New $SU(4)$ symmetry of hadrons after quasi-zero mode removal

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FWF Der Wissenschaftsfonds.

UNI GRAZ

Outline

1. **„Exploring a new $SU(4)$ symmetry of meson interpolators"**
   Phys. Rev. D 92, 01 (2015), 016001; hep-lat/1504.02323

2. **„Evidence for a new $SU(4)$ symmetry with $J = 2$ mesons"**
   Phys. Rev. D 91, 11 (2015), 114512; hep-lat/1505.03285

3. **„Emergence of a new $SU(4)$ symmetry in the baryon spectrum"**
   hep-lat/1508.01413

PRINCIPAL IDEA:

- We remove the chiral condensate from the valence quarks **by hand** and ask, what happens to the hadron spectrum

- Originally we wanted to study the relation between confinement and chiral symmetry breaking $\longrightarrow$ Does confinement persist the unbreaking of chiral symmetry?

- What we observe seems to be a **new symmetry of confinement**

Outline

1. Explanation of $SU(4)$ Symmetry via the Example of Spin-2 Mesons

   \+ Implications of the Symmetry

2. New Results on Baryons and $SU(4)$ Symmetry

3. Conclusions and Outlook

Quasi-zero mode removal

M. Denissenya, L. Glozman, C. B. Lang, M. Pak, M. Schröck

- Our working tool is lattice QCD
- To calculate meson masses, we evaluate correlators:

$$O_{\pi^+}(x) = \overline{d}(x)\gamma_5 u(x) \qquad \langle O_{\pi^+}(t)\overline{O}_{\pi^+}(0)\rangle$$

- In these expectation values the inverse Dirac operator occurs
- We remove the quasi-zero modes from the inverse Dirac operator via the prescription:

$$S_k(x,y) = S_{\text{FULL}}(x,y) - \sum_{i=1}^{k} \frac{1}{\lambda_i} v_i(x) v_i^\dagger(y)$$

- **Banks-Casher:** chiral condensate is connected with density of quasi-zero modes
- We decouple the condensate from the valence quarks
- Only a very small number of eigenvalues removed (10 to 30 out of millions)

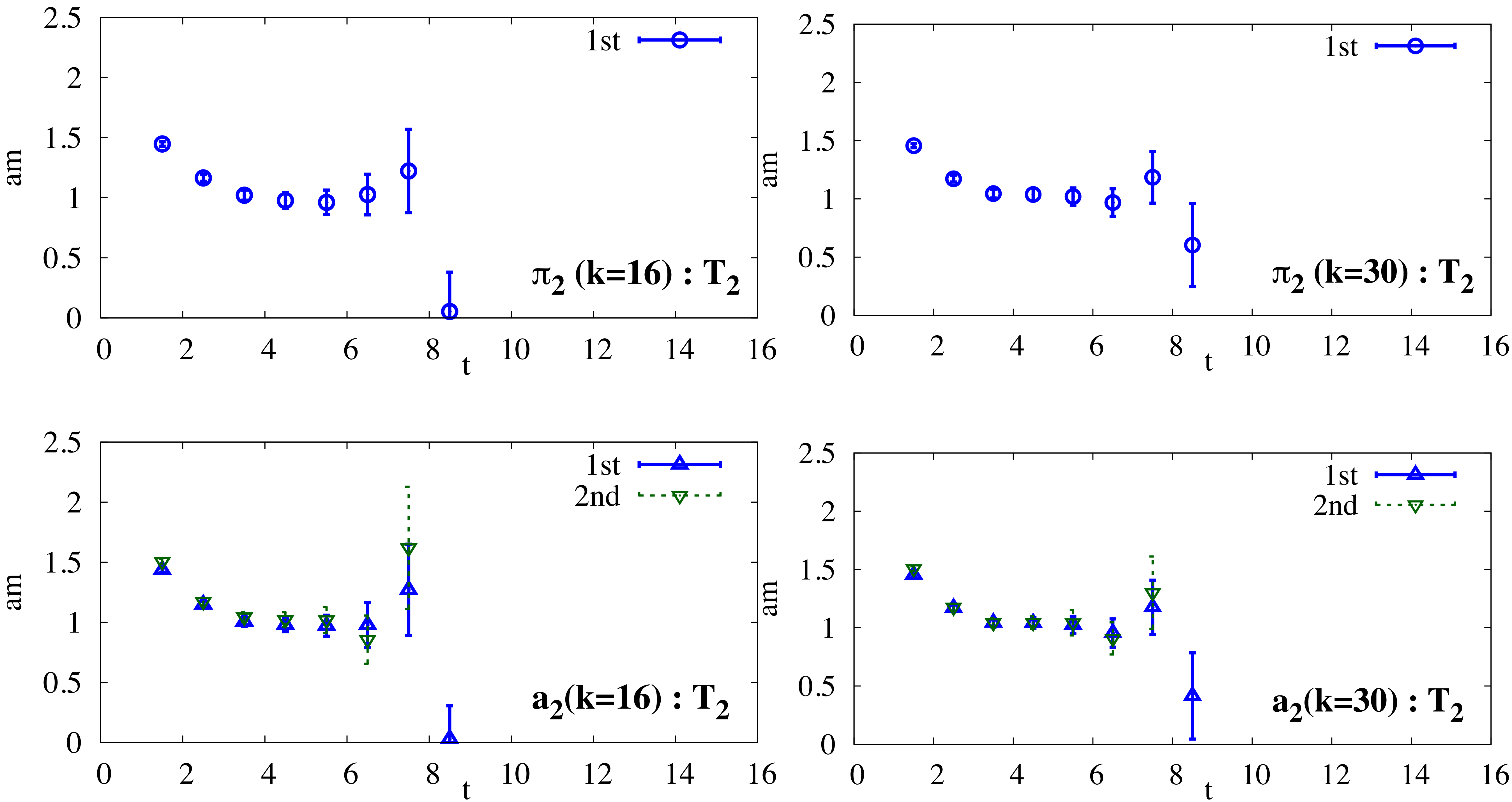
Lattice Setup and Meson Spectroscopy

- Two-flavor dynamical Overlap configurations from JLQCD on $16^3 \times 32$ lattice with $a = 0.118\,\mathrm{fm}$ S. Aoki et. al (2008)
- Pion mass $M_\pi = 289(2)\mathrm{MeV}$
- Topological sector fixed to $Q_T = 0$
- 83 gauge configurations
- Jacobi smeared and derivative based quark propagators with different smearing widths
- Spectroscopy via the variational method $C_{ij}(t) = \langle O_i(t)\overline{O}_j(0)\rangle$

$$C(t)\vec{v} = \lambda_n(t)C(n_0)\vec{v} \qquad \lambda_n(t) \sim e^{-m_n t}$$

Now we evaluate masses of the $J = 2$ iso-vector mesons π_2, a_2, ρ_2 **after quasi-zero mode removal**

Do hadrons survive the quasi-zero mode removal?



