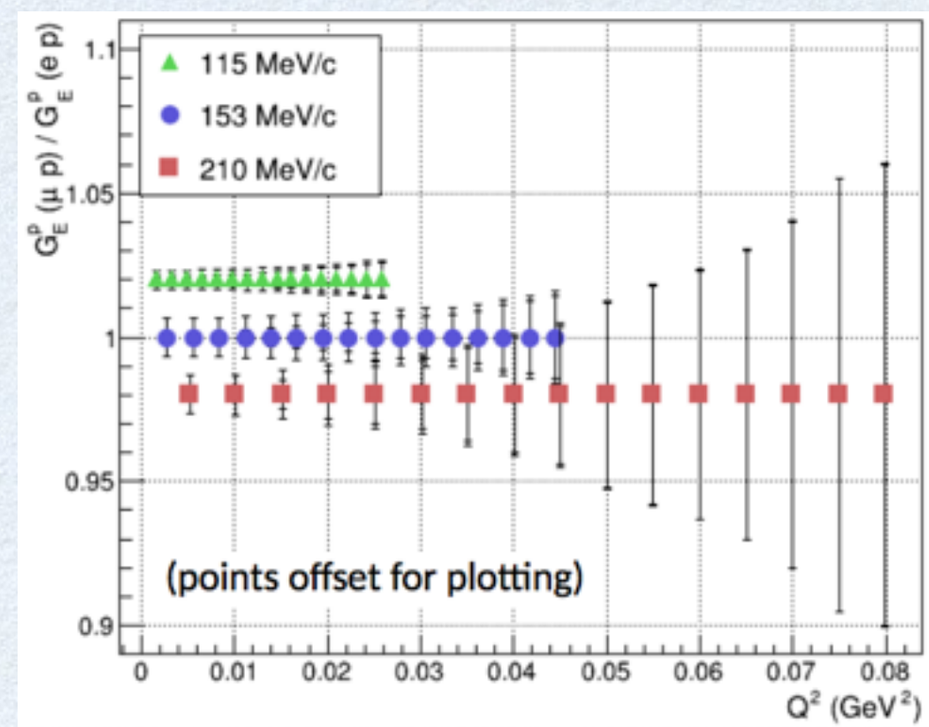
TPE correction in hadronic model

$$\delta_{2\gamma} \sim \Re\mathcal{G}_M, \Re\mathcal{F}_2, \Re\mathcal{F}_3, \Re\mathcal{F}_4, \Re\mathcal{F}_5$$



O. Tomalak and M. Vanderhaeghen (2014)

expected experimental electron over muon ratio



K. Mesick talk (PAVI 2014)

F1F1 contribution dominates

helicity flip lowers correction

T_1 subtraction function TPE correction

Amplitude T_1 reconstructed up to a function

$$T_1\left(\nu_\gamma, Q^2\right) = T_1\left(0, Q^2\right) + \frac{\nu_\gamma^2}{2\pi M} \int\limits_{\nu_{thr}}^{\infty} \frac{F_1\left(\nu', Q^2\right)}{\nu'\left(\nu'^2 - \nu_\gamma^2\right)} \mathrm{d}\nu'$$

