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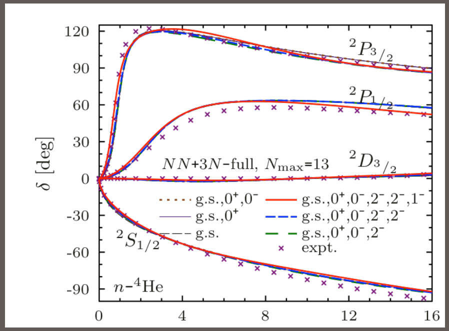
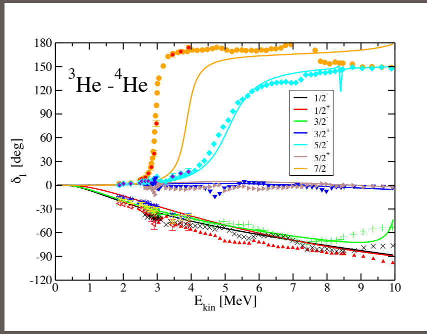
Canada's national laboratory for particle and nuclear physics
Laboratoire national canadien pour la recherche en physique nucléaire
et en physique des particules

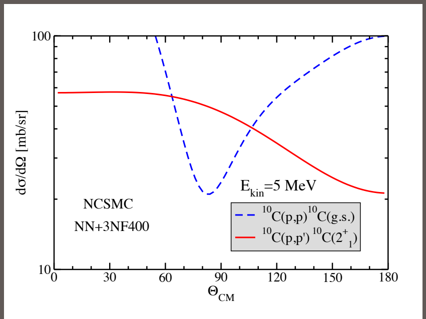
Ab initio treatment of nuclear reactions

International School of Nuclear Physics
36th Course
Nuclei in the Laboratory and in the Cosmos
Erice-Sicily
September 16-24, 2014

Petr Navratil | TRIUMF







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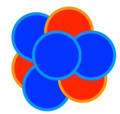
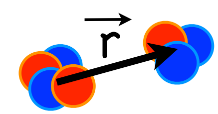
Outline

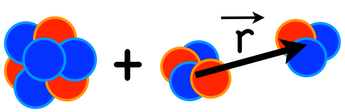
- *Ab initio* calculations in nuclear physics
  - Chiral NN and 3N interactions
- No-core shell model



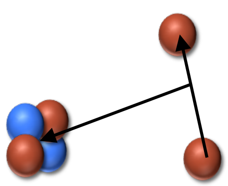
- Including the continuum with the resonating group method
  - NCSM/RGM: n-^{4}He, ^{3}He(d,p)^{4}He, ^{7}Be(p,γ)^{8}B



  - NCSMC: ^{5,7}He, ^{3}He-^{4}He, ^{3}He(α,γ)^{7}Be, ^{11}N (p-^{10}C)



  - Three-body cluster dynamics: ^{6}He



- Outlook

Ab initio Nuclear Structure & Reaction approaches

Ab initio

- ✧ All nucleons are active
- ✧ Exact Pauli principle
- ✧ Realistic inter-nucleon interactions
  - ✧ Accurate description of NN (and 3N) data
- ✧ Controllable approximations

Chiral Effective Field Theory

- **First principles for Nuclear Physics:**
  QCD
  - Non-perturbative at low energies
  - Lattice QCD in the future
- ***For now a good place to start:***
- Inter-nucleon forces from chiral effective field theory
  - Based on the symmetries of QCD
    - Chiral symmetry of QCD ($m_u \approx m_d \approx 0$), spontaneously broken with pion as the Goldstone boson
    - Degrees of freedom: nucleons + pions
  - Systematic low-momentum expansion to a given order (Q/Λ_χ)
  - Hierarchy
  - Consistency
  - Low energy constants (LEC)
    - Fitted to data
    - Can be calculated by lattice QCD

Λ_χ~1 GeV :
Chiral symmetry breaking scale

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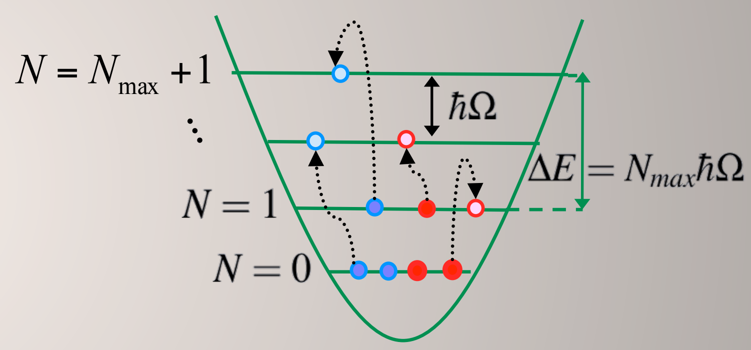
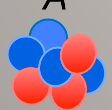
The *ab initio* no-core shell model (NCSM)

- The NCSM is a technique for the solution of the *A*-nucleon bound-state problem
- Realistic nuclear Hamiltonian
  - High-precision nucleon-nucleon potentials
  - Three-nucleon interactions
- Finite harmonic oscillator (HO) basis
  - *A*-nucleon HO basis states
  - complete $N_{max}\hbar\Omega$ model space



- Acceleration of convergence by a **sequence of unitary transformations in momentum space**
  - Similarity-Renormalization-Group (SRG) evolved NN(+NNN) potential

