A description of neutron rich nuclei within the Deformed QRPA

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   - Physical Parameters

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Neutrino Oscillation and SN-Nucleosynthesis

MSW Matter Effect:
Through high-density resonance at $\rho \sim 10^3$ g/cm$^3$

R-process: Heavy Nuclei

$\nu p$-process:
$^{92}\text{Mo}, ^{96}\text{Ru}$?

Explo. Si-burn.: Fe-Co-Ni, $^{60}\text{Co}, ^{55}\text{Mn}, ^{51}\text{V}$ ...

$v$-process
$^{92}\text{Nb}, ^{98}\text{Tc}, ^{180}\text{Ta}, ^{138}\text{La}$ ...

$v$-process
$^6,^7\text{Li}, ^9\text{Be}, ^{10,11}\text{B}$ ...

Taka Kajino

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The r-process abundances

- Two different mass models, FRDM (finite-range droplet mass) and ETFSI (extended Thomas Fermi Strutinsky integral), underestimate the abundances by an order of magnitude or more at $A \approx 110$ region.
- The main effect of the newly measured $\beta$-decay half-lives is an enhancement in the calculated abundance of isotope with $A = 110$ to $120$, relative to abundances calculated using $\beta$-decay half-lives estimated with the FRDM+QRPA. [N. Nishimura et al., PRC. 85, 048801(2012)]
Motivations

- β-decay half-lives for Kr to Tc isotopes

FRDM+QRPA calculation underpredicts the $T_{1/2}$ of the $N=65$ isotones for Rb, Sr, Y, Zr, and Nb. [P. Möller et al., At. Data Nucl. Data Tables 66, 131(1997)]

- The KTUY+GT2 model overestimates the $T_{1/2}$ for Mo and Tc below $N=70$. 

Giuseppe’s talk !!!

Deformation effects ???
Recipe to reproduce solar r-elements


v-Driven Wind Weak R-Process

SUM = 79% (v-SN weak-r)
+16% (MHD Jet) + 5% (NSM)

Abundance $Y_i$

Mass Number $A$

Neutron Star Merger

Magneto-Hydrodynam.
Jet Supernova
Deformation (effects) Nuclei ???


SUPERNOVA NEUTRINO NUCLEOSYNTHESIS OF THE RADIOACTIVE $^{92}$Nb OBSERVED IN PRIMITIVE METEORITES

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Figure 1.2. Various shapes observed or expected in nuclei. Exotic orbitals that appear in regions far from the stability line may provide some new types of deformation. The superdeformation (top) and pear shape (bottom) have been observed experimentally; the oblate superdeformation has been predicted but not observed—less deformed oblate shapes are, however, quite common. The hyperdeformation (second from the top) has been seen in certain nuclei. The octupole banana-type deformation has not been observed in such extreme form, but vibrations of this kind are well known.
Total Hamiltonian of a many body system

In deformed basis Hamiltonian can be written as

\[
H = H_0 + H_{\text{int}},
\]

\[
H_0 = \sum_{\alpha \rho \alpha'} \epsilon_{\alpha \rho \alpha'} c_{\alpha \rho \alpha'}^\dagger c_{\alpha \rho \alpha'},
\]

\[
H_{\text{int}} = \sum_{\alpha \beta \gamma \delta \rho \beta' \gamma' \delta'} V_{\alpha \rho \alpha' \beta \rho \beta' \gamma \gamma' \delta \delta'} c_{\alpha \rho \alpha'}^\dagger c_{\beta \rho \beta' \delta \delta'}^\dagger c_{\gamma \gamma'},
\]

where, \( \alpha \): single particle state.

\( \rho_{\alpha}(\pm 1) \): sign of the angular momentum projection.

\( \Omega_{\alpha} \): projection of the total angular momentum on the nuclear symmetry axis.

