QRPA calculations of stellar weak decay rates

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Weak-decay rates

Problem :

- Weak-decay rates determine late stages of stellar evolution.
- Experimental extrapolations or theoretical predictions:

Reproduce exp information on $T_{1/2}$ and BGT under terrestrial conditions.

Theoretical approach :

- Deformed HF+BCS+QRPA formalism with Skyrme forces and residual interactions in both ph and pp channels.

Results : Weak-decay rates at various (ρ,T) in stellar scenarios

- pf-shell nuclei. Main constituents of stellar core in presupernova formations: Sc, Ti, V, Mn, Fe, Co, Ni, Zn isotopes.

  BGT measured in laboratory (Charge exchange reactions) and compared with benchmark Shell Model calculations.

- Neutron-deficient waiting-point isotopes (Ni-Sn) at (ρ,T) typical of rp-process.
- Neutron rich Zr-Mo isotopes: r process.

Weak decay rates

$$\lambda = \ln 2 \left(T_{1/2}\right)^{-1} = \frac{\ln 2}{D} \sum_{if} P_i(T) B_{if} \Phi_{if}(\rho, T)$$

Initial states thermally populated

$$P_i(T) = \frac{2J_i + 1}{G} e^{-E_i/(kT)}, \quad G = \sum_i (2J_i + 1)\, e^{-E_i/(kT)}$$

$$P_{i=g.s.}(T=0) = 1$$

Nuclear structure

$$B_{if}(GT) = \left(\frac{g_A}{g_V}\right)^2_{\text{eff}} \left\langle f \left\| \sum_k \sigma^k t_{\pm}^k \right\| i \right\rangle^2$$

Phase space factors :

β^+, EC : $\Phi_{if} = \Phi_{if}^{EC} + \Phi_{if}^{\beta^+}$

β^- : $\Phi_{if} = \Phi_{if}^{\beta^-}$

$\lambda(\rho, T)$ are different from laboratory (P_i, cEC)

Phase space factors

$$\Phi_{if}^{\beta^-} = \int_1^{Q_{if}} \omega\sqrt{\omega^2-1}(Q_{if}-\omega)^2 F(Z+1,\omega)\left[1-S_e(\omega)\right]\left[1-S_\nu(Q_{if}-\omega)\right]d\omega$$

$$\Phi_{if}^{\beta^+} = \int_1^{Q_{if}} \omega\sqrt{\omega^2-1}(Q_{if}-\omega)^2 F(-Z+1,\omega)\left[1-S_p(\omega)\right]\left[1-S_\nu(Q_{if}-\omega)\right]d\omega$$

$$\Phi_{if}^{cEC} = \int_{\omega_\ell}^{\infty} \omega\sqrt{\omega^2-1}(Q_{if}+\omega)^2 F(Z,\omega)S_e(\omega)\left[1-S_\nu(Q_{if}+\omega)\right]d\omega$$

$$\Phi^{oEC} = \frac{\pi}{2}\left[q_K^2 g_K^2 B_K + q_{L_1}^2 g_{L_1}^2 B_{L_1} + \cdots\right]$$

- q Neutrino energy
- g Radial components of the e-wf at r=0
- B Exchange and overlap corrections

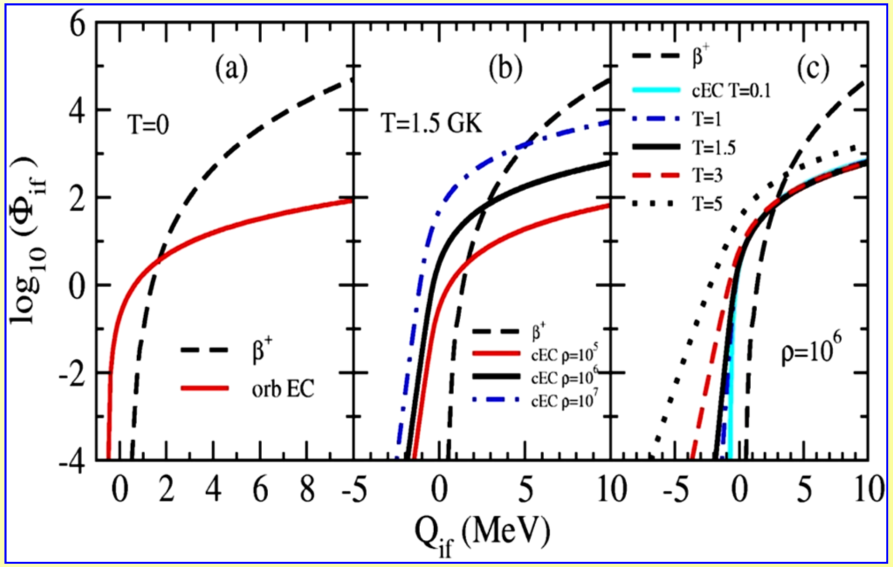
Distribution functions

$S_p=S_\nu=0$

S_e: Fermi-Dirac distribution

$$S_e = \frac{1}{\exp\left[(\omega-\mu_e)/(kT)\right]+1}$$

$$Q_{if} = \frac{1}{m_e c^2}\left(M_p - M_d + E_i - E_f\right)$$



Hartree-Fock method

$$H = \sum_{k=1}^{A} T(k) + \sum_{k<l=1}^{A} W(k,l)$$

How to extract a single-particle potential U(k)

out of the sum of two-body interactions W(k,l)

$$H = \sum_{k=1}^{A} \left[T(k) + U(k) \right] + \left[\sum_{k<l=1}^{A} W(k,l) - \sum_{k=1}^{A} U(k) \right] = H_0 + V_{res}$$

Hartree-Fock: Selfconsistent method to derive the single-particle potential

Variational principle: Search for the best Slater det. minimizing the energy

Assume that the resulting residual interaction is small

Skyrme effective interactions

Two-body interaction: zero-range limit of a finite range force

- leading term: delta-function with strength t_0 and spin exchange x_0
- leading finite range corrections t_1 , x_1 , t_2 , x_2 (finite range = momentum dependence)
- short-range spin-orbit interaction with strength W_0

Short-range density-dependent two-body force t_3, x_3 , α

$$V_{ij} = t_0 (1 + x_0 P_\sigma) \delta(\vec{r}_i - \vec{r}_j) + \frac{1}{2} t_1 (1 + x_1 P_\sigma) \delta(\vec{r}_i - \vec{r}_j)(k^2 + k'^2)$$
$$+ t_2 (1 + x_2 P_\sigma) \vec{k}' \cdot \delta(\vec{r}_i - \vec{r}_j) \vec{k} + i W_0 (\vec{\sigma}_i + \vec{\sigma}_j) \cdot \vec{k}' \times \delta(\vec{r}_i - \vec{r}_j) \vec{k}$$
$$+ \frac{1}{6} t_3 (1 + x_3 P_\sigma) \delta(\vec{r}_i - \vec{r}_j) \rho^\alpha \left(\frac{\vec{r}_i + \vec{r}_j}{2} \right)$$

Parameters (10) fitted using nuclear matter properties and ground state properties of a selected set of nuclei (binding energies, charge radii,…)

Sk3, SG2, SLy4

Skyrme Hartree-Fock

Schroedinger equation

$$\left[-\vec{\nabla}\cdot\frac{\hbar^2}{2m^*(\vec{r})}\vec{\nabla}+U_q(\vec{r})+\vec{W}(\vec{r})\cdot(-i)\left(\vec{\nabla}\times\vec{\sigma}\right)\right]\phi_i=e_i\phi_i$$

$m^*(\vec{r}),\quad U_q(\vec{r}),\quad \vec{W}(\vec{r})$ — Algebraic combinations of the densities $[\rho,\tau,J]$

$$\rho_{st}(\vec{r})=\sum_i\left|\phi_i(\vec{r},s,t)\right|^2,\quad \tau_{st}(\vec{r})=\sum_i\left|\vec{\nabla}\phi_i(\vec{r},s,t)\right|^2,\quad \vec{J}_{st}(\vec{r})=\sum_{i,s'}\phi_i^*(\vec{r},s',t)\left(-i\vec{\nabla}\times\vec{\sigma}\right)\phi_i(\vec{r},s,t)$$

$$\frac{\hbar^2}{2m_q^*(\vec{r})}=\frac{\hbar^2}{2m}+\frac{1}{8}\left[t_1(2+x_1)+t_2(2+x_2)\right]\rho(\vec{r})-\frac{1}{8}\left[t_1(1+2x_1)+t_2(1+2x_2)\right]\rho_q(\vec{r})$$

$$\begin{aligned}U_q(\vec{r})=&\frac{1}{2}t_0\left[(2+x_0)\rho-(1+2x_0)\rho_q\right]+\frac{1}{24}t_3\left\{(2+x_3)(2+\alpha)\rho^{\alpha+1}-(2x_3+1)\left[2\rho^{\alpha}\rho_q+\alpha\rho^{\alpha-1}\left(\rho_p^2+\rho_n^2\right)\right]\right\}\\&+\frac{1}{8}\left[t_1(2+x_1)+t_2(2+x_2)\right]\tau+\frac{1}{8}\left[t_2(1+2x_2)-t_1(1+2x_1)\right]\tau_q+\frac{1}{16}\left[-3t_1(2+x_1)+t_2(2+x_2)\right]\nabla^2\rho\\&+\frac{1}{16}\left[3t_1(1+2x_1)+t_2(1+2x_2)\right]\nabla^2\rho_q+\frac{1}{8}(t_1-t_2)\vec{J}_q-\frac{1}{8}(t_1x_1+t_2x_2)\vec{J}+\delta_{q,p}V_{coul}(\vec{r})\end{aligned}$$

$$\vec{W}_q(\vec{r})=\frac{1}{2}W_0\left(\vec{\nabla}\rho+\vec{\nabla}\rho_q\right)$$

Axially symmetric deformed nuclei

$$\Phi_i^{\Omega_i \pi_i}\left(\vec{R},\sigma,q\right)=\chi_{q_i}(q)\left[\Phi_i^{+}(r,z)e^{i\Lambda^{-}\varphi}\chi_{+1/2}(\sigma)+\Phi_i^{-}(r,z)e^{i\Lambda^{+}\varphi}\chi_{-1/2}(\sigma)\right]$$

Expansion into eigenfunctions of deformed harmonic oscillator

$$V(r,z)=\frac{1}{2}M\omega_\perp^2 r^2+\frac{1}{2}M\omega_z^2 z^2$$

$$\phi_\alpha\left(\vec{R},\sigma\right)=\psi_{n_r}^{\Lambda}(r)\psi_{n_z}(z)\frac{e^{i\Lambda\phi}}{\sqrt{2\pi}}\chi_\Sigma(\sigma)$$

$$\psi_{n_r}^{\Lambda}(r)=N_{n_r}^{\Lambda}\beta_\perp\sqrt{2}\eta^{\Lambda/2}e^{-\eta/2}L_{n_r}^{\Lambda}(\eta)$$

$$\psi_{n_z}(z)=N_{n_z}\beta_z^{1/2}e^{-\xi^2/2}H_{n_z}(\xi)$$

Optimal basis to minimize truncation effects

N= 12 major shells

$$\beta_o=\left[M\left(\omega_\perp^2\omega_z\right)^{1/3}\right]^{1/2}=\left(\beta_\perp^2\beta_z\right)^{1/3}$$

$$q=\frac{\omega_\perp}{\omega_z}=\left(\frac{\beta_\perp}{\beta_z}\right)^2$$

$$\Phi_i\left(\vec{R},\sigma,q\right)=\chi_{q_i}\sum_\alpha C_\alpha^i\phi_\alpha\left(\vec{R},\sigma\right),\qquad \alpha=\{n_r,n_z,\Lambda,\Sigma\}$$

Pairing correlations in BCS approximation

BCS ground state

$$|\varphi_{BCS}\rangle = \prod_{i>0}\left(u_i + v_i a_i^+ a_{\bar{i}}^+\right)|0\rangle$$

$$H = \sum_{k>0} \varepsilon_k \left(a_k^+ a_k + a_{\bar{k}}^+ a_{\bar{k}}\right) - G \sum_{kk'>0} a_k^+ a_{\bar{k}}^+ a_{\bar{k}'} a_{k'}$$

Variational equation constrained by the expectation value of particle number

BCS eqs.

Number eq.

$$2\sum_i v_i^2 = N$$

Gap eq.

$$\Delta = G\sum_{k>0} u_k v_k$$

$$v_i^2 = \frac{1}{2}\left[1 - \frac{e_i - \lambda}{E_i}\right]; \quad E_i = \sqrt{(e_i - \lambda)^2 + \Delta^2}$$

Fixed gaps taken from phenomenology

$$\Delta_n = \frac{1}{8}\left[B(N-2,Z) - 4B(N-1,Z) + 6B(N,Z) - 4B(N+1,Z) + B(N+2,Z)\right]$$

Residual interactions

Particle-hole residual interaction consistent with the HF mean field

$$V_{ph} = \frac{1}{16}\sum_{sts't'}\left[1+(-1)^{s-s'}\vec{\sigma}_1\cdot\vec{\sigma}_2\right]\left[1+(-1)^{t-t'}\vec{\tau}_1\cdot\vec{\tau}_2\right]\frac{\delta^2 E}{\delta\rho_{st}(\vec{r}_1)\delta\rho_{s't'}(\vec{r}_2)}\delta\left(\vec{r}_1-\vec{r}_2\right)$$

$$V_{ph}^{\sigma\tau} = \frac{1}{16}\sum_{sts't'}(-1)^{s-s'}(-1)^{t-t'}\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\tau}_1\cdot\vec{\tau}_2\frac{\delta^2 E}{\delta\rho_{st}(\vec{r}_1)\delta\rho_{s't'}(\vec{r}_2)}\delta\left(\vec{r}_1-\vec{r}_2\right)$$

$$= \frac{1}{16}\left[-4t_0 - 2t_1k_F^2 + 2t_2k_F^2 - \frac{2}{3}t_3\rho^{\alpha}\right]\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\tau}_1\cdot\vec{\tau}_2\delta\left(\vec{r}_1-\vec{r}_2\right)$$

Average over nuclear volume → **Separable forces**

$$V_{GT}^{ph} = 2\chi_{GT}^{ph}\sum_{K=0,\pm1}(-1)^K\beta_K^+\beta_{-K}^-, \quad \beta_K^+ = \sigma_K t^+ = \sum_{\pi\nu}\langle\nu|\sigma_K|\pi\rangle a_\nu^+ a_\pi$$

$$\chi_{GT}^{ph} = -\frac{3}{8\pi R^3}\left\{t_0 + \frac{1}{2}k_F^2\left(t_1 - t_2\right) + \frac{1}{6}t_3\rho^{\alpha}\right\}$$

$$V_{GT}^{pp} = -2\kappa_{GT}^{pp}\sum_{K=0,\pm1}(-1)^K P_K^+ P_K, \quad P_K^+ = \sum_{\pi\nu}\left\langle\nu\left|\left(\sigma_K\right)^+\right|\pi\right\rangle a_\nu^+ a_{\bar{\pi}}^+$$

pnQRPA with separable forces

Phonon operator

$$\Gamma^{+}_{\omega_K} = \sum_{\gamma_K}\left[X^{\omega_K}_{\gamma_K} A^{+}_{\gamma_K} - Y^{\omega_K}_{\gamma_K} A_{\bar{\gamma}_K} \right], \quad A^{+}_{\gamma_K} = \alpha^{+}_{n}\alpha^{+}_{\bar{p}}$$

$$\Gamma_{\omega_K}\left|0\right\rangle = 0, \quad \Gamma^{+}_{\omega_K}\left|0\right\rangle = \left|\omega_K\right\rangle$$

pnQRPA equations

Transition amplitudes

$$\left\langle \omega_K \left| \sigma_K t^{\pm} \right| 0 \right\rangle = \mp M^{\omega_K}_{\pm}$$

$$M^{\omega_K}_{-} = \sum_{\pi\nu}\left(q_{\pi\nu} X^{\omega_K}_{\pi\nu} + \tilde{q}_{\pi\nu} Y^{\omega_K}_{\pi\nu} \right)$$

$$M^{\omega_K}_{+} = \sum_{\pi\nu}\left(\tilde{q}_{\pi\nu} X^{\omega_K}_{\pi\nu} + q_{\pi\nu} Y^{\omega_K}_{\pi\nu} \right)$$

$$\tilde{q}_{\pi\nu} = u_\nu v_\pi \Sigma^{\pi\nu}_{K}, \quad q_{\pi\nu} = v_\nu u_\pi \Sigma^{\pi\nu}_{K}, \quad \Sigma^{\pi\nu}_{K} = \left\langle \nu \left| \sigma_K \right| \pi \right\rangle$$