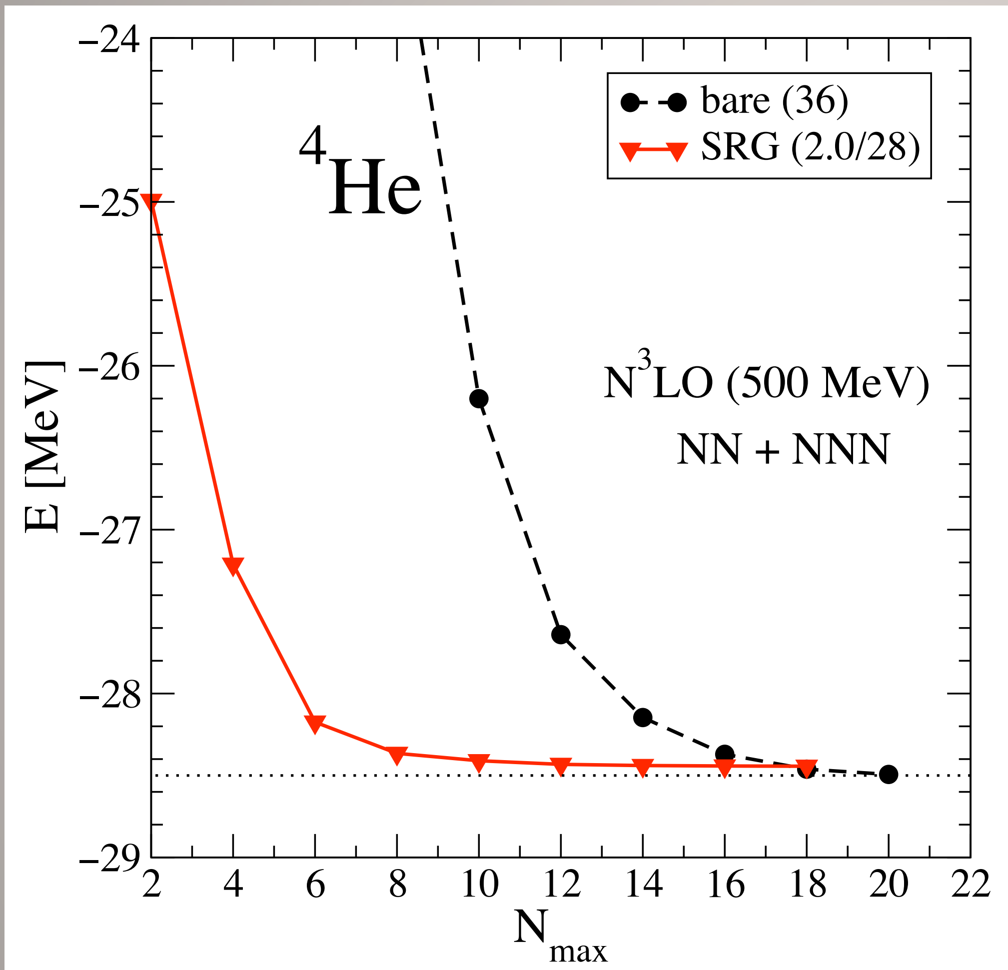
A



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

Convergence to exact solution with increasing $N_{\max}$ for bound states. No coupling to continuum.

Calculations with chiral 3N: SRG renormalization needed



PRL 103, 082501 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

Evolution of Nuclear Many-Body Forces with the Similarity Renormalization Group

E. D. Jurgenson,[1] P. Navrátil,[2] and R. J. Furnstahl[1]

A=3 binding energy and half life constraint
c_D=-0.2, c_E=-0.205, Λ=500 MeV

Chiral N^{3}LO NN plus N^{2}LO NNN potential

- Bare interaction (black line)
  - Strong short-range correlations
    - Large basis needed
- SRG evolved effective interaction (red line)
  - Unitary transformation

$$\boxed{H_\alpha = U_\alpha H U_\alpha^+ \Rightarrow \frac{dH_\alpha}{d\alpha} = \left[\left[T, H_\alpha\right], H_\alpha\right]} \quad \left(\alpha = 1/\lambda^4\right)$$

  - Two- plus *three*-body components, *four*-body omitted
  - Softens the interaction
    - Smaller basis sufficient

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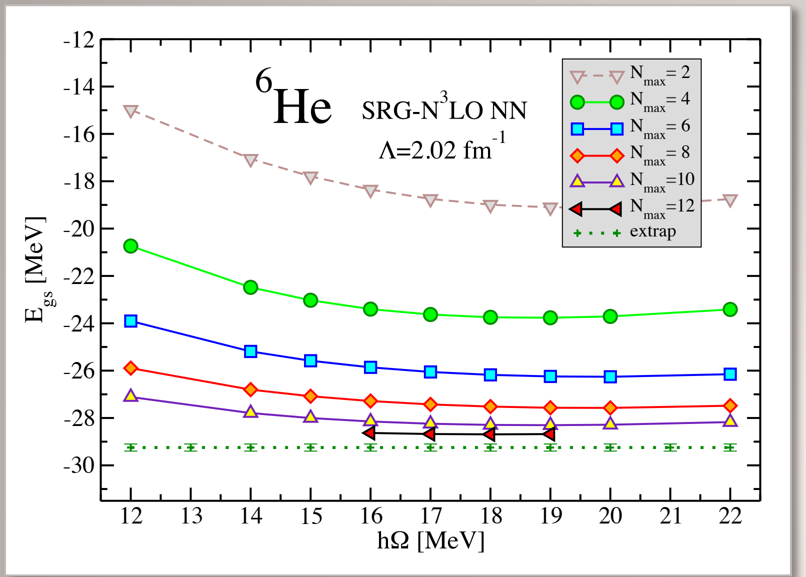
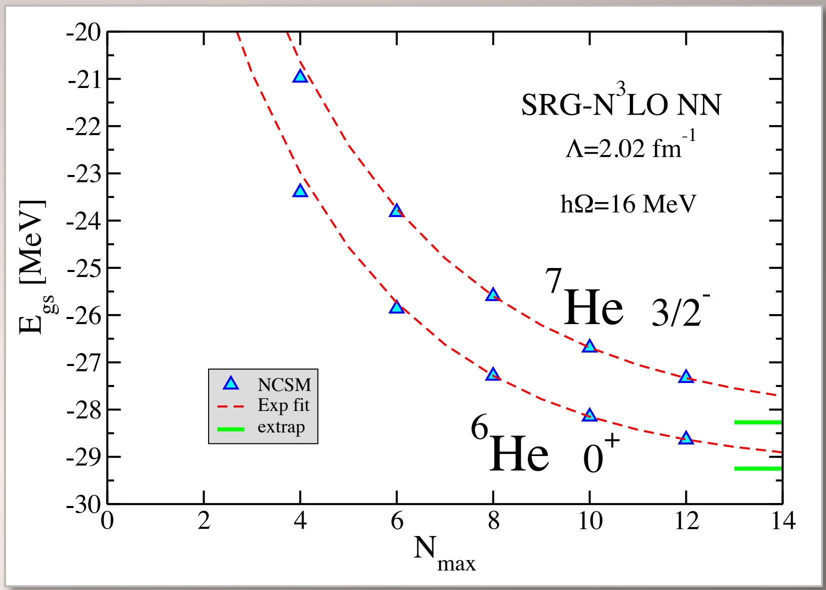
NCSM calculations of ^{6}He and ^{7}He g.s. energies