\( -\Omega_{\alpha} \): time reversal state.
Single particle states (SPSs)

The SPSs are calculated from the eigen-equation of the total Hamiltonian in a deformed (Nilsson) basis obtained by the deformed axially symmetric Woods-Saxon potential.

\[
\begin{align*}
|\alpha\rho_\alpha = +1> &= \sum_{Nn_z}\left[b_{Nn_z\Omega_\alpha}^{(+)}|N, n_z, \Lambda_\alpha, \Omega_\alpha = \Lambda_\alpha + 1/2> \right. \\
&\left. + b_{Nn_z\Omega_\alpha}^{(-)}|N, n_z, \Lambda_\alpha + 1, \Omega_\alpha = \Lambda_\alpha + 1 - 1/2> \right],
\end{align*}
\]

N : main quantum number in deformed basis \\
n_z : the numbers of node the basis function in z direction \\
\Lambda : the projection of the orbital angular momentum onto the z axis

The time –reversed state is

\[
\begin{align*}
|\alpha\rho_\alpha = -1> &= \sum_{Nn_z}\left[b_{Nn_z\Omega_\alpha}^{(+)}|N, n_z, -\Lambda_\alpha, \Omega_\alpha = -\Lambda_\alpha - 1/2> \\
&\left. - b_{Nn_z\Omega_\alpha}^{(-)}|N, n_z, -\Lambda_\alpha - 1, \Omega_\alpha = -\Lambda_\alpha - 1 + 1/2> \right],
\end{align*}
\]
**Deformed BCS for the ground state**

Since the deformed s. p. states are expanded in terms of a spherical s. p. basis, the s. p. states with different orbital and total angular momenta in the spherical basis states would be mixed.
DQRPA eq with neutron-proton pairing correlations.
Realistic two body interaction was taken by Brueckner G-matrix, which is a solution of the Bethe-Goldstone Eq., derived from the Bonn-CD one-boson exchange potential.
**Expansion of the deformed state by a spherical basis**

To exploit **G-matrix elements** and **matrix elements of transition operators**, which are calculated on the spherical basis, the deformed basis is **expanded in terms of a spherical basis**.

\[
|a\Omega\rangle = \sum_{N_n z} b_{N_n z} \left( |N, n_z, \Lambda\rangle \right)
\]

**Sph. harmonic oscillator w. f.**

The expansion coefficient \( B \) is

\[
B_a^\alpha = \sum_{N_n z \Sigma} C_{\Lambda \Lambda_1}^{\Omega \alpha} A_{N n_z \Sigma}^{N_0 l} b_{N n_z \Sigma}.
\]

**C-G coef. of orbital & spin**  **Spatial overlap integral**  **Eigenvalue eq. of the total Hamiltonian**

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 Expansion coefficient B with different $\beta_2$ values

- Number of the spherical s. p. basis increases as the $\beta_2$ value increases.

http://arxiv.org/abs/1205.4561 v4
\textbf{Particle model space $N_{\text{max}}$ & pairing strength $g_{\text{pair}}$}

The particle model space $5\hbar\omega$ is not enough to reproduce the empirical pairing gap. Therefore, the particle model space can be used beyond $6\hbar\omega$ in $G$-matrix. In this calculation we use $N_{\text{max}} = 10\hbar\omega$ in $G$-matrix. ($5\hbar\omega$ in deformed basis)

\begin{equation}
\Delta_{\alpha p \bar{\alpha} p} = -\frac{1}{2} \sum_{J, c} g_{\text{pair}}^p F_{\alpha a \bar{\alpha} a}^J F_{\gamma c \gamma c}^J G(aacc, J)(u_{1pe}^* v_{1pe} + u_{2pe}^* v_{2pe}),
\end{equation}

FIG. 2: (Color online) Dependence of neutron pairing strength $g_{\text{pair}}^n$ in Eq. (11) on particle model space $N_{\text{max}}^{\text{sph}}$ in $G$-matrix. Black dashed, red dotted, and blue solid points are results for $\beta_2 = 0.1$, 0.2, and 0.3, respectively.
\[ A^{\alpha''\beta''\gamma''\delta''}(K) = (E_{\alpha\alpha''} + E_{\beta\beta''})\delta_{\alpha'\alpha''}\delta_{\beta'\beta''} - \sigma_{\alpha''\beta''}\sigma_{\gamma''\delta''} \]
\[ \times \left[ g_{pp}(u_{\alpha''}v_{\beta''}u_{\gamma''}v_{\delta''} + v_{\alpha''}u_{\beta''}v_{\gamma''}u_{\delta''}) V_{\alpha\beta, \gamma\delta}^K \right. \]
\[ + g_{ph}(u_{\alpha''}v_{\beta''}u_{\gamma''}v_{\delta''} + v_{\alpha''}u_{\beta''}v_{\gamma''}u_{\delta''}) V_{\alpha\beta, \gamma\delta}^K \]
\[ \left. + g_{ph}(u_{\alpha''}v_{\beta''}u_{\gamma''}v_{\delta''} + v_{\alpha''}u_{\beta''}v_{\gamma''}u_{\delta''}) V_{\alpha\beta, \gamma\delta}^K \right] \]