Single-particle states from deformed Skyrme Hartree-Fock

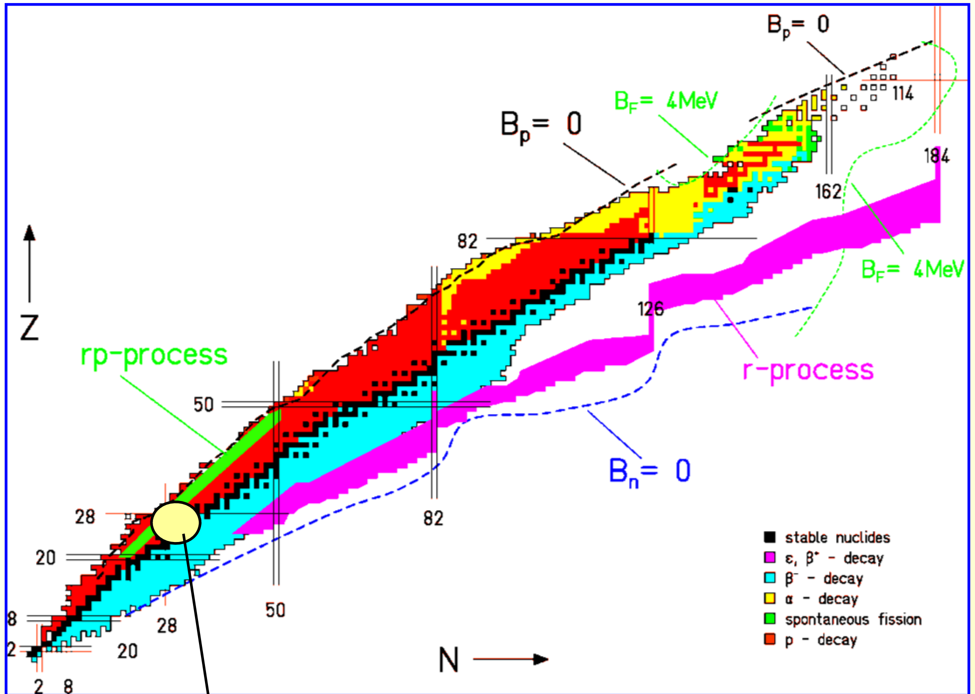
u, v: Occupation amplitudes from BCS pairing

B(GT) in the lab. system

$$I_i K_i \left(0^{+}0\right) \rightarrow I_f K_f \left(1^{+}K\right)$$

$$B_\omega(GT^{\pm}) = \delta\left(\omega_0 - \omega\right)\left\langle \omega_0 \left| \sigma_0 t^{\pm} \right| 0 \right\rangle^2 + 2\delta\left(\omega_1 - \omega\right)\left\langle \omega_1 \left| \sigma_1 t^{\pm} \right| 0 \right\rangle^2$$

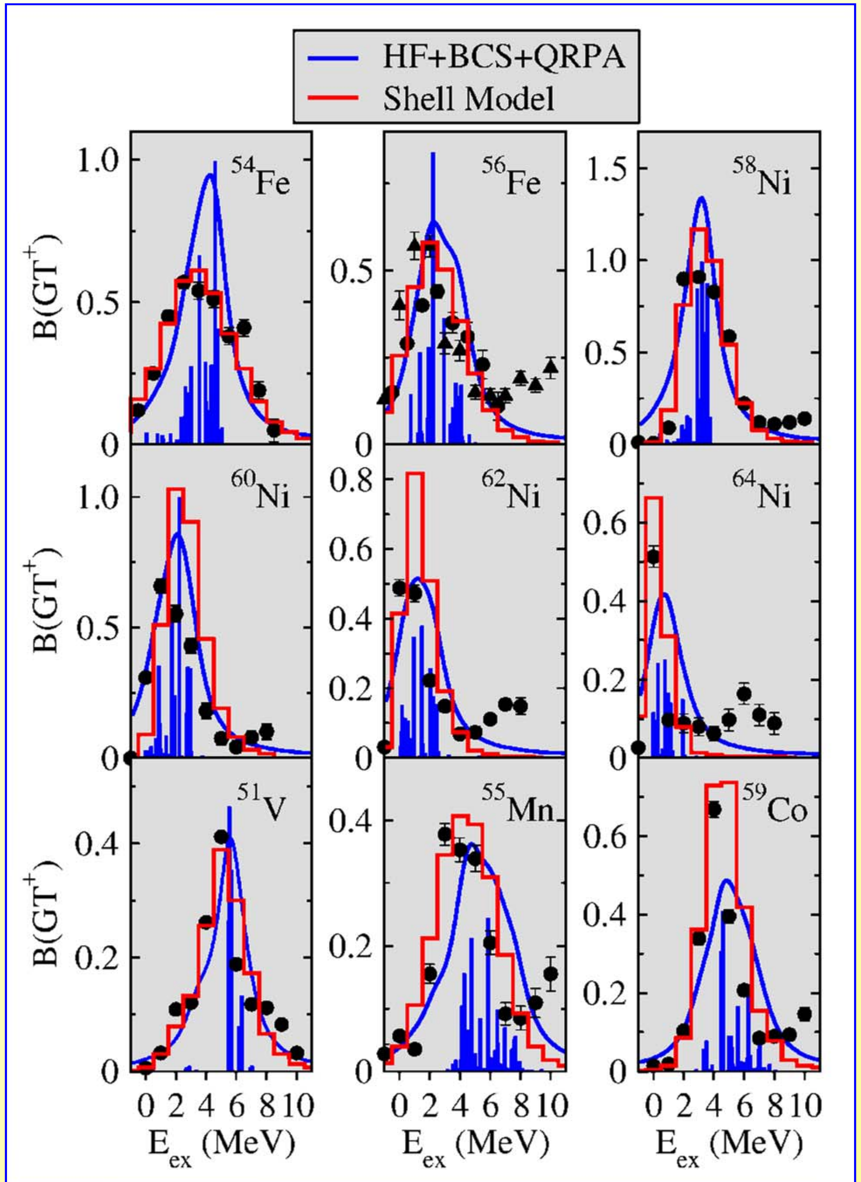
Stable nuclei in Fe-Ni mass region: Theory vs experiment



Main constituents of stellar core in presupernovae. Comparison with :

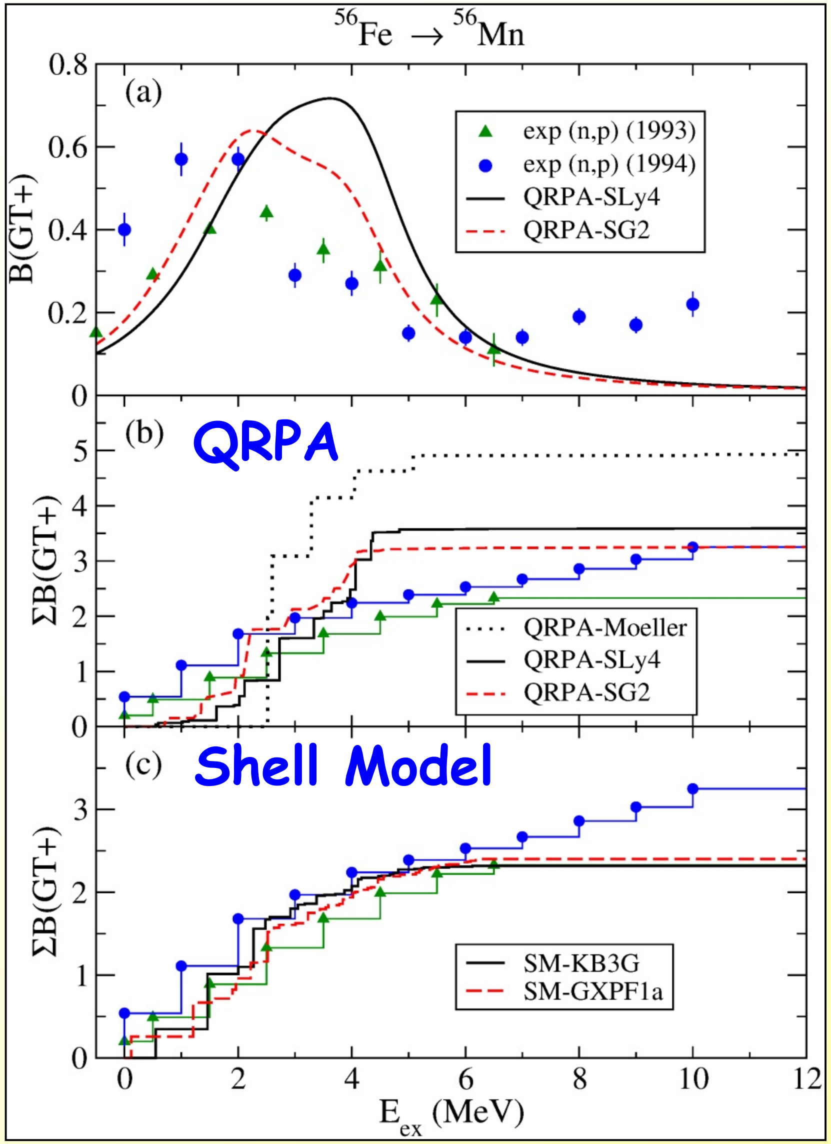
- exp. (n,p), (p,n)
- SM calculations

GT properties: Test of QRPA



SM: NPA 653, 439 (1999)
QRPA: NPA 716, 230 (2003)

Weak decay rates in pf-shell nuclei



Calculations for several isotopes:

Sc, Ti, V, Fe, Mn, Ni, Co, Zn

exp: charge exchange reactions

(n,p)

$$B_{if}(GT) \propto \langle f | \sigma t | i \rangle^2$$

Accumulated GT strength

A.L. Cole PRC 86, 015809 (2012)

P.S. PRC87, 045801 (2013)

Weak decay rates in pf-shell nuclei

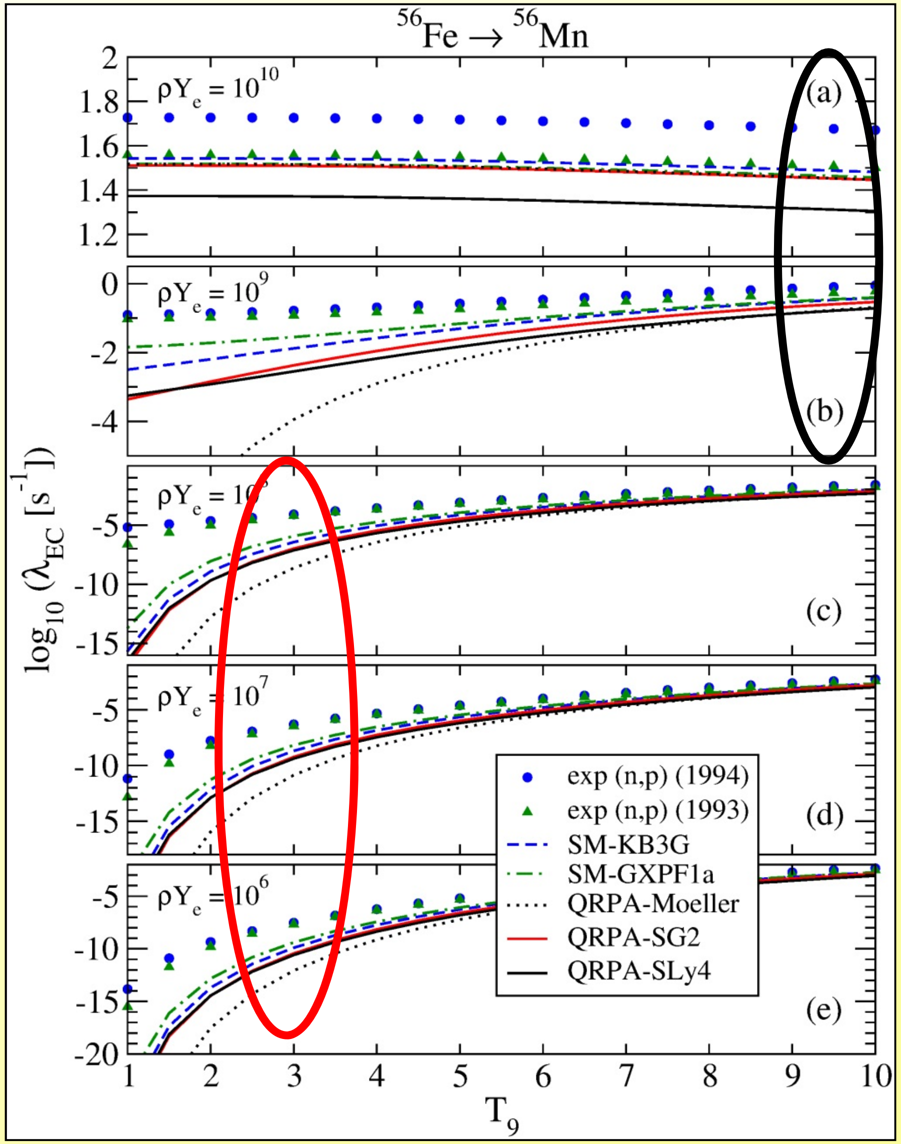
$$\lambda(\rho,T) \propto \sum_{if} P_i(T) B_{if} \Phi_{if}(\rho,T)$$

Electron Capture from e⁻ plasma

$$\Phi_{if}^{cEC} = \int_{\omega_\ell}^{\infty} \omega\sqrt{\omega^2-1}(Q_{if}+\omega)^2 F(Z,\omega)\times S_e(\omega)\left[1-S_\nu(Q_{if}+\omega)\right]d\omega$$

$$S_e = \frac{1}{\exp\left[(\omega-\mu_e)/(kT)\right]+1}$$

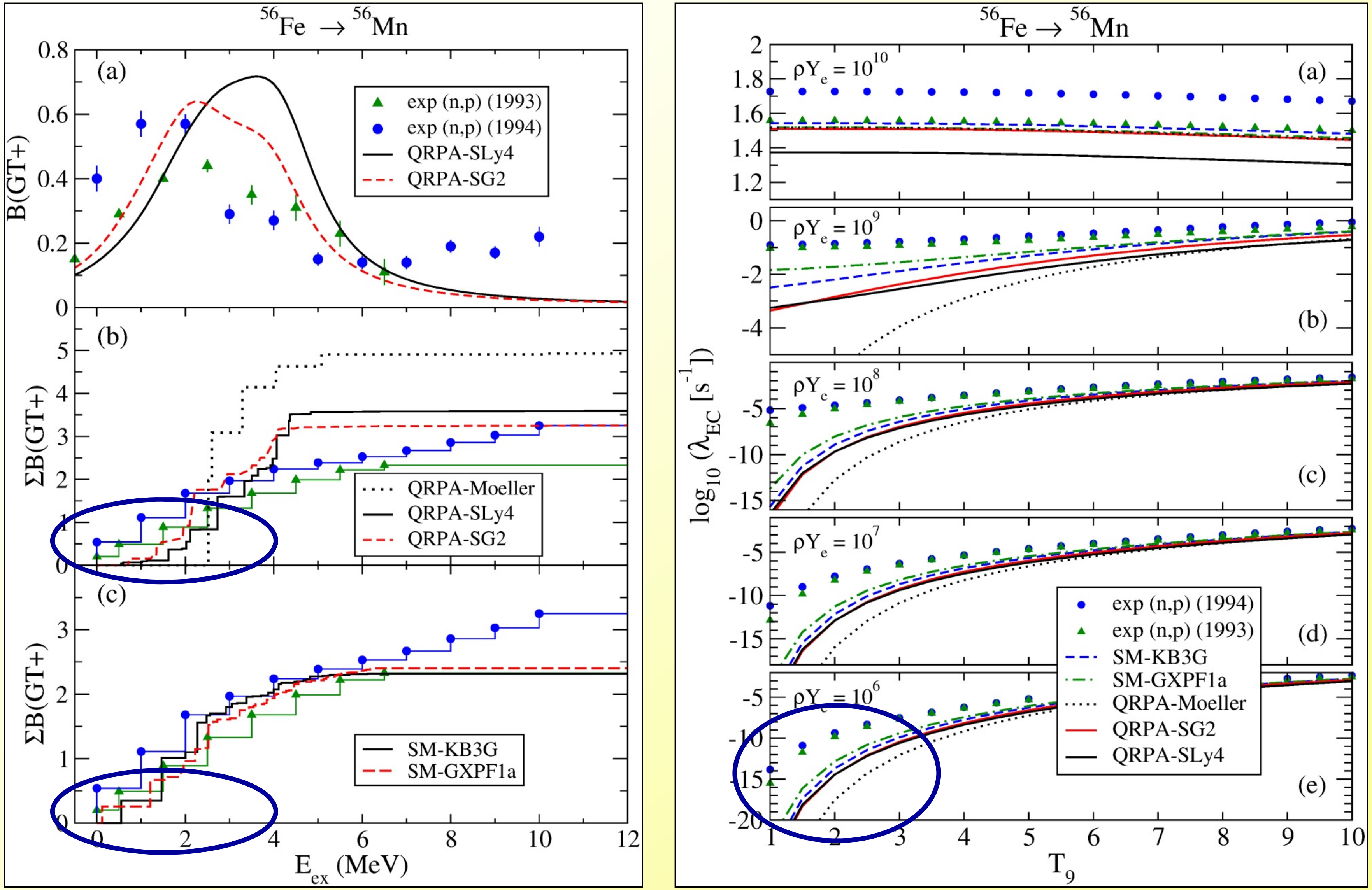
$$Q_{if} = \frac{1}{m_e c^2}\left(M_p - M_d + E_i - E_f\right)$$



