Glueballs and tetraquarks from Dyson-Schwinger equations

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universitätbonn

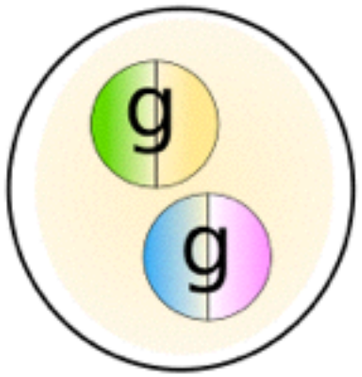
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HIC for FAIR

Helmholtz International Center

Overview

1. Introduction



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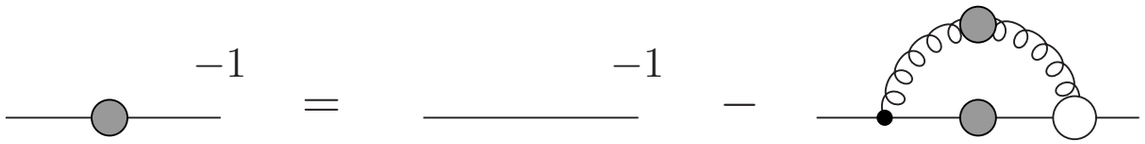
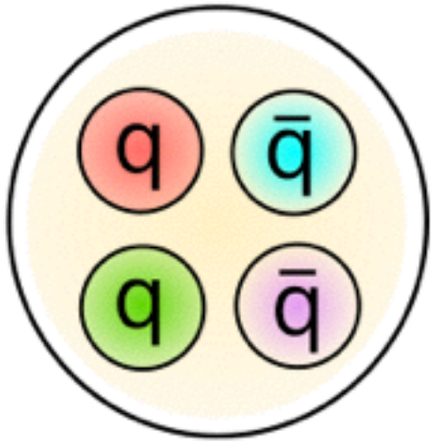


3. Quarks and mesons



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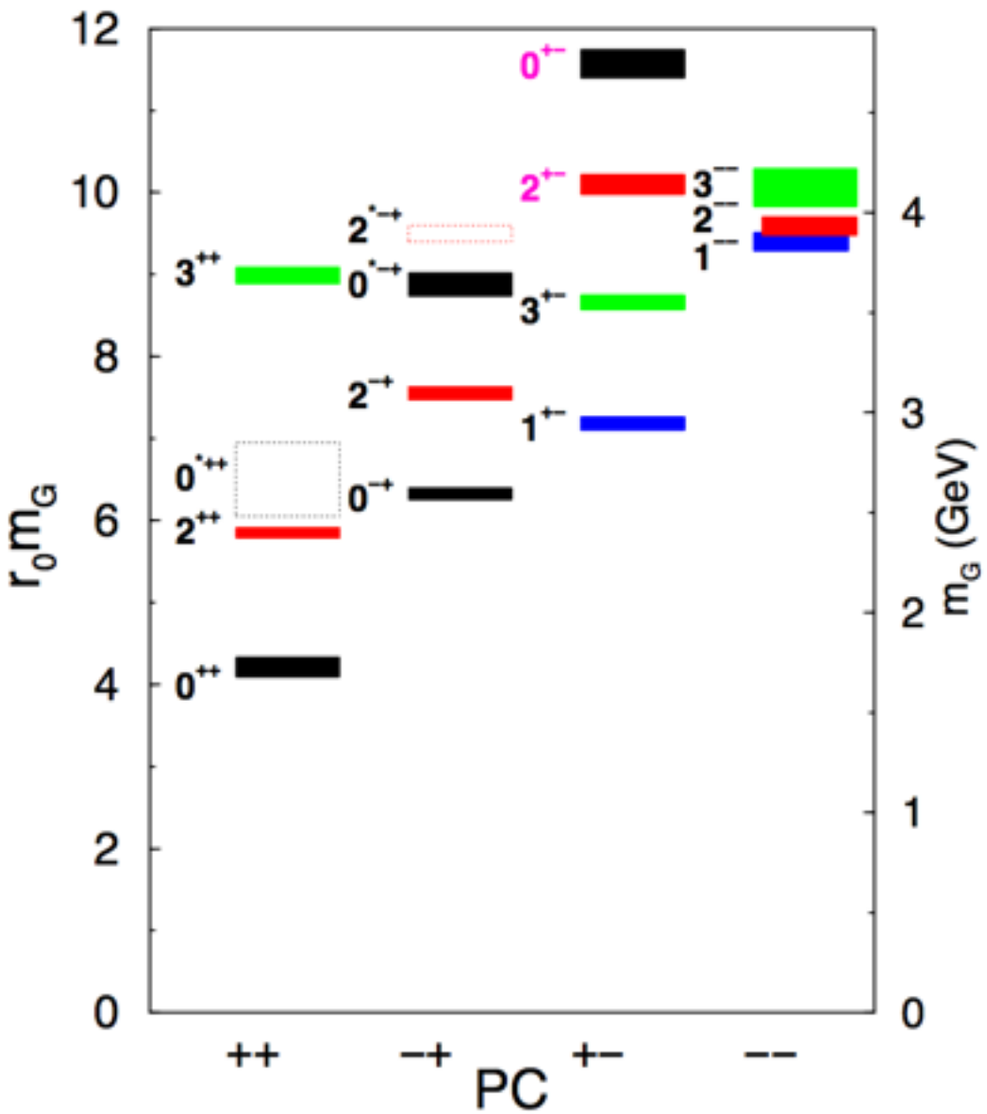
Glueballs



Morningstar and Peardon, PRD 60 (1999) 034509
Y.~Chen et al., PRD 73 (2006) 014516

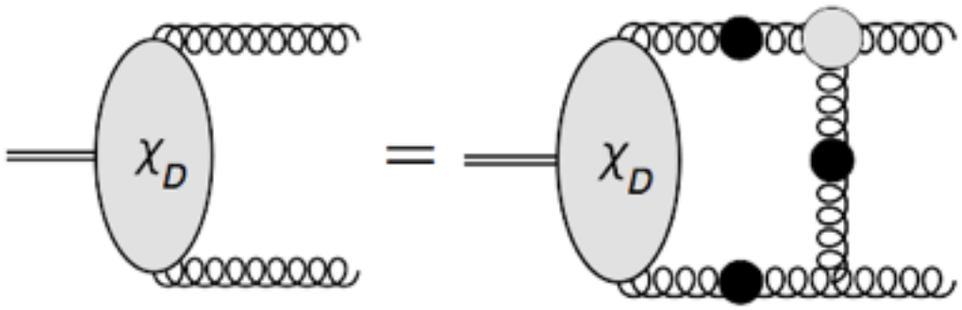
Lattice:

- States in the light and heavy quark energy regions
- Most calculations quenched
- Preliminary unquenched results: larger masses

Gregory et al., JHEP 1210 (2012) 170

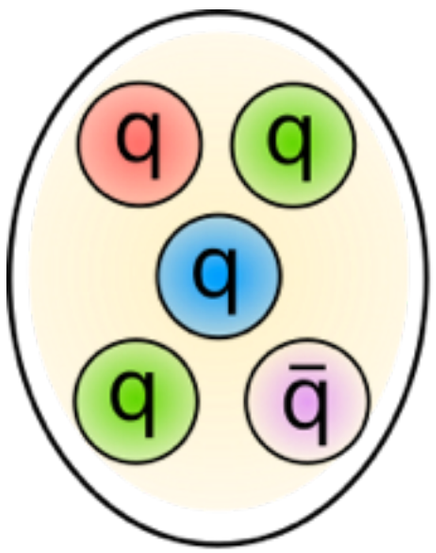
DSE:

- structural information

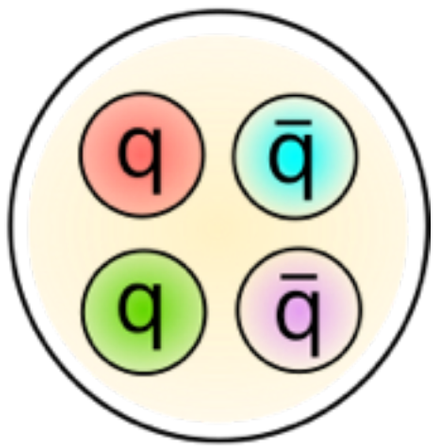


Meyers, Swanson, PRD 87 (2013) 3, 036009
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

Tetraquarks in the light meson sector



Pentaquark



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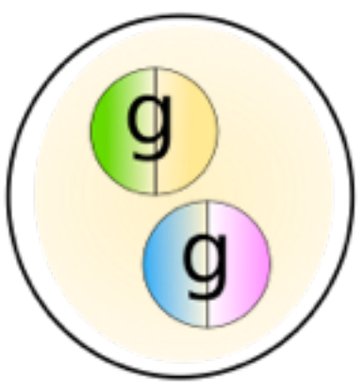

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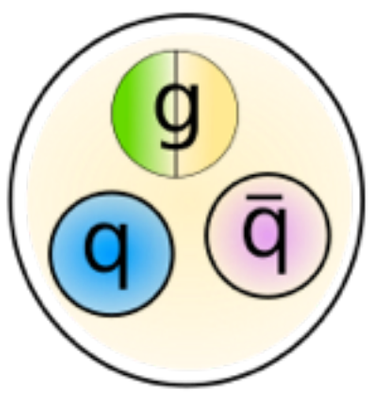
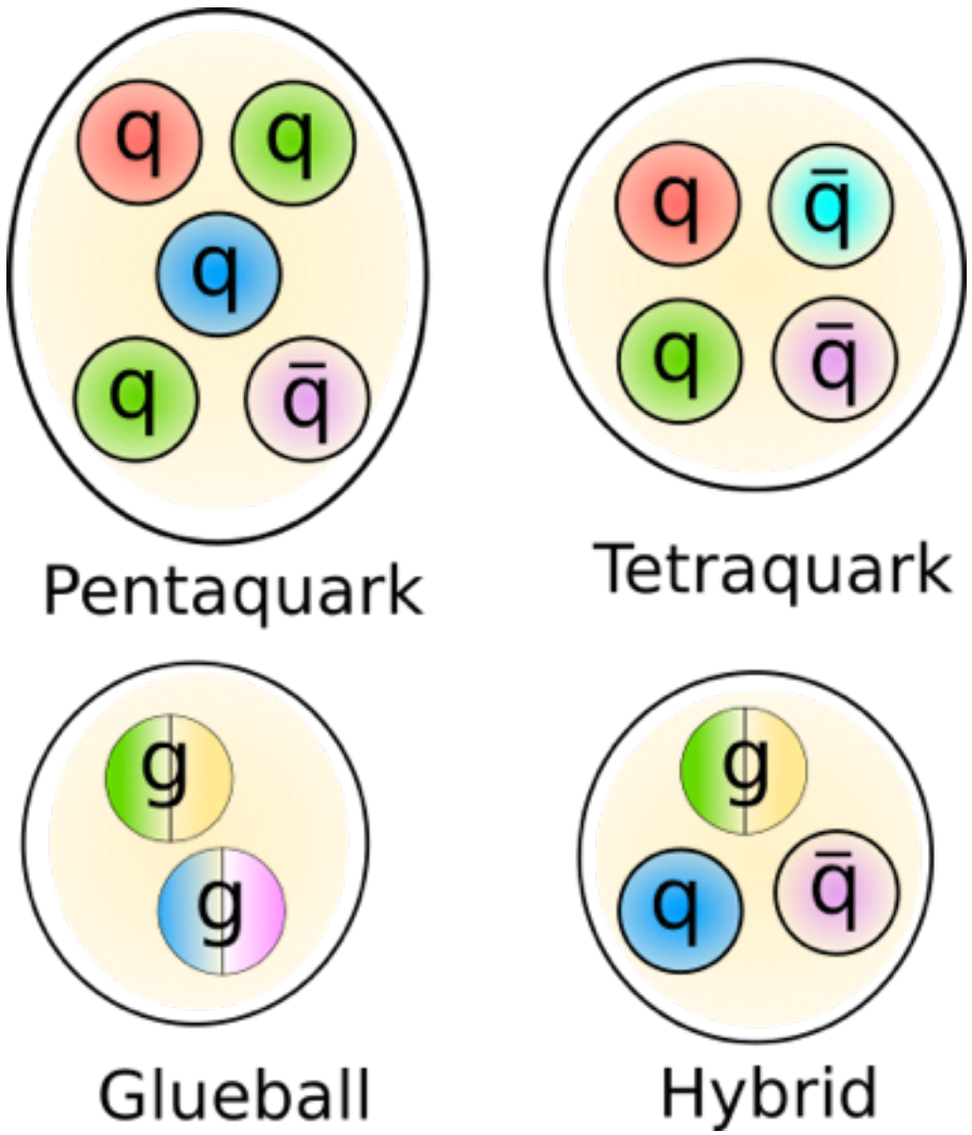
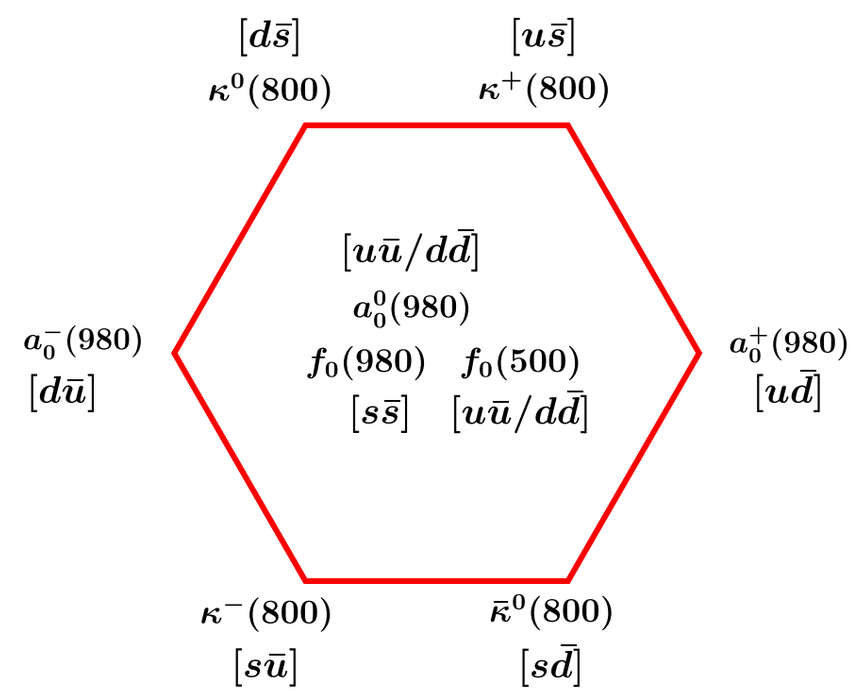