- States exist ⟶ confinement is intact ⟶ masses can be extracted
- Masses are too large to come from a system of free or weakly interacting quarks

Chiral symmetry predictions for spin-2 mesons

- Classification of states in the (I_L, I_R) irreps of $SU(2)_L \times SU(2)_R \times \mathcal{C}_i$

$(0,0)$		$\omega_2(0,2^{--})$		$f_2(0,2^{++})$	
$(1/2,1/2)_a$	$U_{\mathrm{A}}(1)$	$\pi_2(1,2^{-+})$	$\xleftrightarrow{SU_{\mathrm{A}}(2)}$	$f_2'(0,2^{++})$	$U_{\mathrm{A}}(1)$
$(1/2,1/2)_b$		$a_2'(1,2^{++})$	$\xleftrightarrow{SU_{\mathrm{A}}(2)}$	$\eta_2(0,2^{-+})$	
$(1,0) \oplus (0,1)$		$a_2(1,2^{++})$	$\xleftrightarrow{SU_{\mathrm{A}}(2)}$	$\rho_2(1,2^{--})$	

- Predictions from $SU(2)_L \times SU(2)_R \times U(1)_A$:

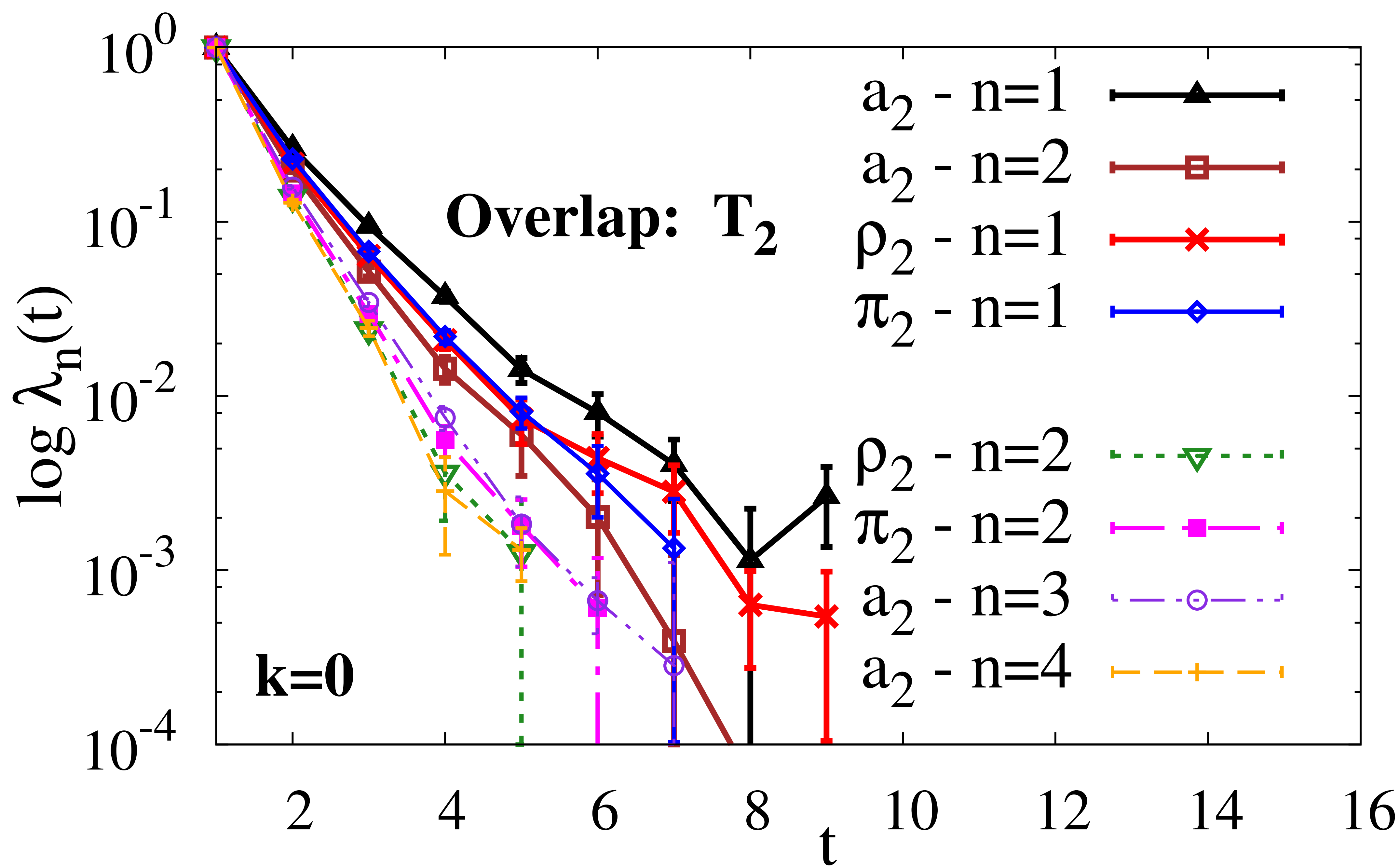
$$\pi_2 \longleftrightarrow f_2' \longleftrightarrow a_2' \longleftrightarrow \eta_2$$

$$a_2 \longleftrightarrow \rho_2$$

- **No degeneracy between these two multiplets**
- **Not all iso-vectors are mass degenerate**