Si burning

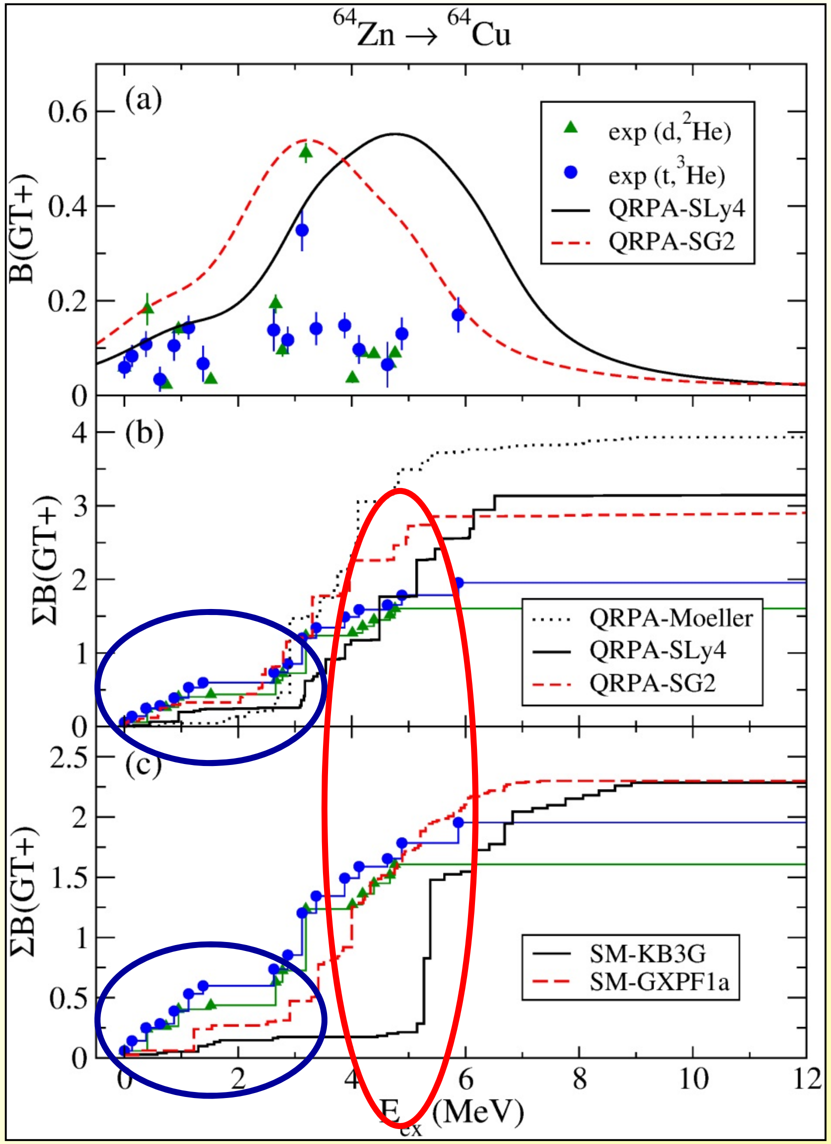
Ia Supernova

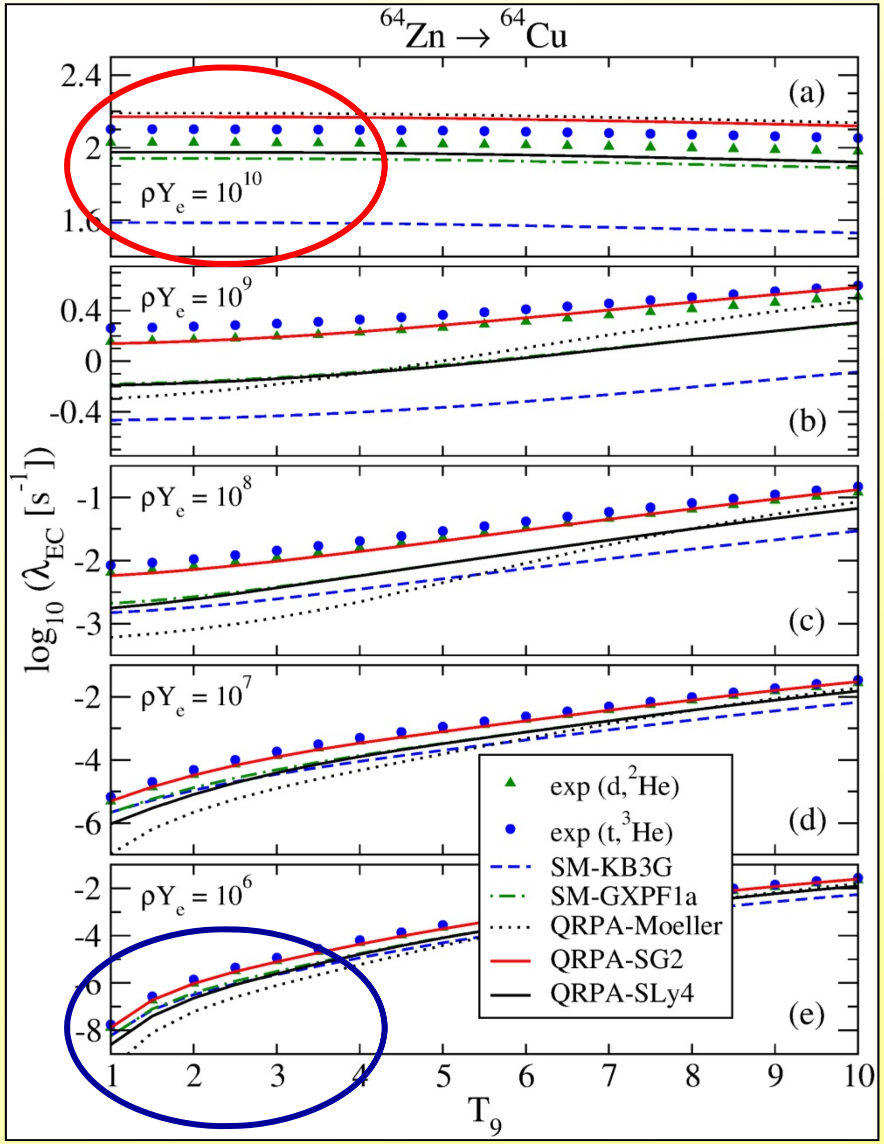
Weak decay rates in pf-shell nuclei



(Q<0) large: sensitive low-lying exc. **P.S. PRC87, 045801 (2013)**

Weak decay rates in pf-shell nuclei



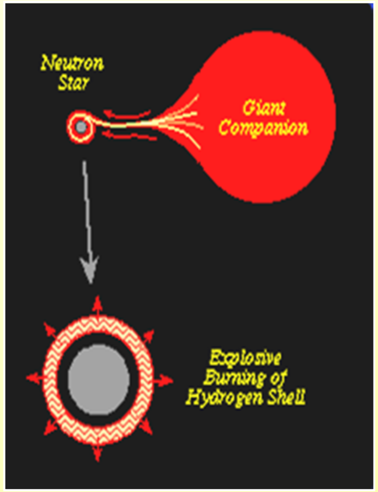


Q small

Exotic Nuclei : Nuclear Astrophysics

X-ray bursts: Source of intense X-ray emissions generated by thermonuclear runaway in the H-rich environment of an accreting n-star, fed from a binary red giant companion.

Nucleosynthesis mechanism: rp capture process



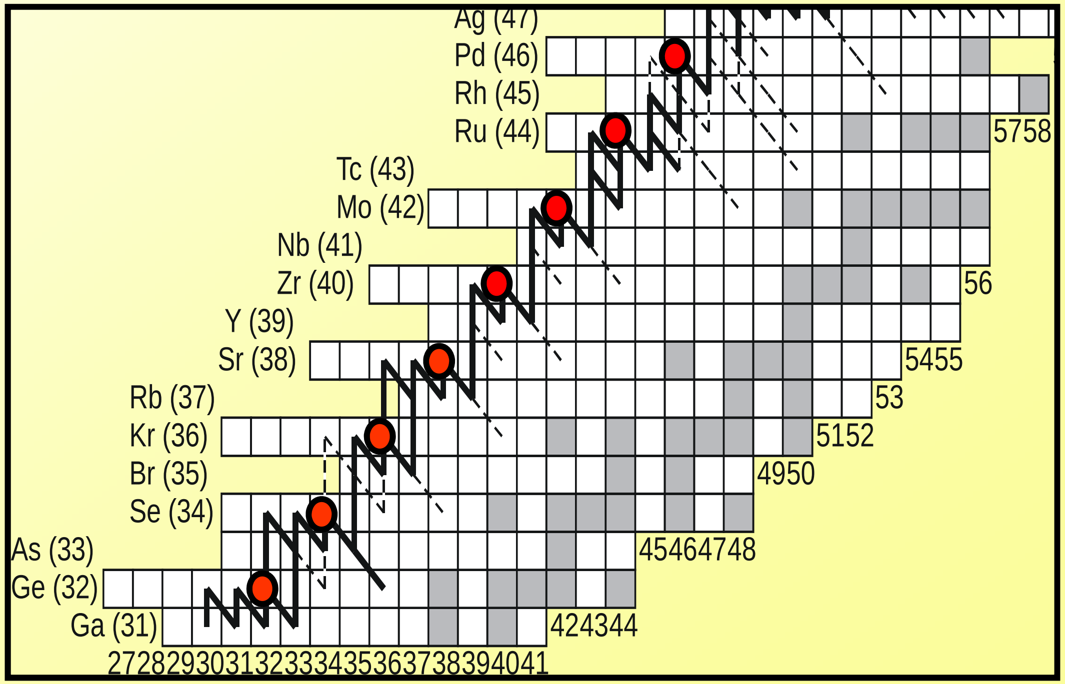
rp-process: Proton capture reaction rates are orders of magitude faster than the competing β^+-decays.

Waiting point nuclei: When the dominant p-capture is inhibited, the reaction flow waits for a slow beta-decay to proceed further.