Hybrid

Tetraquarks in the light meson sector

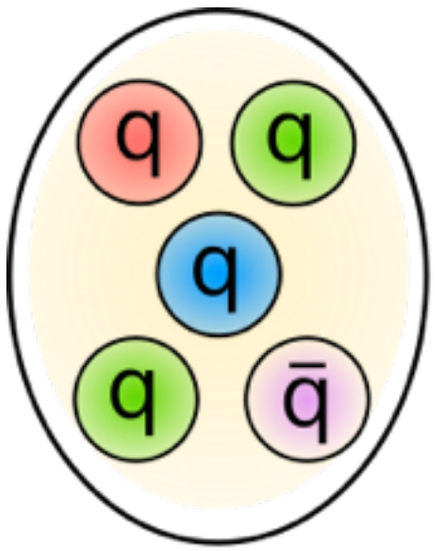


Light meson sector:
Scalars!

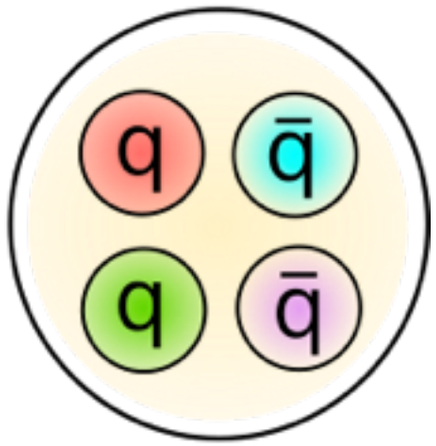


$$f_0(980) \to \pi\pi, K\bar{K}$$
$$a_0(980) \to \pi\pi, K\bar{K}$$

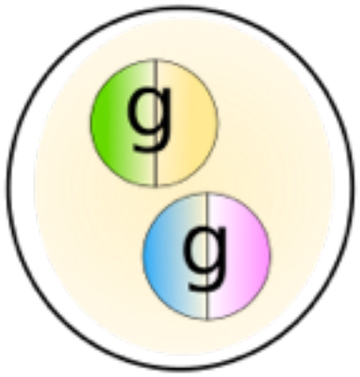
Tetraquarks in the light meson sector



Pentaquark



Tetraquark



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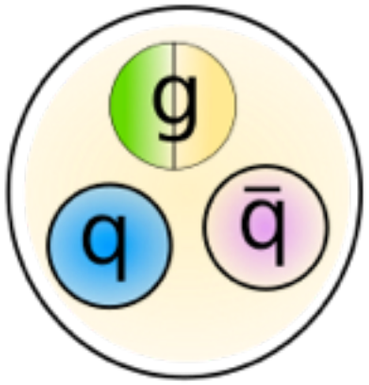

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Light meson sector:
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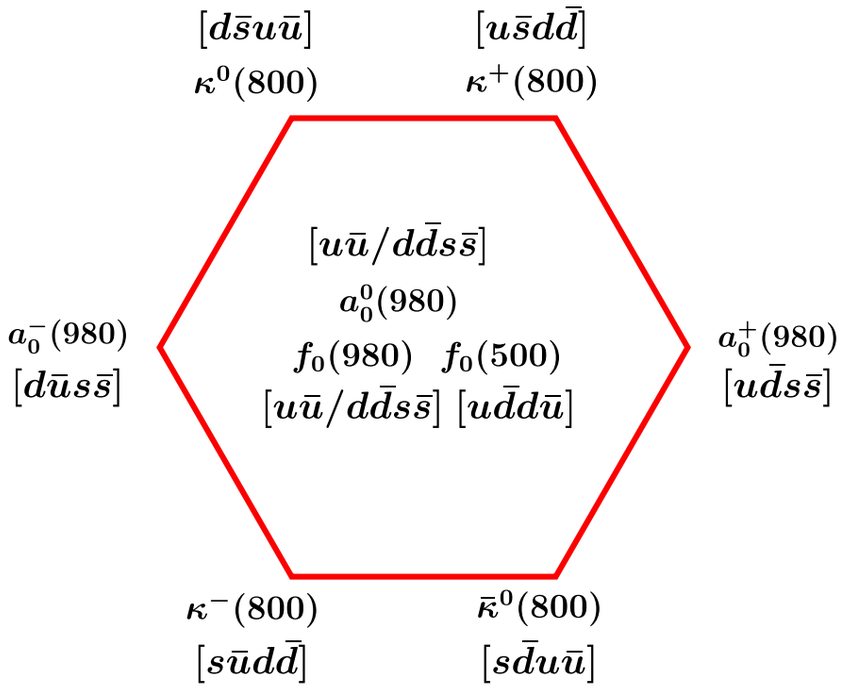

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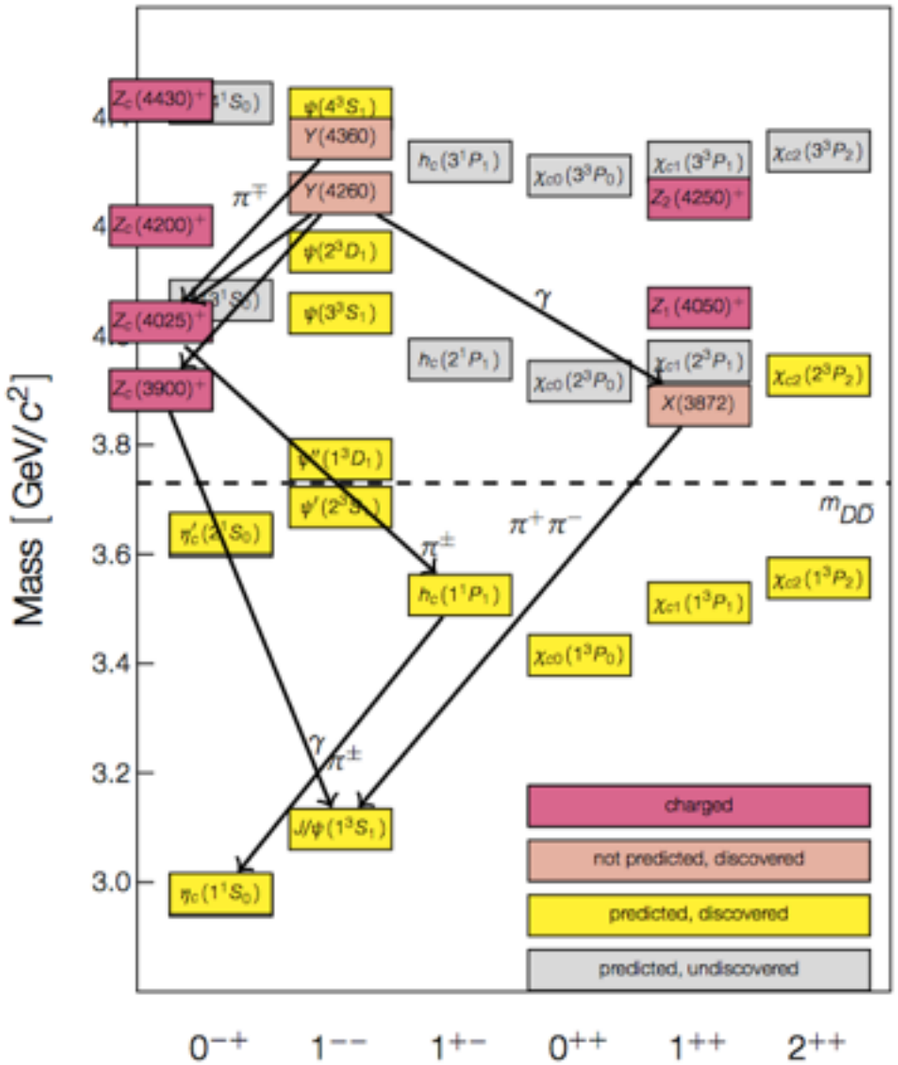
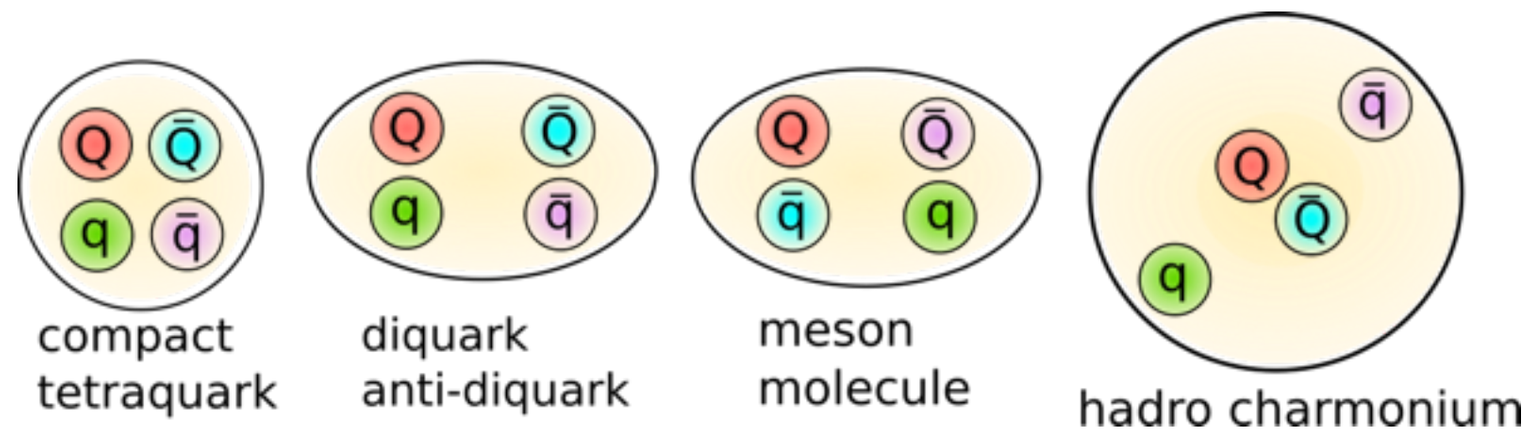
Tetraquark candidates in charmonium region



Wolfgang Gradl, BESIII, St Goar 2015

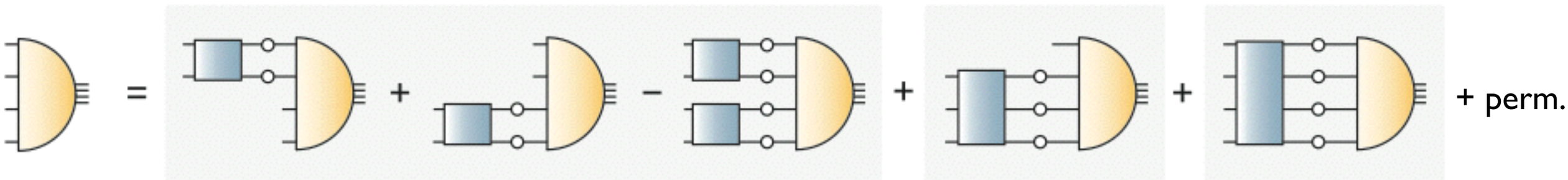
Internal structure ??



