# Baryons as relativistic three-quark bound states

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International School of Nuclear Physics, 37th Course,
*Probing Hadron Structure with Lepton and Hadron Beams*
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# Motivation: Why Functional Approaches to QCD?

Where to look for the **nucleon in QCD?**
Free propagation of lowest three-quark bound state:

**Six-quark Green function!**

Calculating it requires either

- to employ a lattice (*i.e.*, give up Poincaré invariance)
- to use Monte-Carlo algorithms (*i.e.*, use a statistical method)
- to run programs on supercomputers

or

- to fix a gauge (*i.e.*, sacrifice gauge invariance)
- to truncate equations in a way which is verified *á posteriori*
- to perform a lot of (computer) algebra

Method 1: Numerical, partly excellent, results for hadron properties!
NB: Based on extrapolations $a \to 0, V \to \infty$ & $m_\pi \to 0$!
Method 2: Qualitative insight!
E.g. relation of observables to confinement, D$\chi$SB, axial anomaly, ...

# Motivation: Why Functional Approaches to QCD?

**QCD correlation functions** contribute to the understanding of

- ★ confinement of gluons, quarks, and colored composites.
- ★ D$\chi$SB, *i.e.*, generation of the quarks' ***constituent masses*** and ***chirality-changing quark-gluon interactions.***
- ★ $U_A(1)$ anomaly and topological properties.

**Functional Methods**

(Exact Renorm. Group, Dyson-Schwinger eqs., *n*PI methods, . . . ):
Input into hadron phenomenology via **QCD bound state eqs.**.

- Bethe-Salpeter equations for **mesons**
  form factors, decays, reactions, ...
- covariant Faddeev equations for **baryons**
  nucleon form factors, Compton scattering, meson production, ...

**Functional approaches to Landau gauge QCD:**

- in principle ab initio ☺
- perturbation theory included ☺
- multi-scale problems accessible ☺
- some elementary Green's functions quite well-known ☺
- in other gauges complicated . . . ☹
- truncations for numerical solutions necessary ☹

# QCD bound state equations

State-of-the-art:

- **rainbow-ladder truncation** (= dressed gluon exchange) for mesons **and** baryons in an unified approach.

  NB: Chirality-changing interactions of lesser importance in

  $$0^-, 1^-, \ldots, \frac{1}{2}^+, \frac{3}{2}^+, \ldots \text{ ground states.}$$

  Results include:
  Masses, form factors, decays, Compton scattering, meson prod., …

- recently some results (masses, decay constants, $\sigma$ term) ***beyond rainbow-ladder.***

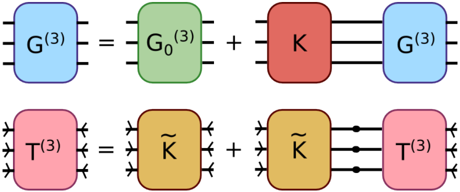
  see, *e.g.*, H. Sanchis-Alepuz and R. Williams,
  Phys. Lett. B **749** (2015) 592 [arXiv:1504.07776 [hep-ph]];
  J. Phys. Conf. Ser. **631** (2015) 012064 [arXiv:1503.05896 [hep-ph]].

# Outline

1. Motivation: Why Functional Approaches to QCD?
2. Relativistic Three-Fermion Bound State Equations
3. Structure of Baryonic Bound State Amplitudes
4. Quark Propagator and Rainbow Truncation
5. Interaction Kernels and Rainbow-Ladder Truncation
6. Coupling of E.M. Current and Quark-Photon Vertex
7. Some Selected Results
8. Summary and Outlook

# Relativistic Three-Fermion Bound State Equations

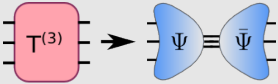
**Dyson-Schwinger eq. for 6-point fct. $\Longrightarrow$ 3-body bound state eq.:**