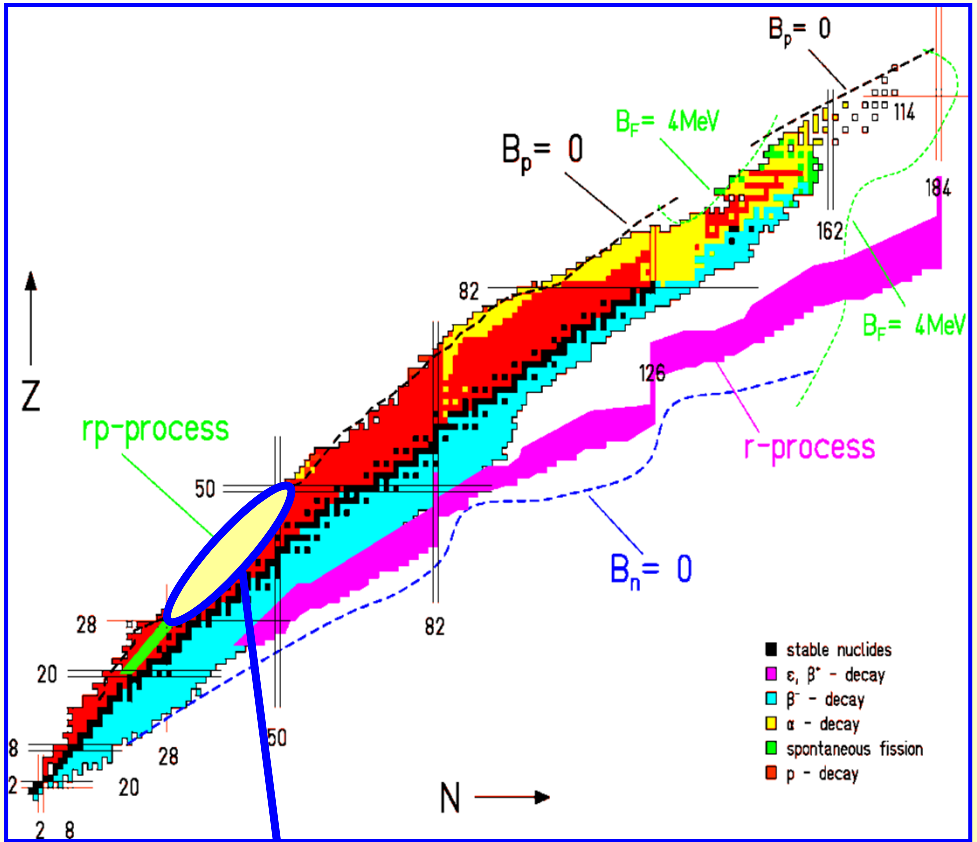
Time scale

Isotope abundances

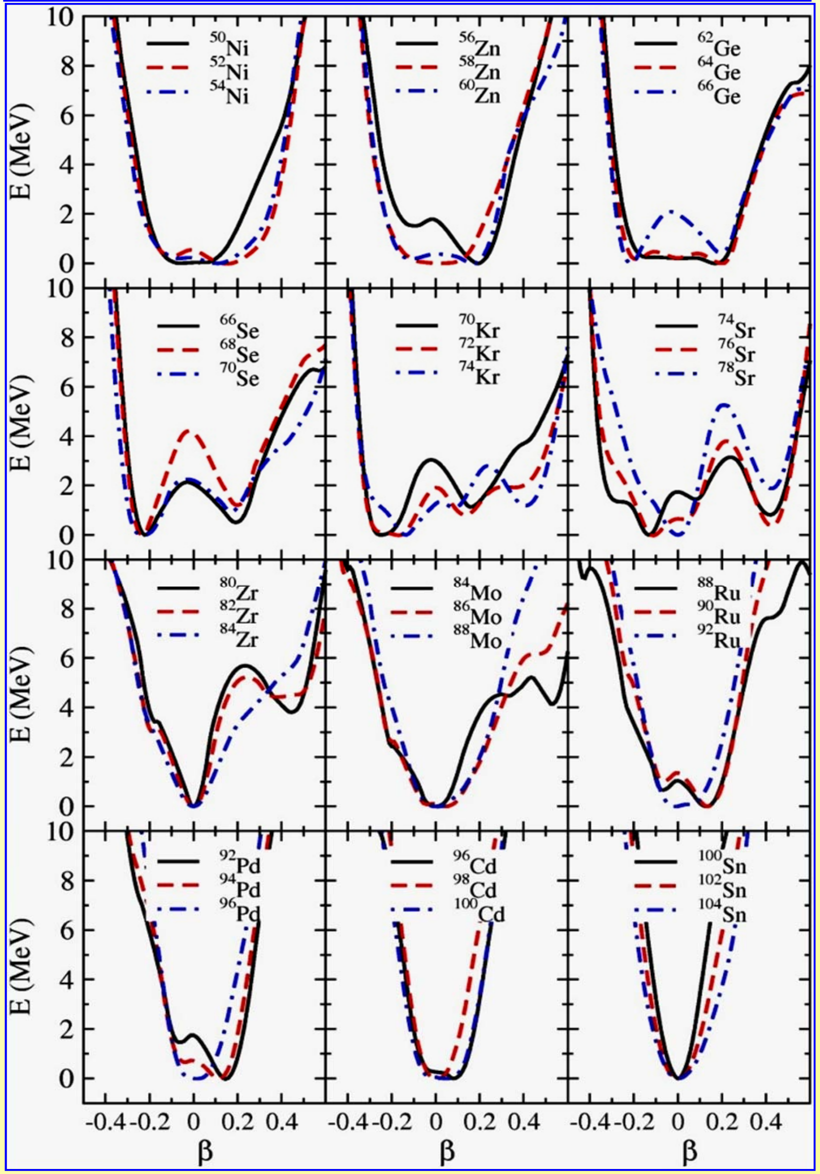
Light curve profile



Medium mass neutron deficient isotopes



Constrained HF+BCS

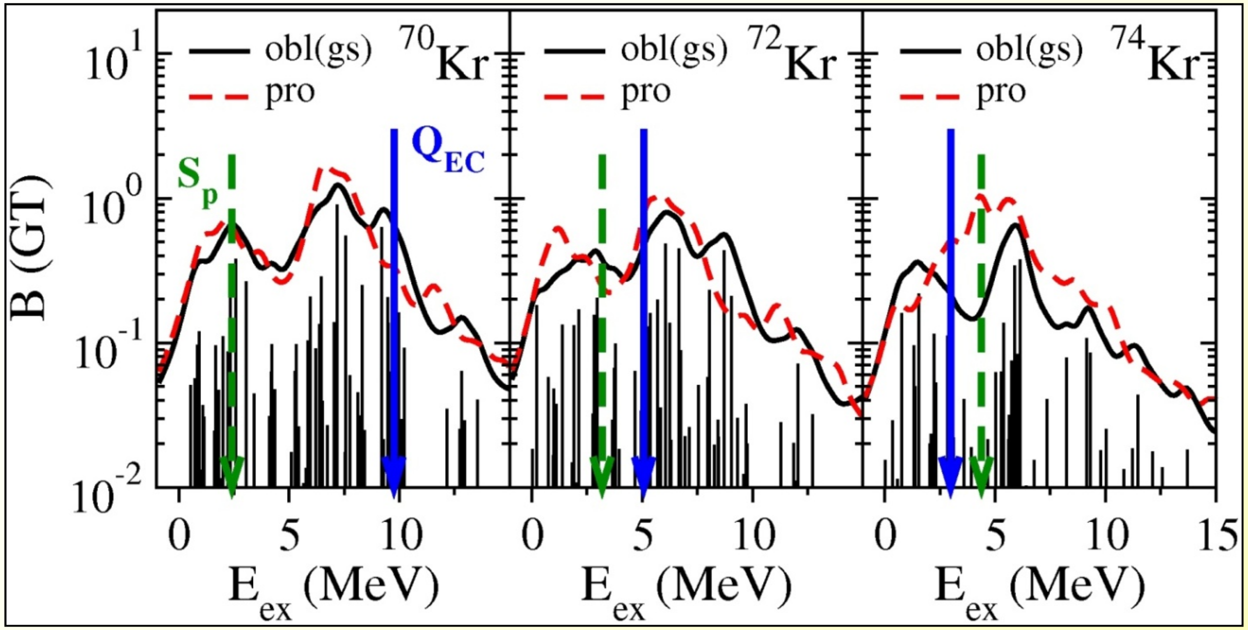


Waiting point nuclei in rp-processes

Shape coexistence

Beyond full Shell Model

Gamow-Teller strength distribution



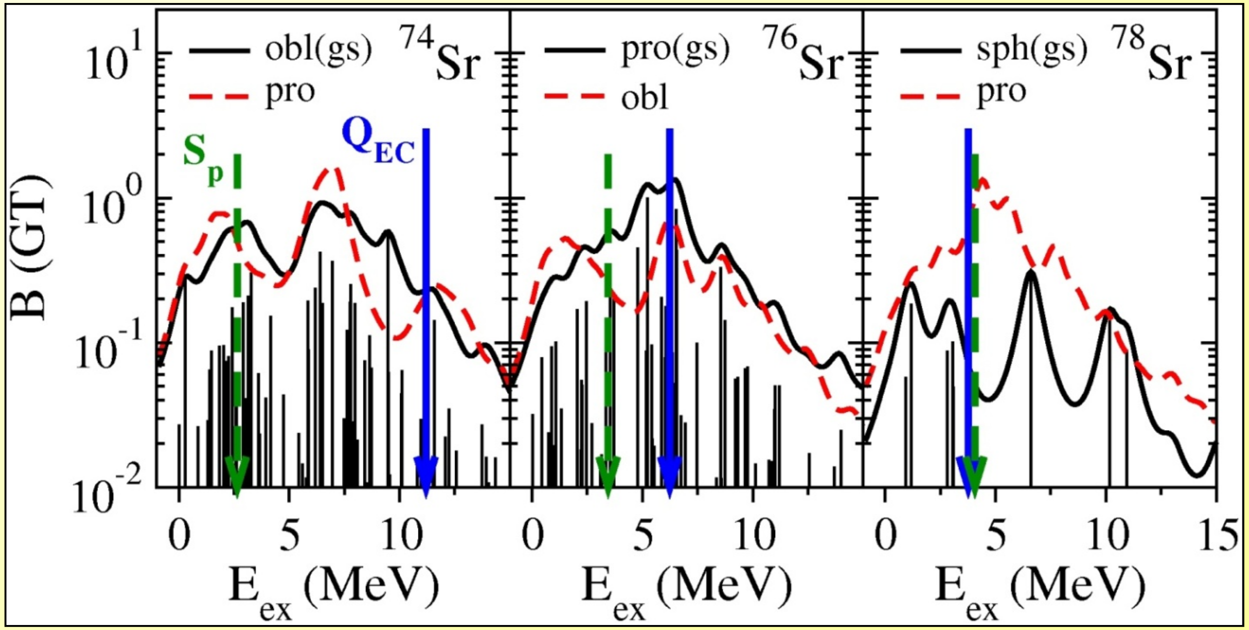
Calculations for

Ni, Zn, Ge, Se,

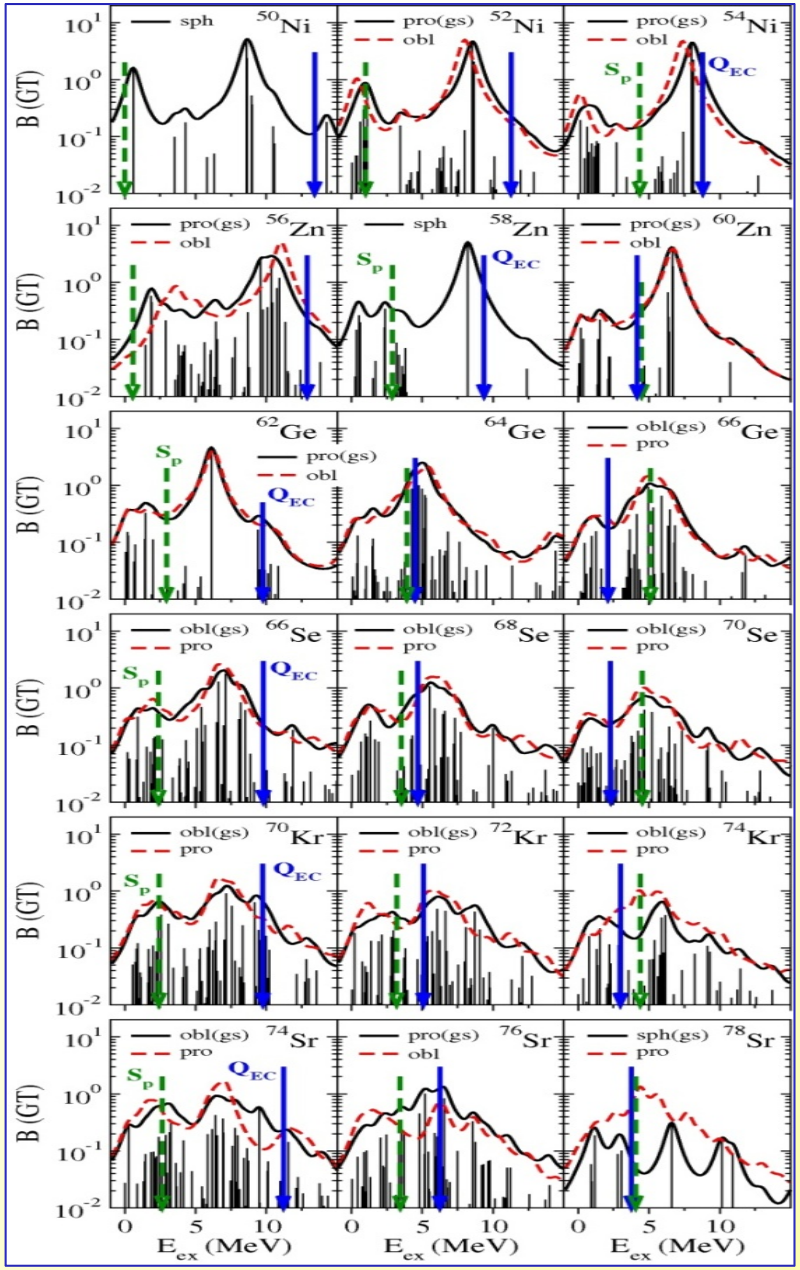
Kr, Sr, Zr, Mo,

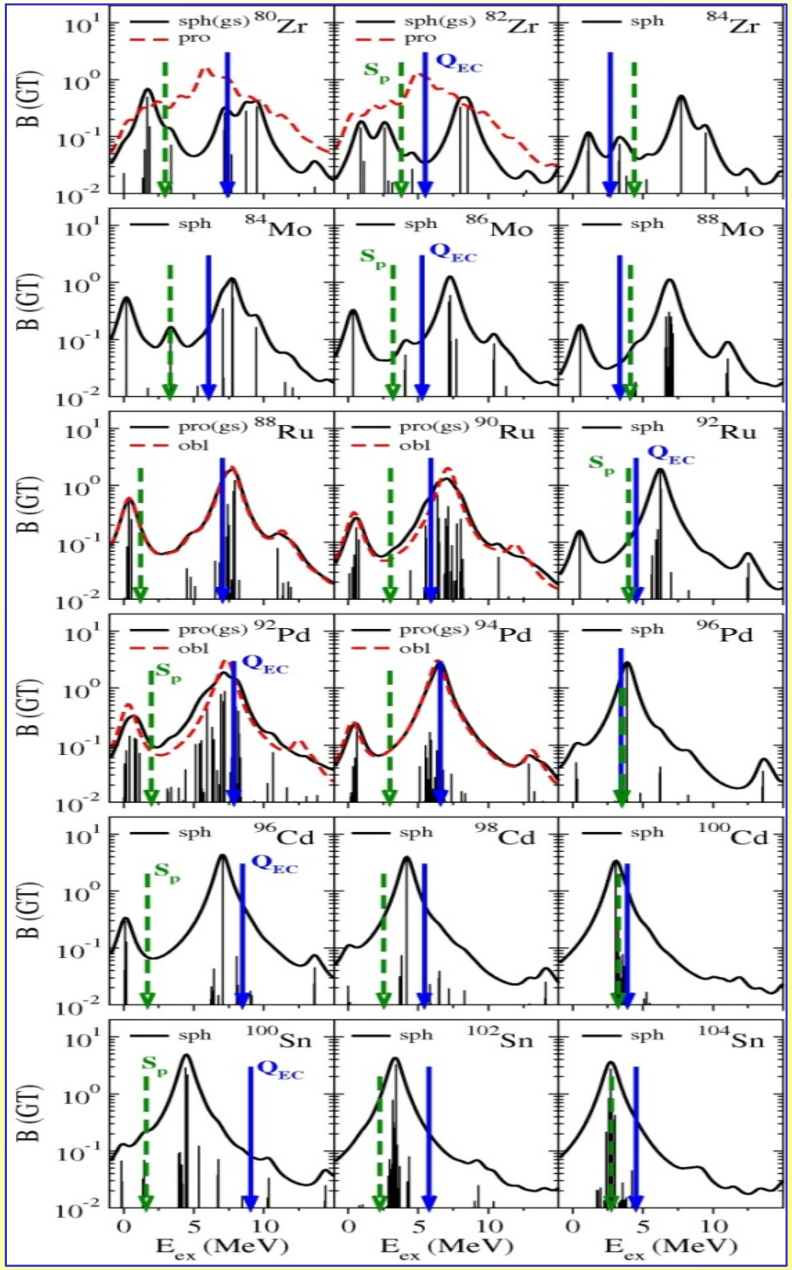
Ru, Pd, Cd, Sn

(N=Z and neighbors)