Related to details of underlying QCD forces between quarks

Tetraquarks from the four-body interaction

Exact equation:



Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichman, CF, PLB 718 (2012) 545-549

Eichman, CF, Heupel, 1508.07178

- Basic idea:
  solve four-body equation without any assumption on internal clustering
- Key elements: quark propagator and interaction kernels

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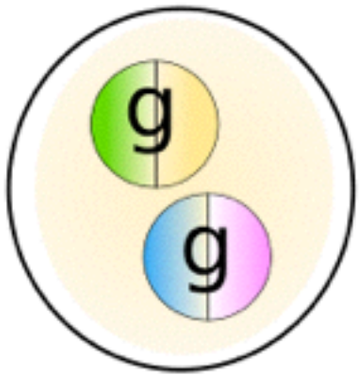

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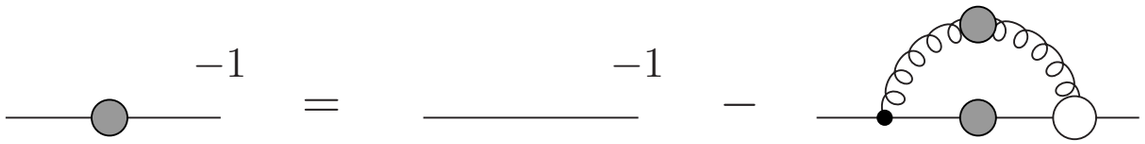

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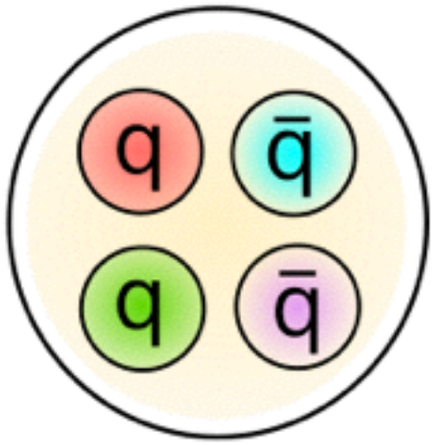


4. Tetraquarks

QCD in covariant gauge

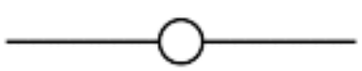
Quarks, gluons and ghosts

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \exp\left\{ -\int d^4x \left(\bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}(F^a_{\mu\nu})^2 \right.\right.$$

$$\left.\left. + \text{gauge term} + \bar{c}(-\partial D)c) \right\}\right.$$

Landau gauge propagators in momentum space,

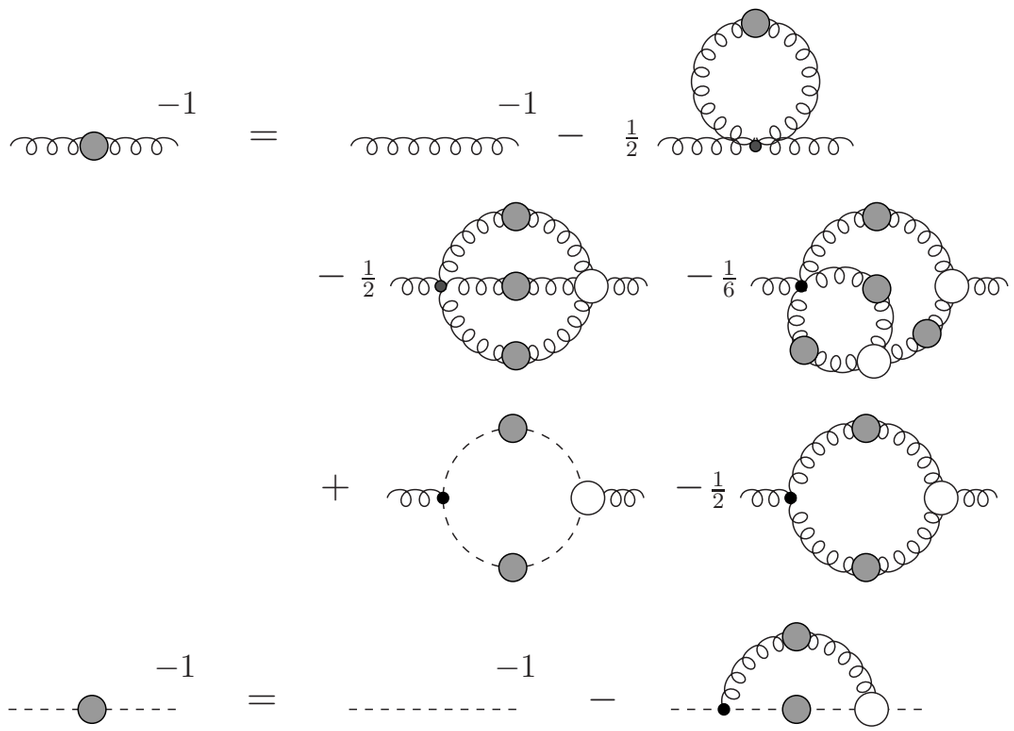
$$D^{Gluon}_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$

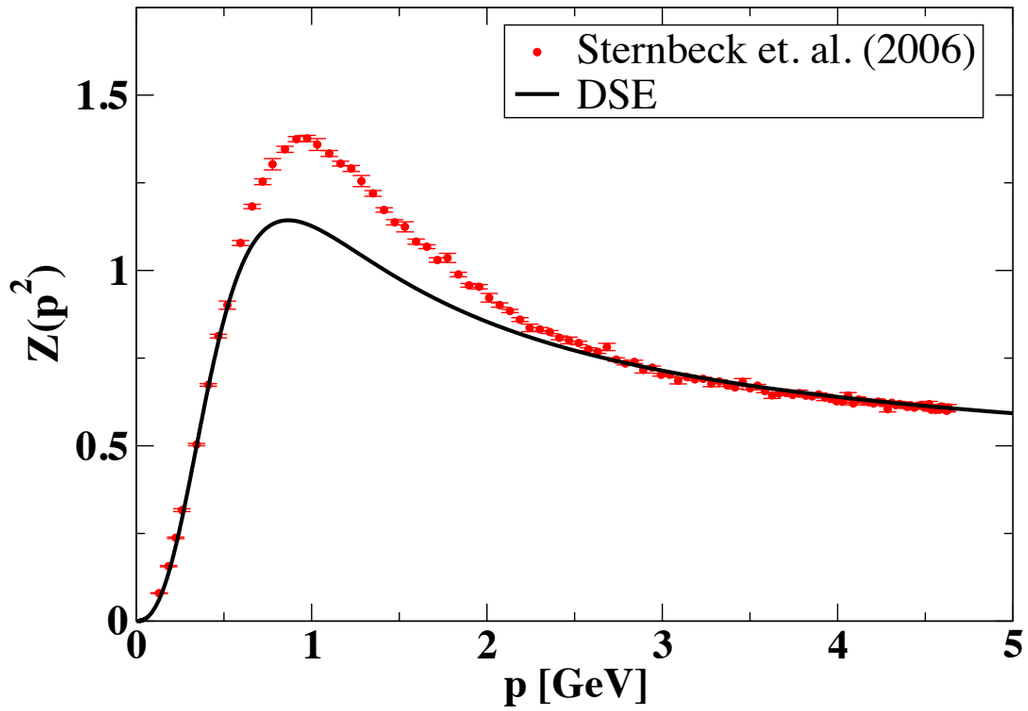


$$S^{Quark}(p) = Z_f(p^2)[-i\not{p} + M(p^2)]^{-1}$$

The Goal: gauge invariant information in a gauge fixed approach.

Landau gauge gluon propagator





$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

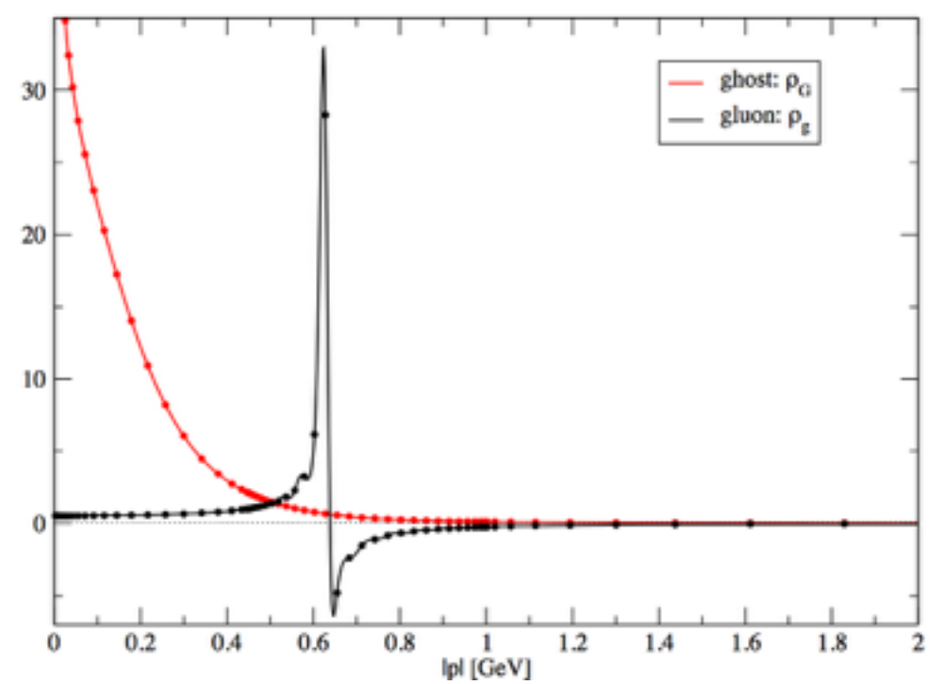
CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408.
Huber and von Smekal, JHEP 1304 (2013) 149

- spacelike momenta: excellent agreement with lattice
- spectral function: positivity violations

$$600\,\mathrm{MeV} < m_g < 700\,\mathrm{MeV}$$

Cornwall, Papavassiliou,...