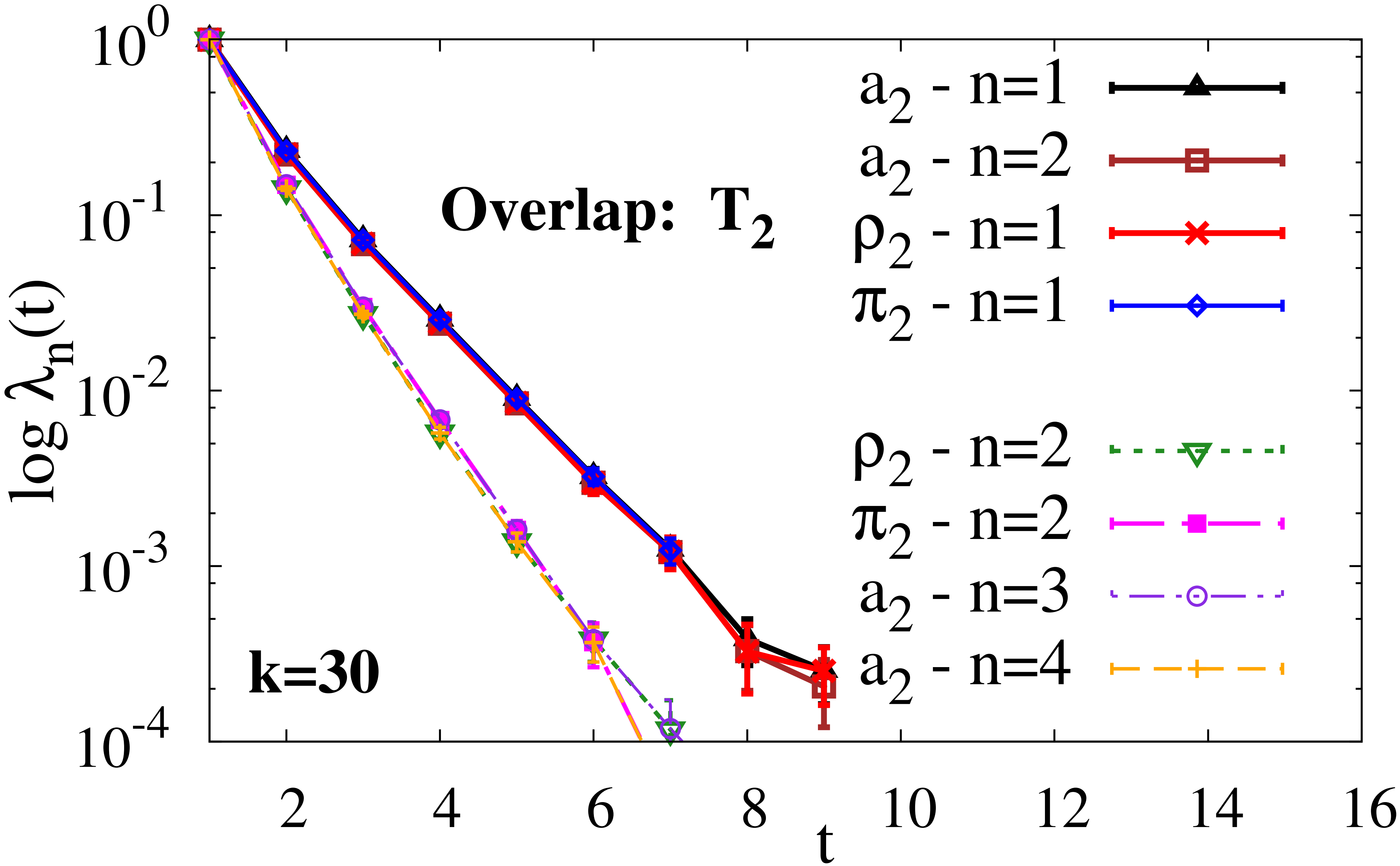
Eigenvalues of the correlation matrix

- Before chiral symmetry restoration:



Eigenvalues of the correlation matrix

- After chiral symmetry restoration:

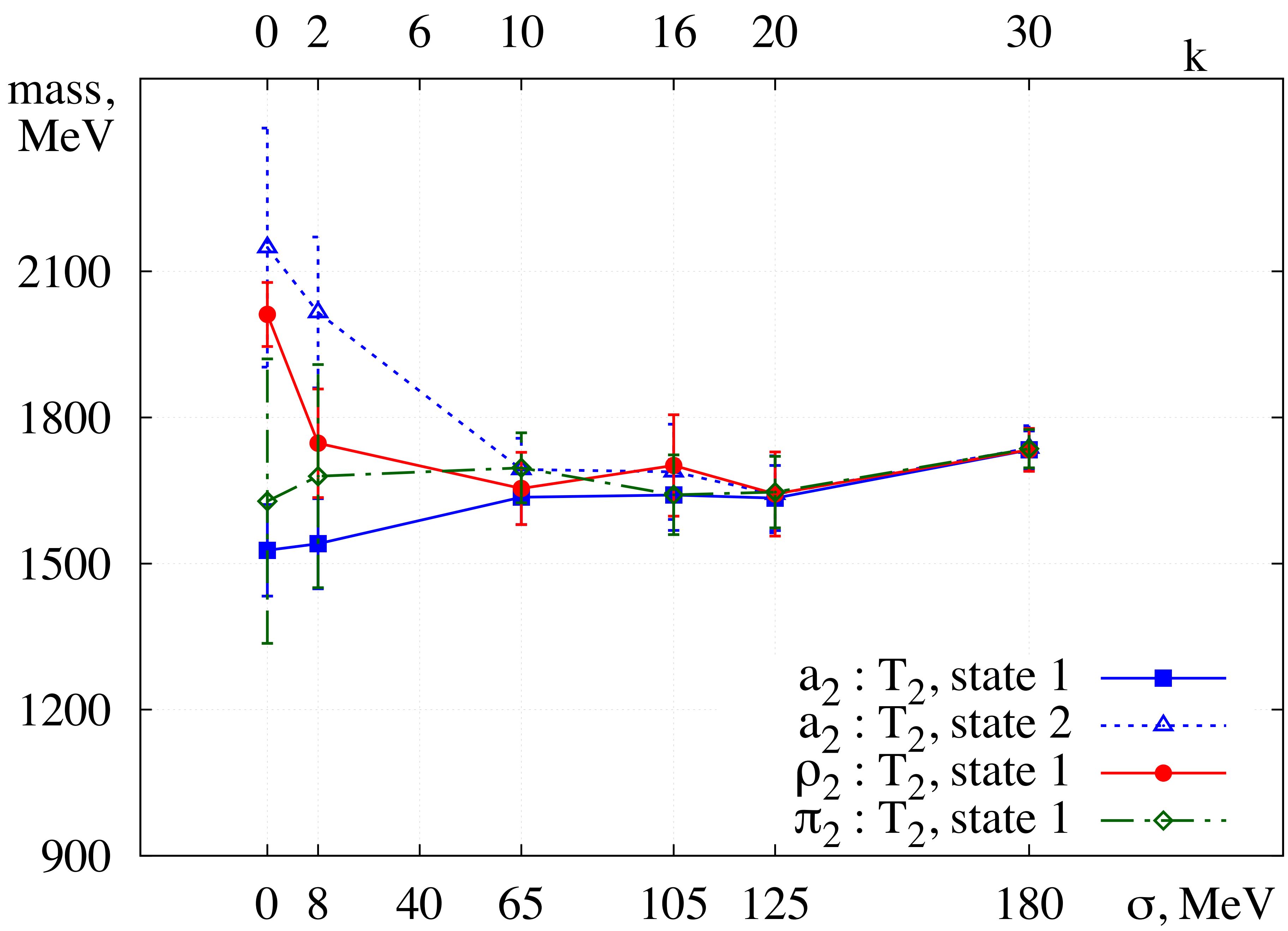


$J = 2$ meson spectrum after quasi-zero mode removal

All iso-vector states become mass degenerate

M Denissenya, L. Glozman, M.Pak;
Phys. Rev. D91 (2015) 3, 034505

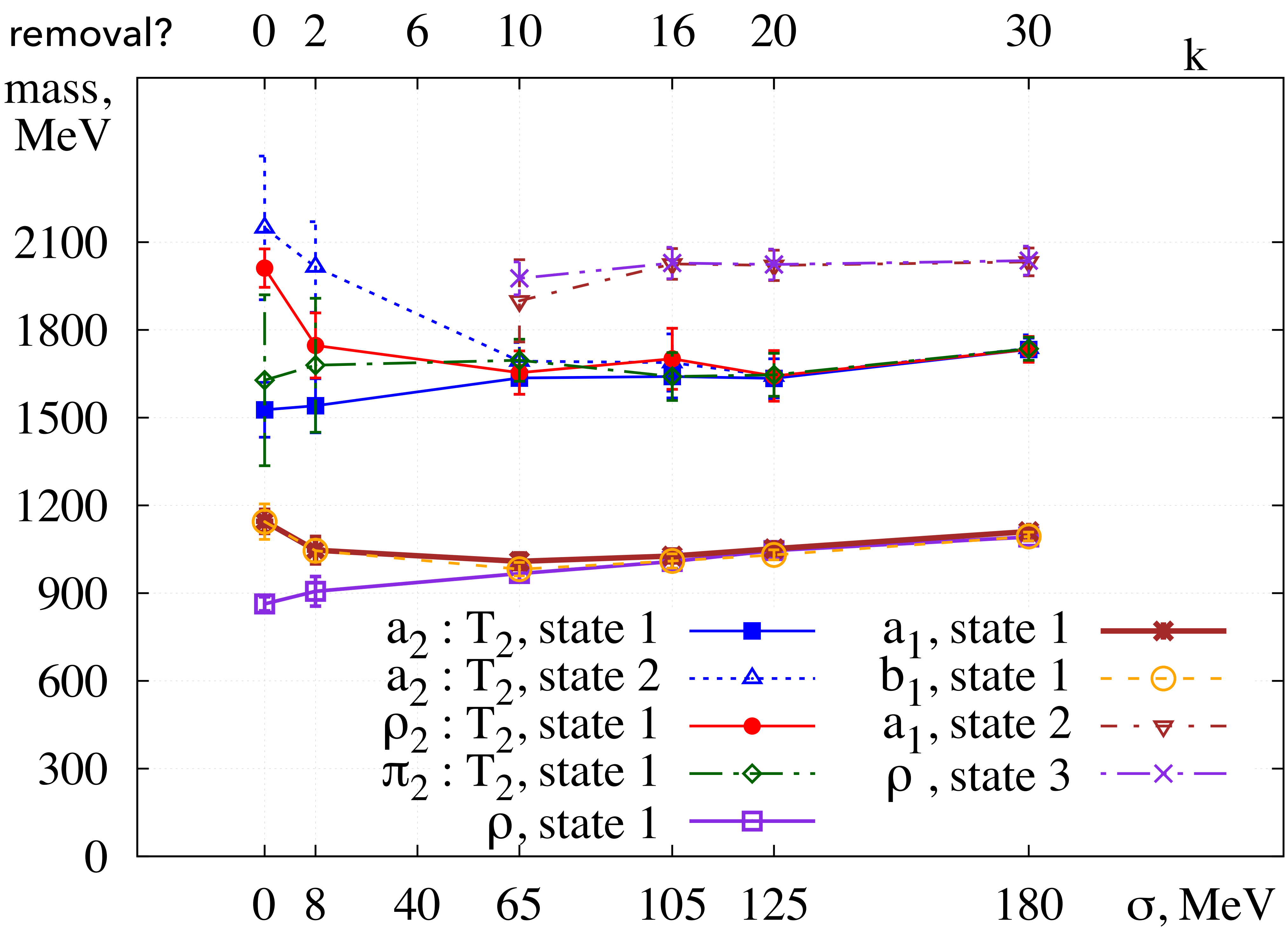
Higher symmetry than chiral symmetry is observed



Which symmetry is it? $\longrightarrow$ We find, that $SU(4)$ can explain this degeneracy

Higher symmetry?

- Degeneracy of **ground state spin-2 mesons** with **excited spin-1 mesons** after quasi-zero mode removal?



- **No evidence** for an even higher degeneracy, but for a precise statement repeat calcuation on **larger volumes**

$SU(4)$ symmetry for spin-2 mesons

$(0,0)$	$\omega_2(0,2^{--})$	$SU(4)$	$f_2(0,2^{++})$
$(1/2,1/2)_a$	$\pi_2(1,2^{-+})$		$f_2'(0,2^{++})$
$(1/2,1/2)_b$	$a_2'(1,2^{++})$		$\eta_2(0,2^{-+})$
$(1,0)\oplus(0,1)$	$a_2(1,2^{++})$		$\rho_2(1,2^{--})$

- Predictions from $SU(4)$:

$$f_2 \longleftrightarrow \pi_2 \longleftrightarrow f_2' \longleftrightarrow a_2' \longleftrightarrow \eta_2 \longleftrightarrow a_2 \longleftrightarrow \rho_2$$

- **All iso-vectors are mass degenerate**
- No constraints on mass of $\omega_2(0,2^{--})$

$SU(4)$ - symmetry

L. Glozman; Eur. Phys. J. A51 (2015) 3, 034505

L. Glozman, M. Pak; Phys. Rev. D92 (2015) 1, 016001

- **Not** a symmetry of the QCD Lagrangian; **emerges** after quasi zero-mode removal
- Is the **symmetry of hadrons** after quasi-zero mode removal
- The fundamental vector is $\Psi = \begin{pmatrix} u \\ d \end{pmatrix}$ with $\Psi \rightarrow \Psi' = e^{i\boldsymbol{\epsilon}\cdot\boldsymbol{T}/2}\Psi \equiv W\,\Psi$

$$\begin{pmatrix} u'_L \\ u'_R \\ d'_L \\ d'_R \end{pmatrix} = \begin{pmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{pmatrix} \begin{pmatrix} u_L \\ u_R \\ d_L \\ d_R \end{pmatrix}$$

Not only quarks of fixed chirality mix, but also the **left- and right-handed components**

- All states of given J **except one isoscalar state** become mass degenerate via $SU(4)$
- Subgroups: $SU(4) \supset SU(2)_{CS} \times SU(2)_L \times SU(2)_R \times U(1)_A$

What does the symmetry mean?