\[ B^{\alpha''\beta''\gamma''\delta''}(K) = -\sigma_{\alpha''\beta''}\sigma_{\gamma''\delta''} \]
\[ \times \left[ -g_{pp}(u_{\alpha''}v_{\beta''}u_{\gamma''}v_{\delta''} + v_{\alpha''}u_{\beta''}v_{\gamma''}u_{\delta''}) V_{\alpha\beta, \gamma\delta}^K \right. \]
\[ + g_{ph}(u_{\alpha''}v_{\beta''}u_{\gamma''}v_{\delta''} + v_{\alpha''}u_{\beta''}v_{\gamma''}u_{\delta''}) V_{\alpha\beta, \gamma\delta}^K \]
\[ \left. + g_{ph}(u_{\alpha''}v_{\beta''}u_{\gamma''}v_{\delta''} + v_{\alpha''}u_{\beta''}v_{\gamma''}u_{\delta''}) V_{\alpha\beta, \gamma\delta}^K \right] \]

\[ V_{\alpha\beta, \gamma\delta}^K = \sum_{J} \sum_{abcd} F_{\alpha\alpha''b}^{JK} F_{\gamma\delta c}^{JK} G(abcd, J) \]
\[ V_{\alpha\beta, \gamma\delta}^{K'} = \sum_{J} \sum_{abcd} F_{\alpha\alpha''d}^{JK'} F_{\gamma\delta b}^{JK'} G(adcb, J) \]
\[ V_{\alpha\gamma, \delta\beta}^{K} = \sum_{J} \sum_{abcd} F_{\alpha\alpha''e}^{JK} F_{\gamma\delta b}^{JK} G(acdb, J) \]

\[ g_{ph} \text{ is determined by adjusting the calculated positions of the GT giant resonances for } ^{48}\text{Ca}, ^{90}\text{Zr}, \text{ and } ^{208}\text{Pb}. \]
\[ g_{pp} \text{ is determined by a fitting procedure to } \beta\text{-decay half-lives of nuclei with } Z \leq 40. \]
Particle-hole strength $g_{\text{ph}}$

The energy of the GTGR is roughly reproduced.
- Particle-particle strength $g_{pp}$

All GT peaks get shifted to smaller energies as $g_{pp}$ increase.
In deformed bases [ case I ]

In spherical bases [ case II ]

\[
< K^+, m | \beta_K^- | \text{QRPA} > = \sum_{\alpha \rho \rho} < \alpha p \rho | \tau^+ \sigma_K | \beta_n \rho \beta > [u_{\alpha \rho} v_{\beta_n} X_{(\alpha \beta_n)K}^m + u_{\alpha \rho} u_{\beta_n} Y_{(\alpha \beta_n)K}^m],
\]

\[
< \alpha p \rho | \tau^+ \sigma_K = 0 | \beta_n \rho \beta > = \delta_{\Omega_p \Omega_n} \rho_{\alpha} \sum_{N n_z} [b_{N n_z \Omega_p}^{(+)} b_{N n_z \Omega_n}^{(+)} - b_{N n_z \Omega_p}^{(-)} b_{N n_z \Omega_n}^{(-)}],
\]

\[
< \alpha p \rho \alpha | \tau^+ \sigma_K = 1 | \beta_n \rho \beta > = - \sqrt{2} \delta_{\Omega_p \Omega_n + 1} \sum_{N n_z} b_{N n_z \Omega_p}^{(+)} b_{N n_z \Omega_n}^{(-)} (\rho_\alpha = \rho_\beta = +1)
\]

\[
= + \sqrt{2} \delta_{\Omega_p \Omega_n + 1} \sum_{N n_z} b_{N n_z \Omega_p}^{(-)} b_{N n_z \Omega_n}^{(+)} (\rho_\alpha = \rho_\beta = -1)
\]

\[
= - \sqrt{2} \delta_{\Omega_p \Omega_n + 1} \delta_{\Omega_n \Omega_p} \frac{1}{2} \sum_{N n_z} b_{N n_z \Omega_p}^{(+)} b_{N n_z \Omega_n}^{(+)} (\rho_\alpha = +1, \rho_\beta = -1),
\]