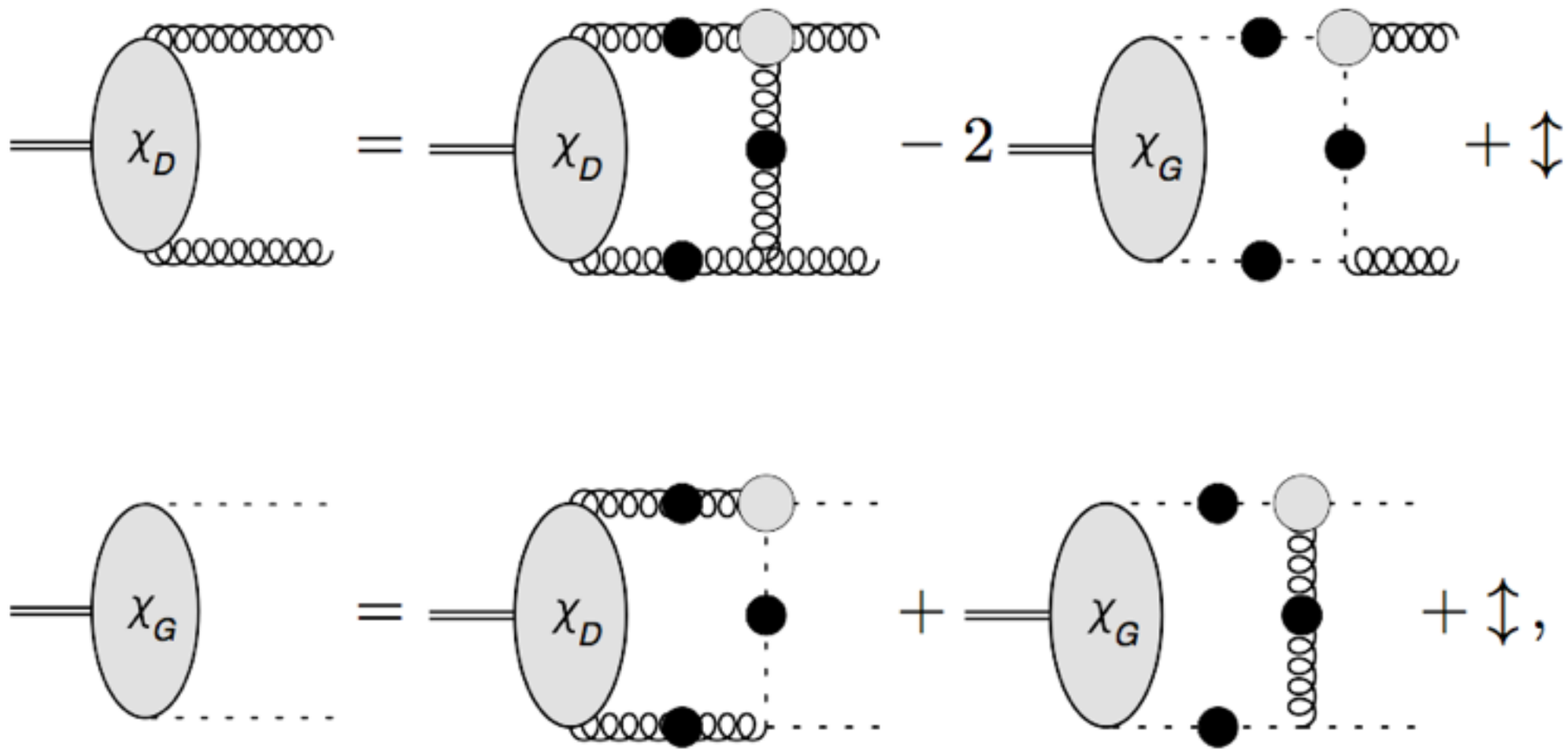
Gluon cannot appear in detector!



spectral function

Strauss, CF, Kellermann, Phys. Rev. Lett. 109, (2012) 252001

Glueballs from DSE/BSEs



- Mixing of two-gluon amplitudes with ghost-antighost
- Probes analytical structure of gluons and ghosts

Results: $M(0^{++}) = 1.64\,\mathrm{GeV}$

$M(0^{-+}) = 4.53\,\mathrm{GeV}$

← ghost do not contribute !

Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

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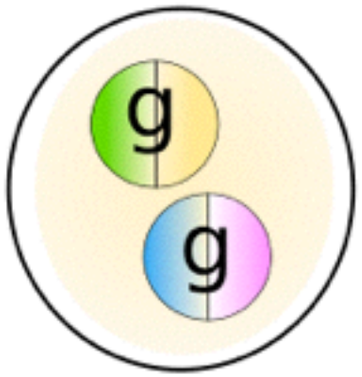

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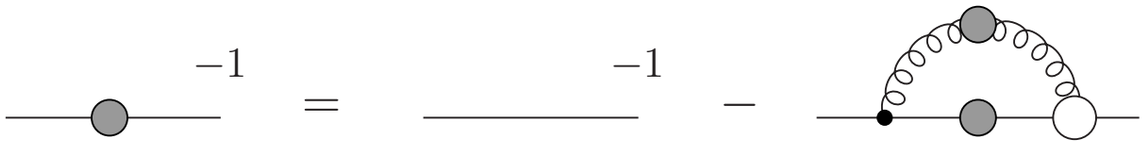

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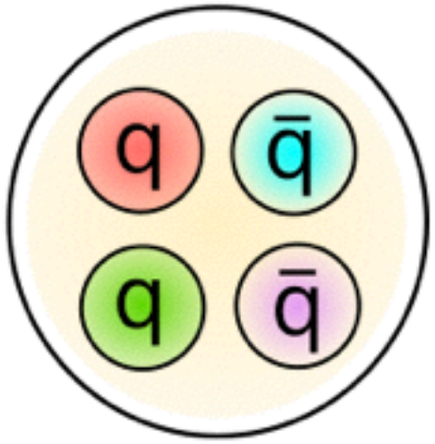
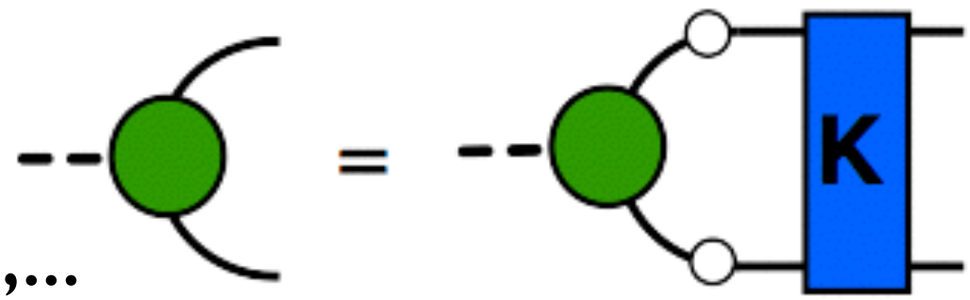


DSEs and Bethe-Salpeter equation



π, ρ,...

Kernel K uniquely related to quark-DSE via axWTI

→Pion is bound state **and** Goldstone boson

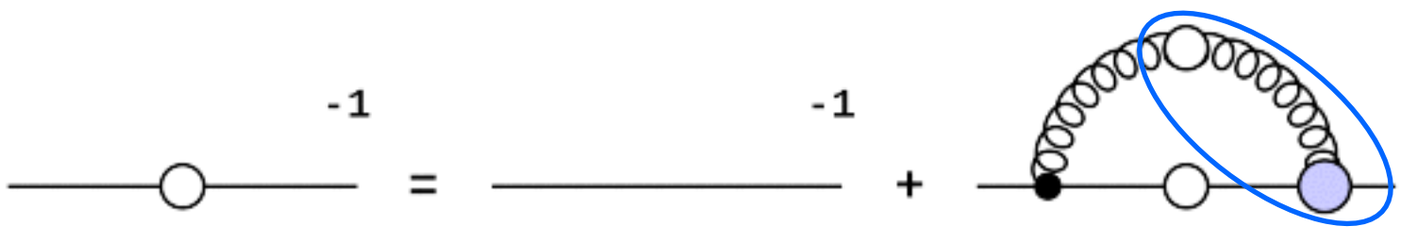
Maris, Roberts, Tandy, PLB 420 (1998) 267

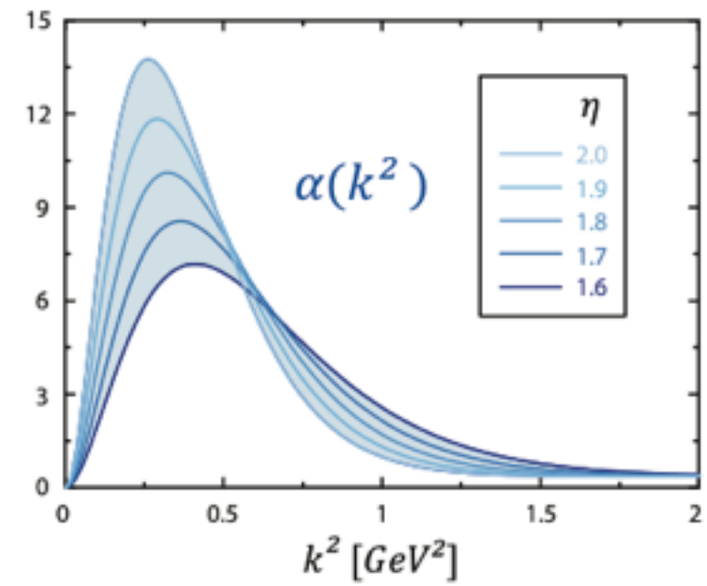
Two strategies:

I. use rainbow-ladder model for quark-gluon interaction
→ok for some phenomenological applications

II. calculate gluon and vertex from their DSEs

Strategie I: Model for quark-gluon interaction





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

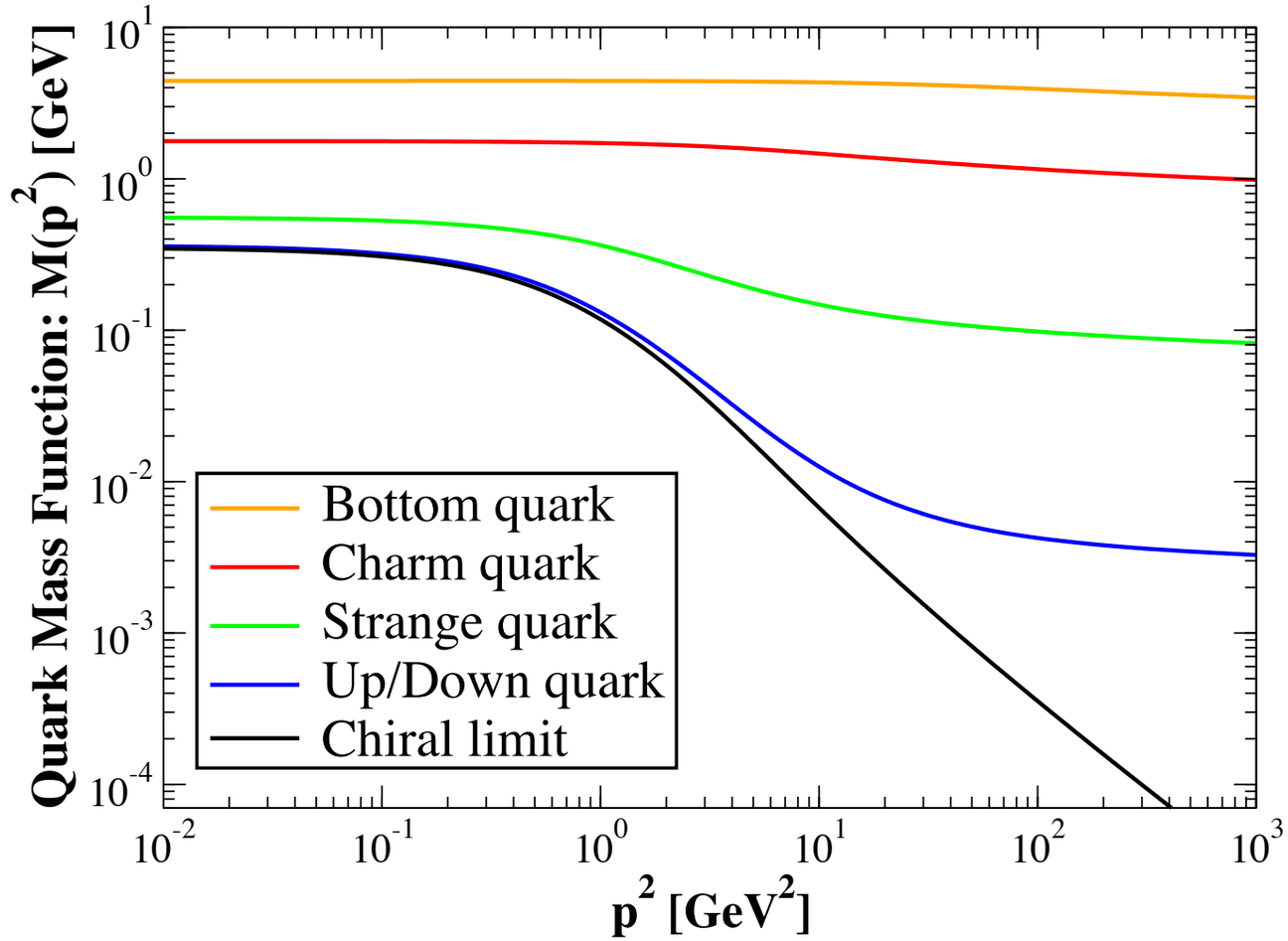
$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