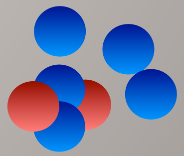




$E_{g.s.}$ [MeV]	^{4}He	^{6}He	^{7}He
NCSM N_{max}=12	-28.05	-28.63	-27.33
NCSM extrap.	-28.22(1)	-29.25(15)	-28.27(25)
Expt.	-28.30	-29.27	-28.84

- Soft SRG evolved NN potential
- ✓ N_{max} convergence OK
- ✓ Extrapolation feasible

- ^{7}He unbound
  - Expt. E_{th}=+0.430(3) MeV: NCSM E_{th}≈ +1 MeV
  - Expt. width 0.182(5) MeV: **NCSM no information about the width**

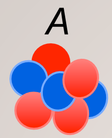


^{7}He unbound

Extending no-core shell model beyond bound states

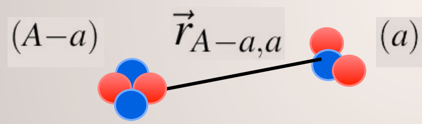
Include more many nucleon correlations…

NCSM →

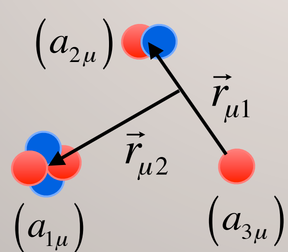


$$\Psi^{A} = \sum_{N=0}^{N_{\max}} \sum_{i} c_{Ni} \Phi_{Ni}^{A}$$

+



+



$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

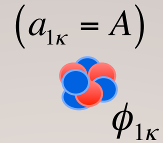
+

…

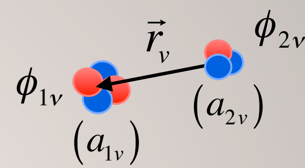
…using the Resonating Group Method (RGM) ideas

Trial function: generalized cluster wave function

$$\psi^{(A)} = \sum_{\kappa} c_\kappa \phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)$$



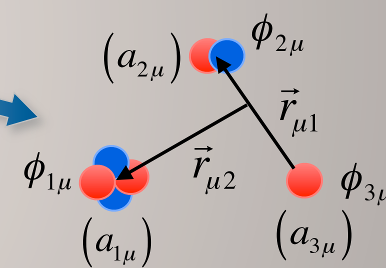
$$+ \sum_{\nu} \hat{A}_\nu \phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right)\phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right) g_\nu(\vec{r}_\nu)$$



$$a_{1\nu} + a_{2\nu} = A$$

$$+ \sum_{\mu} \hat{A}_\mu \phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right)\phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right)\phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right) G_\mu(\vec{r}_{\mu 1}, \vec{r}_{\mu 2})$$

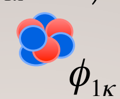
$$+ \cdots$$



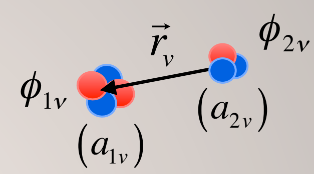
$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

Trial function: generalized cluster wave function

$$\psi^{(A)} = \sum_{\kappa} c_\kappa \boxed{\phi_{1\kappa}}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)$$



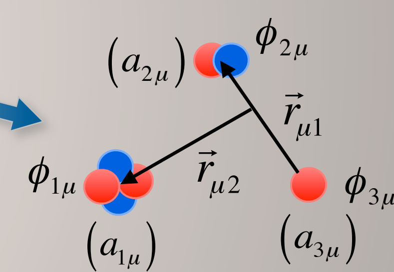
$$+\sum_{\nu} \hat{A}_\nu \boxed{\phi_{1\nu}}\left(\left\{\vec{\xi}_{1\nu}\right\}\right)\boxed{\phi_{2\nu}}\left(\left\{\vec{\xi}_{2\nu}\right\}\right) g_\nu(\vec{r}_\nu)$$



$$a_{1\nu} + a_{2\nu} = A$$

$$+\sum_{\mu} \hat{A}_\mu \boxed{\phi_{1\mu}}\left(\left\{\vec{\xi}_{1\mu}\right\}\right)\boxed{\phi_{2\mu}}\left(\left\{\vec{\xi}_{2\mu}\right\}\right)\boxed{\phi_{3\mu}}\left(\left\{\vec{\xi}_{3\mu}\right\}\right) G_\mu(\vec{r}_{\mu 1}, \vec{r}_{\mu 2})$$

$$+ \cdots$$

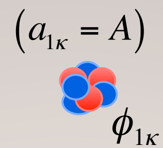


$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

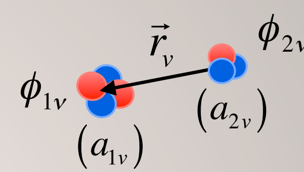
- ϕ : antisymmetric cluster wave functions
  - $\{\xi\}$: Translationally invariant internal coordinates (Jacobi relative coordinates)
  - These are known, they are an input

Trial function: generalized cluster wave function

$$\psi^{(A)} = \sum_{\kappa} c_\kappa \phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)$$