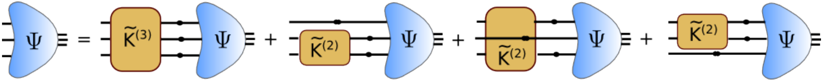

**BOUND STATE:**

Pole in $G^{(3)}$
or (equiv.) for $P^2 = -M_B^2$
Pole in $T^{(3)}$

**bound state amplitudes:**

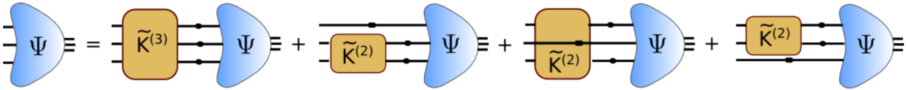



**covariant 3-body bound state eq.** (*cf.*, Bethe-Salpeter for 2-body BS)**:**



# Relativistic three-fermion bound state equations

**3-body bound state eq.:**




NB: With 3-particle-irreducible interactions $\tilde{K}^{(3)}$ neglected:
Poincaré-covariant Faddeev equation.

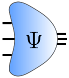
**Elements needed for bound state equation:**

- Tensor structures (color, flavor, Lorentz / Dirac) of the BS ampl.
- Full quark propagators for *complex* arguments
- Interaction kernels $K_{2,3}$

**Needed for coupling to e.m. current:**

- Full quark-photon vertex

# Structure of Baryonic Bound State Amplitudes




$\equiv \quad \sim \langle 0|q_\alpha q_\beta q_\gamma|B_\mathcal{I}\rangle \propto \Psi_{\alpha\beta\delta\mathcal{I}}$ (with multi-indices $\alpha = \{x, D, c, f, \ldots\}$)

and $\mathcal{I}$ baryon (multi-)index $\Longrightarrow$ baryon quantum numbers

C. Carimalo, J. Math. Phys. **34** (1993) 4930.

For a solution with all tensor components:

G. Eichmann, RA, A. Krassnigg, D. Nicmorus, PRL **104** (2010) 201601

Comparison to mesonic BS amplitudes $\langle 0|q_\alpha \bar{q}_\beta|M_\mathcal{I}\rangle \propto \Phi_{\alpha\beta\mathcal{I}}$ :

- scalar and pseudoscalar mesons: 4 tensor structures each
- vector and axialvector mesons: 12 tensor struct. each, 8 transv.
- tensor and higher spin mesons: 8 transverse struct. each

which are functions of two Lorentz-invariant variables.

C. H. Llewellyn-Smith, Annals Phys. **53** (1969) 521.

# Structure of Baryonic Bound State Amplitudes

**Facts about the decomposition:**

- Independent of any truncation of the bound state equation.
- Only Poincaré covariance and parity invariance exploited.
- It includes all possible internal spin and orbital angular momenta.
- For positive-parity, positive-energy (particle) baryons it consists of

spin-$\frac{1}{2}$ particle: 64 elements

| | # elements |
|---|---|
| s-wave | 8 |
| p-wave | 36 |
| d-wave | 20 |

G. Eichmann et al., PRL 104 (2010) 201601

spin-$\frac{3}{2}$ particle: 128 elements

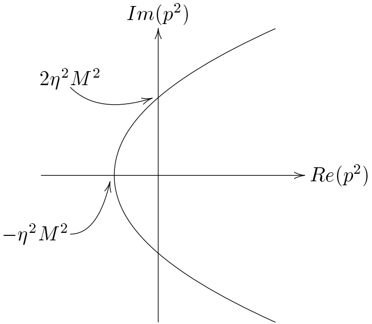
| | |
|---|---|
| s-wave | 4 |
| p-wave | 36 |
| d-wave | 60 |
| f-wave | 28 |

H. Sanchis Alepuz et al. PRD 84 (2011) 096003

Each tensor structure is multiplied by a function of five Lorentz-invariant variables!

K.F. Liu's talk: Quark angular momentum contribute $\sim$ 47% of proton spin!

All (non-perturbative) approaches to QFT employ Euclidean momenta:
Connection to the world of real particles requires analytic continuation!