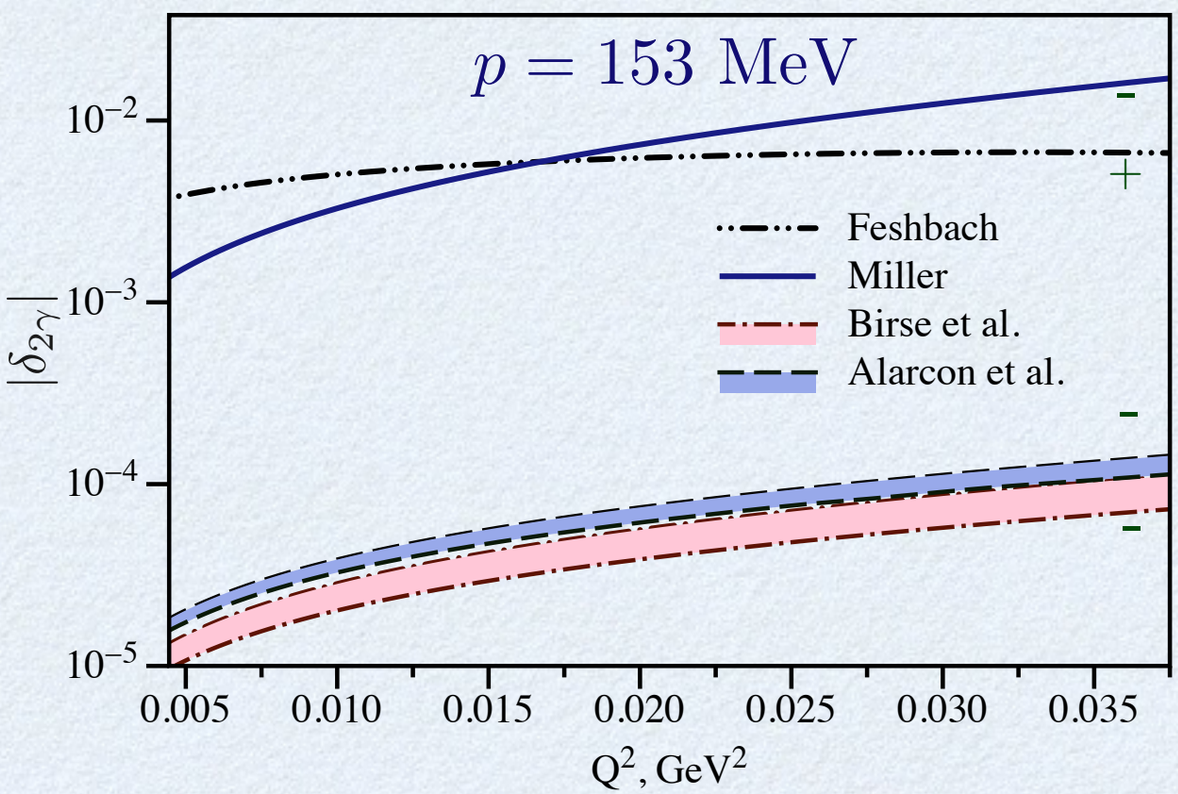
with

$$T_1(0, Q^2) = \beta_M Q^2 F_{loop}(Q^2)$$

Subtraction function contributes only to $\mathcal{F}_4$ amplitude

In the limit of small electron mass TPE correction vanishes

$$\delta_{2\gamma,0}^{subt} \approx -\frac{\beta_M Q^2 m^2}{E} \int\limits_0^{\infty} f\left(x, \frac{Q^2}{m^2}\right) F_{loop}\left(\frac{Q^2\left(x^2-1\right)}{4}\right) \mathrm{d}x^2$$

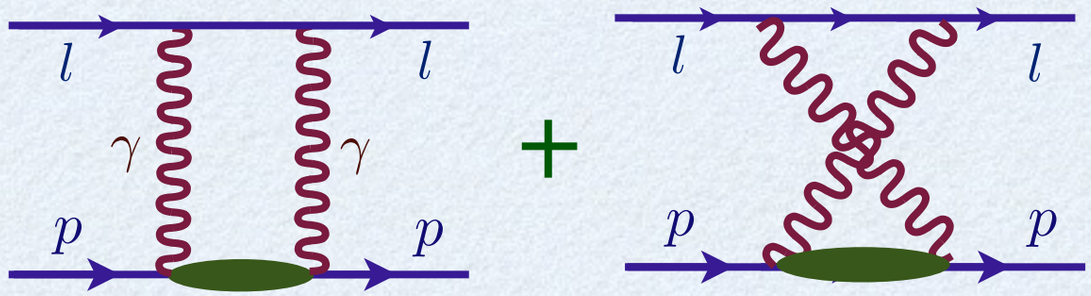


Valid only for small Q^2

For enhanced at HE function

$$\delta_{2\gamma,0}^{subt} \approx -\frac{3\beta_M Q^2 m^2}{2\pi E} \int\limits_0^{\infty} F_{loop}\left(\tilde{Q}^2\right) \frac{\mathrm{d}\tilde{Q}^2}{\tilde{Q}^2}$$