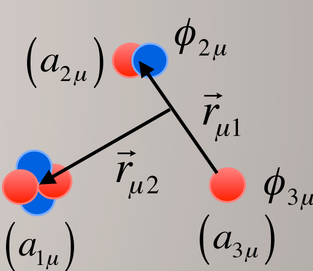
$$+ \sum_{\nu} \boxed{\hat{A}_\nu} \phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right) \phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right) g_\nu(\vec{r}_\nu)$$



$$a_{1\nu} + a_{2\nu} = A$$

$$+ \sum_{\mu} \boxed{\hat{A}_\mu} \phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right) \phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right) \phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right) G_\mu(\vec{R}_{\mu 1}, \vec{R}_{\mu 2})$$

$$+ \cdots$$



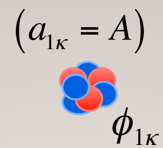
$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

- ***A_ν, A_μ*** : intercluster antisymmetrizers
  - Antisymmetrize the wave function for exchanges of nucleons between clusters
  - Example:

$$a_{1\nu} = A-1,\ a_{2\nu} = 1 \quad \Rightarrow \quad \hat{A}_\nu = \frac{1}{\sqrt{A}}\left[1 - \sum_{i=1}^{A-1} \hat{P}_{iA}\right]$$

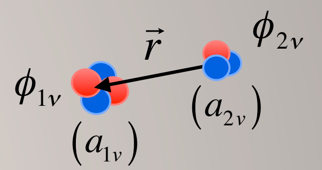
Trial function: generalized cluster wave function

$$\psi^{(A)} = \sum_{\kappa} \boxed{c_{\kappa}} \phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)$$



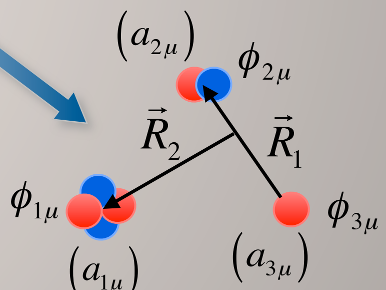
$$+ \sum_{\nu} \int \boxed{g_{\nu}(\vec{r}\,)} \hat{A}_{\nu} \left[\phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right) \phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right) \delta(\vec{r} - \vec{r}_{\nu}) \right] d\vec{r}$$

$a_{1\nu} + a_{2\nu} = A$



$$+ \sum_{\mu} \iint \boxed{G_{\mu}(\vec{R}_1, \vec{R}_2)} \hat{A}_{\mu} \left[\phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right) \phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right) \phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right) \delta(\vec{R}_1 - \vec{R}_{\mu 1}) \delta(\vec{R}_2 - \vec{R}_{\mu 2}) \right] d\vec{R}_1 d\vec{R}_2$$

$$+ \cdots$$



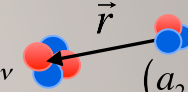
$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$

- ***c***, ***g*** and ***G***: discrete and continuous linear variational amplitudes
  - Unknowns to be determined

Trial function: generalized cluster wave function

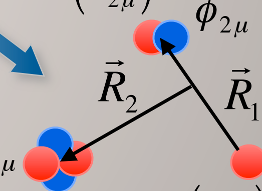
$$\psi^{(A)} = \sum_{\kappa} c_{\kappa} \boxed{\phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)}$$

$$+\sum_{\nu} \int g_{\nu}(\vec{r}\,)\ \hat{A}_{\nu} \boxed{\left[\phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right)\phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right)\delta(\vec{r}-\vec{r}_{\nu})\right]} d\vec{r}$$



$$+\sum_{\mu} \iint G_{\mu}(\vec{R}_1,\vec{R}_2)\ \hat{A}_{\mu} \boxed{\left[\phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right)\phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right)\phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right)\delta(\vec{R}_1-\vec{R}_{\mu 1})\delta(\vec{R}_2-\vec{R}_{\mu 2})\right]} d\vec{R}_1 d\vec{R}_2$$

$$+\ \cdots$$



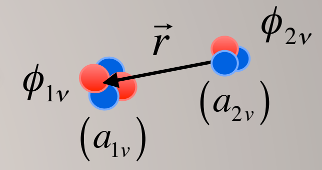
- Discrete and continuous set of basis functions
  - Non-orthogonal
  - Over-complete

Binary cluster wave function

$$\psi^{(A)} = \cancel{\sum_{\kappa} c_{\kappa} \phi_{1\kappa}\left(\left\{\vec{\xi}_{1\kappa}\right\}\right)}$$

$$a_{1\nu} + a_{2\nu} = A$$

$$+ \sum_{\nu} \int g_{\nu}(\vec{r}\,)\, \hat{A}_{\nu}\left[\phi_{1\nu}\left(\left\{\vec{\xi}_{1\nu}\right\}\right)\phi_{2\nu}\left(\left\{\vec{\xi}_{2\nu}\right\}\right)\delta(\vec{r} - \vec{r}_{\nu})\right] d\vec{r}$$



$$\cancel{+ \sum_{\mu} \iint G_{\mu}(\vec{R}_1, \vec{R}_2)\, \hat{A}_{\mu}\left[\phi_{1\mu}\left(\left\{\vec{\xi}_{1\mu}\right\}\right)\phi_{2\mu}\left(\left\{\vec{\xi}_{2\mu}\right\}\right)\phi_{3\mu}\left(\left\{\vec{\xi}_{3\mu}\right\}\right)\delta(\vec{R}_1 - \vec{R}_{\mu 1})\delta(\vec{R}_2 - \vec{R}_{\mu 2})\right] d\vec{R}_1 d\vec{R}_2}$$

$$+ \cdots$$

- In practice: function space limited by using relatively simple forms of Ψ chosen according to physical intuition and energetical arguments
  - Most common: expansion over binary-cluster basis

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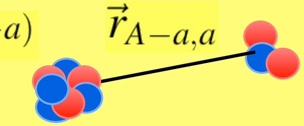
The *ab initio* NCSM/RGM in a snapshot

- Ansatz: $\Psi^{(A)} = \sum_{\nu} \int d\vec{r}\, \varphi_{\nu}(\vec{r})\, \hat{\mathcal{A}}\, \Phi^{(A-a,a)}_{\nu\vec{r}}$