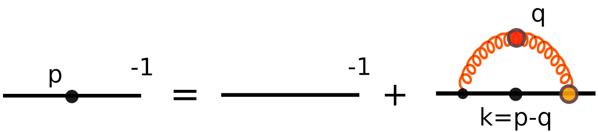

**In bound state eqs.:**

- Knowledge of the quark propagator inside parabolic region required.
- Parabola limited by nearest quark singularities: $M < 2m_q(3m_q)$ for mesons (baryons)
- ground states unaffected by singularities.

- Lattice: Values for real $p^2 \geq 0$ only.
- Dyson-Schwinger / ERG eqs.: complex $p^2$ accessible.

# Quark Propagator and Rainbow Truncation

**Dyson-Schwinger eq. for Quark Propagator:**




$$S^{-1}(p) = Z_2 S_0^{-1} + g^2 Z_{1f} \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S(k) \Gamma^\nu(k,p;q) D_{\mu\nu}(q)$$

**Rainbow** truncation

Projection onto tree-level tensor $\gamma_\mu$, restrict momentum dependence

$$Z_{1f}\frac{g^2}{4\pi} D_{\mu\nu}(q)\Gamma_\nu(k,p;q) \to \begin{cases} Z_{1f}\frac{g^2}{4\pi} T_{\mu\nu}(q)\frac{Z(q^2)}{q^2}\left(Z_{1f} + \Lambda(q^2)\right)\gamma_\nu \\ =: Z_2^2 T_{\mu\nu}(q)\frac{\alpha_{eff}(q^2)}{q^2}\gamma_\nu \end{cases}$$

# Interaction Kernels and Rainbow-Ladder Truncation

- Truncation of the quark-gluon vertex in the quark DSE.
- The BSE interaction kernel must be truncated accordingly.
- **Physical requirement: Chiral symmetry**
  axial WT id. relates quark DSE and bound-state eq. kernel.

**Ladder** truncation

$q\bar{q}$ kernel compatible with rainbow truncation and axial WT id.:

$$K^{q\bar{q}} = 4\pi Z_2^2 \frac{\alpha_{eff}(q^2)}{q^2} T_{\mu\nu}(q)\gamma^\mu \otimes \gamma^\nu$$

Together constitute the DSE/BSE **Rainbow-Ladder truncation.**

**Note: the truncation can and should be systematically improved!**

H. Sanchis-Alepuz, C. S. Fischer and S. Kubrak, Phys. Lett. B **733** (2014) 151;
H. Sanchis-Alepuz and R. Williams, Phys. Lett. B **749** (2015) 592.

# Interaction Kernels and Rainbow-Ladder Truncation

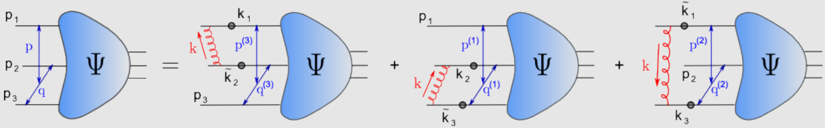
**Rainbow-Ladder truncated three-body BSE:**

- Previous studies used successfully the quark-diquark ansatz (reduction to a two-body problem).
- pNRQCD: 3-body contribution $\sim$ 25 MeV for heavy baryons.

Supported by this, **the three-body irreducible kernel $K^{(3)}$ is neglected** (Faddeev approximation).

- Quark-quark interaction $K^{(2)}$: **same as quark-antiquark truncated kernel.** (!Different color factor!)

Rainbow-Ladder truncated **covariant Faddeev equation**



# Interaction Kernels and Rainbow-Ladder Truncation

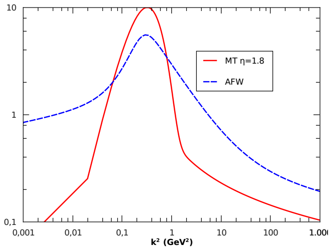
**Models for effective interaction:**

Maris-Tandy model (Maris & Tandy PRC60 1999)

$$\alpha(k^2) = \alpha_{IR}(k^2; \Lambda, \eta) + \alpha_{UV}(k^2)$$

- Purely phenomenological model.
- $\Lambda$ fitted to $f_\pi$.
- Ground-state pseudoscalar properties *almost* insensitive to $\eta$ around 1.8

Describes very succesfully hadron properties.