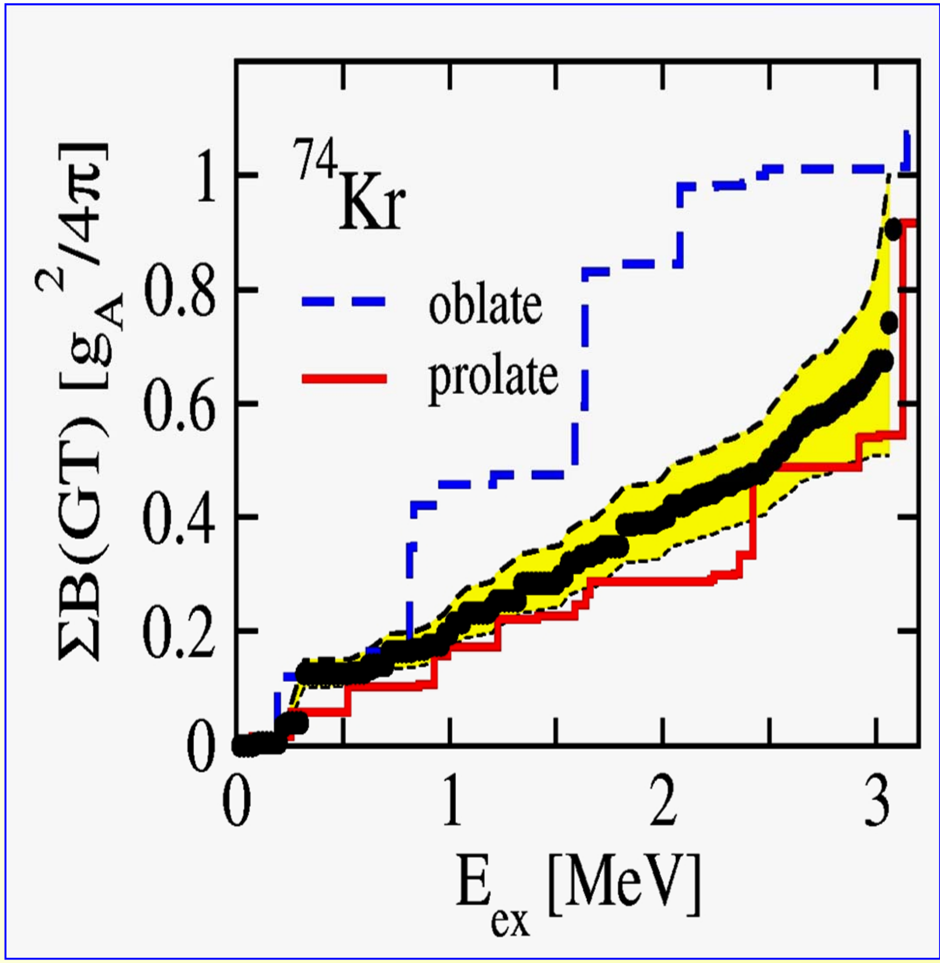
Gamow-Teller strength distribution



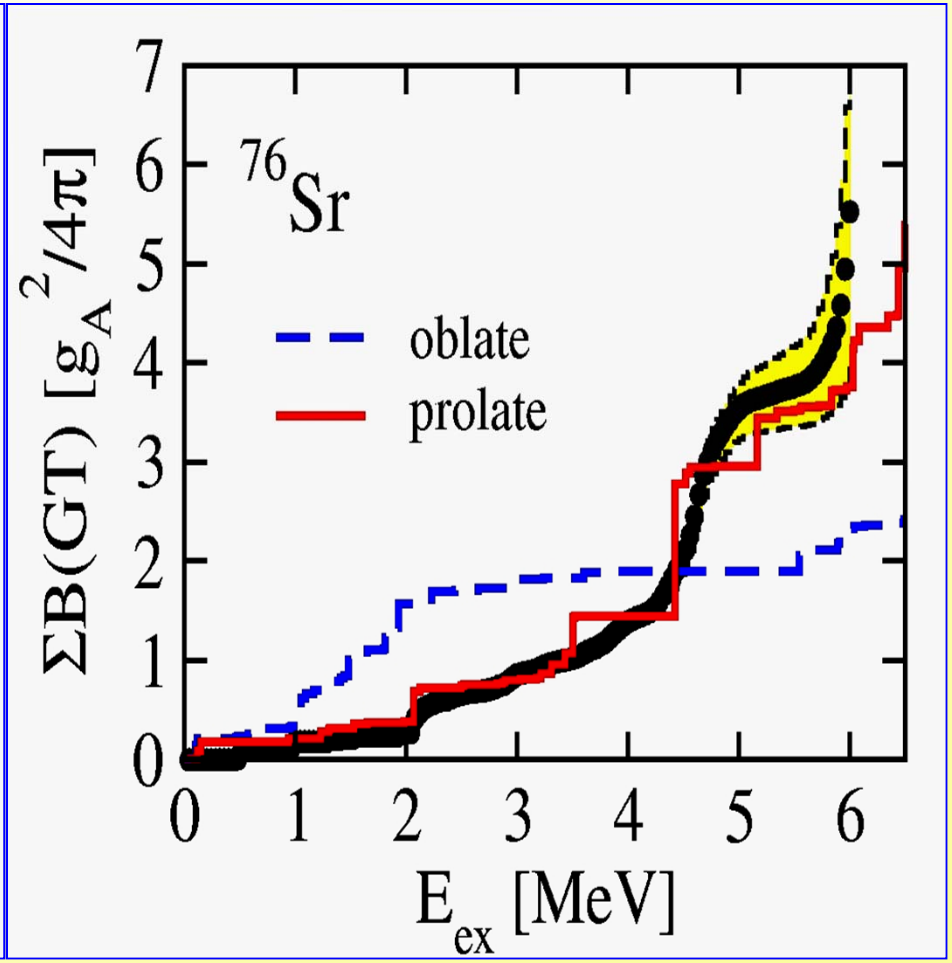


Gamow-Teller strength: Theory and Experiment

ISOLDE: Total absorption spectroscopy



Exp: Poirier et al. PRC69, 034307 (2004)



Exp: Nacher et al. PRL92, 232501 (2004)

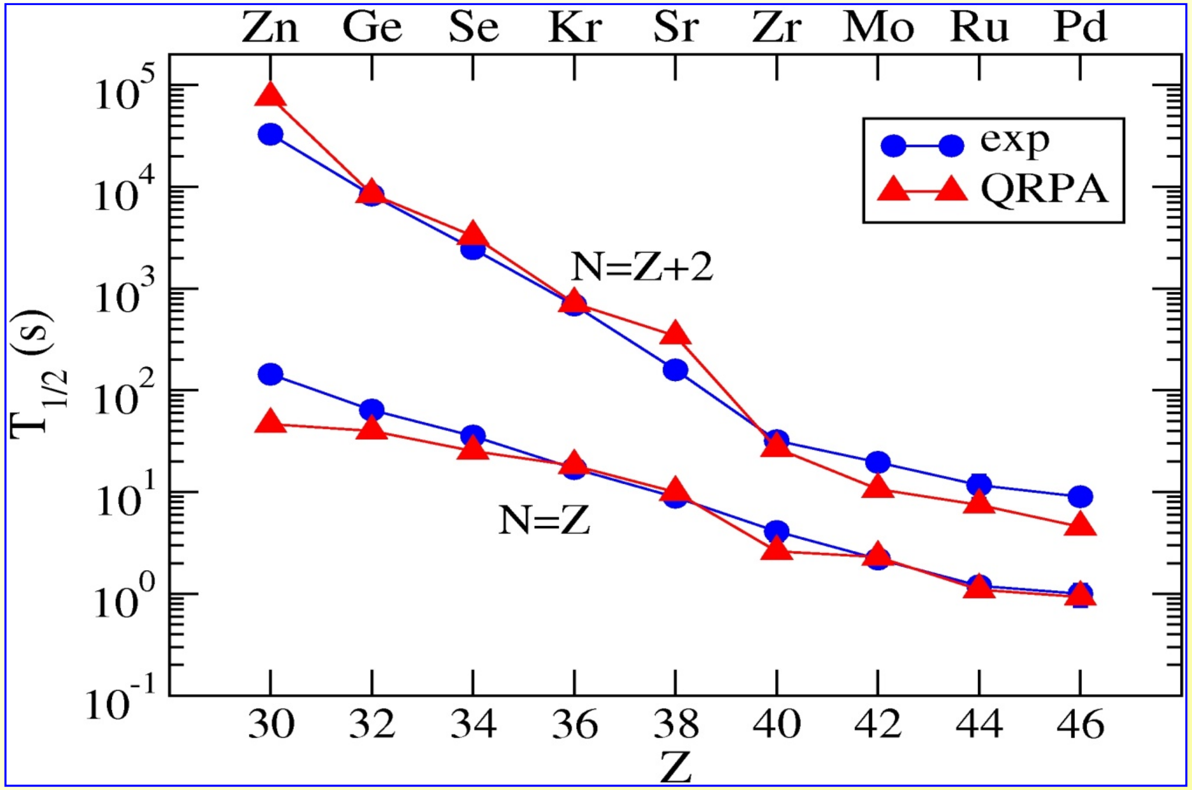
β⁺/EC half-lives: Theory and Experiment

$$T_{1/2}^{-1} = \frac{(g_A/g_V)_{\text{eff}}^2}{6200} \sum_f \Phi^{\beta^+/EC} \left| \left\langle f \| \beta^+ \| i \right\rangle \right|^2$$

$$\Phi_{if}^{\beta^+} = \int_1^{Q_{if}} \omega \sqrt{\omega^2 - 1} (Q_{if} - \omega)^2 F(Z,\omega) d\omega$$

$$\Phi^{EC} = \frac{\pi}{2} \left[q_K^2 g_K^2 B_K + q_{L_1}^2 g_{L_1}^2 B_{L_1} + \cdots \right]$$

$$(g_A/g_V)_{\text{eff}} = 0.74 (g_A/g_V)_{\text{bare}}$$

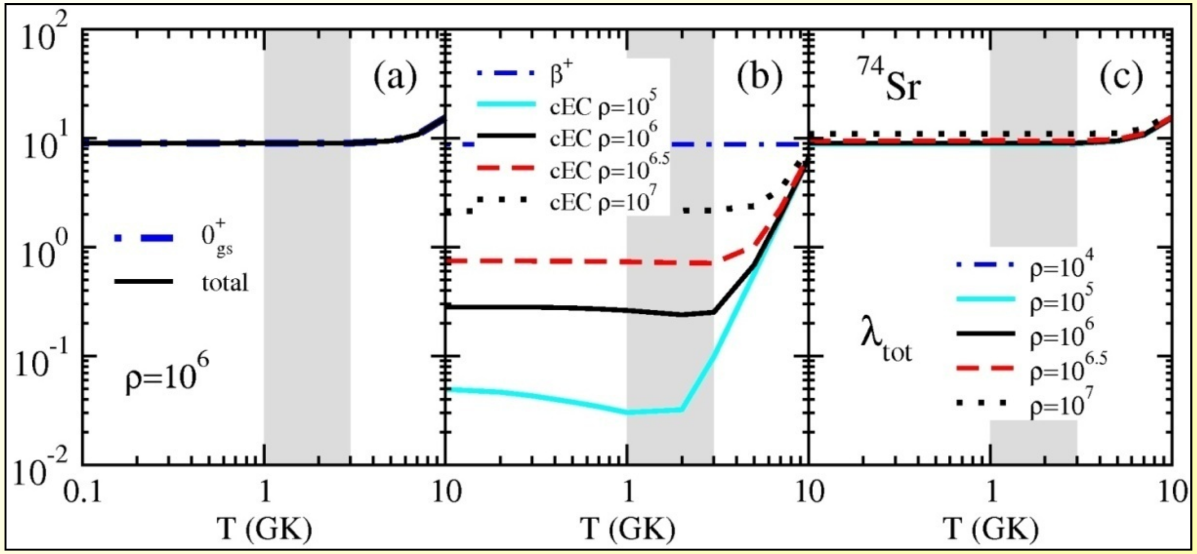


Good agreement with experiment:

Reliable extrapolations to high ρ and T

Weak decay rates in rp-process

^{74}Sr

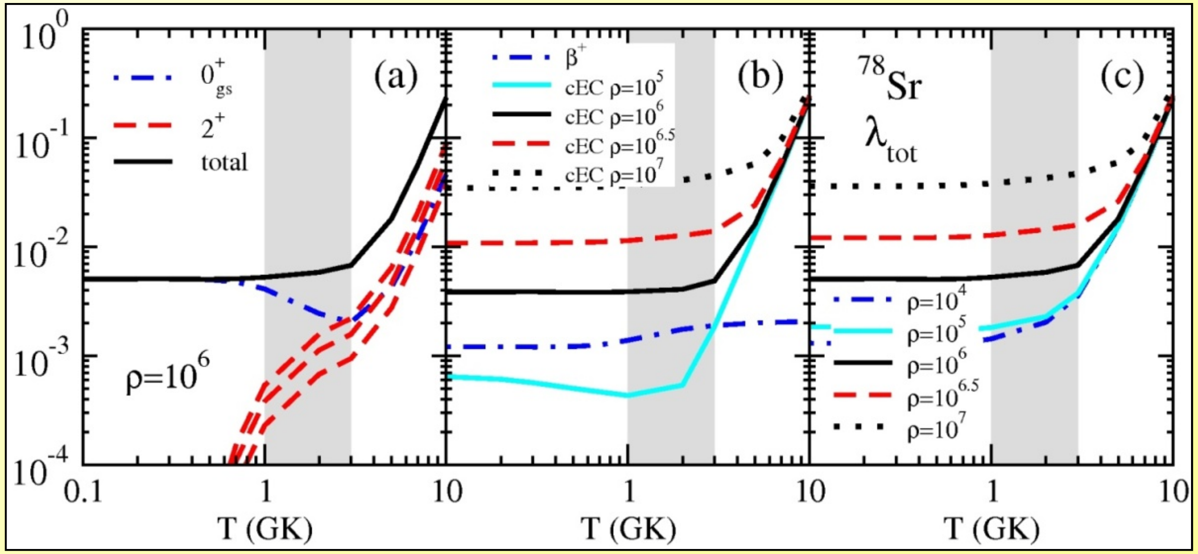


Deformed pn-QRPA

B(GT) and $T_{1/2}$: Good agreement with experiment

Competition β+/EC

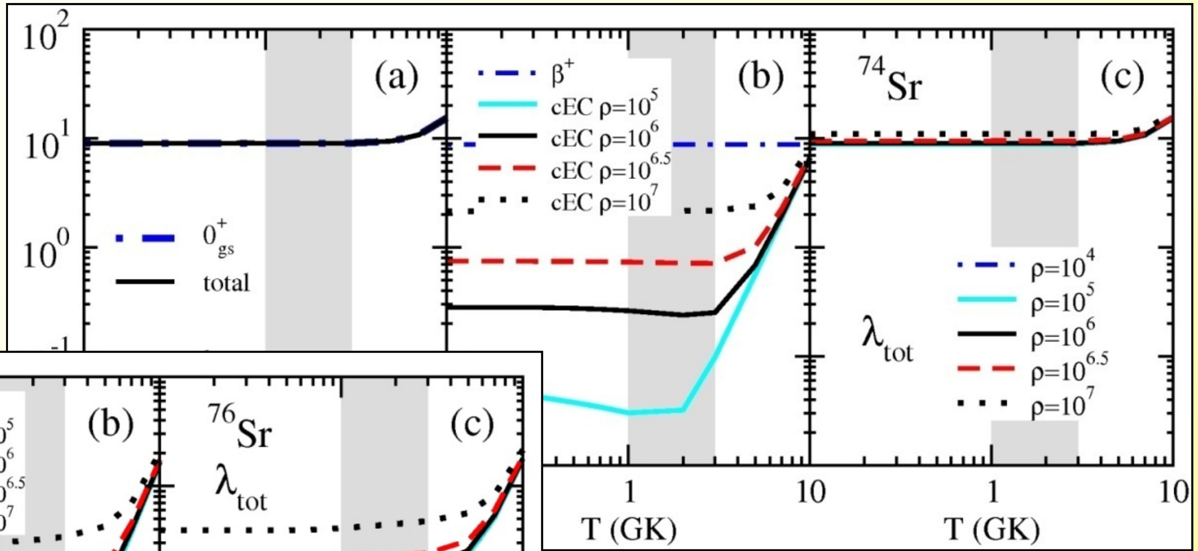
P.S. PRC83, 025801 (2011)