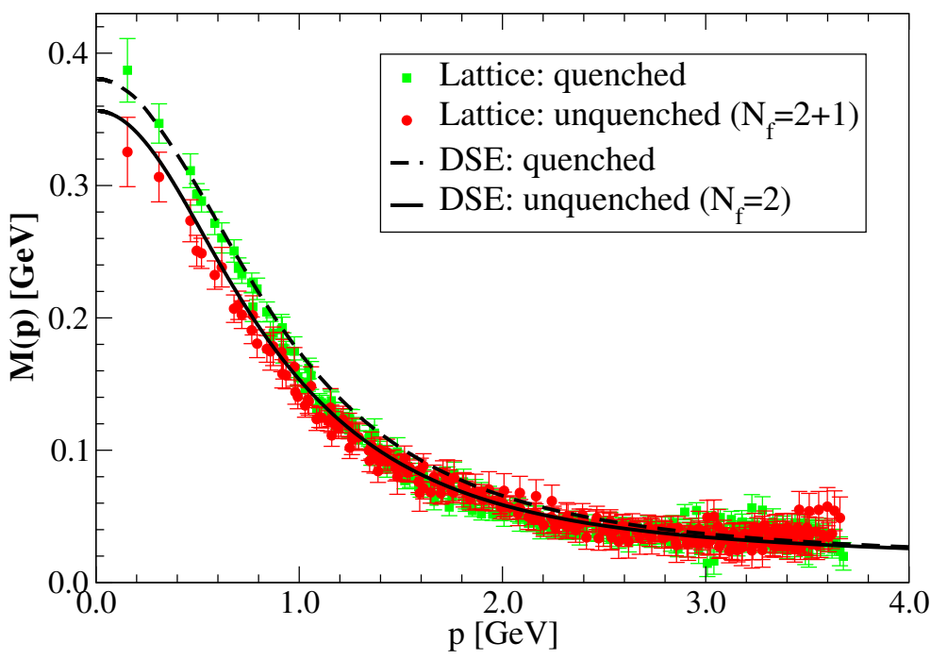
- fix Λ from f_π; small dependence of many results on η
- masses m_u=m_d, m_s, m_c, from π, K, J/ψ
- Renormalizable and momentum dependent !
- Qualitatively similar to results from explicit calculation

CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408.
Williams, EPJA 51 (2015) 5, 57.

Quark mass: flavor dependence

Typical solution:





CF, Nickel, Williams, EPJ C 60 (2009) 47

- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: $\langle \bar{\Psi}\Psi \rangle \approx (250\,\mathrm{MeV})^3$

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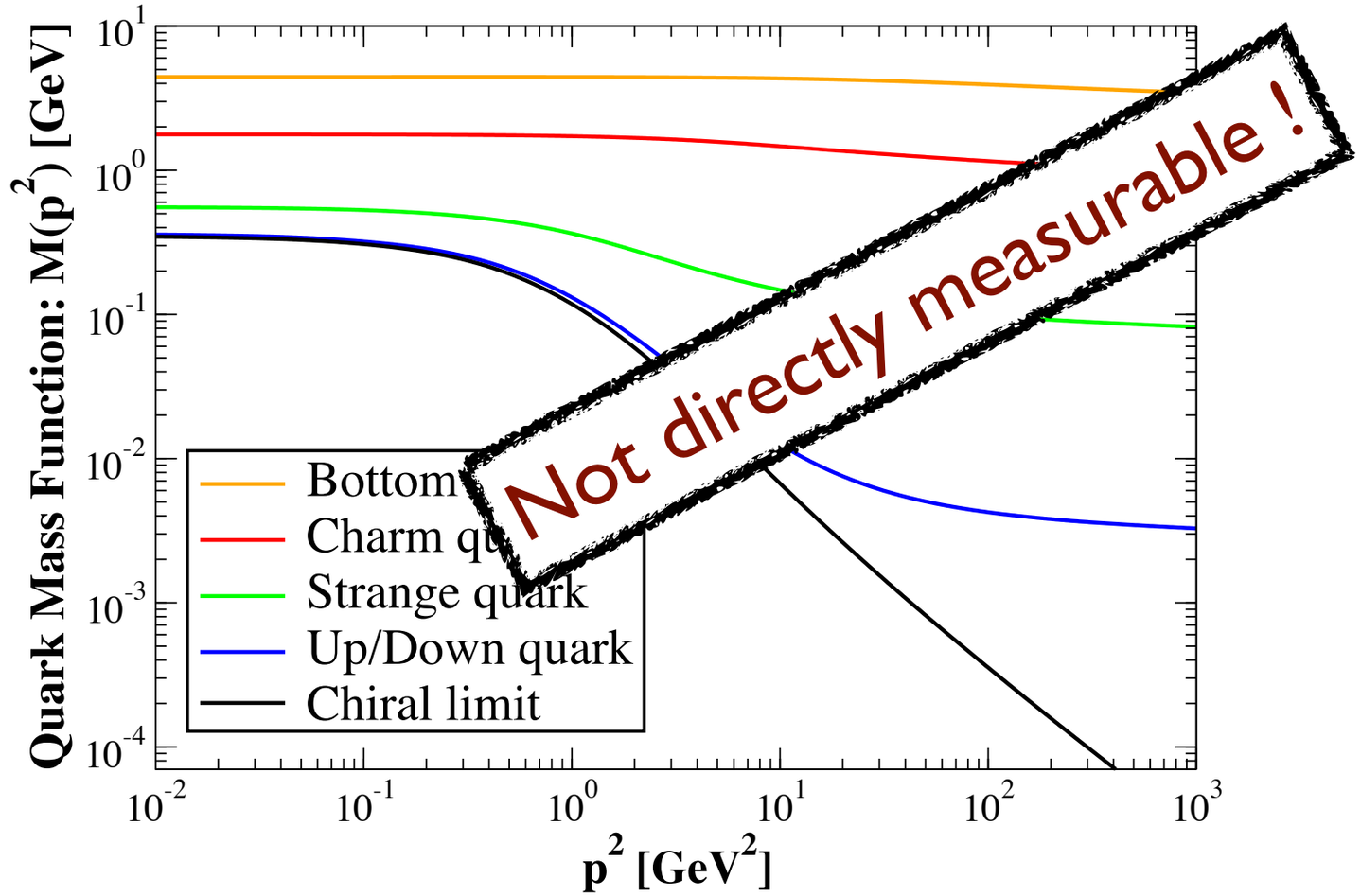