DSE motivated model (R.A.,C.S. Fischer,R. Williams EPJ A38 2008)

$$\alpha(k^2; \Lambda_S, \Lambda_B, \Lambda_{IR}, \Lambda_{YM})$$

- DSE-based in the deep IR.
- Designed to give correct masses of $\pi$, $\rho$ and $\eta'$ *($U_A(1)$ anomaly!)*.
- 4 energy scales! Fitted to $\pi$, $K$ and $\eta'$.

# Interaction Kernels and Rainbow-Ladder Truncation

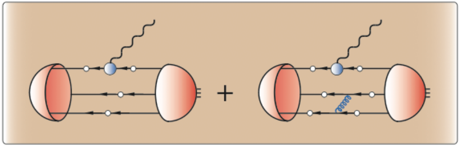
**Note: The resulting qq-interaction is chirality-conserving, flavour-blind and current-quark mass independent.**

## Beyond Rainbow-Ladder

- "Corrections beyond-RL" refers to corrections to the effective coupling but also to additional structures beyond vector-vector interaction.
- They can induce a different momentum dependence of the interaction.
- They can also **induce a quark-mass and quark-flavour dependence of the interaction**
- Question: how important are beyond-RL effects?

# Coupling of E.M. Current and Quark-Photon Vertex

## Electromagnetic current in the three-body approach:



by “gauging of equations”
M. Oettel, M. Pichowsky and L. von Smekal, Eur. Phys. J. A **8** (2000) 251;
A. N. Kvinikhidze and B. Blankleider, Phys. Rev. C **60** (1999) 044003.

Impulse appr. + Coupling to spectator q + Coupling to 2-q kernel + Coupling to 3-q kernel

Coupling to 2-q kernel: not present in RL appr.

Coupling to 3-q kernel: not present in Faddeev appr.

## Additional Input: Quark-Photon Vertex

# Coupling of E.M. Current and Quark-Photon Vertex

Quark-Photon Vertex:

- Vector WT id. determines vertex up to purely transverse parts:
  "Gauge" part (Ball-Chiu vertex)
  completely specified by dressed quark propagator.
- Can be straightforwardly calculated in Rainbow-Ladder appr.:
  - important for renormalizibility (Curtis-Pennington term),
  - anomalous magnetic moment,
  - contains $\rho$ meson pole!

**The latter property is important to obtain the correct physics!**

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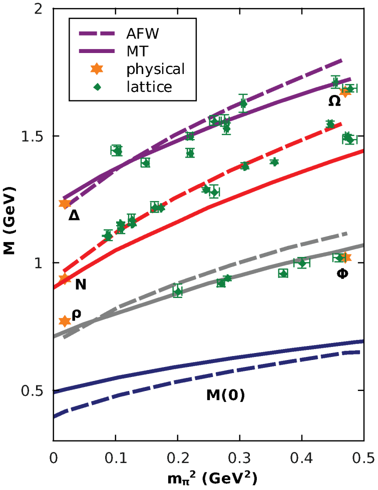
All elements specified to calculate baryon amplitudes and properties:
Use computer with sufficient RAM ($\sim$ tens of GB) and run for a few hours . . .