$(A-a)$ $\vec{r}_{A-a,a}$ (a)



$\psi^{(A-a)}_{1\nu}\, \psi^{(a)}_{2\nu}\, \delta(\vec{r} - \vec{r}_{A-a,a})$

eigenstates of $H_{(A\text{-}a)}$ and $H_{(a)}$ in the *ab initio* NCSM basis

- Many-body Schrödinger equation:

$$H\Psi^{(A)} = E\Psi^{(A)}$$

$T_{\mathrm{rel}}(r) + \mathcal{V}_{\mathrm{rel}} + \bar{V}_{\mathrm{Coul}}(r) + H_{(A-a)} + H_{(a)}$

realistic nuclear Hamiltonian

$$\sum_{\nu} \int d\vec{r} \left[\mathcal{H}^{(A-a,a)}_{\mu\nu}(\vec{r}\,', \vec{r}) - E\, \mathcal{N}^{(A-a,a)}_{\mu\nu}(\vec{r}\,', \vec{r}) \right] \varphi_{\nu}(\vec{r}) = 0$$

$\langle \Phi^{(A-a,a)}_{\mu\vec{r}\,'} | \hat{\mathcal{A}} H \hat{\mathcal{A}} | \Phi^{(A-a,a)}_{\nu\vec{r}} \rangle$

Hamiltonian kernel

$\langle \Phi^{(A-a,a)}_{\mu\vec{r}\,'} | \hat{\mathcal{A}}^2 | \Phi^{(A-a,a)}_{\nu\vec{r}} \rangle$

Norm kernel

Norm kernel (Pauli principle)

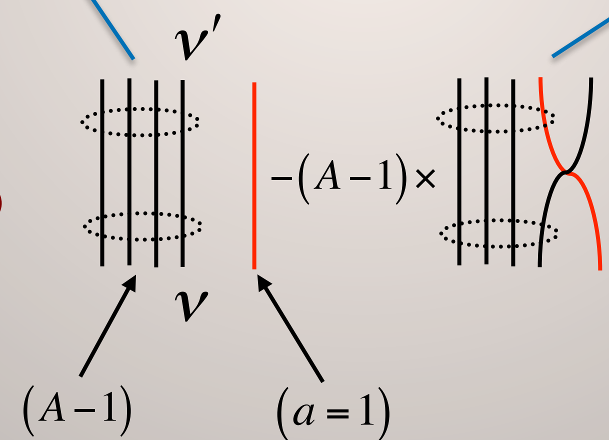
Single-nucleon projectile

$$\left\langle \Phi^{J^\pi T}_{\nu' r'} \middle| \hat{A}_{\nu'} \hat{A}_{\nu} \middle| \Phi^{J^\pi T}_{\nu r} \right\rangle = \left\langle (A-1) \; r' \; (a'=1) \middle| 1 - \sum_{i=1}^{A-1} \hat{P}_{iA} \middle| (a=1) \; r \; (A-1) \right\rangle$$

$$N^{J^\pi T}_{\nu'\nu}(r', r) = \underbrace{\delta_{\nu'\nu} \frac{\delta(r'-r)}{r' r}} - \underbrace{(A-1) \sum_{n'n} R_{n'\ell'}(r') R_{n\ell}(r) \left\langle \Phi^{J^\pi T}_{\nu' n'} \middle| \hat{P}_{A-1,A} \middle| \Phi^{J^\pi T}_{\nu n} \right\rangle}$$

$${}_{\mathrm{SD}}\left\langle \psi^{(A-1)}_{\mu_1} \middle| a^{+} a \middle| \psi^{(A-1)}_{\nu_1} \right\rangle_{\mathrm{SD}}$$

Direct term:
Treated exactly!
(in the full space)

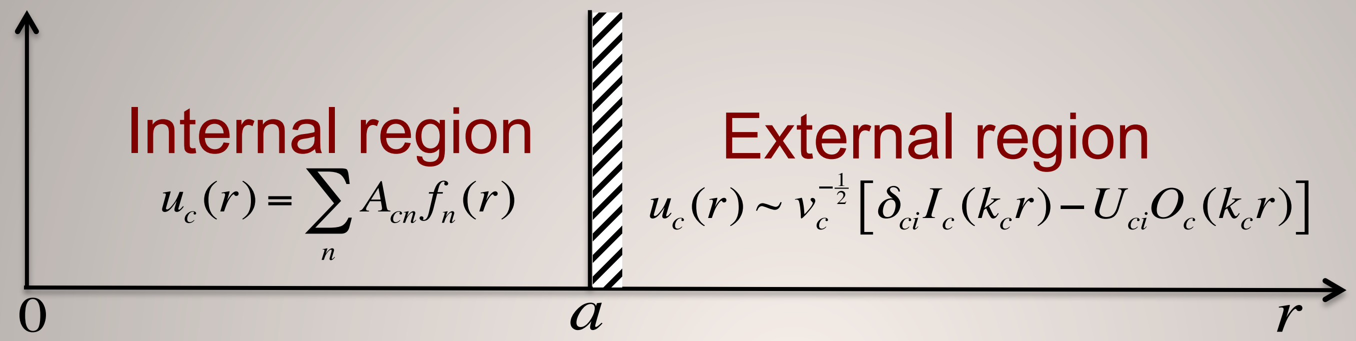


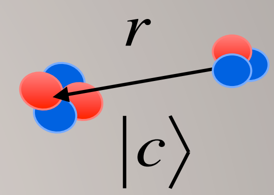
Exchange term:
Obtained in the model space!
(Many-body correction due to the exchange part of the inter-cluster antisymmetrizer)

$$\frac{\delta(r - r_{A-a,a})}{r \, r_{A-a,a}} = \sum_n R_{n\ell}(r) R_{n\ell}(r_{A-a,a})$$