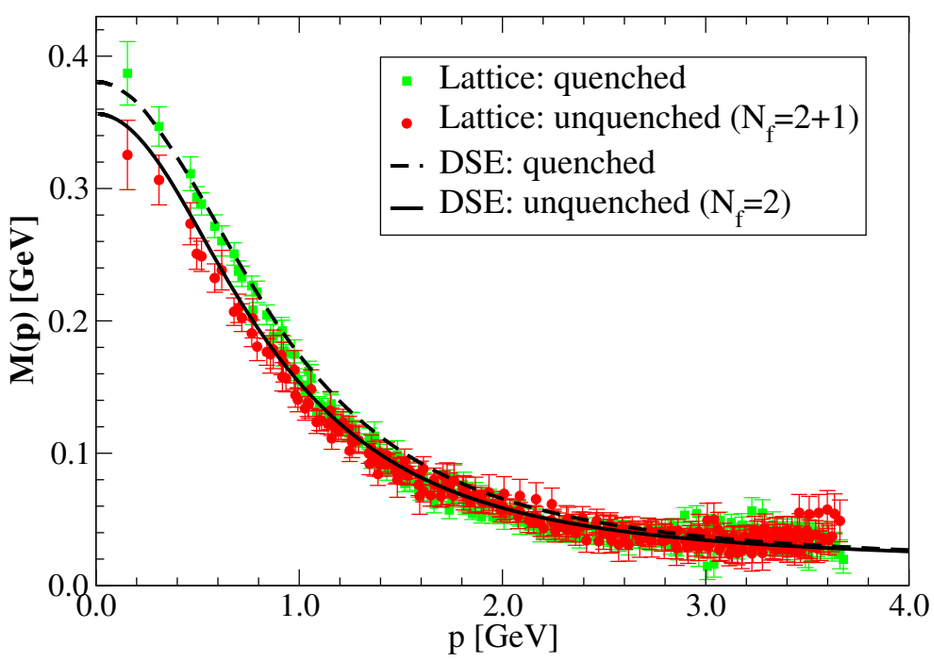

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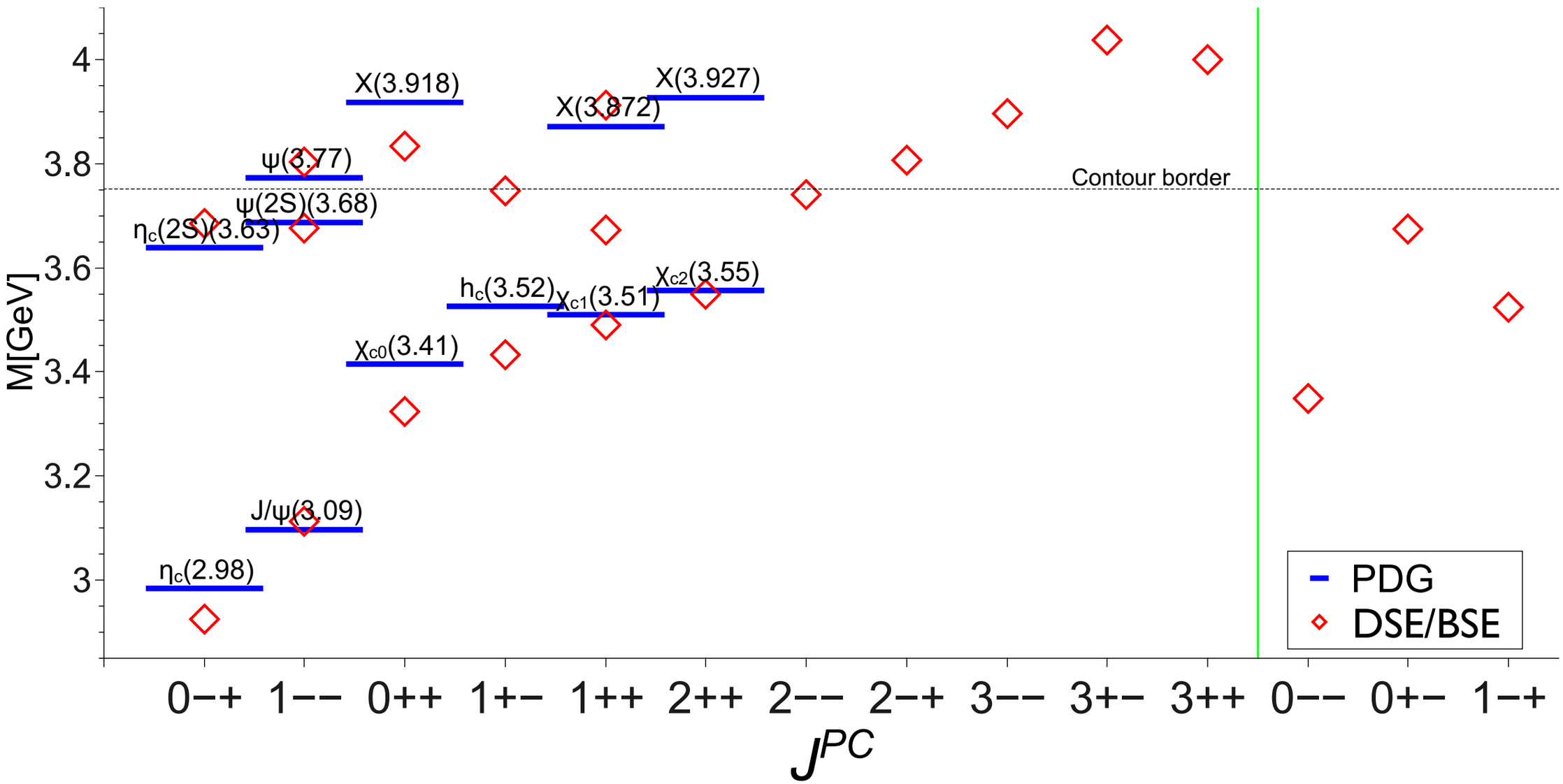
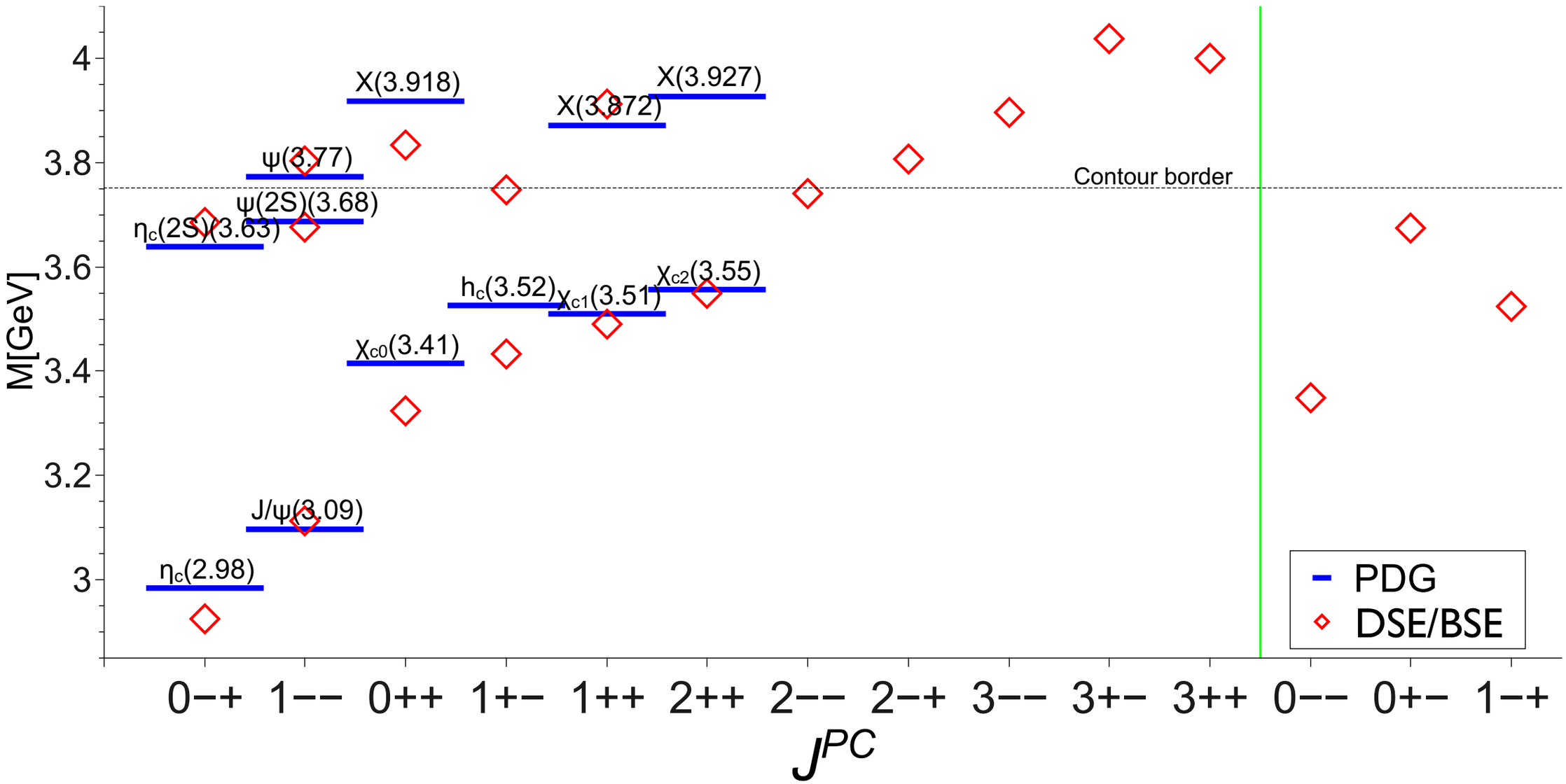
Charmonium spectrum



CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

- good channels: $1^{--}, 2^{++}, 3^{--}, \ldots$
- acceptable channels: 0^{-+}
- clear deficiencies in other channels: missing spin-structure

Charmonium spectrum



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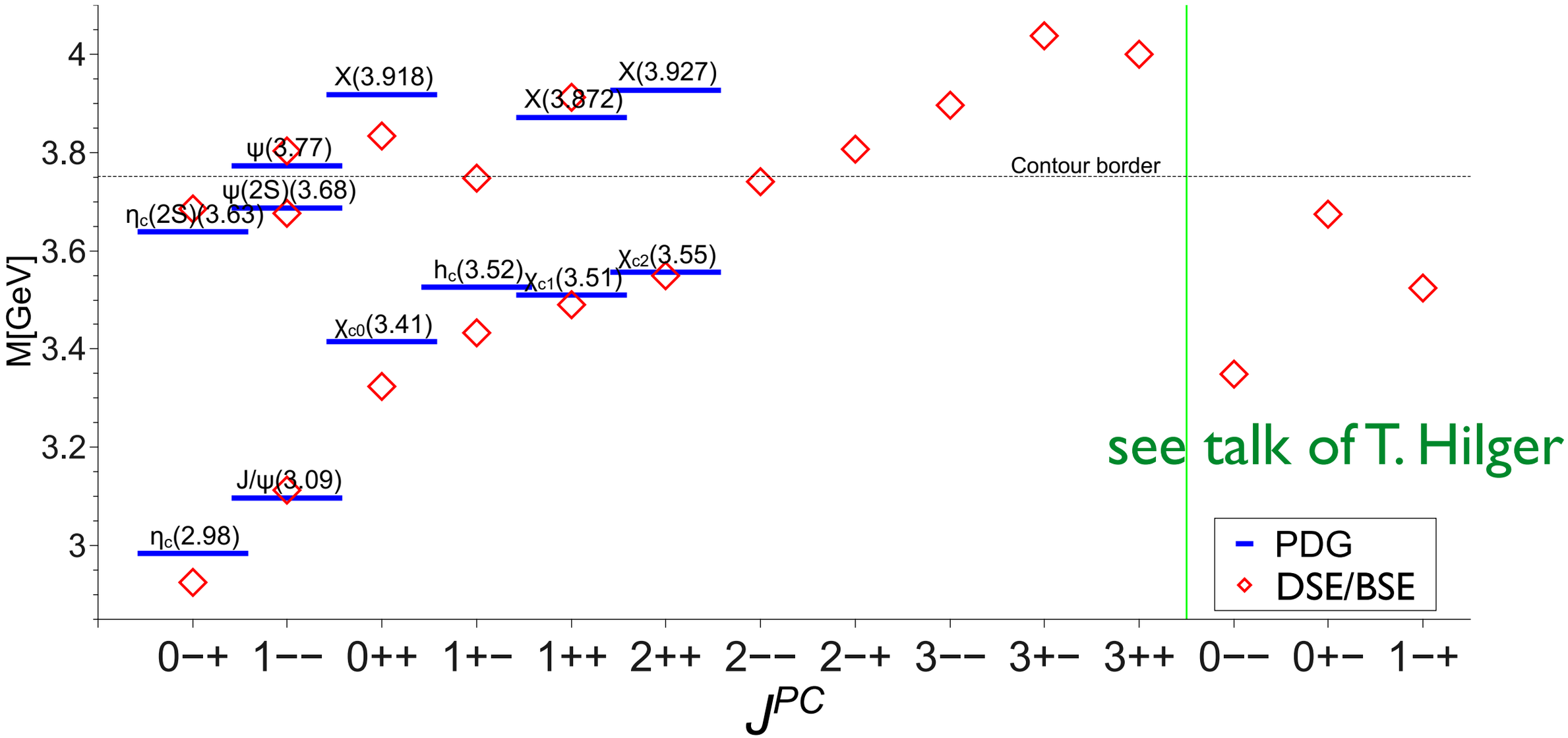

see talk of T. Hilger

CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

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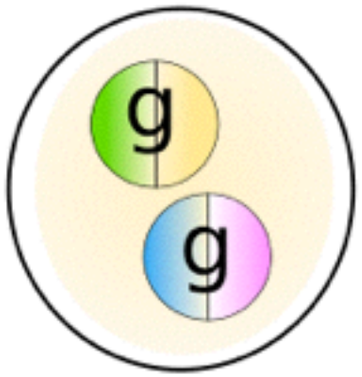

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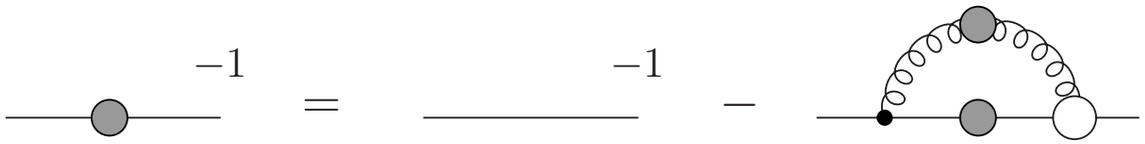

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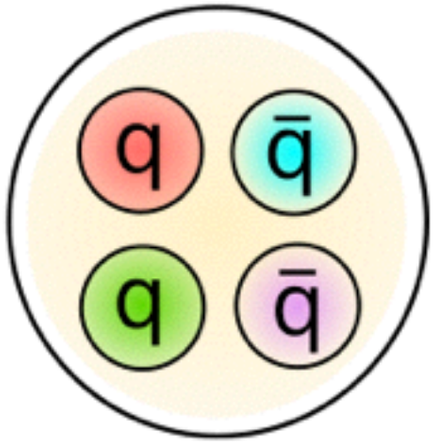
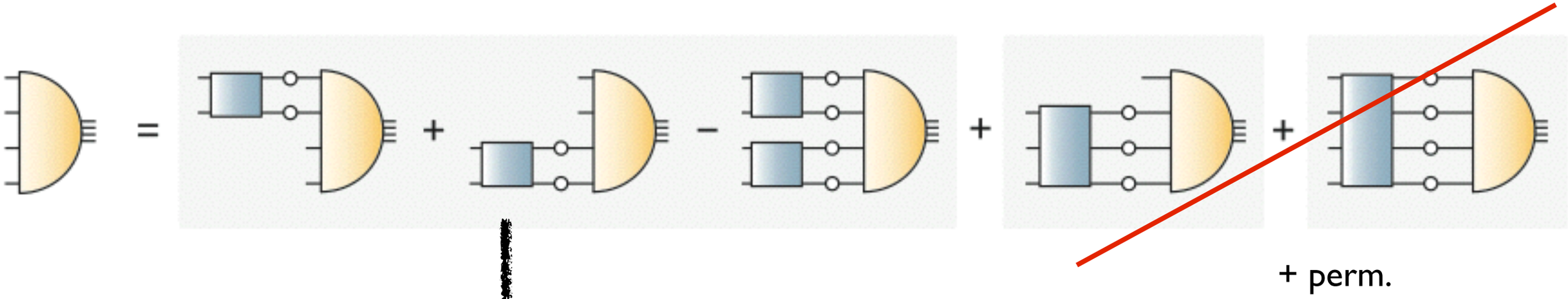
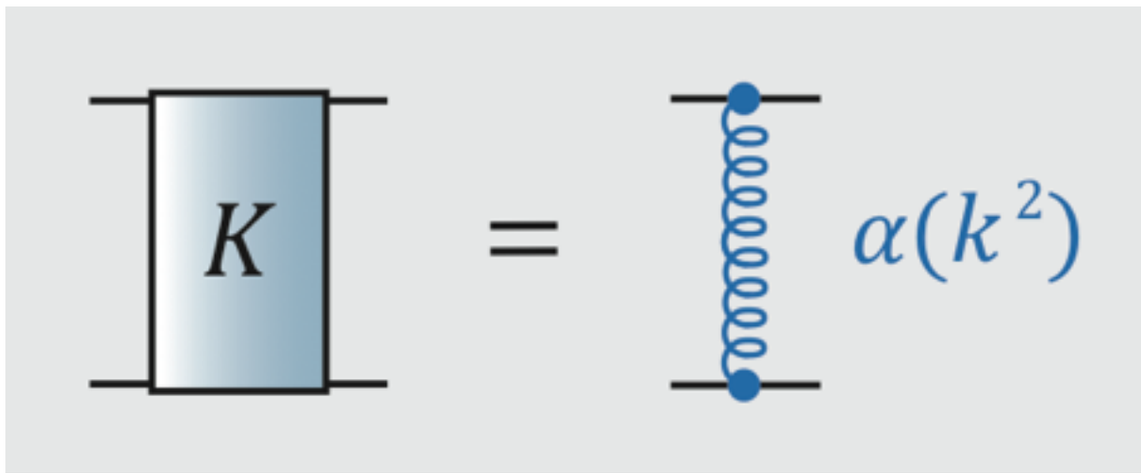
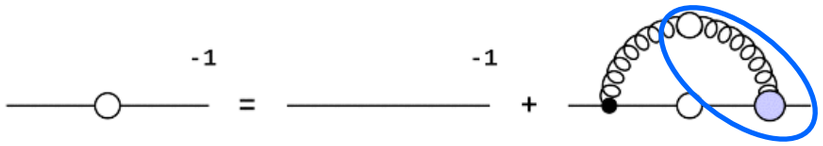