# Some Selected Results




PoS QNP2012 (2012) 112

- Both models designed to reproduce correctly D$\chi$SB and pion properties within RL. **They capture beyond-RL effects at this quark-mass.**
- This behaviour extends to other light states ($\rho$, $N$, $\Delta$), one gets a good description.
- Both interactions similar at inter-mediate momentum region: **$\sim 0.5 - 1$ GeV is the relevant momentum region for D$\chi$SB & ground-state hadron props.**
- Slight differences at larger current masses, however, **qualitative model indep.**

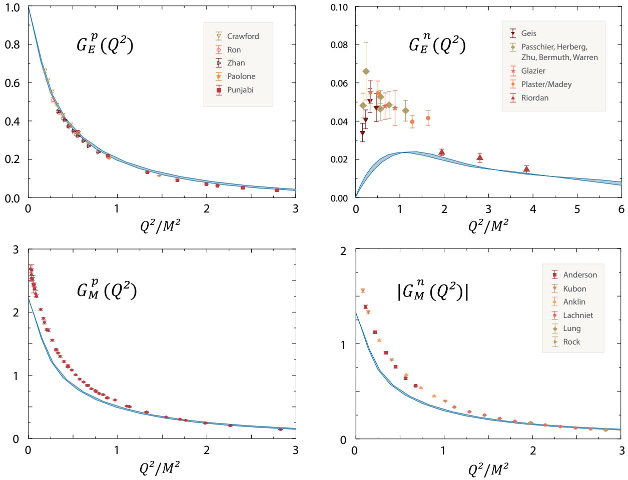
# Some Selected Results

## Nucleon electromagnetic form factors

**Nucleon em. FFs**
vs. momentum transfer
Eichmann, PRD 84 (2011)

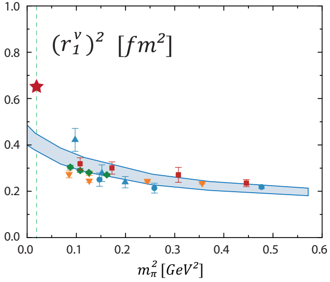
- Good agreement with recent **data** at large $Q^2$
- Good agreement with **lattice** at large quark masses
- **Missing pion cloud** below ~2 GeV$^2$, in chiral region

$\sim$ **nucleon quark core** without pion effects



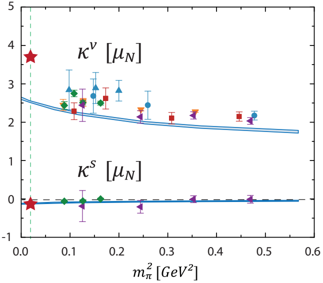
## Nucleon electromagnetic form factors

**Nucleon charge radii:**
isovector (p-n) Dirac (F1) radius




- **Pion-cloud effects** missing in chiral region (⇒ divergence!), agreement with lattice at larger quark masses.

**Nucleon magnetic moments:**
isovector (p-n), isoscalar (p+n)



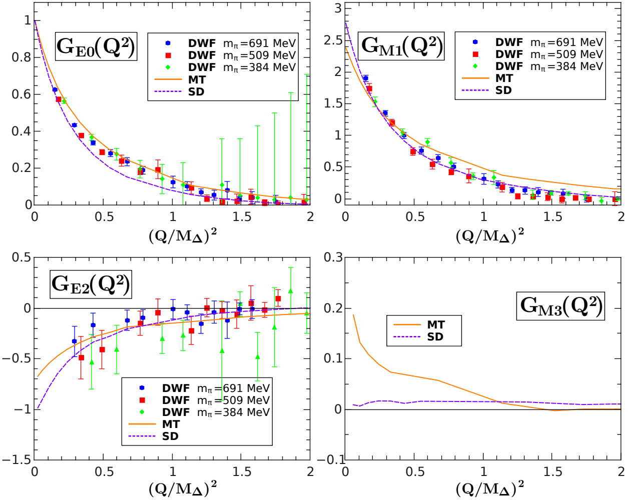

Eichmann, PRD 84 (2011)