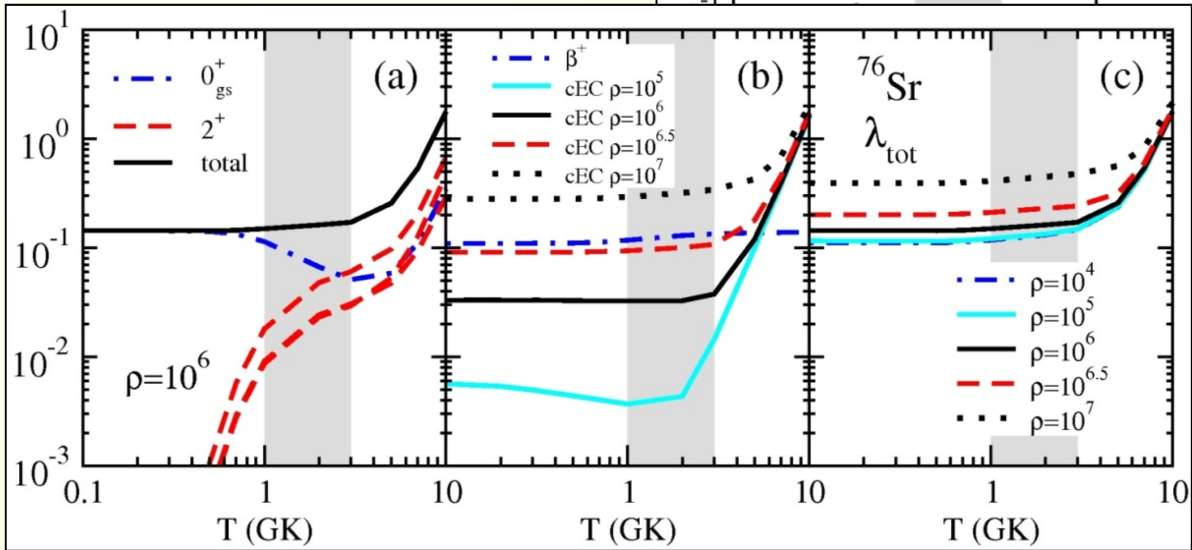


^{78}Sr

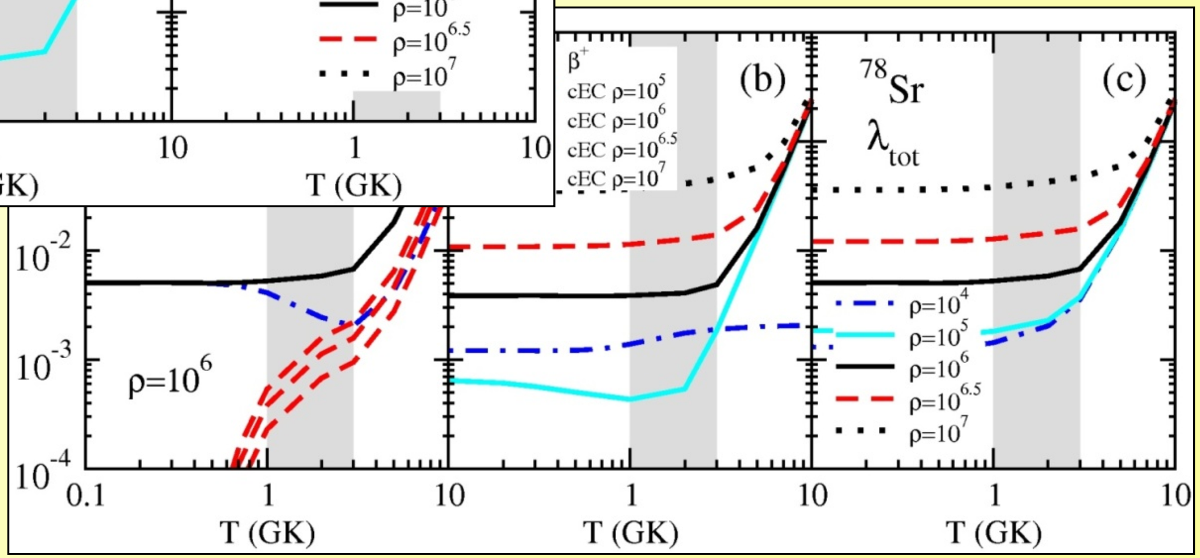
Weak decay rates in rp-process

^{74}Sr





^{76}Sr (WP)

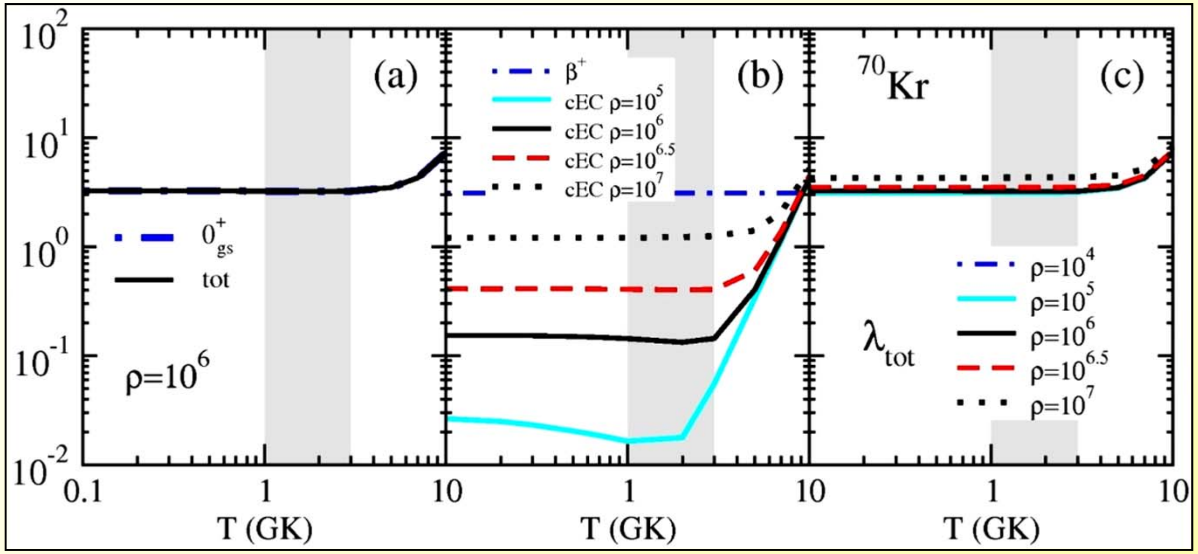


^{78}Sr

Weak decay rates in rp-process

^{70}Kr

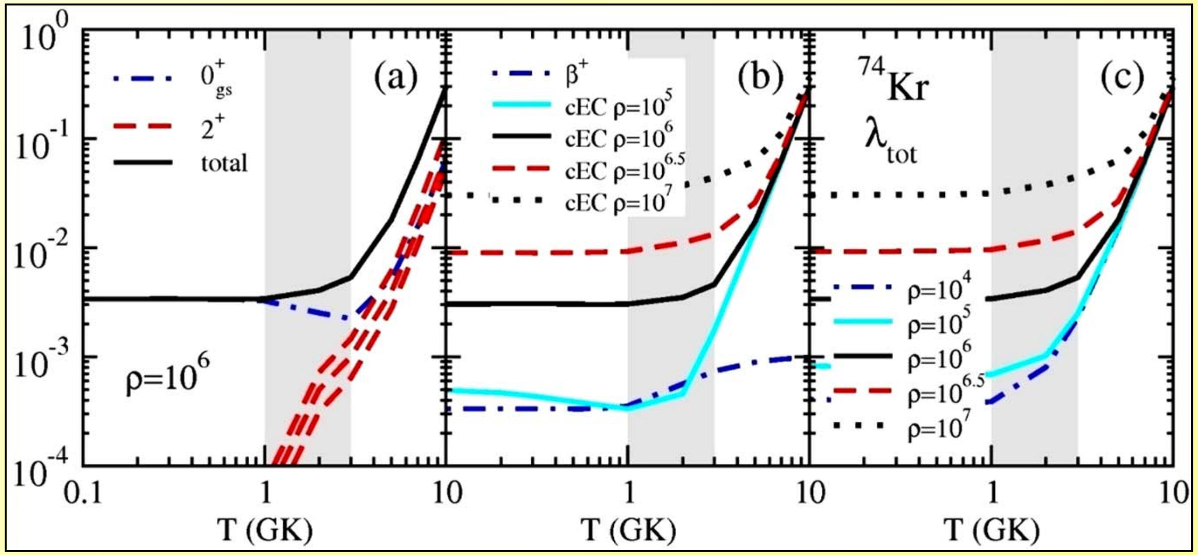
(Large Q)



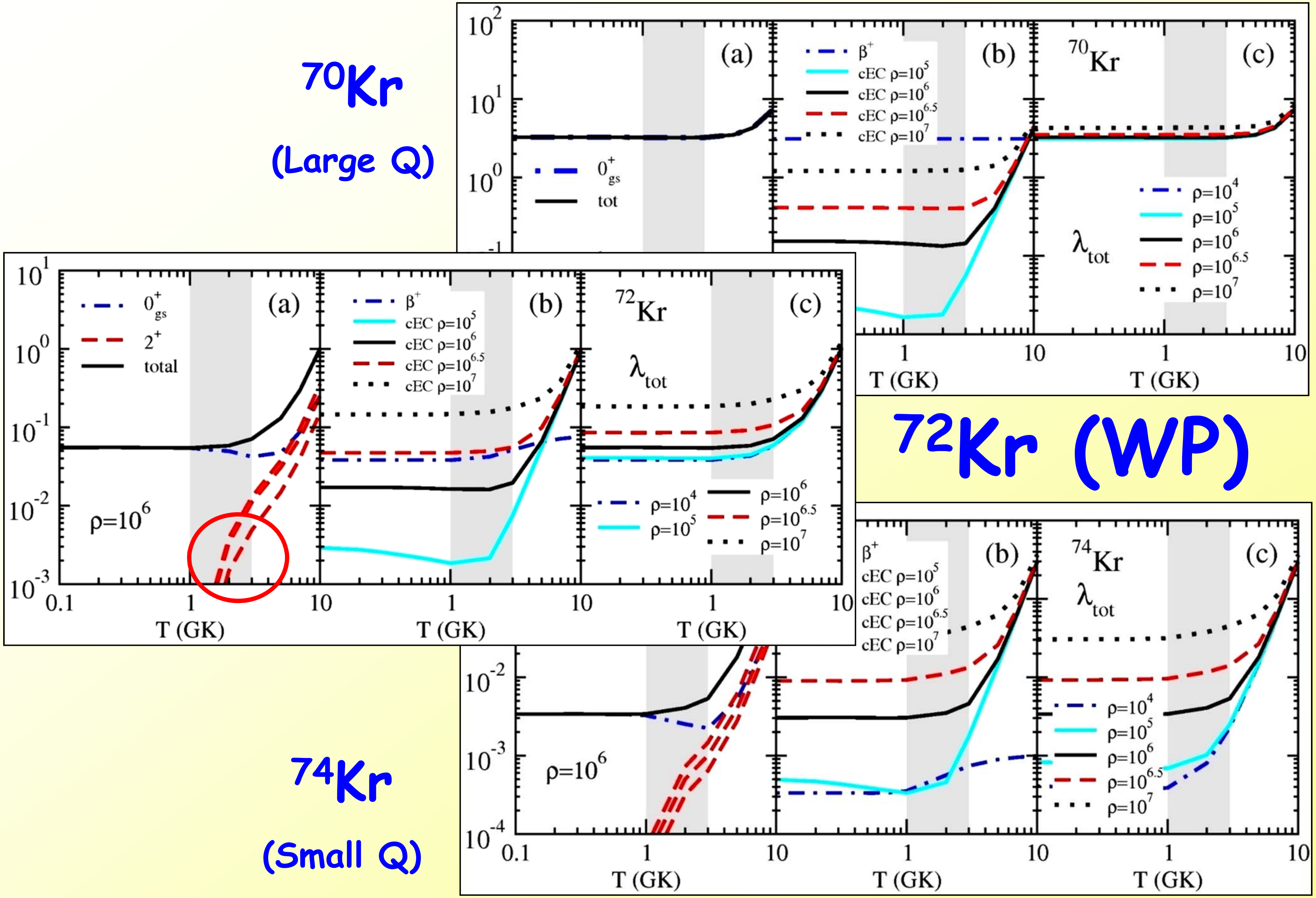
P.S. Phys. Rev. C 83, 025801 (2011)

^{74}Kr

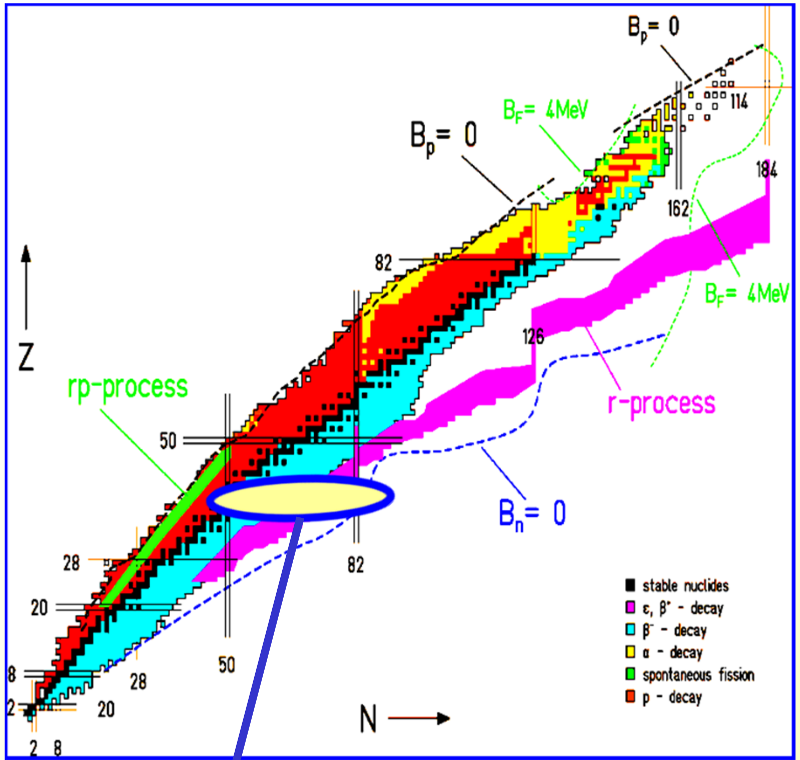
(Small Q)



