36th INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS Nuclei in the Laboratory and in the Cosmos Erice, Italy, September 16 -24, 2014

He-4 from BBN and Cosmic Neutrino

Daniela Kirilova

IA, Bulgarian Academy of Sciences, Sofia, Bulgaria

BBN and Neutrino

Big Bang Nucleosynthesis milestone of Big Bang cosmology

Theoretically well established

Precise data on nuclear processes rates from lab expts at low E (10 KeV – MeV) Observational data on D, He, Li

Baryon fraction measured by CMB

BBN precision probe for early Universe and for new physics at BBN energies.

BBN is sensitive baryometer, speedometer and leptometer.

BBN probes any non-standard physics at BBN epoch.

He-4 - most sensitive radiometer and leptometer at RD stage BBN and CNB formation are nearby epochs, hence we expect considerable influence on primordial He-4 and neutrino

Neutrino experimental data firmly established physics BSMs

Neutrino oscillations expt challenged SMs assumptions:

m=0, N_{eff}=3, L=0, equilibrium FD distribution

Effects of BSM neutrino characteristics on He-4 and

cosmological constraints on them

Outline

He-4 in SBBN Primordial He-4 as a test for BSM neutrino effective number of relativistic particles neutrino oscillations in presence of L in presence of v_s L asymmetry

Le4 in standard BBN



BBN predicts the abundances of D, He-3, He-4, Li-7 produced during the early hot stage, 1 s - 20 m1 - 0.1 MeV, of the Universe evolution.

 $H_0, \Omega_B, \Omega_V, N_{eff}, L, etc$

The primordially produced abundances depend on:

- baryon-to-photon ratio n \checkmark
- relativistic energy density \checkmark
- n lifetime (880.1+-1.1 s) \checkmark

neutrino characteristics



1. Weak interactions freeze out

- 2. D forms
- 3. Nuclear chain (>400 reactions)

Observational data verse calculated abundances of primordially produced elements.



Observational data (2σ error). Vertical band give η_{CMB} Main problem: Primordial abundances are not observed directly (chemical evolution after BBN).

Observations in systems least contaminated by stellar evolution Account for galactic chemical evolution

(424 reactions code) Coc et al, 2014 run BBN code (PArthENoPE) to get Y_p, X_D Li-7, Li-6, Be, B, CNO

D abundance

D measurements towards QSA in high redshift z, low metalicity Z (0.1-0.001 of the Solar Z) H-rich clouds absorbing light from background QSA.

- Improved precision during last years Cooke et al, 2014
- ✤ Among light elements D is the best baryometer.

 $\eta = (5.7 \pm 0.3) \times 10^{-10}$ $\Omega_b h^2 = 0.021 \pm 0.001$

$$\Omega_b h^2 = 3.65 \times 10^7 \eta, \quad \Omega_b = \frac{\rho_b}{\rho_c}, \quad \rho_c = \frac{3H^2}{8\pi G_N}$$

D/H=(2.53±0.04) 10⁻⁵



✤ Baryon density is measured with very high precision by BBN.

5.7 x 10⁻¹⁰ < η_{BBN} < 6.7 x 10⁻¹⁰ 95% CL 0.021 < $\Omega_{h}h^{2}$ < 0.025

CMB anisotropy measurements at ~380 000 y are consistent with BBN D measurements

 $\eta_{\text{CMB}} = 6.047 \times 10^{-10} \pm 0.074 \ 95\% \ \text{CL}$ $\Omega_b h^2 = 0.02207 \pm 0.00027$

Baryon density is ~ 0.05 of the total density \rightarrow much bigger than the luminous matter (0.005) \rightarrow considerably less than the gravitating matter (0.3) \rightarrow Hubble ST + CMB + clusters



not enough to close the Universe. most of the baryons are optically dark. nonbaryonic DM needed

The Abundance of Primordial He-4

- *Observations* in systems least contaminated by stellar evolution: in clouds of ionized H (H II regions), the most metal-poor blue compact galaxies
- * Account for galactic chemical evolution extrapolated towards zero metalicity
- Small statystical error but large systematics (interstellar reddening, clouds T, e density)



 $Y_p = 0,2421 \pm 0,0021$ $Y_p = 0,2429 \pm 0,009$ $Y_p = 0,245 \pm 0,013$ $Y_p = 0,252 \pm 0,001$