- **But:** pion-cloud **cancels** in $\kappa^s$ ⇔ **quark core**

Exp: $\kappa^s = -0.12$
Calc: $\kappa^s = -0.12(1)$ **!!**

## Δ electromagnetic form factors

H. Sanchis-Alepuz *et al.*, Phys. Rev. D **87** (2013) 095015 [arXiv:1302.6048 [hep-ph]].

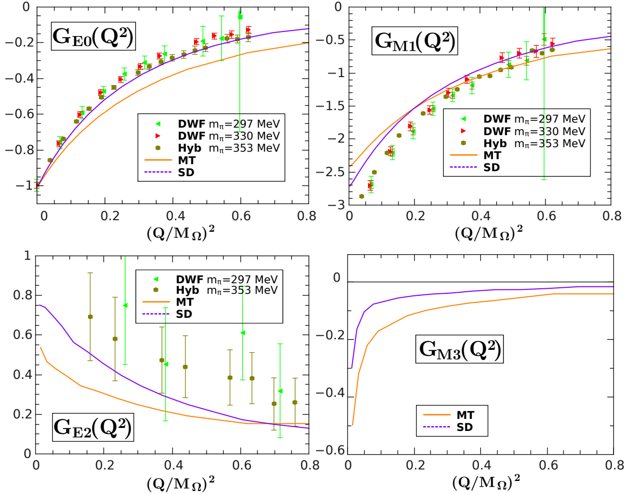



- $G_{E2}$ and $G_{M3}$: Deviation from sphericity!
- Important: Difference to quark-diquark model in $G_{E2}$ and $G_{M3}$.
- Large $G_{E2}$ for small $Q^2$!
- "Small" $G_{M3}$?

# Some Selected Results

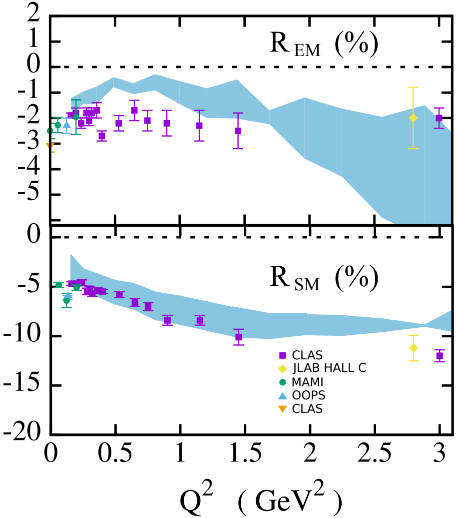
## Ω electromagnetic form factors

H. Sanchis-Alepuz *et al.*, Phys. Rev. D **87** (2013) 095015 [arXiv:1302.6048 [hep-ph]].




- Again deviation from sphericity!
- Only weak quark mass dependence!

**$\Delta \to N\gamma$ electromagnetic transition form factors**




H. Sanchis-Alepuz *et al.*, in preparation

Slight deviation from corresponding results in diquark-quark model!

Good agreement with experimental results.

Deviation from sphericity is an effect from sub-leading components required by Poincaré invariance, *i.e.*, relativistic physics!

# Summary

**Hadrons from QCD bound state equations:**

- QCD bound state equations:
  Unified approach to mesons and baryons feasible!
- So far:
  In rainbow-ladder appr. meson observables
  and octet / decuplet masses and (e.m., axial, . . . ) form factors.
  full octet/decuplet (*i.e.*, incl. hyperons): H. Sanchis-Alepuz *et al.*, in preparation
- Under consideration:
  In rainbow-ladder appr. 2-photon processes as,
  *e.g.*, nucleon Compton scattering;
  G. Eichmann and C. S. Fischer, Phys. Rev. D **87** (2013) 036006
  [arXiv:1212.1761 [hep-ph]]; G. Eichmannn's talk .
  and hadronic contribution to light-light scattering and thus $(g-2)_\mu$
  T. Goecke, C. S. Fischer and R. Williams, Phys. Rev. D **87** (2013) 034013.

# Outlook

- Even in ground state form factors beyond rainbow-ladder effects at small $Q^2$!
  There: Likely hadronic (pionic) effects!
- Chirality-violating BRL effects ("spin-flip interactions"):
  - scalar and axialvector mesons,
  - excited mesons and baryons (spectrum)
  - exotic mesons and baryons (tetraquarks, pentaquarks, . . . )
  - . . .
- Systematic approach: Include knowledge on quark-gluon vertex!