Weak decay rates in rp-process

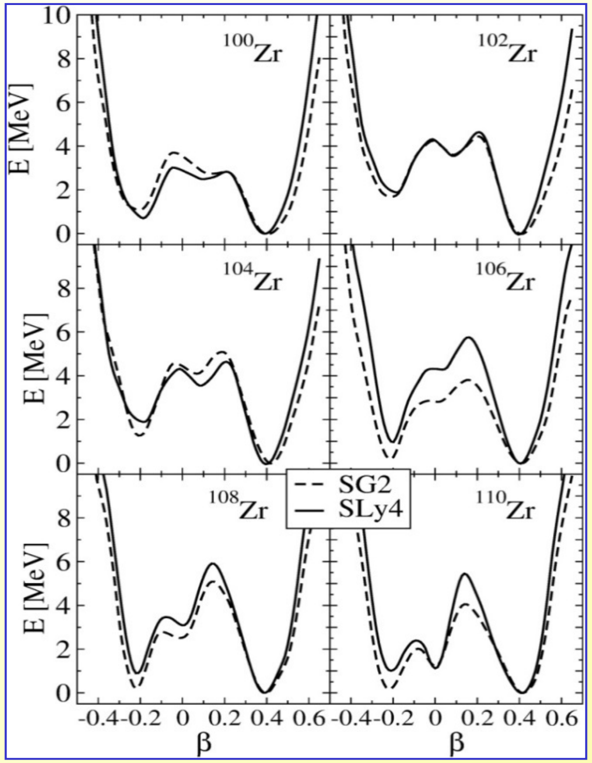


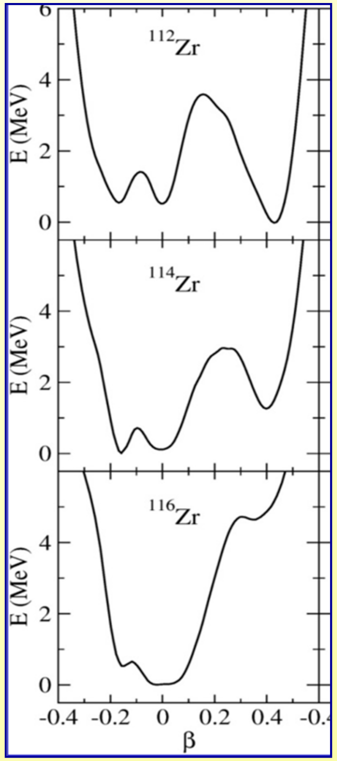
Zr-Mo neutron-rich isotopes

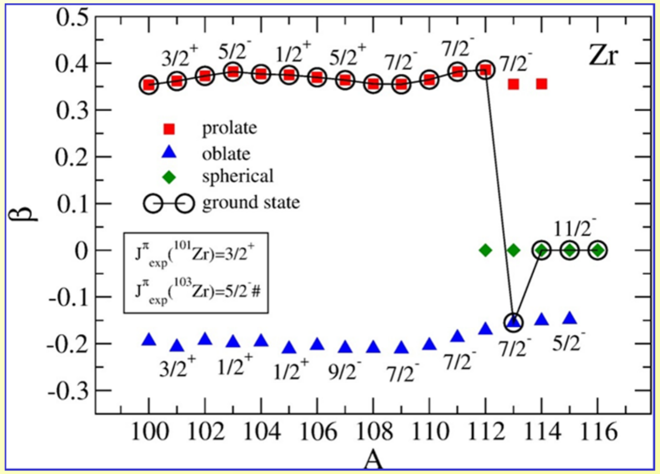


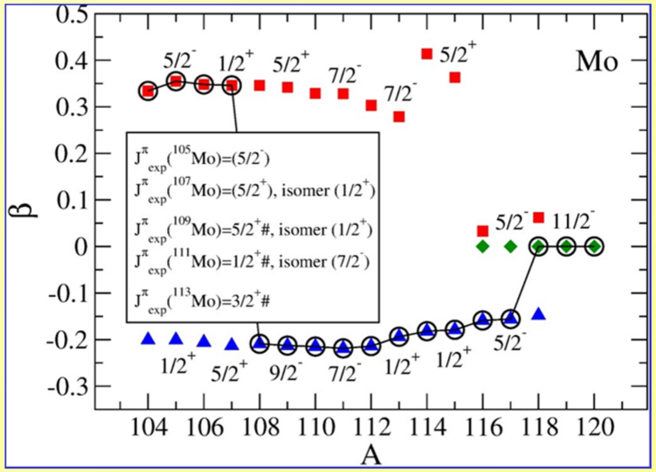
$^{100-116}Zr$

$^{104-120}Mo$

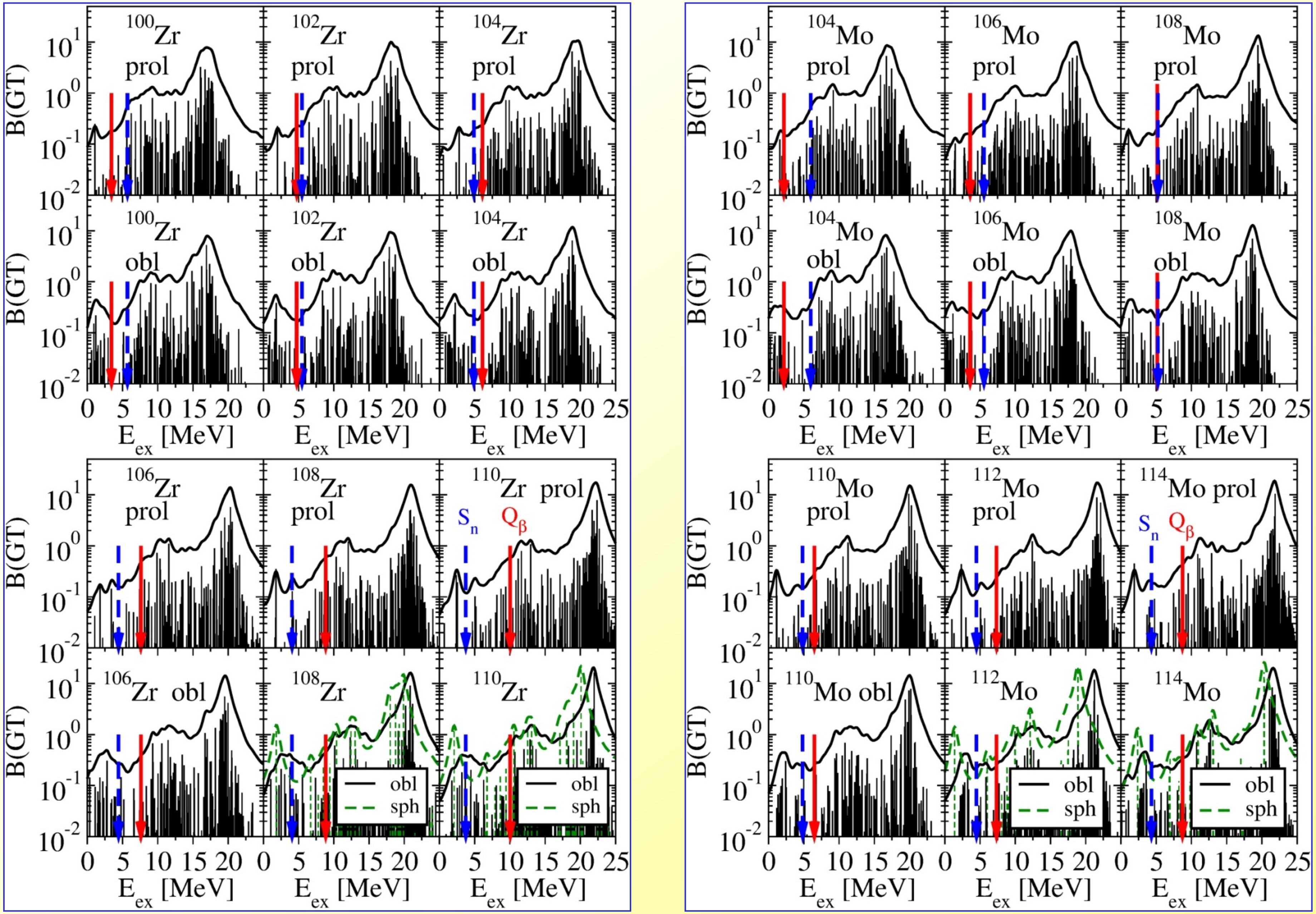






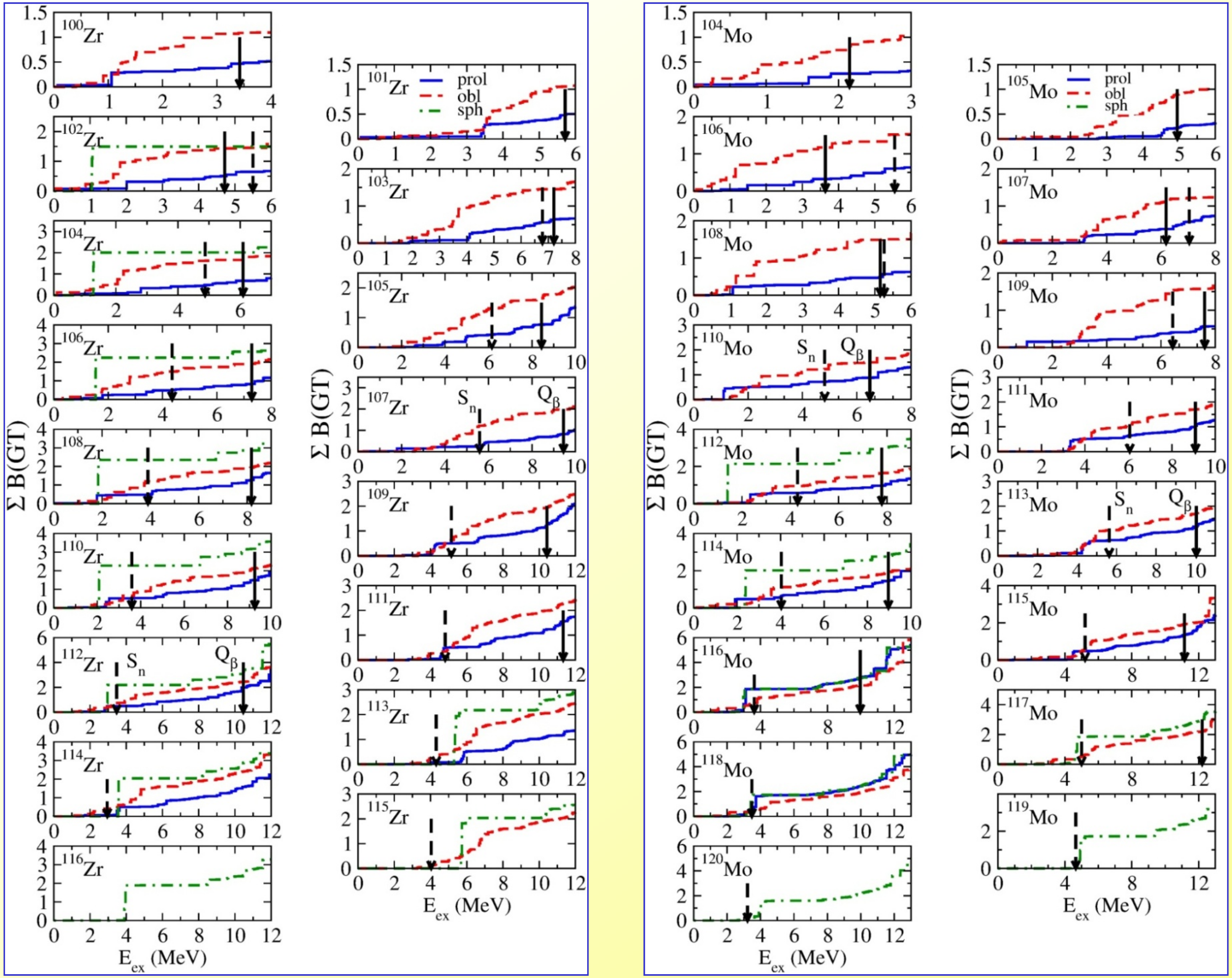


Zr-Mo B(GT)



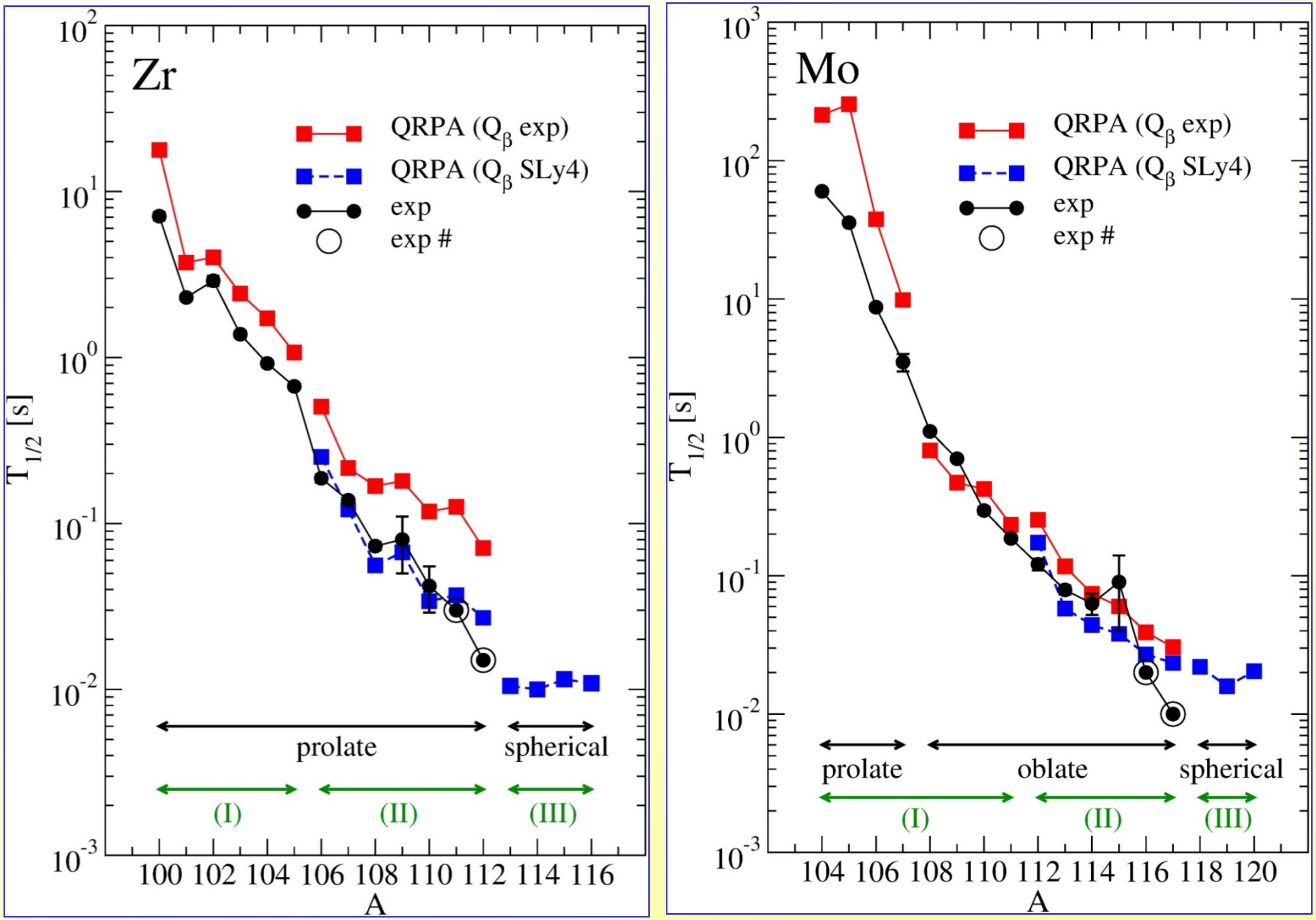
100Zr prol
102Zr prol
104Zr prol
100Zr obl
102Zr obl
104Zr obl
106Zr prol
108Zr prol
110Zr prol
S n
Q β
106Zr obl
108Zr
110Zr
obl
sph
104Mo prol
106Mo prol
108Mo prol
104Mo obl
106Mo obl
108Mo obl
110Mo prol
112Mo prol
114Mo prol
110Mo obl
112Mo
114Mo
B(GT)
E ex [MeV]

Zr-Mo B(GT)

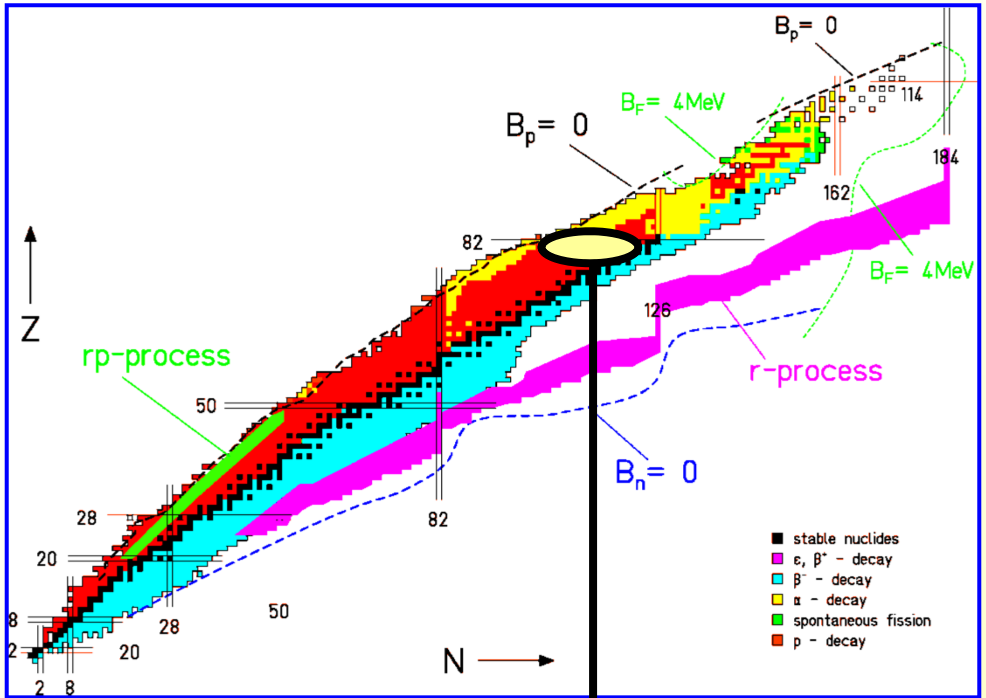


Σ B(GT)
E_ex (MeV)
prol
obl
sph
S_n
Q_β
100Zr
101Zr
102Zr
103Zr
104Zr
105Zr
106Zr
107Zr
108Zr
109Zr
110Zr
111Zr
112Zr
113Zr
114Zr
115Zr
116Zr
104Mo
105Mo
106Mo
107Mo
108Mo
109Mo
110Mo
111Mo
112Mo
113Mo
114Mo
115Mo
116Mo
117Mo
118Mo
119Mo
120Mo