\[
< \alpha p \rho \alpha | \tau^+ \sigma_K = -1 | \beta_n \rho \beta > = - \sqrt{2} \delta_{\Omega_p \Omega_n - 1} \sum_{N n_z} b_{N n_z \Omega_p}^{(-)} b_{N n_z \Omega_n}^{(+)} (\rho_\alpha = \rho_\beta = +1)
\]

\[
= + \sqrt{2} \delta_{\Omega_p \Omega_n - 1} \sum_{N n_z} b_{N n_z \Omega_p}^{(+)} b_{N n_z \Omega_n}^{(-)} (\rho_\alpha = \rho_\beta = -1)
\]

\[
= + \sqrt{2} \delta_{\Omega_p \Omega_n - 1} \delta_{\Omega_n \Omega_p} \frac{1}{2} \sum_{N n_z} b_{N n_z \Omega_p}^{(+)} b_{N n_z \Omega_n}^{(+)} (\rho_\alpha = +1, \rho_\beta = -1).
\]

\[
< \alpha p \rho \alpha | \tau^+ \sigma_K | \beta_n \rho \beta > = \sum_{ab} F_{\alpha_p \beta_n}^{1K} \frac{< a_p | \tau^+ \sigma_K | b_n >}{\sqrt{3}},
\]

\[
< a_p | \tau^+ \sigma_K | b_n > = \sqrt{6} \delta_{n a n b} \delta_{l_a l_b} \sqrt{2 j_a + 1} \sqrt{2 j_b + 1} (-1)^{l_a + j_a + \frac{3}{2}} \left\{ \frac{1}{2}, \frac{1}{2}, 1 \right\} \left\{ j_b, j_a, l_a \right\}.
\]
Test of the expansion method for the Gamow-Teller strength

\[ B(GT^-) \]

\[ B(GT^-) \]

\[ E_{ex} \, [MeV] \]

\[ \beta_2 = 0.3 \, \text{deformed} \]

\[ \beta_2 = 0.3 \, \text{expanded} \]

FIG. 5: (Color online) GT strength distributions for \(^{82}\text{Se}\) at \( \beta_2 = 0.3 \). They are calculated in the deformed basis by Eq. (30) \( \sim \) (32) (a) and in the spherical basis by Eq. (33) (b).

Test of the DQRPA by the IKEDA sum rule w and w/o the closure relation

\[ (S^-_{GT} - S^+_{GT})_{ISR \, II} = \sum_{K=0,\pm 1} \sum_m \left[ |<K^+,m|\hat{\beta}_K|QRPA>|^2 - |<K^+,m|\hat{\beta}_K^+|QRPA>|^2 \right] \]

\[ = \sum_{K=0,\pm 1} \sum_{\alpha\beta} \sum_m \sum_{\alpha\beta} |<\alpha\rho_\alpha|\tau^+\sigma_K|\beta_\alpha \beta_\beta >|^2 (v_{\alpha\rho_\alpha}^2 v_{\beta_\alpha}^2 - v_{\alpha\rho_\alpha}^2 v_{\beta_\beta}^2) \left[ (X_{(\alpha\beta)K})^2 - (Y_{(\alpha\beta)K})^2 \right] \]

\[ (S^-_{GT} - S^+_{GT})_{ISR \, I} = \sum K=0,\pm 1 \sum_{\alpha\rho_\alpha} \sum_{\beta_\beta} |<\alpha\rho_\alpha|\tau^+\sigma_K|\beta_\alpha \beta_\beta >|^2 (v_{\alpha\rho_\alpha}^2 v_{\beta_\alpha}^2 - v_{\alpha\rho_\alpha}^2 v_{\beta_\beta}^2) \left[ (X_{(\alpha\beta)K})^2 - (Y_{(\alpha\beta)K})^2 \right] \]
Results of the Gamow–Teller strength distributions

http://arxiv.org/abs/1205.4561 v4

GT(-) strength for $^{76}\text{Ge}$ with different $\beta_2$ value

$\beta_2 = 0.157$ by RMF

$0.262$ from $B(E2)$
Running sum of GT(-) strength for $^{76}\text{Ge}$

$\Sigma B(\text{GT}^-)$

$^{76}\text{Ge}(p, n)^{76}\text{As}$

- Thies et al.
- Madey et al.
- Sarriguren

3(N-Z)

Results by DQRPA reproduce well experimental data.