Izotov, Thuan 2000 Izotov, Thuan 2004 Olive, Skillman 2004 Izotov et al 2007

$Y_p = 0,2565 \pm 0,001 \pm 0,005 (system)$	yst) Izotov, Thuan 2010
•	93 Sp of 86 low Z HII
Y _p =0,2561±0,0108	Aver et al. 2010
Y _p =0,2465±0,0097	Aver et al. 2013
$Y_{p}=0,254\pm0,00016\pm0,003$	Izotov et al 2013 (111 HII)

 Y_p =0,266± 0,021 from Planck (consistent with HII determination)

He-4 most abundant (25%), precisely measured (3%), simple post-BBN evolution. Good agreement b/n observational determinations and BBN predictions \rightarrow prefered element for cosmological tests and constraints on BSM physics

He production

 $\begin{array}{c} v_e + n \leftrightarrow p + e^- \\ e^+ + n \leftrightarrow p + \widetilde{v}_e \end{array} \quad \begin{array}{c} n \\ p \end{array} \sim e^{-\frac{\Delta m}{T}} \quad \Delta m = 1.293 MeV \end{array}$ T > 1 MeV $n \rightarrow p + e^- + \widetilde{v}$ p T < 1 MeV $\Gamma \sim G_F^2 T^5$ $H \sim \sqrt{g_{eff} G} T^2$ $g_{eff} = \frac{11}{2} + \frac{7}{4} N_v = 10,75$ $T_f \sim \left(\frac{g_{eff}G}{G_{r}}\right)^{1/6} \sim 0,7 MeV \qquad \left(\frac{n}{p}\right)_f \sim e^{-\frac{\Delta m}{T_f}} \sim \frac{1}{6}$ • T < 80 KeV $p + n \rightarrow D + \gamma \qquad D + D \rightarrow^4 He + \gamma$ $(X_n)_f = \left(\frac{N_n}{N_{nuc}}\right)_f = \frac{\left(\frac{n}{p}\right)_f}{1 + \left(\frac{n}{p}\right)} \qquad Y_p = 2(X_n)_f e^{-\frac{t}{\tau_n}} \sim 0.24$ $\tau_{n} = 880s$ δY_{кн}~0.013 δN_{eff}

Primordial He-4 Y_p , predicted by BBN, is calculated with great precision. Assuming CMB measured baryon density: : 0.2461-0.2466 (424 reactions code) Cok et al, 2014

He4 and BSN Neutrino

⁴He – powerful test for BSM neutrino physics

♦ BBN produced He-4 – the most precise radiometer: provides stringent bounds on excess radiation due to its dependence on $H \sim \sqrt{g_{eff}GT^2}$:

$$\delta \mathbf{Y}_{\mathsf{KH}} \sim 0.013 \ \delta \mathbf{N}_{\mathsf{eff}} \qquad \rho_{\mathrm{r}} = \rho_{\gamma} + \rho_{\nu} + \rho_{x} = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \mathbf{N}_{\mathsf{eff}} \right] \rho_{\gamma}$$

✤ BBN produced He-4 is sensitive to neutrino characteristics (n, N, sp, L..) $\Gamma \sim G_F^2 E_v^2 N_v$

- Number of generations (types of relativistic neutrino)
- Neutrino Oscillations
- Lepton Asymmetry
- Neutrino decays

He-4 speedometer

BBN constrains the effective number of relativistic species

 $\rho_{\rm r} = \rho_{\gamma} + \rho_{\nu} + \rho_x = \left[1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right] \rho_{\gamma}$

δΥ~0.013 δΝ_{eff}

Non-zero ΔN_{eff} will indicate any extra relativistic component: like sterile neutrino, neutrino oscillations, lepton asymmetry, neutrino decays, nonstandard thermal history, etc