Near-forward inelastic TPE correction($\mu^- p$)

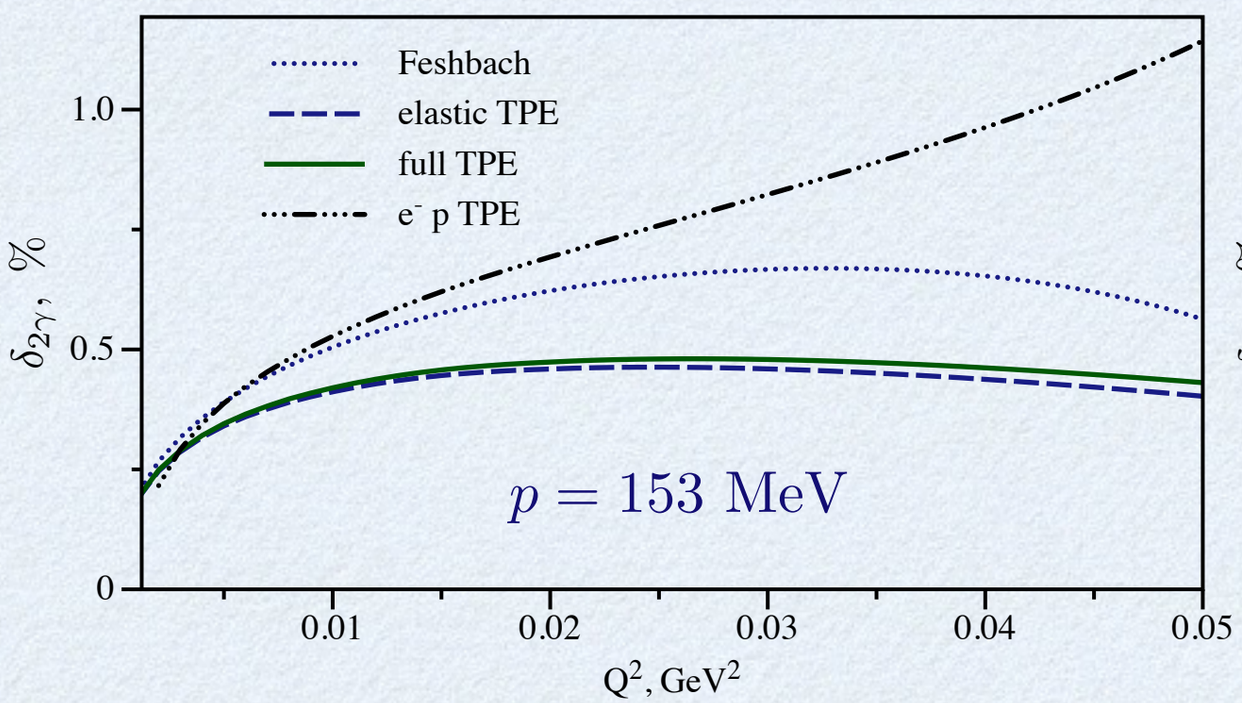


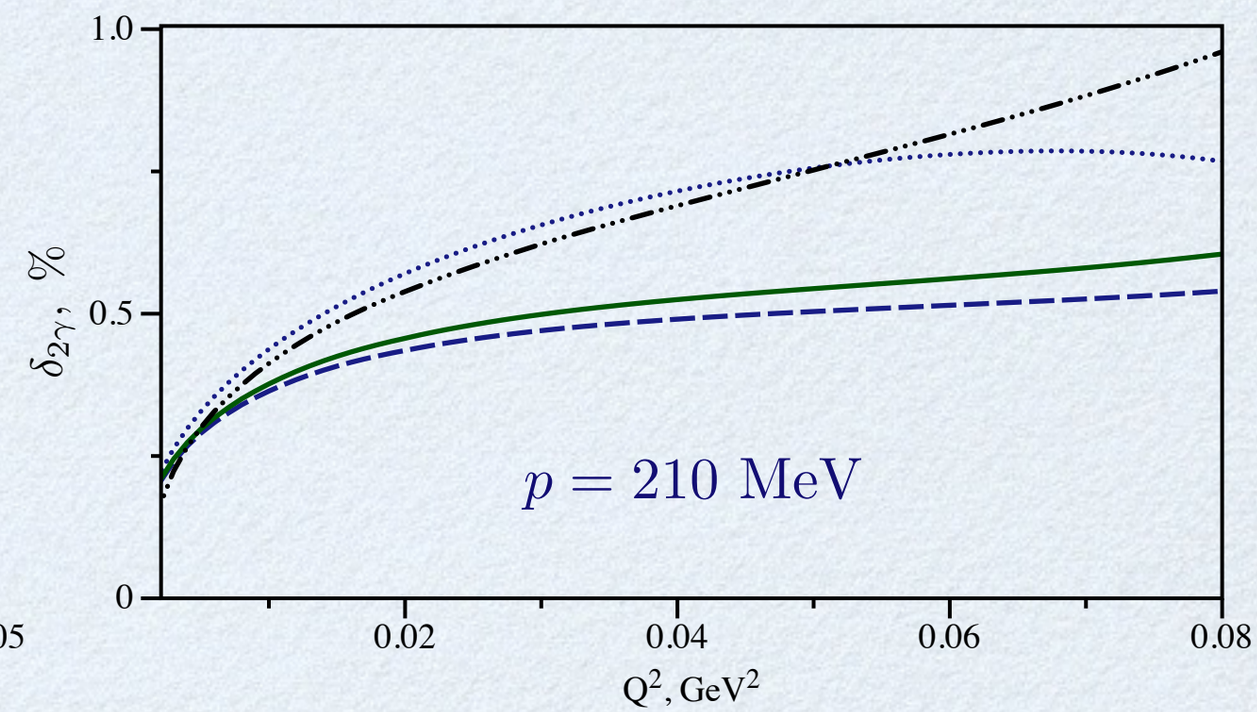
TPE blob - forward Compton scattering

perform the Wick rotation

$$\delta_{2\gamma} = \int \mathrm{d}\nu_\gamma \mathrm{d}\tilde{Q}^2 (w_1(\nu_\gamma, \tilde{Q}^2) \cdot F_1(\nu_\gamma, \tilde{Q}^2) + w_2(\nu_\gamma, \tilde{Q}^2) \cdot F_2(\nu_\gamma, \tilde{Q}^2))$$

- No $Q^2 \ln Q^2$ term in limit $Q^2 \ll m^2, M^2, ME$
  - Uncertain Q^2 region of applicability





O. Tomalak and M. Vanderhaeghen (2015)

small effect for MUSE kinematics

Conclusions

- Dispersive relations for lepton-proton scattering provide alternative method for HFS correction derivation

- Subtracted DR formalism for ep scattering proposed

- DR checked vs. hadronic model calculation (ep):
F_1F_1, F_1F_2: agreement F_2F_2 : on-shell model violates DR

- DR checked vs. hadronic model calculation (μp):
F_1F_1 : agreement F_1F_2, F_2F_2 : on-shell model violates DR

- T_1 subtraction function TPE correction studied

- Theoretical estimates for elastic (ep and μp) cross section

- Low-Q expansion reproduced (ep) and obtained (μp)

Plans

- Inclusion of inelastic intermediate states (πN)

Thanks for your attention !!!