$\text{ISR}_{\text{exp}} = 55\%$ (up to 12MeV)

There may be a possibility of the high-lying GT state above 12.0 MeV.

$\text{ISR}_{\text{DQRPA}} \approx 98\%$
GT(+) strength for $^{76}\text{Se}$ with different $\beta_2$ value

- $\beta_2 = 0.1$
- $\beta_2 = 0.2$
- $\beta_2 = -0.1$
- $\beta_2 = -0.2$
- $\beta_2 = 0.3$
- $\beta_2 = -0.3$

$\beta_2 = -0.244$ by RMF, 0.309 from B(E2)
Running sum of GT(+) strength for $^{76}$Se
GT(−,+)/ strength for $^{90}$Zr with different $\beta_2$ value

- $^{90}$Zr(p,n)$^{90}$Nb
  - $B(GT^+ + IVSM)$(30~38 MeV)
  - $E_{ex}$ [MeV]

- $^{90}$Zr(n,p)$^{90}$Y
  - $B(GT^+ + IVSM)$(17~25 MeV)
  - $E_{ex}$ [MeV]

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$\beta_2 = 0.001$ by RMF
$0.089$ from $B(E2)$

**Is DQRPA proper at $\beta_2=0$?***
Does DQRPA go back to QRPA at $\beta_2=0$??

If we take the limit, the deformed basis goes back to the spherical basis. But, for $\beta_2 \neq 0$, the deformed basis has many components because the angular momentum projection $\Omega_j$ may have angular momenta higher $j$.

$\Omega_j$ can be composed of different $j$ values in the $\beta_2 = 0.3$.

$|Nn_z\Lambda : 000 > = |0s\frac{1}{2} > ; \beta_2 = 0,\,
|Nn_z\Lambda : 000 > = 0.98|0s\frac{1}{2} > +0.0005|1s\frac{1}{2} > +0.094|0d\frac{5}{2} > +0.0116|0d\frac{3}{2} > +\cdots ; \beta_2 = 0.3.$

Single particle states by eigenequation of total Hamiltonian are linear combination of the deformed basis even if we take the $\beta_2 = 0$ limit.

$|\frac{1}{2} > = 0.93|0s\frac{1}{2} > -0.36|1s\frac{1}{2} > +0.04|2s\frac{1}{2} > +0.01|3s\frac{1}{2} > +\cdots ; \beta_2 = 0,\,
|\frac{1}{2} > = 0.92|0s\frac{1}{2} > -0.36|1s\frac{1}{2} > -0.03|0d\frac{3}{2} > +0.04|0d\frac{5}{2} > +\cdots ; \beta_2 = 0.1.$

One may notice $|\Omega_j = 1/2 >$ state has other components $|ns1/2 >$ although main component is $|0s1/2 >$.

Therefore, the extension to $\beta_2=0$ values may not be exact, but approximate treatment.
Fig. 1. GT(−) strength distributions for $^{90}$Zr with $np$ pairing (right) and without $np$ pairing (left) with respect to the parent nucleus $^{90}$Zr. The experimental Q value between $^{90}$Zr and $^{90}$Nb is 6.111 MeV. Experimental data are from Refs. 20 and 21.

Fig. 2. GT(−) strength distributions for $^{92}$Zr with $np$ pairing (right) and without $np$ pairing (left) and with respect to the parent nuclei $^{92}$Zr. The experimental Q value between $^{92}$Zr and $^{92}$Nb is 2.005 MeV.
Results of the nuclear beta decays for Zr and Mo isotopes

Table II. \( Q_{\beta}^{exp}(Q_{\beta}^{FRDM}) \) used in this work with \( \beta_2 \) values.