Planck Collaboration $N_{eff} = 3.36^{+0.34}_{-0.32}$

Main Oscillations Effects on He-4 $v_a \leftrightarrow v_s$ **Dynamical effect:** Production of additional neutrino species enhances the energy density increase $H \sim \sqrt{g_{eff}GT^2} \rightarrow 4$ He overproduction Dolgov ,1981 1 additional $\nu \rightarrow \delta Y_p / Y_p = 5 \%$ $\Gamma_{osc} \sim \frac{\delta m^2}{E}$ Kinetic effect: v_e energy spectrum distortion due to v_e depletion, neutrino-antineutrino asymmetry change, deviation from FD distribution decrease $\Gamma \sim G_F^2 E_{\nu}^2 N_{\nu}$, n/p freezes earlier \rightarrow ⁴He overproduction *D.K.*, 1988; Barbieri, Dolgov, 1990; DK M. Chizhov, 1996

Fast $v_a \leftrightarrow v_s$ effective before v_a decoupling increase δN_s

Nonequilibrium $v_e \leftrightarrow v_s$ effective after v_a decoupling and $\delta N_s < 1$ cause distortion of v_e distribution neutrino-antineutrino asymmetry growth for $|\delta m^2|\sin^4 2\theta \le 10^{-9.5} eV^2$

He-4 produced in BBN with late $v_e \leftrightarrow v_s$

► Evolution of nucleons in the presence of late $v_e \leftrightarrow v_s$ $\delta m^2 \sin^4 2\theta \le 10^{-7}$

$$\begin{split} \frac{\partial n_p}{\partial t} &= Hp_n \frac{\partial n_n}{\partial p_n} + \int d\Omega(e^-, p, v) \Big| A(e^- p \to vn) \Big|^2 (n_{e^-} n_p - n_n \rho_{LL}) \\ &- \int d\Omega(e^+, p, \tilde{v}) \Big| A(e^+ n \to p \tilde{v}) \Big|^2 (n_{e^+} n_n - n_p \overline{\rho}_{LL}) \\ &\delta m^2 \leq 10^{-7} eV^2 \quad all \ mixing \ angles \ \theta \quad 0 \leq \delta N_s \leq 1 \\ &2 \ MeV \geq T \geq 0.3 \ MeV \qquad 10^{-10} < L < 0.01 \end{split}$$

 $\blacktriangleright \text{ Interplay b/n dynamical and kinetic effect of oscillations on He-4 was explored.} \\ \delta N = \delta N_{k,0} - \delta N_{k,0} \delta N_s + \delta N_s \qquad \delta Y_p \sim 0.013 \, \delta N \qquad D.K, IJMPD 2004$

Dependences of He-4 production on oscillation parameters, L and population of inert neutrino $Y_p(\delta m^2, \theta, L, \delta N_s)$ were numerically calculated.
D.K. Chizhov, NPB 98, 2000; D.K. ApPhys 2003; JCAP 2012

➤ Maximum He-4 overproduction was found.

He-4 from BBN with late $v_e \leftrightarrow v_s$



growth (solid line) and in case asymmetry growth is neglected



Maximum He-4 overproduction in BBN with oscillations



Dependence of maximum overproduction on the mixing

- \succ He-4 decreases at small mixing due to L growth.
- > BBN with nonequilibrium $v_e \leftrightarrow v_s$ constraints v oscillation for He-4 uncertainty up to 32%





Cosmological constraints on neutrino oscillations





Fits to BBN constraints

 $\delta m^2 > 10^{-6} eV^2$ nonres case Dolgov, Villante,03 $\delta m_{es}^2 \sin^4 2\theta_{es} \le 3.16 \times 10^{-5} eV^2 \ln^2(1 - \Delta N_v)$ $\delta m_{\mu s}^2 \sin^4 2\theta_{\mu s} \le 1.74 \times 10^{-5} eV^2 \ln^2(1 - \Delta N_{\nu})$ $\delta m^2 \sin^4 2\theta \leq 10^{-7}$ DK, Chizhov NPB 2001 $\delta m^2 \left(\sin^2 2\theta\right)^4 \le 1.5 \times 10^{-9} eV^2 \quad \delta m^2 > 0$ $\delta m^2 < 8.2 \times 10^{-10} eV^2$ large θ , $\delta m^2 < 0$ $\delta m^2 \rangle 0$ δm² $\log(\delta m^{2} [eV^{2}])$ -7.5 -8.0-8.5 -9.0-9.5-'o log(sin²2v)

 \checkmark BBN constraints are more stringent than experimental ones

 $\checkmark \quad \text{Excluded 2 of the possible solutions of the solar neutrino}$

problem : LMA and LOW active-sterile solutions (1990, 1999) years before experimental results.