Zr-Mo Half-lives

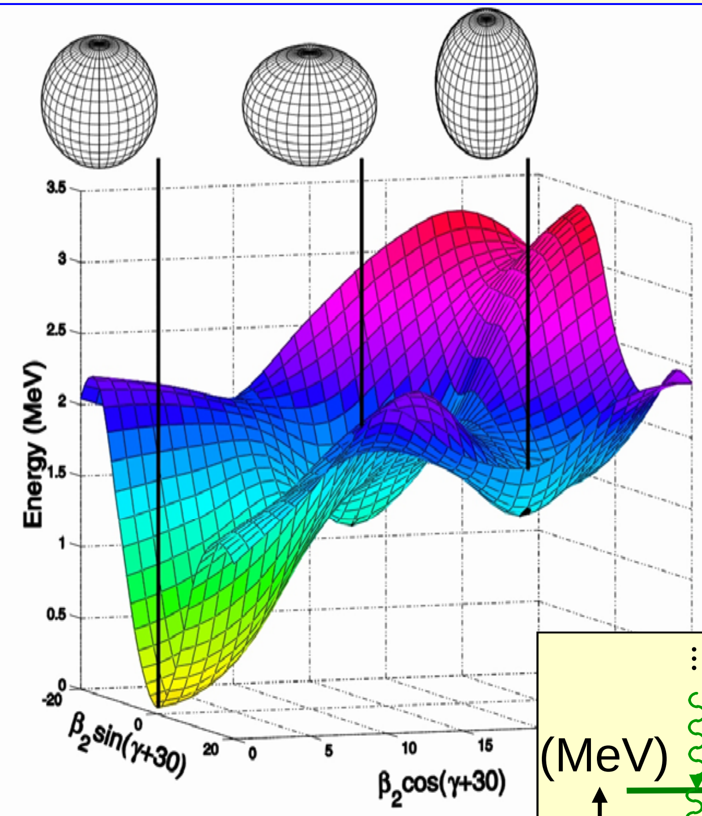


Shape dependence of GTdistributions in neutron-deficient Hg, Pb, Po isotopes

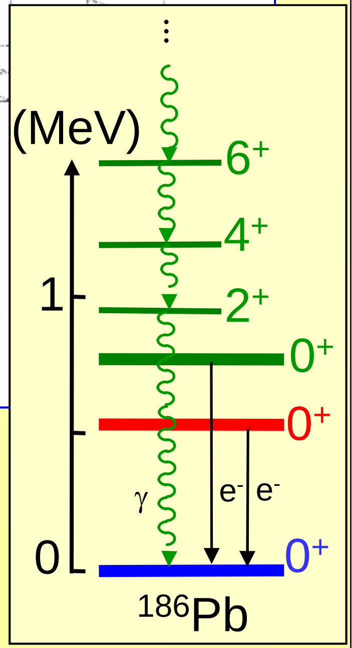


- Triple shape coexistence at low excitation energy

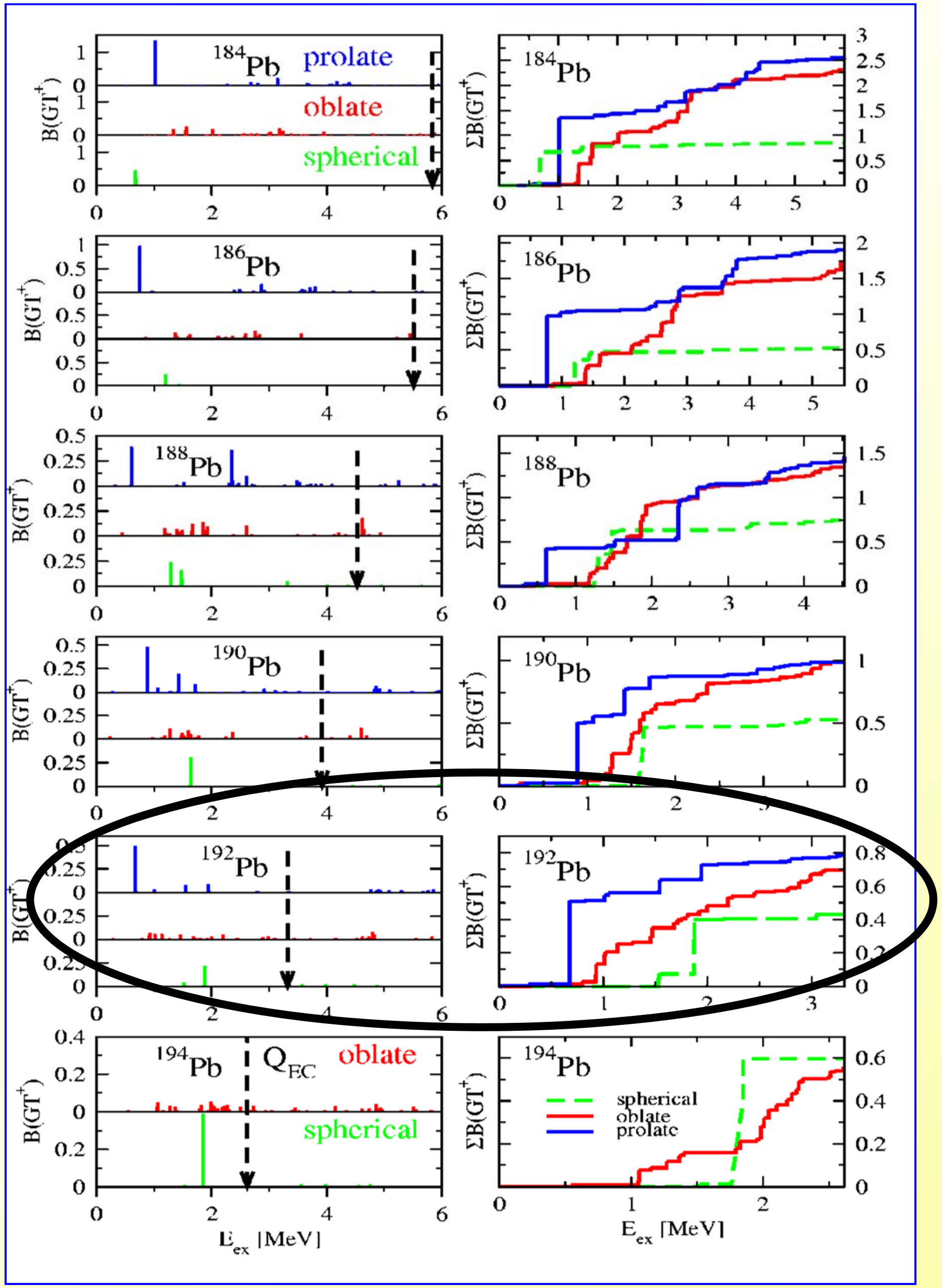
- Search for signatures of deformation on their beta-decay patterns



Potential Energy Surface



Shape dependence of GTdistributions in neutron-deficient: Pb isotopes

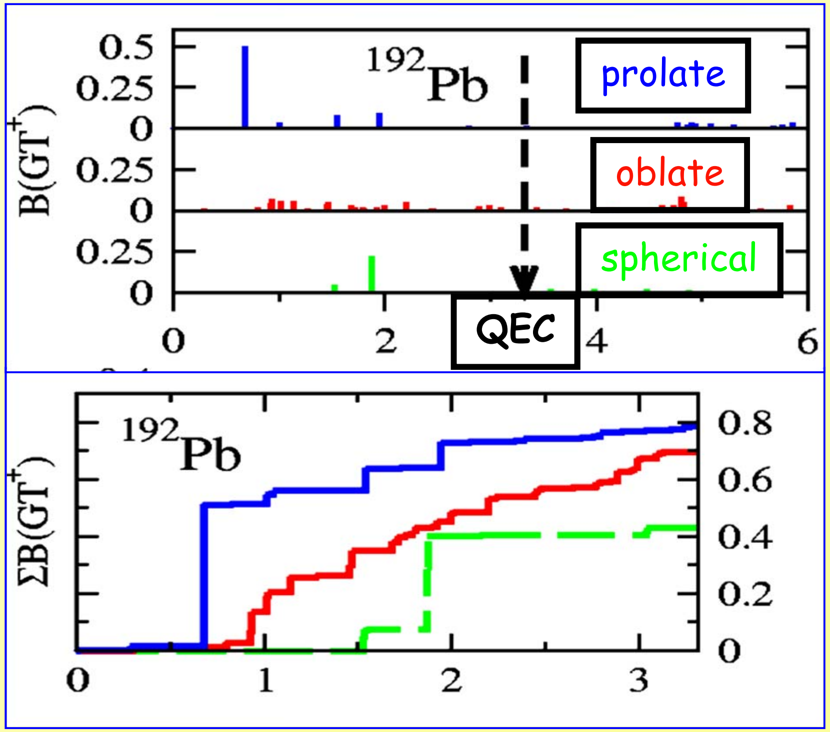


B(GT) strength distributions

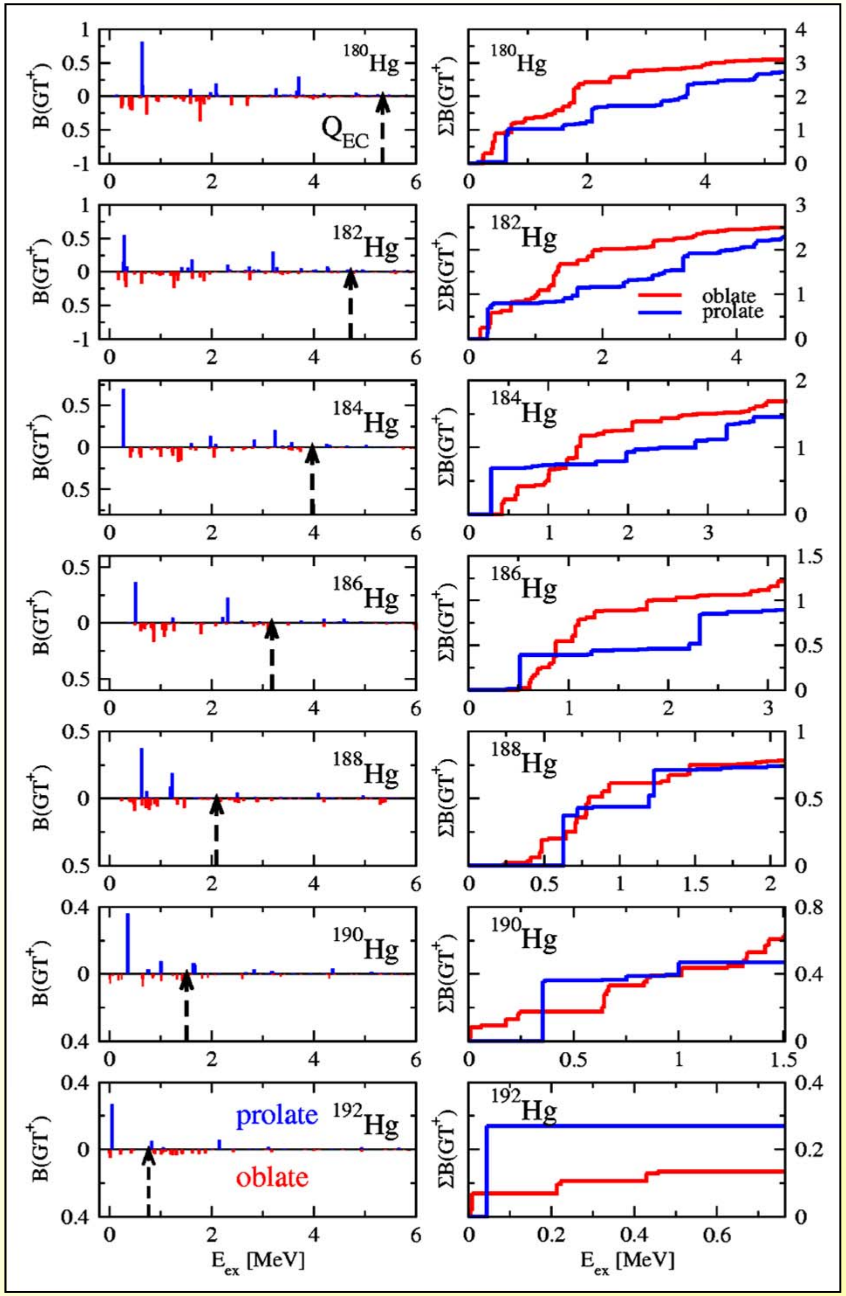
- Not very sensitive to : Skyrme force and pairing treatment
- Sensitive to : Nuclear shape

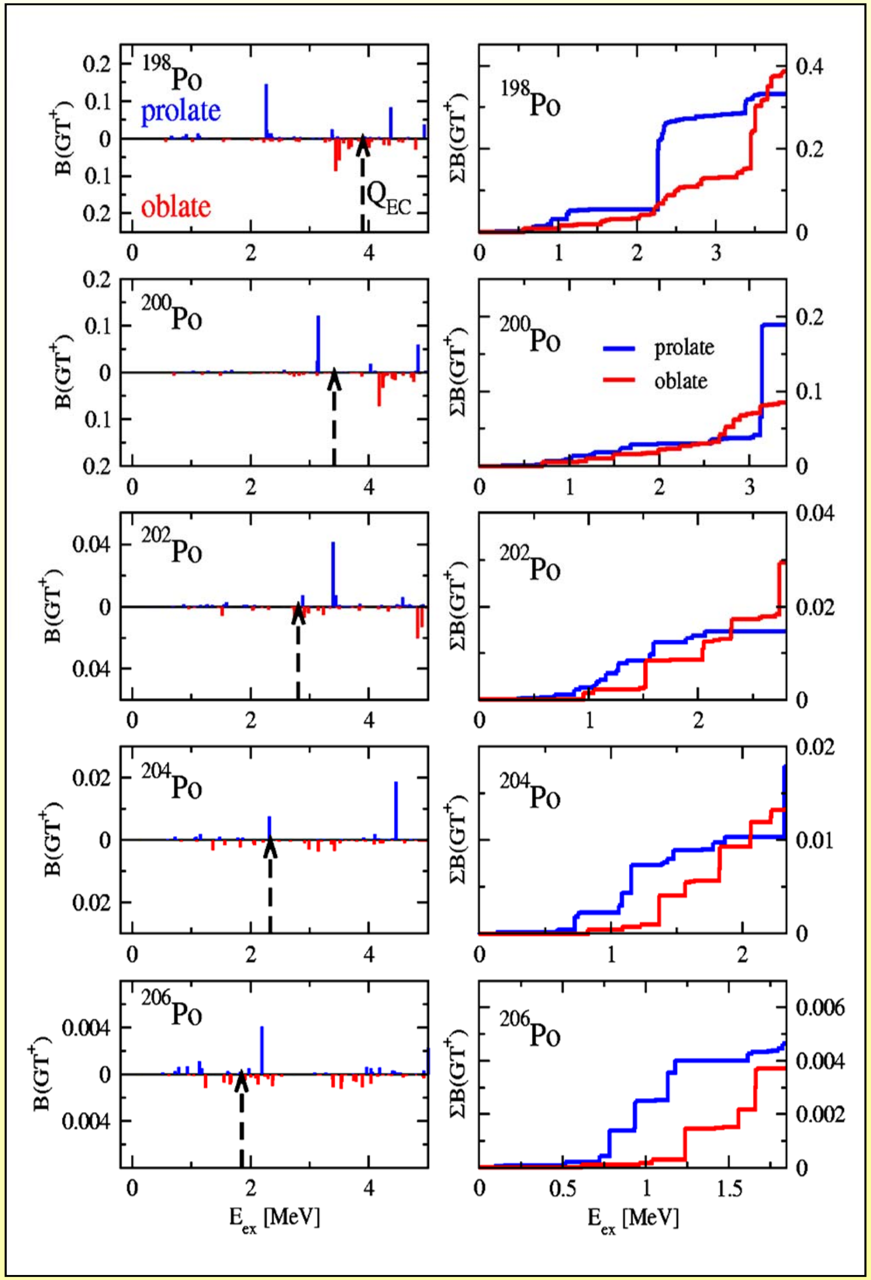
Signatures of deformation

PRC 72, 054317 (2005), PRC 73, 054302 (2006)



Shape dependence of GTdistributions in neutron-deficient Hg, Po isotopes





Conclusions

Nuclear structure model (deformed Skyrme HF+BCS+QRPA)

- Nuclear structure in different mass regions, astrophysical applications.
- Reproduce half-lives and main features of GT strength distributions extracted from beta-decay and/or charge exchange reactions.
- Weak-decay rates at (ρ,T) typical of astrophysical scenarios:
  - pf-shell nuclei (presupernova): EC rates from QRPA comparable quality to benchmark SM calculations.
  - Neutron-deficient WP nuclei (rp-process): EC/β^+ compete
  - Neutron-rich Zr-Mo isotopes (r process).