- We look at the QCD Hamiltonian in **Coulomb gauge**
- There are **two interactions** of quarks with gluons:
  - Interaction with **color-electric field** (via color-Coulomb interaction)
  - Interaction with **spatial gluons**
- **Confinement**: two static quarks mediated by color-Coulomb interaction **give a linear rising potential**
- Here we consider dynamical quarks
- The interaction with the **spatial gluons is forbidden** due to $SU(4)$
- After removing the quasi-zero modes, we have a situation where quarks interact with the color-electric field only $\longrightarrow$ **dynamical string**

Implications (QCD in Coulomb Gauge)

- **Coulomb interaction:** (comes from the color-electric field; is the confining part)

$$H_C = \frac{g^2}{2} \int d^3x d^3y J^{-1} \, \rho^a(\boldsymbol{x}) F^{ab}(\boldsymbol{x}, \boldsymbol{y}) \, J \, \rho^b(\mathbf{y})$$

  - This part is $SU(4)$ symmetric

Color-Coulomb potential

- **Coupling to transverse gluons:**

$$H_T = -g \int d^3x \, \Psi^\dagger(\boldsymbol{x}) \boldsymbol{\alpha} \cdot \boldsymbol{A}(\boldsymbol{x}) \, \Psi(\boldsymbol{x})$$

  - This part is **not** $SU(4)$ symmetric

- The only interaction left in the system is via the color-Coulomb potential

Symmetry of Confinement

- After removing the quasi-zero modes $SU(4)$ becomes the **symmetry of confinement**

- The hadrons can be viewed to be primaly $SU(4)$ energy levels, before the dynamics of the quasi-zero modes are switched on

- It could be used to construct a **new order parameter** for the confinement-deconfinement transition

L.Glozman; hep-ph/1508.02885

- It could be important for **highly excited hadrons**, where it is conjectured, that states are less affected by the chiral condensate

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2. **New Results on Baryons and $SU(4)$ Symmetry**

3. **Conclusions and Outlook**

Chiral symmetry predictions for baryons

- Nucleon and Delta Interpolators are of the form:

$$N_{\pm}^{(i)} = \varepsilon_{abc} \mathcal{P}_{\pm} \Gamma_1^{(i)} u_a \left(u_b^T \Gamma_2^{(i)} d_c - d_b^T \Gamma_2^{(i)} u_c \right)$$

$$\Delta_{\pm}^{(i)} = \varepsilon_{abc} \mathcal{P}_{\pm} \Gamma_1^{(i)} u_a \left(u_b^T \Gamma_2^{(i)} u_c \right)$$

I, J^P	$\Gamma_1^{(i)}$	$\Gamma_2^{(i)}$	r
$N(\frac{1}{2}, \frac{1}{2}^{\pm})$	$\mathbb{1}$	$C\gamma_5$	$(\frac{1}{2}, 0) + (0, \frac{1}{2})$
	γ_5	C	$(\frac{1}{2}, 0) + (0, \frac{1}{2})$
	$i\mathbb{1}$	$C\gamma_5\gamma_0$	$(1, \frac{1}{2}) + (\frac{1}{2}, 1)$
$\Delta(\frac{3}{2}, \frac{1}{2}^{\pm})$	$i\gamma_i\gamma_5$	$C\gamma_i$	$(1, \frac{1}{2}) + (\frac{1}{2}, 1)$
$N(\frac{1}{2}, \frac{3}{2}^{\pm})$	$i\gamma_5$	$C\gamma_i\gamma_5$	$(1, \frac{1}{2}) + (\frac{1}{2}, 1)$
$\Delta(\frac{3}{2}, \frac{3}{2}^{\pm})$	$i\mathbb{1}$	$C\gamma_i$	$(1, \frac{1}{2}) + (\frac{1}{2}, 1)$

$U_{\mathrm{A}}(1)$

$SU(4)$

$SU_{\mathrm{A}}(2)$

Does this symmetry apply for baryons as well?

- We now take the two $J = \frac{1}{2}$ nucleon correlators, which are **not** related via chiral symmetry

$$\mathcal{O}_{N^\pm} = \varepsilon^{abc}\mathcal{P}_\pm u^a \left[u^{bT} C\gamma_5 d^c - d^{bT} C\gamma_5 u^c\right]$$

$$\mathcal{O}_{N^\pm} = \varepsilon^{abc}\mathcal{P}_\pm u^a \left[u^{bT} C\gamma_5\gamma_0 d^c - d^{bT} C\gamma_5\gamma_0 u^c\right]$$