✓ Exclude electron-sterile solution to LSND

BBN constraints on $v_e \leftrightarrow v_s$ and non-zero δN_s



DK&Panayotova JCAP2006; DK IJMPD 07

Kinetic effect dominates, hence He-4 overproduction decreases with δN_s increase.

Additional v_s population may strengthen or relax BBN constraints depending on the interplay b/n dynamical and kinetic effect on He-4.

He-4 constraints on oscillations parameters changed by L



- ✤ Relic L may relax He-4 constraints on neutrino mass differences at large mixing and strengthen them at small mixing. Large enough L $\delta m^2 (eV^2) < L^{3/2}$ may suppress oscillations. *Kirilova JCAP 2012*
- ✤ Generated neutrino-antineutrino asymmetry by resonant oscillations leads to relaxation of the He-4 constraints on neutrino mass differences at small mixings.

He-4 and L

$$L = (n_l - n_{\bar{l}}) / n_{\gamma} \qquad L \sim \sum_{i} L_{\nu_i} \qquad L = \sum_i \frac{1}{12\zeta(3)} \frac{T_{\nu_i}^3}{T_{\gamma}^3} (\xi_{\nu_i}^3 + \pi^2 \xi_{\nu_i}) \qquad \xi = \mu/T$$

- Non-zero L increases the radiation energy density $\Delta N_{eff} = \frac{15}{7} \left[\frac{(\mu/T)}{\pi} \right]^4 + 2 \left[\frac{(\mu/T)}{\pi} \right]^2$
- $|L_{ve}| > 0.01$ effect neutron-proton kinetics in pre-BBN epoch ٠ Simhael Steigman, 2008: $Y_p \sim (0.2482 \pm 0.0006) + 0.0016\eta_{10} + 0.013\Delta N_{eff} - 0.3\xi_v$
- Indirect kinetic $0.01 > L \ge 10^{-8}$ effects n/p kinetics and BBN through neutrino ۲ oscillations DK & Chizhov NPB98; DK JCAP, 2012.
 - Primordially produced He-4 provides the most stringent constraint on L *

> Accounting for flavor neutrino oscillations and v decoupling |L| < 0.1Neutrino degeneracies equilibrate due to oscillations before BBN $\sin^2 \theta_{13} > 0.03$ Dolgov et al., 2002; Serpico IRaffelt, 2005; Miele et al., 2011 *Castorini et al. 2012* -0.071<L<0.054 ; *Steigman, 2012* ξ =0.038 ±0.026 from He-4

Mangano et al., 2013 |L| < 0.2 big θ_{13}

Constraints on L from He-4 in case of BBN with electron-sterile oscillations tiny L may be constrained L~10⁻⁸ $L < (\delta m^2)^{2/3}$

DK, JCAP 2012;Hyperfine Int. 2013

He-4 in BBN with late neutrino oscillations and relic L>10⁻¹⁰

Kirilova JCAP 2012

 \checkmark

 $Y_{p}(\delta m^{2},\theta,L)$



- ✓ Small L change primordial production of He by enhancing or suppressing oscillations.
- ✓ L is able to strengthen, relax and eliminate He-4 cosmological constraints on v oscillations. Then approximate bound holds: $\delta m^2 (eV^2) < L^{3/2}$

 $L < (\delta m^2)^{2/3}$

Excess radiation: BBN with relic L – to suppress oscillations, so that new neutrinos are not thermalized. \mathcal{DK} , CEIP 2014



Primordial He-4 is one of the most precise probes of physics at the RD stage and the most reliable cosmological test of physics beyond SM.

He-4 is very sensitive to radiation density, neutrino spectrum distortion and L It is the most sensitive cosmological probe of the number of neutrino species, of distortions in the energy distribution of neutrinos, lepton asymmetry, neutrino mixing parameters, non-standard interactions, etc.

He-4 is overproduced in BBN with neutrino oscillations effective after neutrino decoupling. BBN constraints strengthen by orders of magnitude when distortion effect of oscillations is accounted for. BBN with nonequilibrium $v_e \leftrightarrow v_s$ constraints v oscillation parameters for He-uncertainty up to 32%(14%) in resonant (non-resonant) case. He constraints on neutrino oscillations depend nontrivially on the population of sterile neutrino and L.