<table>
<thead>
<tr>
<th>nuclide</th>
<th>( Q_{\beta}^{exp}(Q_{\beta}^{FRDM}) ) [MeV]</th>
<th>( \beta_2^{exp}(\beta_2^{RDF}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{102})Zr</td>
<td>4.61</td>
<td>0.427</td>
</tr>
<tr>
<td>(^{104})Zr</td>
<td>5.9</td>
<td>0.371</td>
</tr>
<tr>
<td>(^{106})Zr</td>
<td>7.2</td>
<td>0.375</td>
</tr>
<tr>
<td>(^{108})Zr</td>
<td>8.6</td>
<td>0.381</td>
</tr>
<tr>
<td>(^{110})Zr</td>
<td>9.3</td>
<td>0.401</td>
</tr>
<tr>
<td>(^{112})Zr</td>
<td>(10.77)</td>
<td>0.421</td>
</tr>
<tr>
<td>(^{104})Mo</td>
<td>2.16</td>
<td>0.362</td>
</tr>
<tr>
<td>(^{106})Mo</td>
<td>3.52</td>
<td>0.354</td>
</tr>
<tr>
<td>(^{108})Mo</td>
<td>4.65</td>
<td>(-0.27)</td>
</tr>
<tr>
<td>(^{110})Mo</td>
<td>5.5</td>
<td>(-0.278)</td>
</tr>
<tr>
<td>(^{112})Mo</td>
<td>7.2</td>
<td>(-0.264)</td>
</tr>
<tr>
<td>(^{114})Mo</td>
<td>8.4</td>
<td>(-0.229)</td>
</tr>
</tbody>
</table>
Beta decay in the neutron rich side

If $S_n < Q_\beta$ and the decay proceeds to states above $S_n$, neutron emission competes and can dominate over $\gamma$-ray de-excitation.

The process will dominate far from stability on the n-rich side. To have a full picture of the strength ...
The angular momentum ($L$) of the systems $(e + nu)$ can be non-zero (in the center-of-mass frame of the system).

In reality, isospin is violated by the electromagnetic force, but the violation is weak.

$$\langle J_f, M_f, T_f, T_{0_f} | T^\pi | J_i, M_i, T_i, T_{0_i} \rangle = \sqrt{T_i(T_i + 1) - T_{0_i}(T_{0_i} + 1)} \delta_{J_f, J_i} \delta_{M_f, M_i} \delta_{T_f T_i} \delta_{T_{0_f} + T_{0_i}}$$

$T^\pi$ has rank unity!

$$\Delta J = L \text{ or } L \pm 1, \quad \Delta \pi = (-1)^L$$

<table>
<thead>
<tr>
<th>Decay type</th>
<th>$\Delta J$</th>
<th>$\Delta T$</th>
<th>$\Delta \pi$</th>
<th>$\log_{10} f_{t_{1/2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superallowed</td>
<td>$0^+ \rightarrow 0^+$</td>
<td>0</td>
<td>no</td>
<td>3.1–3.6</td>
</tr>
<tr>
<td>Allowed</td>
<td>0, 1</td>
<td>0, 1</td>
<td>no</td>
<td>2.9–19</td>
</tr>
<tr>
<td>First forbidden</td>
<td>0, 1, 2</td>
<td>0, 1</td>
<td>yes</td>
<td>5–19</td>
</tr>
<tr>
<td>Second forbidden</td>
<td>1, 2, 3</td>
<td>0, 1</td>
<td>no</td>
<td>10–18</td>
</tr>
<tr>
<td>Third forbidden</td>
<td>2, 3, 4</td>
<td>0, 1</td>
<td>yes</td>
<td>17–22</td>
</tr>
<tr>
<td>Fourth forbidden</td>
<td>3, 4, 5</td>
<td>0, 1</td>
<td>no</td>
<td>22–24</td>
</tr>
</tbody>
</table>

The absolute values of GT matrix elements are generally smaller than those for Fermi transitions.

$$fT = \frac{\text{const}}{(F)^2 + g_A^2 (GT)^2}$$

squared matrix elements
GT(-) strength for $^{106\sim114}\text{Mo}$
β-decay half-life of $^{104-114}\text{Mo}$ with QRPA
Summary

0. R- and nu- processes in a view point of nuclear models, QRPA.

1. We used the deformed WS potential and then performed the deformed BCS and deformed QRPA with a realistic two-body interaction calculated by Brueckner G-matrix based on Bonn potential.

2. Results of the Gamow-Teller strength, B(GT±), for ⁷⁶Ge, ⁷⁶,⁸²Se, and ⁹²Zr show that the deformation effect leads to a fragmentation of the GT strength into high-lying GT excited states.

3. Our results show that the running sum of the GT strength distributions for ⁷⁶Ge and ⁷⁶,⁸²Se reproduce well experimental data.

4. Preliminary results of the beta decay of neutron rich nuclei show the importance of the deformation.

5. We are preparing to calculate data relevant to heavy nucleus, A>110, by using a super computer.

6. Future plan 1: apply to the M1 and E2 transitions of even-odd or odd-odd nuclei, and consider the continuum states in DQRPA.

7. Future plan 2: New Mass Model
Thanks for your attention !!