Solving the four-body equation





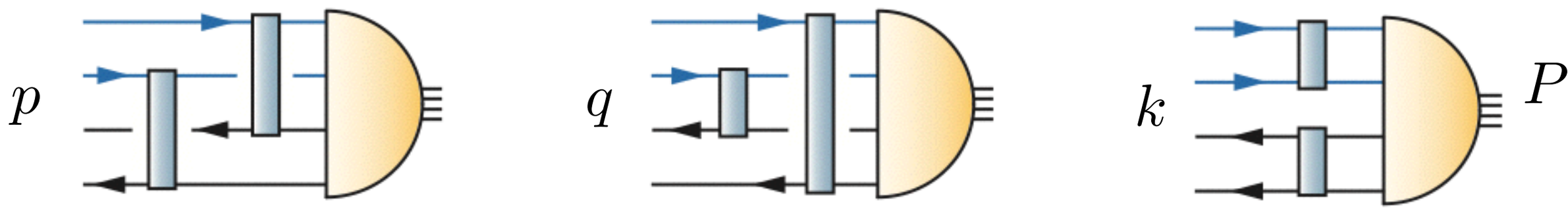
- Input: Non-perturbative quark, quark-gluon interaction



$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,...,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

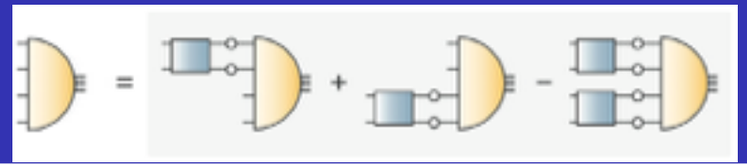
9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

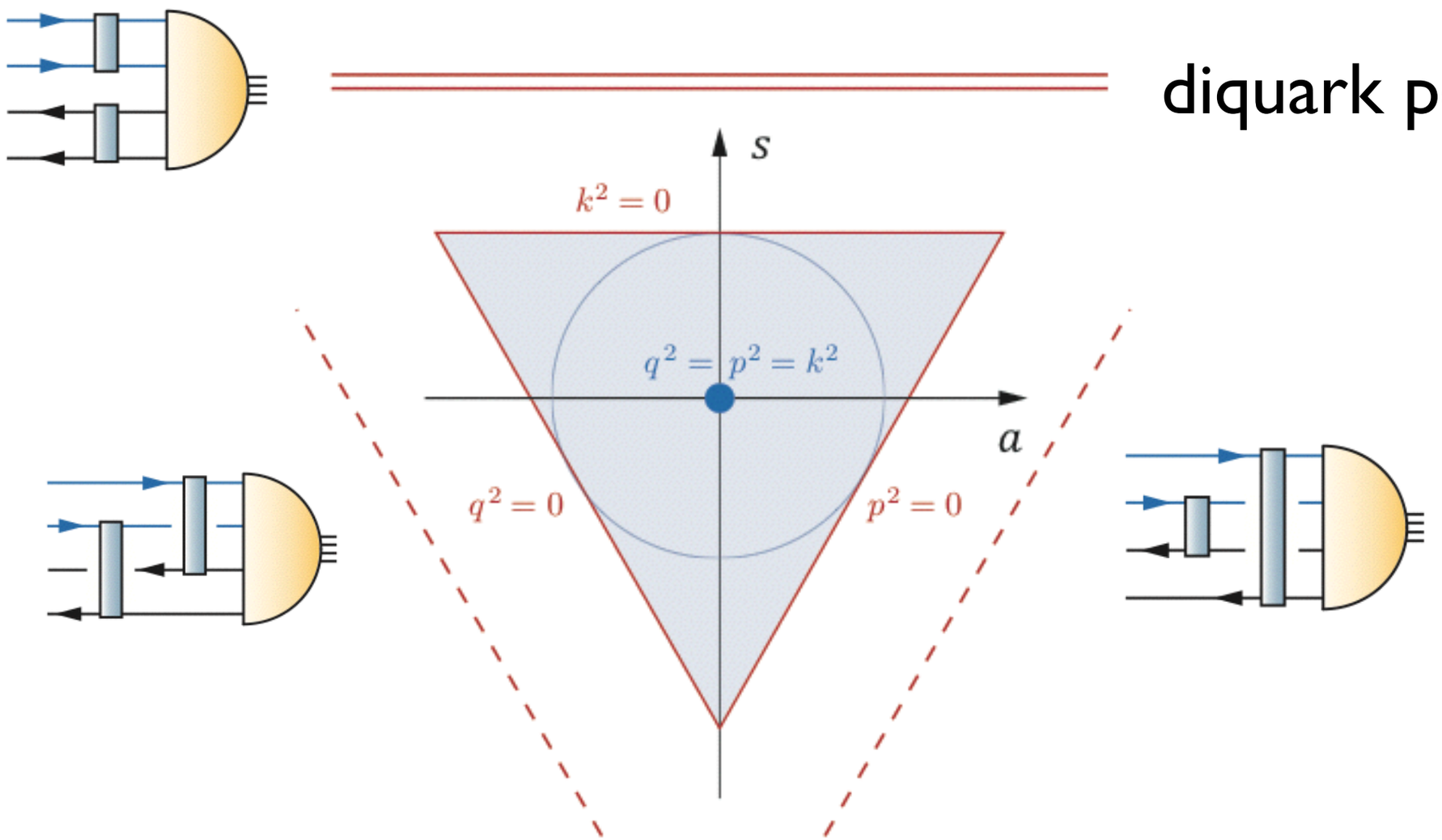
- good approximation: keep s-waves only; 16 tensor structures

Four-body equation:



Organise Dirac-Lorentz-tensors into multiplets of **S4**

- Singlet: $S_0 = (p^2 + q^2 + k^2)/4$, carries overall scale
- Doublet: $a = \sqrt{3}(q^2 - p^2)/(4S_0); \quad s = (p^2 + q^2 - 2k^2)/(4S_0)$

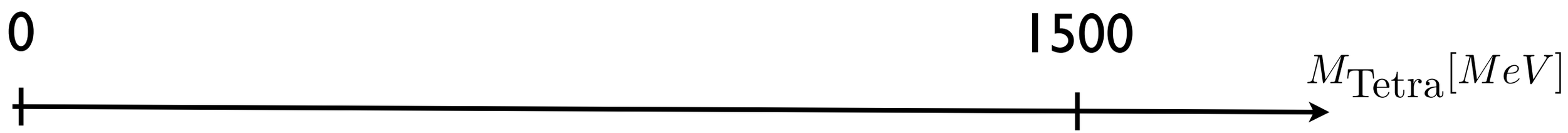


Eichman, CF, Heupel, arXiv: 1505.06336

- Two triplets

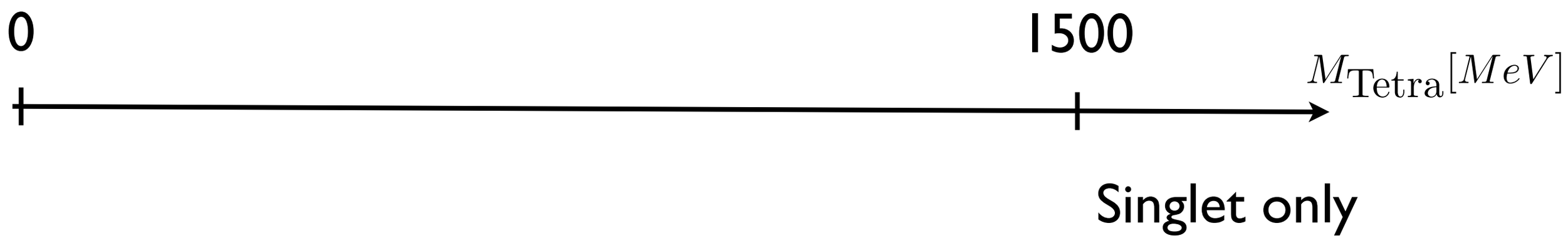
Bound state masses

- Different levels of approximations:



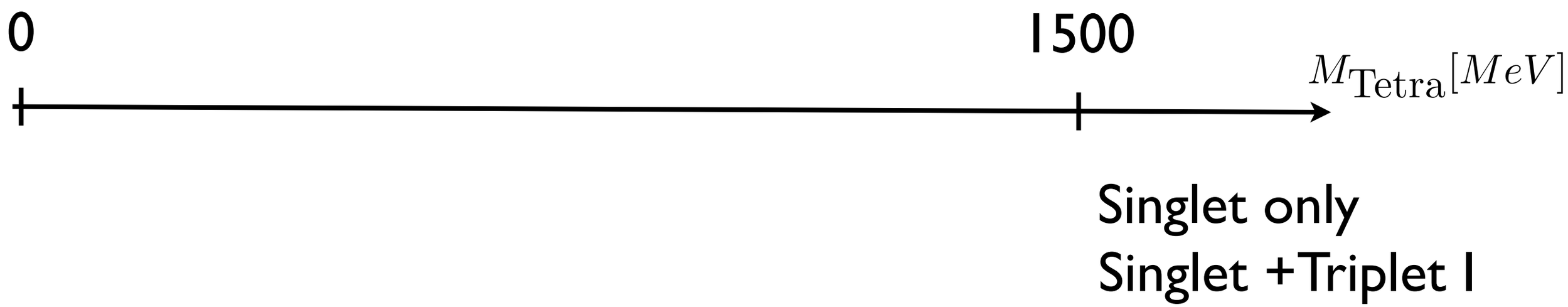
Bound state masses

- Different levels of approximations:



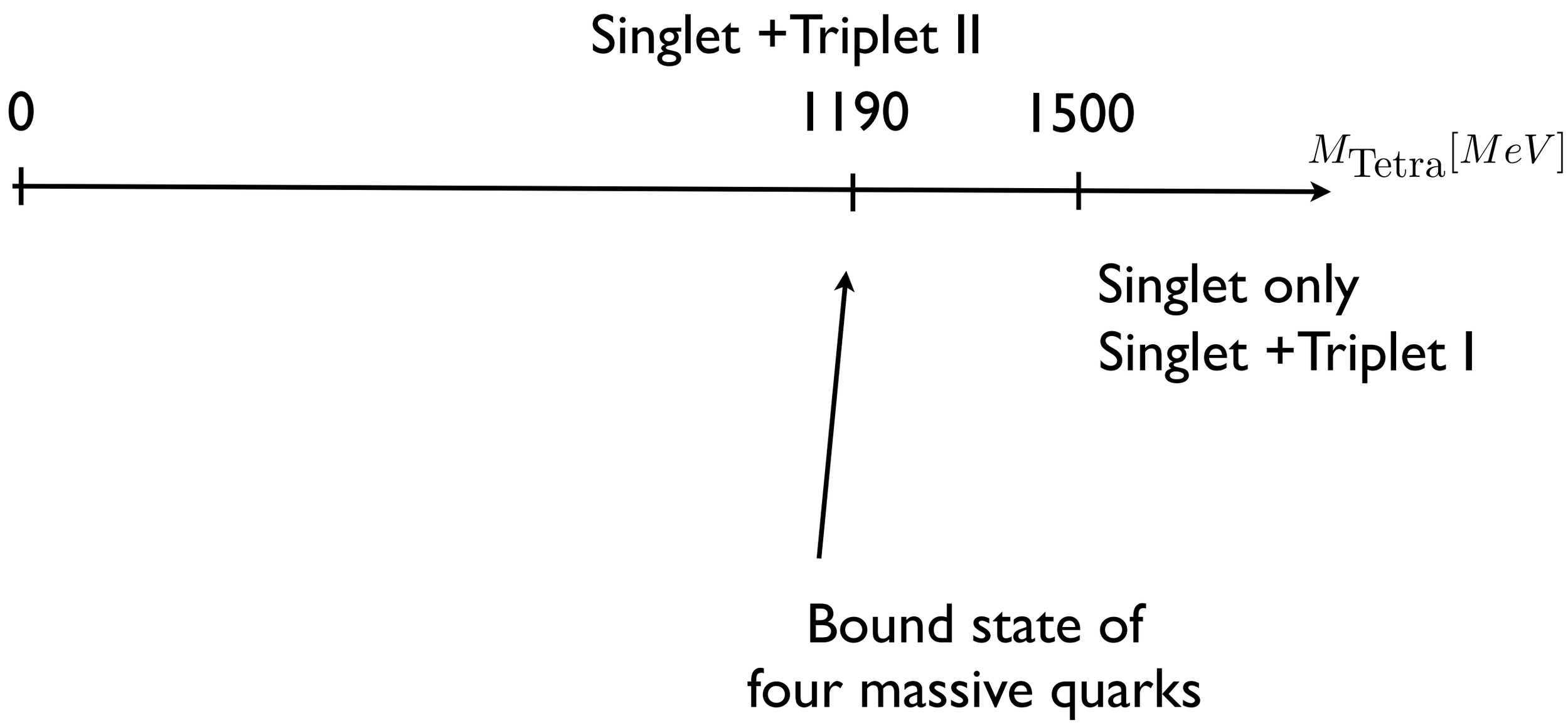
Bound state masses

- Different levels of approximations:



Bound state masses

- Different levels of approximations:



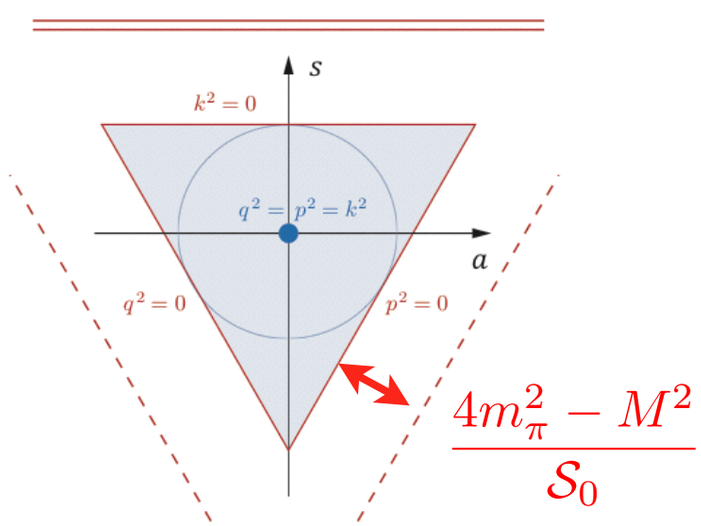
Bound state masses

- Different levels of approximations:

Singlet + Triplet II

0 350 1190 1500 $M_{\mathrm{Tetra}}[MeV]$

Singlet + Doublet



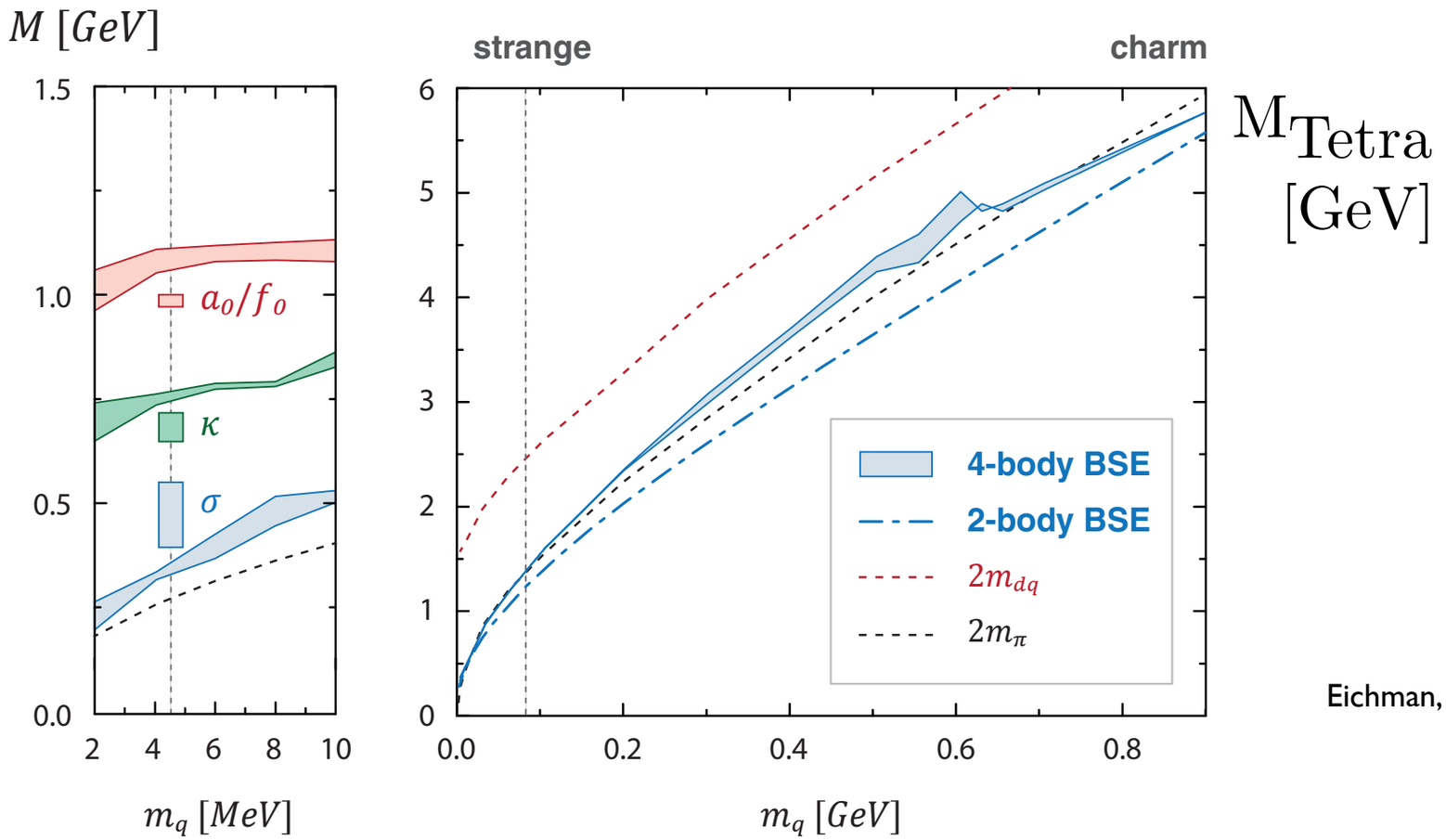
Two-pion resonance

Singlet only

Singlet + Triplet I

Bound state of
four massive quarks

Mass evolution of tetraquark



Eichman, CF, Heupel, 1508.07178

- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results:

$$m_\sigma \sim 350\,\mathrm{MeV}$$

$$m_\kappa \sim 750\,\mathrm{MeV} \qquad m_{ss\bar{s}\bar{s}} \sim 1.5\,\mathrm{GeV}$$

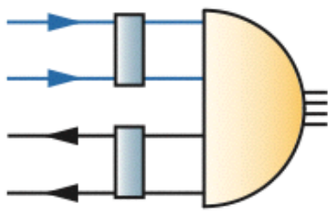
$$m_{a_0,f_0} \sim 1080\,\mathrm{MeV} \qquad m_{cc\bar{c}\bar{c}} \sim 5.7\,\mathrm{GeV}$$

qualitatively similar to two-body framework

Heupel, Eichman, CF, PLB 718 (2012) 545-549

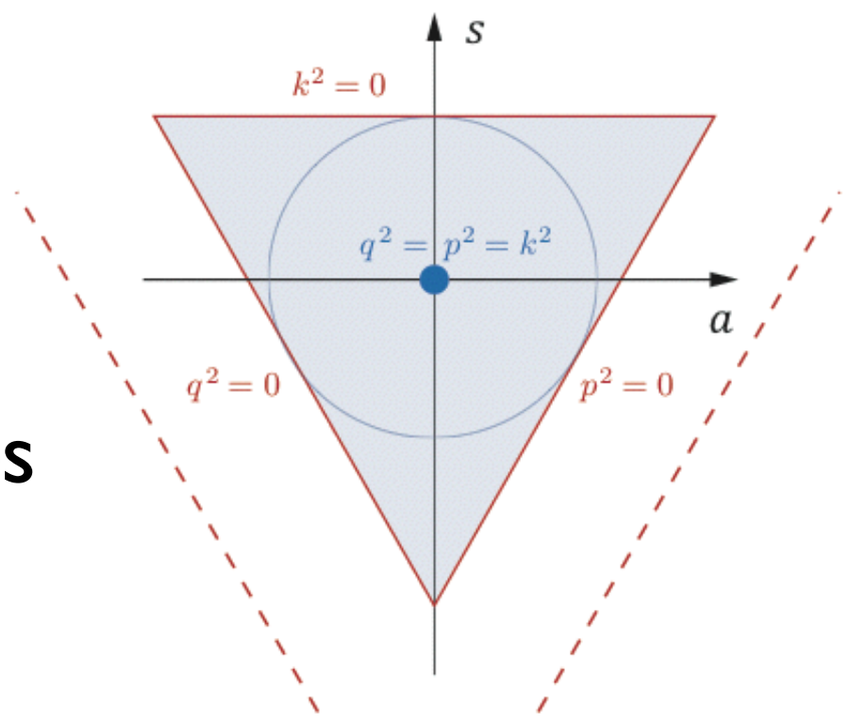
Outlook: heavy-light systems

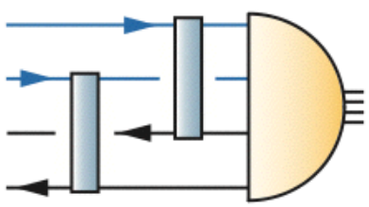
Dynamical situation in **S4**-doublet:



heavy-light diquarks

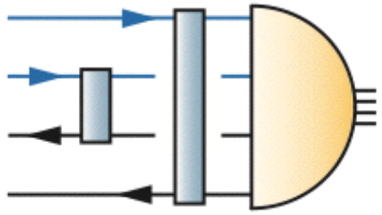
$$cq - \bar{c}\bar{q}$$





meson-molecules

$$c\bar{q} - \bar{c}q$$



hadro-charmonium

$$c\bar{c} - q\bar{q}$$

Dynamical decision of most important clustering!

Summary and outlook

Summary

- Mass gap in YM-theory: scalar glueball mass
- Tetraquarks dominated by internal meson-meson configurations
- Dynamical description of σ as π-π resonance
- Bound cccc-tetraquark....

Outlook

- Improve numerical framework: precision, systematics
- Implement quark mass dependence of quark-gluon interaction
- Explore heavy-light systems

Williams, EPJA 51 (2015)