Microscopic *R*-matrix on a Lagrange mesh

Separation into "internal" and "external" regions at the channel radius a





- This is achieved through the Bloch operator: $$L_c = \frac{\hbar^2}{2\mu_c} \delta(r-a) \left(\frac{d}{dr} - \frac{B_c}{r} \right)$$
- System of Bloch-Schrödinger equations:

$$\left[\hat{T}_{rel}(r) + L_c + \overline{V}_{Coul}(r) - (E - E_c) \right] u_c(r) + \sum_{c'} \int dr' \, r' W_{cc'}(r,r') u_{c'}(r') = L_c u_c(r)$$

$$u_c(r) = \sum_n A_{cn} f_n(r)$$

- Internal region: expansion on square-integrable Lagrange mesh basis
- External region: asymptotic form for large r

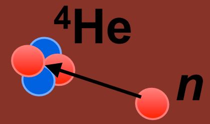
$$\{ a x_n \in [0,a] \}$$
$$\int_0^1 g(x)dx \approx \sum_{n=1}^{N} \lambda_n g(x_n)$$
$$\int_0^a f_n(r) f_{n'}(r) dr \approx \delta_{nn'}$$

$$u_c(r) \sim C_c W(k_c r) \quad \text{or} \quad u_c(r) \sim v_c^{-\frac{1}{2}} \left[\delta_{ci} I_c(k_c r) - U_{ci} O_c(k_c r) \right]$$

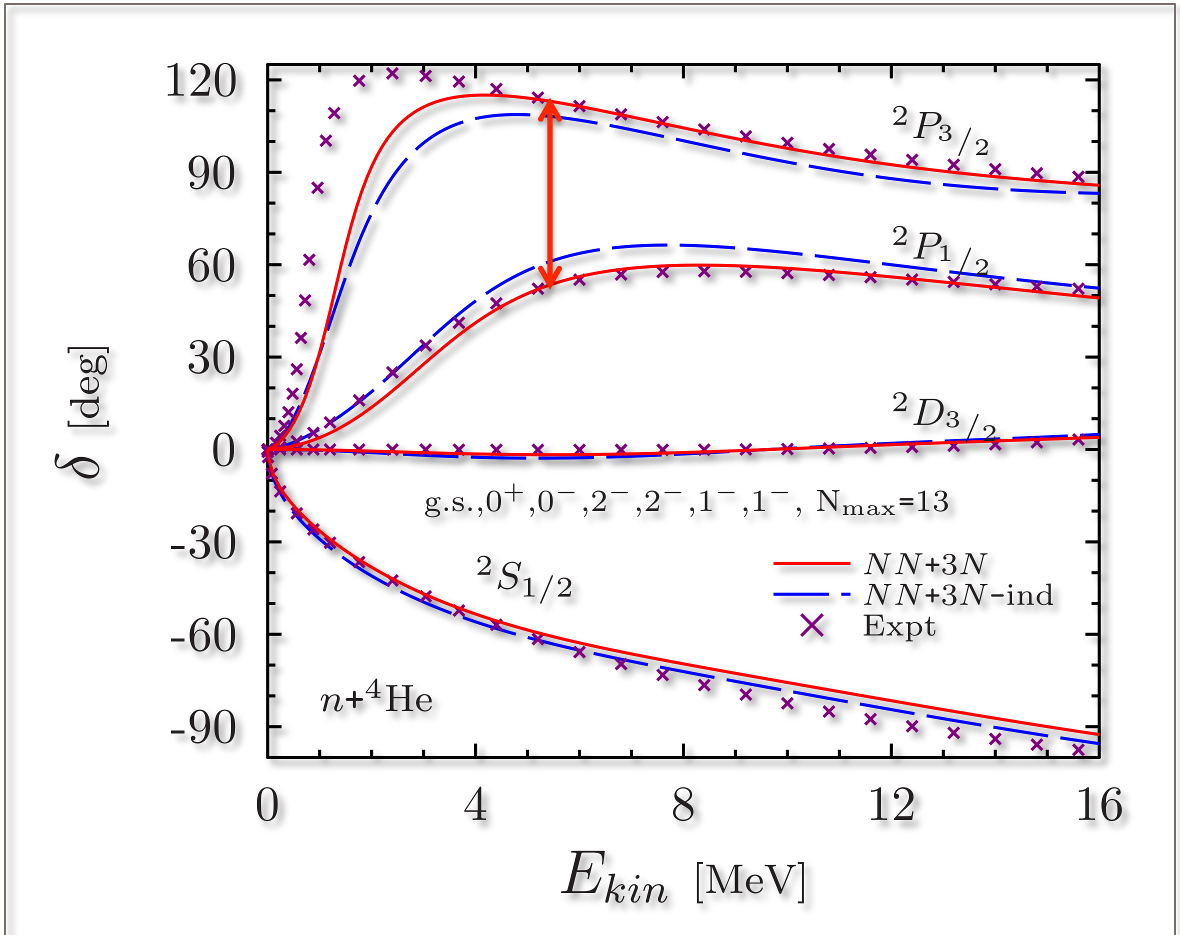
Bound state

Scattering state

Scattering matrix



n-^{4}He scattering: NN vs. NN+NNN interactions



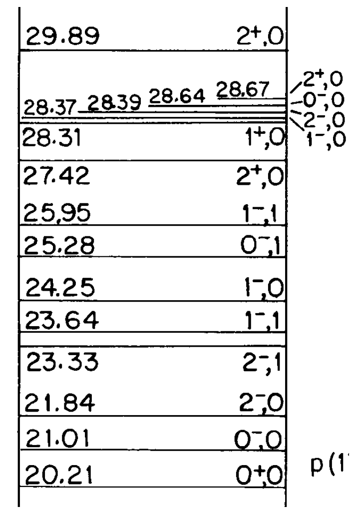
PHYSICAL REVIEW C **88**, 054622 (2013)

***Ab initio* many-body calculations of nucleon-^{4}He scattering with three-nucleon forces**

Guillaume Hupin,[1,*] Joachim Langhammer,[2,†] Petr Navrátil,[3,‡] Sofia Quaglioni,[1,§] Angelo Calci,[2,∥] and Robert Roth[2,¶]

chiral NN+NNN(500)
chiral NN+NNN-induced
SRG λ=2 fm^{-1}
HO N$_{max}$=13, hΩ=20 MeV