- However, they are in the same irreducible rep of $SU(4)$

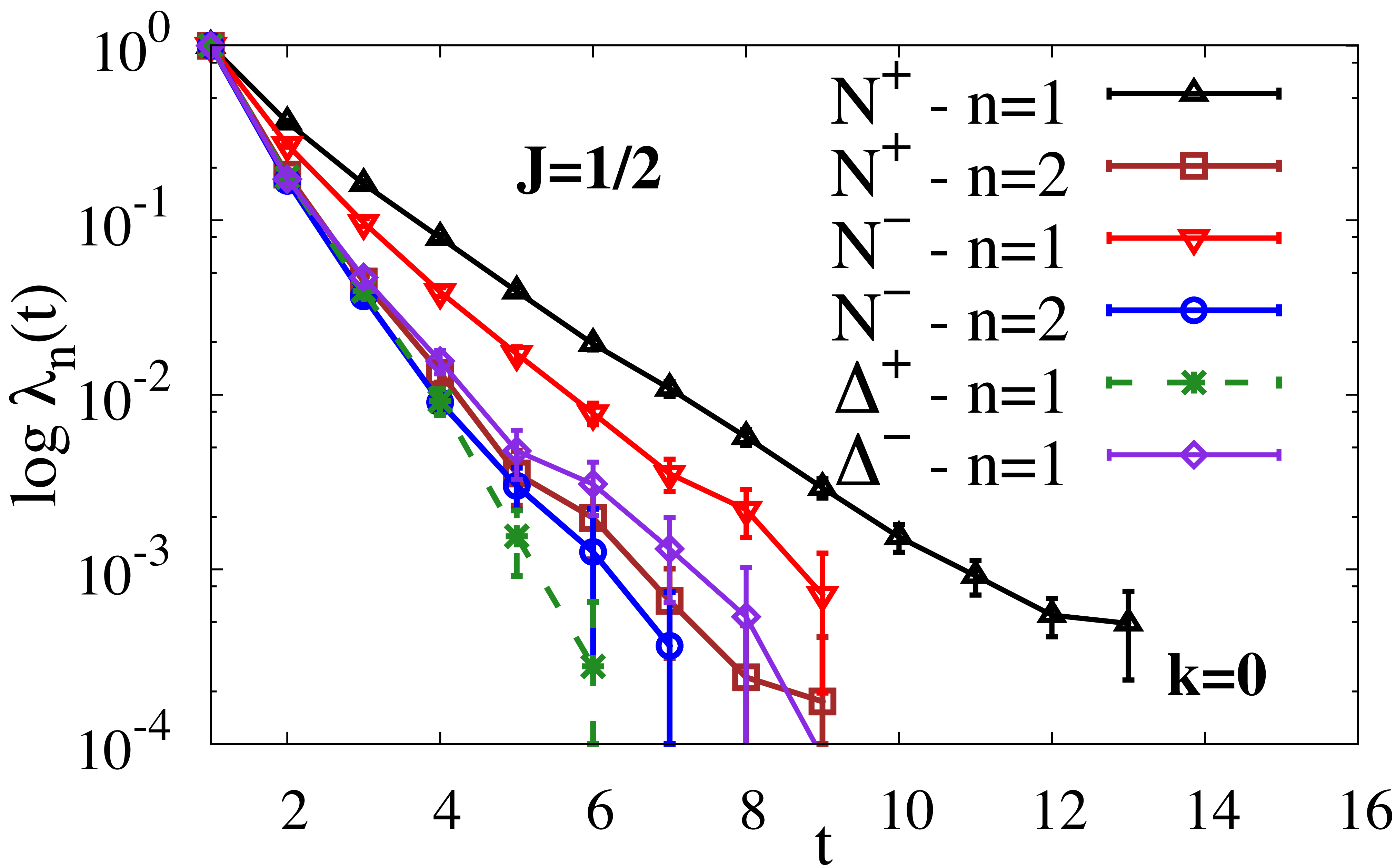
- If their correlators coincide, then $SU(4)$ is a symmetry of baryons as well

- We also take a $J = \frac{1}{2}$ delta correlator into account

$$\mathcal{O}_{\Delta^\pm} = \varepsilon^{abc}\mathcal{P}_\pm \gamma_i u^a \left[u^{bT} C\gamma_i u^c\right]$$

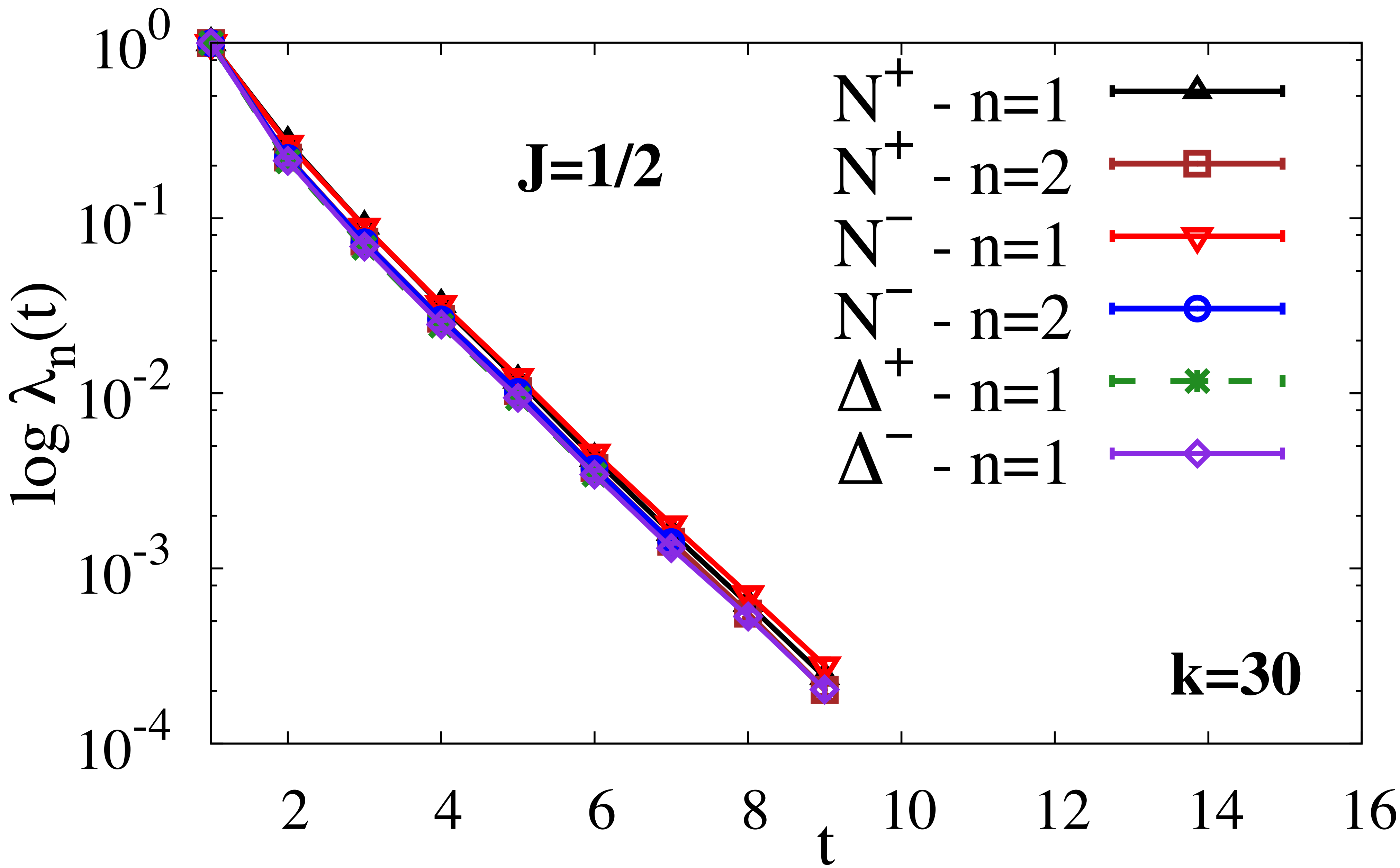
$J = 2$ Correlators

- Before chiral symmetry restoration:



$J = 2$ Correlators

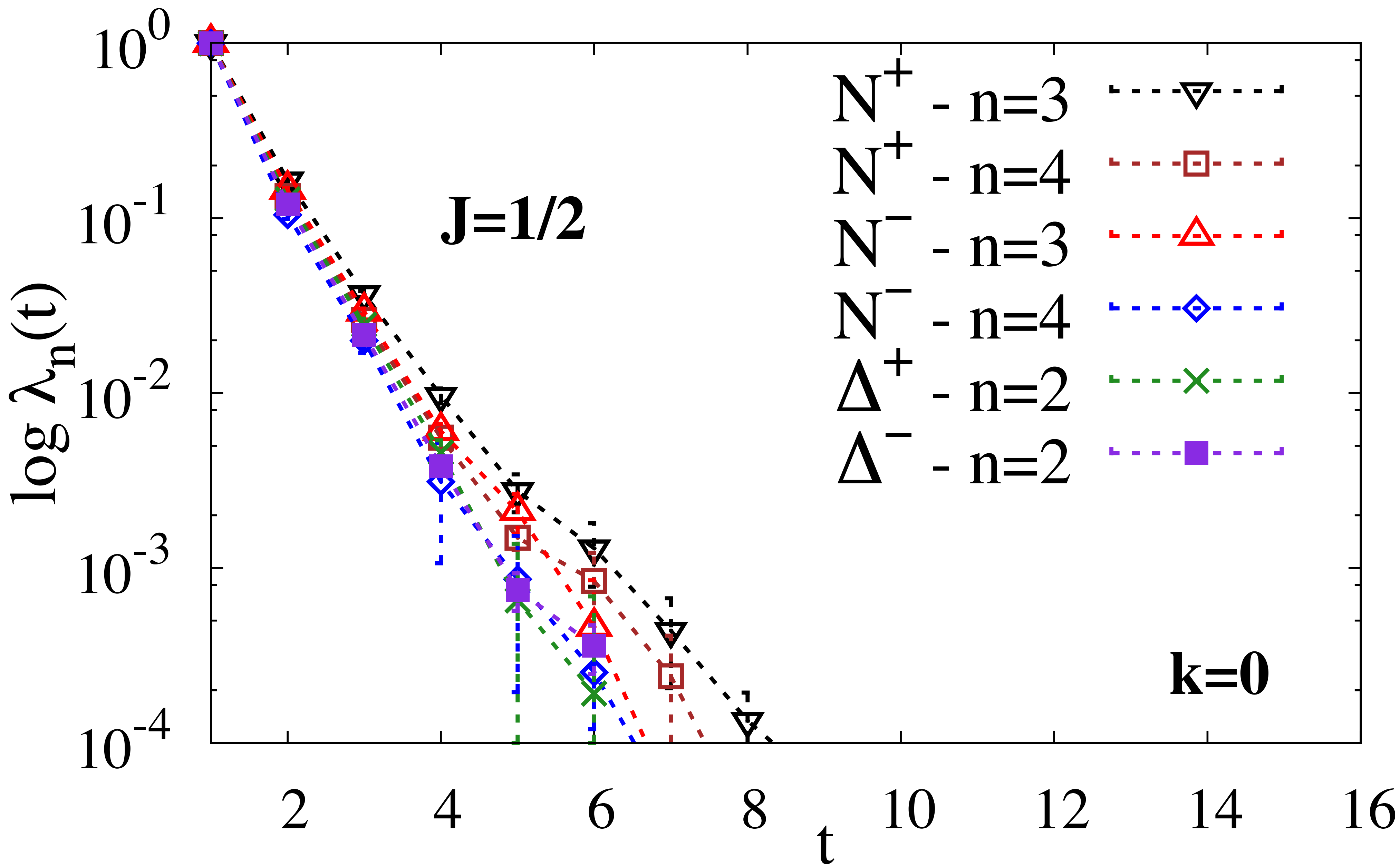
- After chiral symmetry restoration:



All correlators are degenerate! $SU(4)$ is a symmetry of baryons after quasi-zero mode removal

$J = 2$ Correlators

- Before chiral symmetry restoration:



$J = 2$ Correlators

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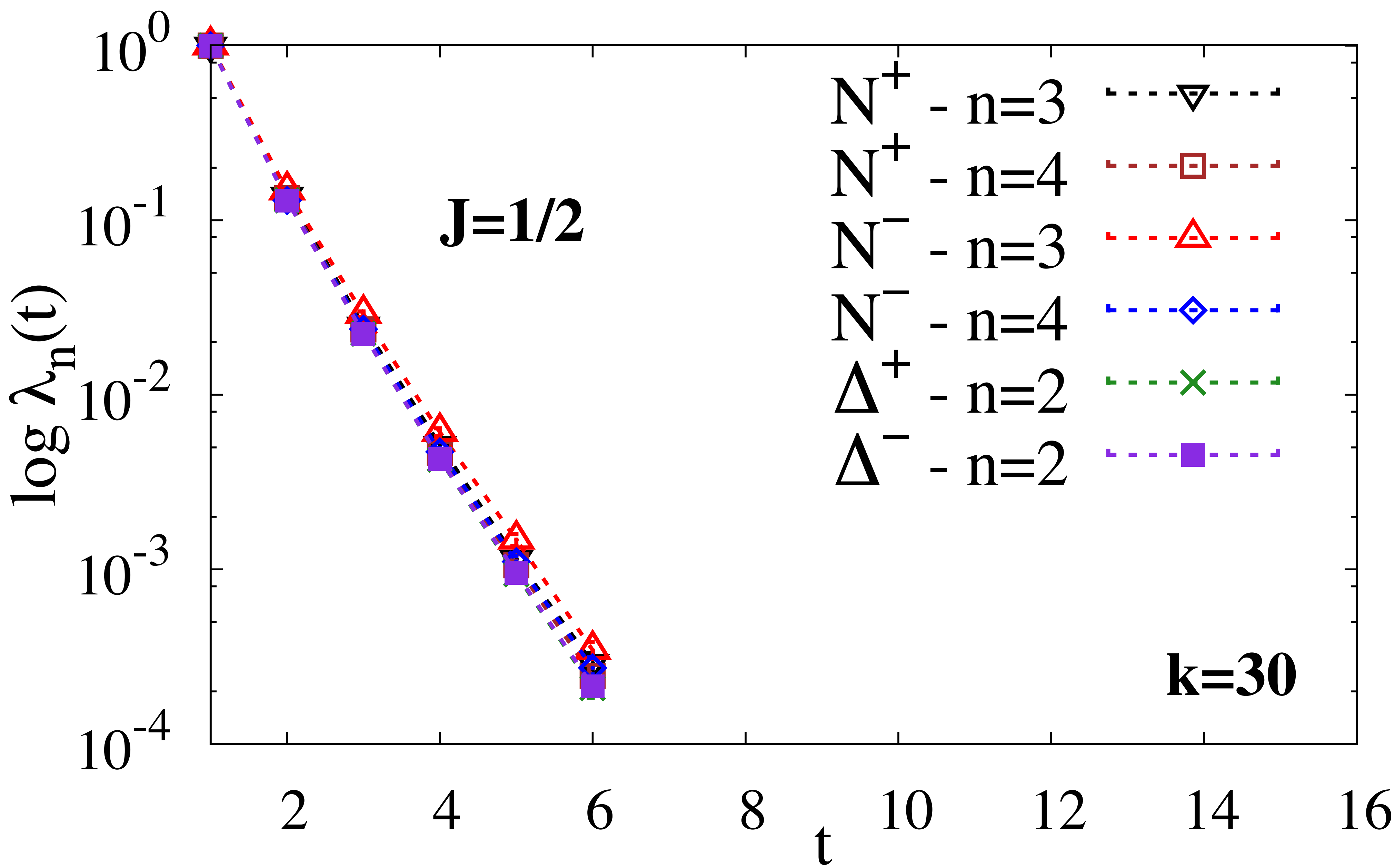
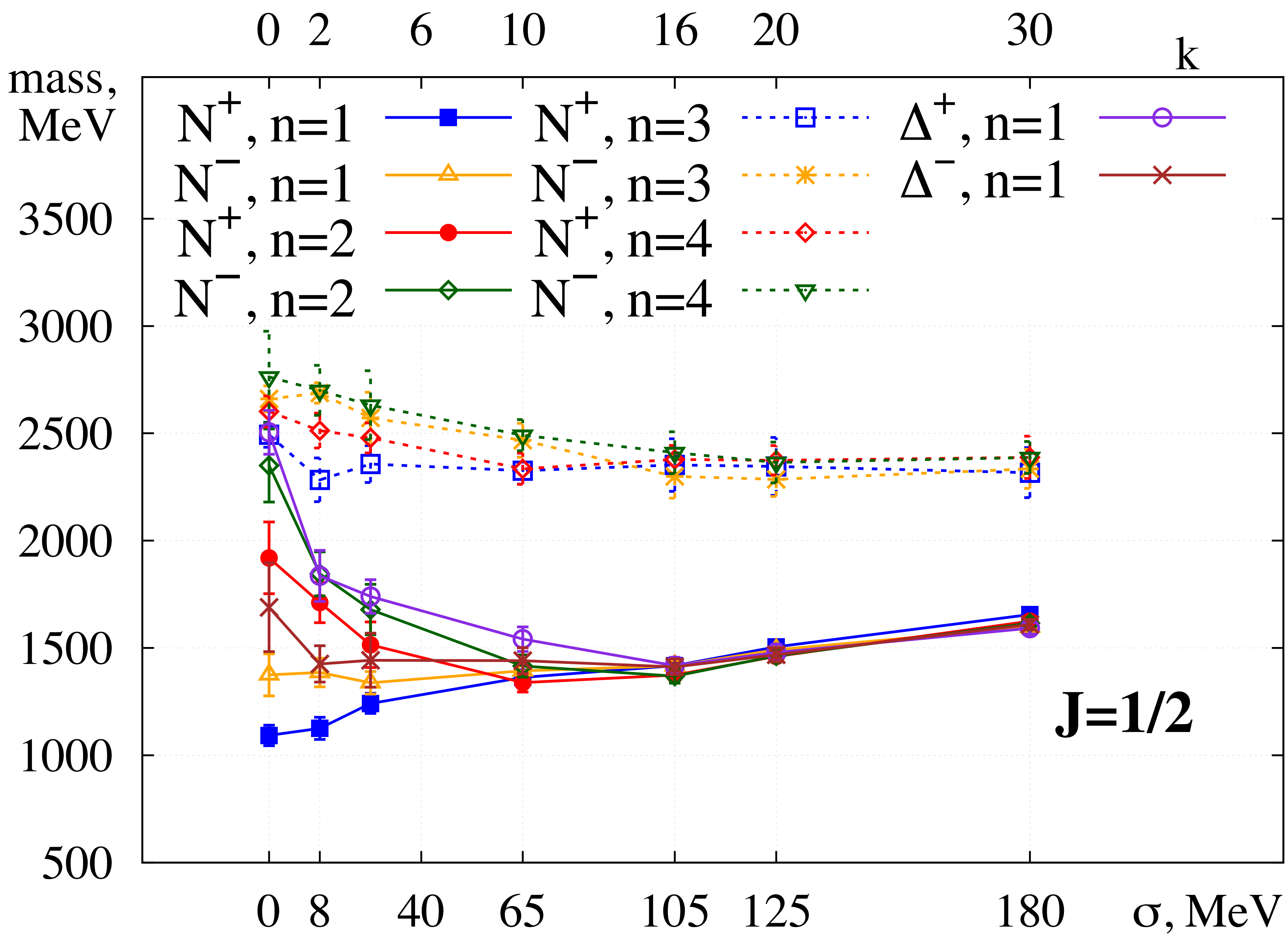


Baryon mass evolution



- Nucleon and Delta states from different $SU(4)$ multiplets are degenerate - higher degeneracy? Currently under investigation!

Summary and Conclusions

MESON SECTOR:

- Spin-2 mesons show emergent $SU(4)$ degeneracy pattern after quasi-zero mode removal
- We expect, that this is general and $SU(4)$ applies for all $J \geq 1$ mesons

BARYON SECTOR:

- With $J = \frac{1}{2}$ baryons we see the symmetry; for $J = \frac{3}{2}$ baryons more interpolators have to be included (under construction)
- It is speculated that an **even higher symmetry** is seen, because interpolators from two different irreducible reps are mass degenerate

THEORETICAL OBSERVATIONS:

- **Only interaction** left in the system after quasi-zero mode removal is color-Coulomb interaction