Additional sterile state may lead to enhancement or reduction of the He overporoduction by oscillations, thus strengthen or relax cosmological constraints.

L is able to enhance, suppress or inhibit oscillations leading to changes of the cosmological constraints on oscillation parameters.

BBN constraints on L strengthen considerably in case of presence of $\nu_e \leftrightarrow \nu_s$ oscillations: He-4 produced in BBN with active-sterile oscillations may feel $L \ge 10^{-8}$.

Благодаря за вниманието! Shanks for the attention!



Solving numerically BBN dynamics

 Weak interactions freeze out at T ~1 MeV

2. Deuterium forms via
$$p n \rightarrow D \gamma$$
 at T ~ 0.1 MeV

3. Nuclear chain

$$\dot{X}_{i} = \sum_{j,k,l} N_{i} \left(\Gamma_{kl \to ij} \frac{X_{k}^{N_{k}} X_{l}^{N_{l}}}{N_{k}! N_{l}!} - \Gamma_{ij \to kl} \frac{X_{i}^{N_{i}} X_{j}^{N_{j}}}{N_{i}! N_{j}!} \right) \equiv \Gamma_{i} \quad ,$$

$$n_{B} \sum_{j} Z_{j} X_{j} = n_{e^{-}} - n_{e^{+}} \equiv L \left(\frac{m_{e}}{T}, \phi_{e} \right) \equiv T^{3} \hat{L} \left(\frac{m_{e}}{T}, \phi_{e} \right) \quad ,$$

$$\left(\frac{\partial}{\partial t} - H \left|\mathbf{p}\right| \frac{\partial}{\partial \left|\mathbf{p}\right|}\right) f_{\nu_{\alpha}}(\left|\mathbf{p}\right|, t) = I_{\nu_{\alpha}} \left[f_{\nu_{e}}, f_{\bar{\nu}_{e}}, f_{\nu_{x}}, f_{\bar{\nu}_{x}}, f_{e^{-}}, f_{e^{+}}\right]$$

•Run BBN code (PArthENoPE) to get $Y_P(N_v,\eta)$, $X_D(N_v,\eta)$ Miele et al. 2011 ${}^{2}H/H = 2.87^{+0.22}_{-0.21} \times 10^{-5}$, $Y_p = 0.247 \pm 0.002_{stat} \pm 0.004_{syst}$

 $\frac{\dot{n}_B}{=} - 3H \quad ,$

 $\dot{a} = -3H(a+P)$

 n_B

Neutrino in SM

• massless

• Neutrino spectra have equilibrium Fermi-Dirac distribution, L=0 at T>1 MeV $n_{\nu}^{eq} = \exp(-E/T)/(1 + \exp(-E/T))$ $T_{\nu} = T_{e} = T_{\gamma}$ $v_{a}v_{\beta} \leftrightarrow v_{a}v_{\beta}$ at T~1 MeV neutrino decouple $\Gamma \sim G_{F}^{2}E_{\nu}^{2}N_{\nu} \leq H$ $v_{a}\overline{v}_{\beta} \leftrightarrow v_{a}\overline{v}_{\beta}$ $T \sim m_{e}, e^{+}e^{-} \rightarrow \gamma\gamma$ photons were heated $T_{\nu}=(4/11)^{1/3}T_{cmb}$ $v_{a}e^{-} \leftrightarrow v_{a}e^{-}$ $v_{a}\overline{v}_{a} \leftrightarrow e^{+}e^{-}$

Neutrino contribution to the energy density of the Universe

$$\rho_{\rm r} = \rho_{\gamma} + \rho_{\nu} + \rho_x = \left[1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right] \rho_{\gamma}$$

Effective number of relativistic neutrino species

 $N_{\rm eff} = 3.046$ $n_{\nu} = 339.3 \, {\rm cm}^{-3}$

Neutrino oscillations found \rightarrow neutrino non-zero mass and mixing, change in N_{eff} n(E) L

Sterile Neutrinos Status

Wellcomed by cosmology:

- may play subdominant role as DM component (eV, KeV)
- may play a role in LSS formation (when constituting few % of the DM it suppresses small scale power in the matter power spectrum and better fits the observational data from SDSS, cluster abundance, weak lensing, Lyman Alpha forest, CMB)
- plays major role in natural baryogenesis through leptogenesis
- The X ray photons from sterile neutrino decays may catalize the production of molecular H and speed up the star formation, causing earlier reionization – observational feature predicted
- CMB feels the increase in the density due to additional particles
- Sterile neutrino is constrained by BPM because it increases the expansion rate and hence dynamically influences He production, in case it is brought into equilibrium. Its decoupling temperature must be $T_R > 130$ MeV.
- In case of oscillations with active neutrino it exerts major effect on nucleons kinetics during pre-BBN and its mixing parameters are constrained by BBN+CMB
- Et cetera.....

Neutrino Oscillations Overview

 \checkmark

 $v_m = U_{mf} v_f$, $(f = e, \mu, \tau)$

It has been observationally and experimentally proved that *neutrinos oscillate*.

The basic idea of oscillations is that mass eigenstates are distinct from the flavor eigenstates.

Solar neutrino problem, atmospheric neutrino anomaly and the results of terrestrial neutrino oscillations experiments were resolved by *flavor neutrino oscillations*.

✓ Combined neutrino oscillations data
 including reactor exps+LSND+MiniBooNe+Gallium:
 hint to 1 or 2 additional light 2 v_s with sub-eV mass
 (in eq. before BBN),

Neutrino oscillations influence Universe processes. Cosmology constrains oscillations b/n light $v_s \leftrightarrow v_e$. P (θ, δm^2 , E, t)

Oscillations imply

non-zero neutrino mass and mixing

 $\delta m^2 \neq 0$ at least 2 neutrino with $m_v \neq 0$

$$\Omega_{\nu} = \frac{3m_0}{93.14h^2 \text{ eV}} \qquad \Longrightarrow \qquad 0.001 < \Omega_{\nu}$$

Flavor neutrino is HDM and hinders the formation of structure at small scales: $0.001 < \Omega_{\nu} < 0.02$

 $\checkmark \quad \text{distribution n(E)} \quad n_v^{cnb} \neq n_v^{eq} = \exp(-E/T)/(1 + \exp(-E/T))$



 $N_e < N_{eq}$

L change

additional species may be brought into equillibrium sterile neutrino

Active-sterile oscillations proceeding after decoupling $\delta m^2 \sin^4 2\theta \le 10^{-7}$ ۰ may strongly distort neutrino distribution and deplete electron neutrino.

Kirilova 88, Kirilova Chizhov PLB,97



The distortion due to active-sterile oscillations and the kinetic effect caused



 δN_k depends on the degree of initial population of v_s.

The effect decreases with δN_s . Precise description of neutrino momenta distribution:

1000 bins used to describe it in non-resonant case up to 10 000 in the resonant case.

Active-sterile oscillations before neutrino decoupling slightly influence active neutrino distributions, because the states are refilled due to interactions with the plasma and bring sterile neutrino into equilibrium.

Evolution of neutrino in presence of $v_e \leftrightarrow v_s$ oscillations and L

• Equations governing the evolution of the oscillating v and v_s , accounting simultaneously for Universe expansion, neutrino oscillations and neutrino forward scattering.

$$\frac{\partial \rho(t)}{\partial t} = Hp_{\nu} \frac{\partial \rho(t)}{\partial p_{\nu}} + i \left[\boldsymbol{H}_{0}, \rho(t) \right] + i \sqrt{2} G_{F} \left(L - \frac{Q}{M_{W}^{2}} \right) N_{\gamma} \left[\alpha, \rho(t) \right] + O \left(G_{F}^{2} \right)$$
$$\frac{\partial \overline{\rho}(t)}{\partial t} = Hp_{\nu} \frac{\partial \overline{\rho}(t)}{\partial p_{\nu}} + i \left[\boldsymbol{H}_{0}, \overline{\rho}(t) \right] + i \sqrt{2} G_{F} \left(-L - \frac{Q}{M_{W}^{2}} \right) N_{\gamma} \left[\alpha, \overline{\rho}(t) \right] + O \left(G_{F}^{2} \right)$$

Non-zero L term leads to coupled integro-differential equations and hard numerical task . L term leads to different evolution of neutrino and antineutrino.