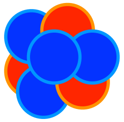
^{4}He g.s. and 6 excited states



The largest splitting between the P-waves obtained with the chiral NN+NNN interaction

New developments: NCSM with continuum

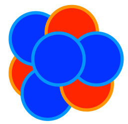
NCSM



$$\left|\Psi_A^{J^\pi T}\right\rangle = \sum_{Ni} c_{Ni} \left|ANiJ^\pi T\right\rangle$$

New developments: NCSM with continuum

NCSM



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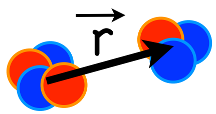
NCSM/RGM



$$|\Psi_A^{J^\pi T}\rangle = \sum_\nu \int d\vec{r}\chi_\nu(\vec{r})\hat{A}\Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\mathcal{H}\chi = E\mathcal{N}\chi$$

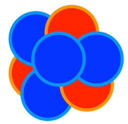
$$\bar{\chi} = \mathcal{N}^{+\frac{1}{2}}\chi$$

$$\boxed{(\mathcal{N}^{-\frac{1}{2}}\mathcal{H}\mathcal{N}^{-\frac{1}{2}})\bar{\chi} = E\bar{\chi}}$$

TRIUMF

New developments: NCSM with continuum

NCSM



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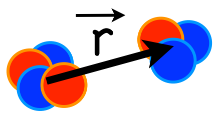
NCSM/RGM



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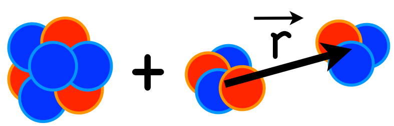
NCSMC

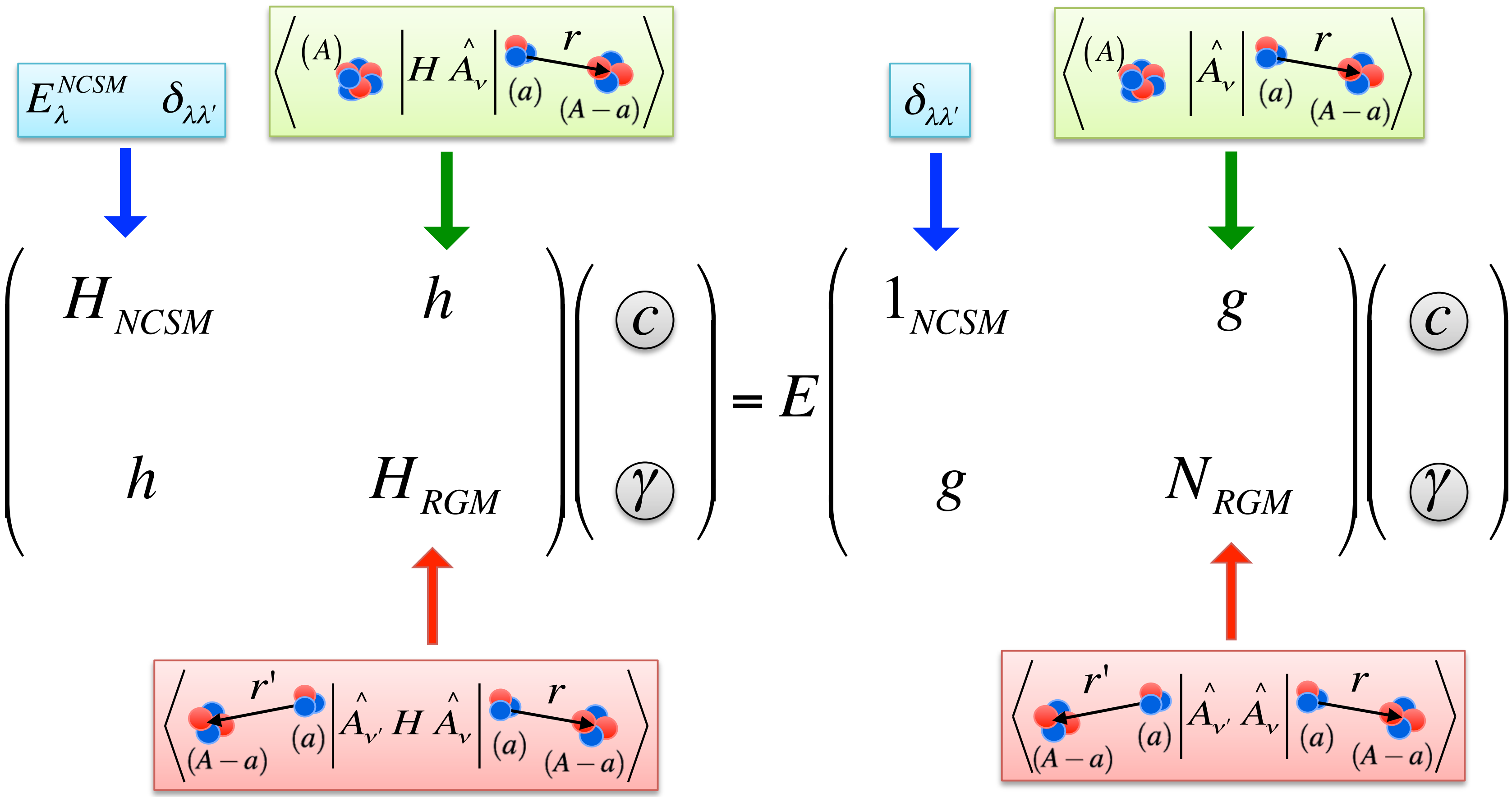


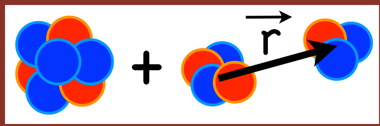
S. Baroni, P. N., and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

$$|\Psi_A^{J^\pi T}\rangle = \sum_\lambda c_\lambda |A\lambda J^\pi T\rangle + \sum_\nu \int d\vec{r} \left(\sum_{\nu'} \int d\vec{r}\,' \mathcal{N}_{\nu\nu'}^{-\frac{1}{2}}(\vec{r},\vec{r}\,') \bar{\chi}_{\nu'}(\vec{r}\,') \right) \hat{A}\Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\boxed{\begin{pmatrix} H_{NCSM} & \bar{h} \\ \bar{h} & \mathcal{N}^{-\frac{1}{2}}\mathcal{H}\mathcal{N}^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} c \\ \bar{\chi} \end{pmatrix} = E \begin{pmatrix} 1 & \bar{g} \\ \bar{g} & 1 \end{pmatrix} \begin{pmatrix} c \\ \bar{\chi} \end{pmatrix}}$$