- To be used:

  • Bethe-Salpeter /covariant Faddeev eq.
  with kernels at consistent 3PI level.

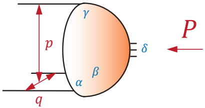
  • Dynamical hadronization in the
  Exact Renormalization Group approach.

# Structure of Baryonic Bound State Amplitudes

| $s$ | $l$ | $T_{ij}$ | |
|---|---|---|---|
| $1/2$ | $0$ | $\mathbb{1}\otimes\mathbb{1}$ | **s waves** |
| $1/2$ | $0$ | $\gamma^\mu_T\otimes\gamma^\mu_T$ | **(8)** |
| $1/2$ | $1$ | $\mathbb{1}\otimes\frac{1}{2}[\not p,\not q]$ | **p waves** |
| $1/2$ | $1$ | $\mathbb{1}\otimes\not p$ | **(36)** |
| $1/2$ | $1$ | $\mathbb{1}\otimes\not q$ | |
| $1/2$ | $1$ | $\gamma^\mu_T\otimes\gamma^\mu_T\,\frac{1}{2}[\not p,\not q]$ | |
| $1/2$ | $1$ | $\gamma^\mu_T\otimes\gamma^\mu_T\,\not p$ | |
| $1/2$ | $1$ | $\gamma^\mu_T\otimes\gamma^\mu_T\,\not q$ | |
| $3/2$ | $1$ | $3(\not p\otimes\not q-\not q\otimes\not p)-\gamma^\mu_T\otimes\gamma^\mu_T[\not p,\not q]$ | |
| $3/2$ | $1$ | $3\not p\otimes\mathbb{1}-\gamma^\mu_T\otimes\gamma^\mu_T\,\not p$ | |
| $3/2$ | $1$ | $3\not q\otimes\mathbb{1}-\gamma^\mu_T\otimes\gamma^\mu_T\,\not q$ | |
| $3/2$ | $2$ | $3\not p\otimes\not p-\gamma^\mu_T\otimes\gamma^\mu_T$ | **d waves** |
| $3/2$ | $2$ | $\not p\otimes\not p+2\not q\otimes\not q-\gamma^\mu_T\otimes\gamma^\mu_T$ | **(20)** |
| $3/2$ | $2$ | $\not p\otimes\not q+\not q\otimes\not p$ | |
| $3/2$ | $2$ | $\not q\otimes[\not q,\not p]-\frac{1}{2}\gamma^\mu_T\otimes[\gamma^\mu_T,\not p]$ | |
| $3/2$ | $2$ | $\not p\otimes[\not p,\not q]-\frac{1}{2}\gamma^\mu_T\otimes[\gamma^\mu_T,\not q]$ | |

$$\chi(x_1,x_2,x_3)=\langle 0|\,T\,\psi(x_1)\,\psi(x_2)\,\psi(x_3)\,|N\rangle$$

**Momentum space:**
Jacobi coordinates $p, q, P$
⇒ 5 Lorentz invariants
⇒ 64 Dirac basis elements




$$\chi(p,q,P)=\sum_k \; f_k(p^2,\,q^2,\,p\cdot q,\,p\cdot P,\,q\cdot P) \;\; \textbf{Momentum}$$
$$\tau^k_{\alpha\beta\gamma\delta}(p,q,P) \;\; \textbf{Dirac} \;\; \otimes \; \textbf{Flavor} \; \otimes \; \textbf{Color}$$

Complete, orthogonal **Dirac tensor basis**
(partial-wave decomposition in nucleon rest frame):
Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

$$T_{ij}\,(\Lambda_\pm\gamma_5 C\otimes\Lambda_+)$$
$$(\gamma_5\otimes\gamma_5)\,T_{ij}\,(\Lambda_\pm\gamma_5 C\otimes\Lambda_+)$$

$$(A\otimes B)_{\alpha\beta\gamma\delta}=A_{\alpha\beta}\,B_{\gamma\delta}$$