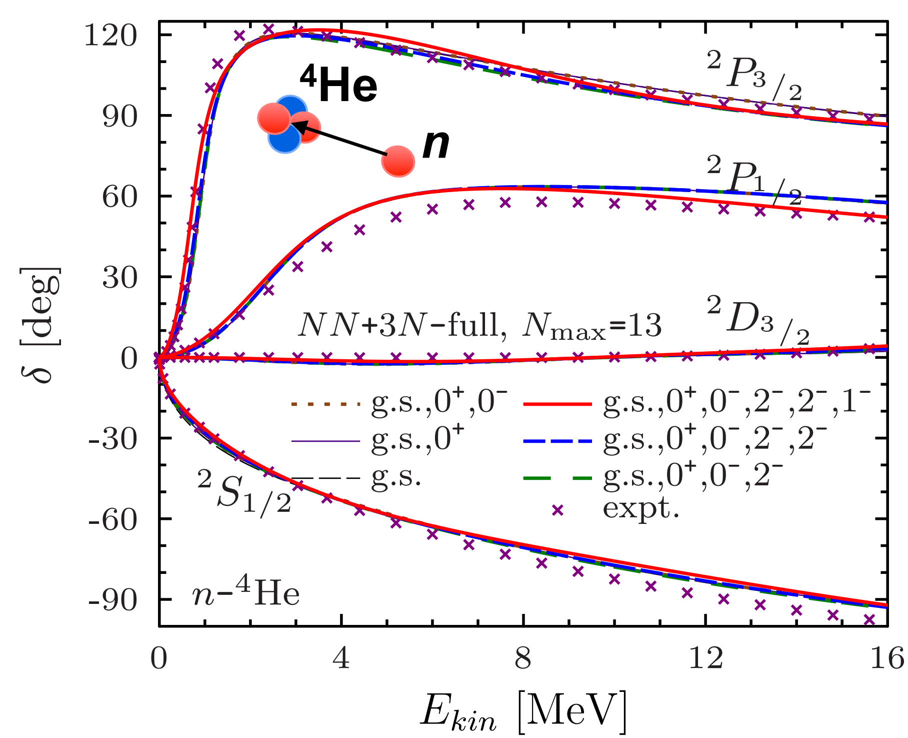
Building blocks of the NCSMC equations





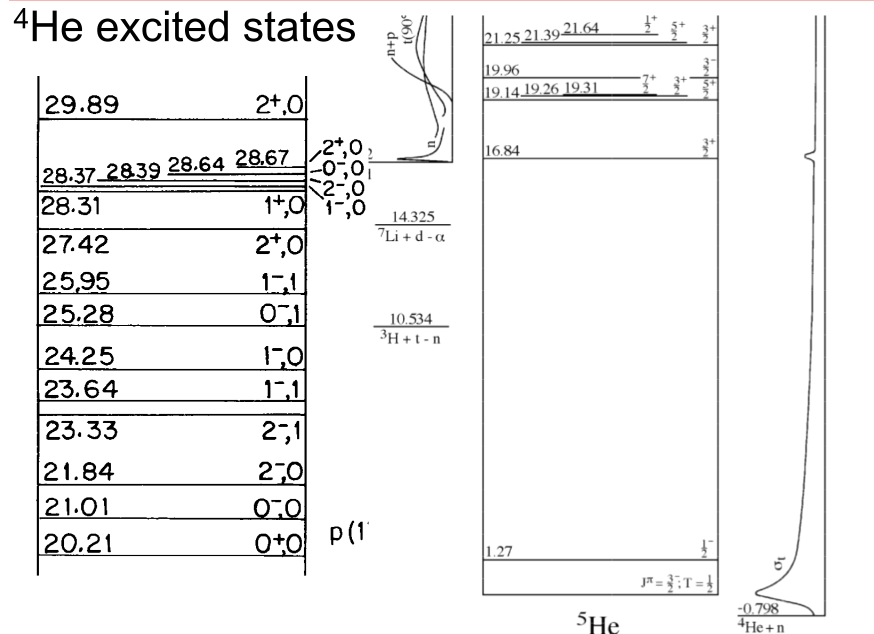
n-^{4}He & p-^{4}He scattering within NCSMC

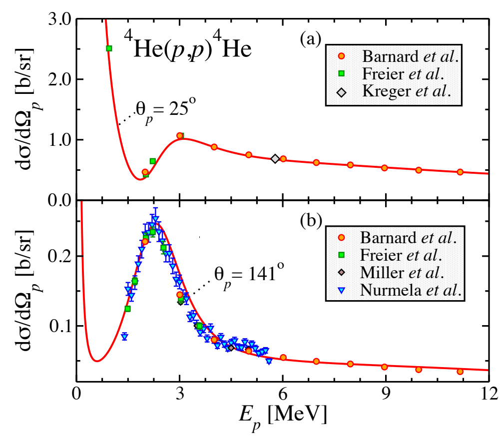
Study of the convergence with respect to the # of ^{4}He low-lying NCSM states



n-^{4}He scattering phase-shifts for NN+NNN potential with λ=2.0 fm^{-1} and 8 low-lying NCSM states of ^{5}He.

Experimental low-lying states of the A=5 nucleon systems.





G. Hupin, S. Quaglioni and P. Navrátil, arXiv:1409.0892 [nucl-th]

TRIUMF

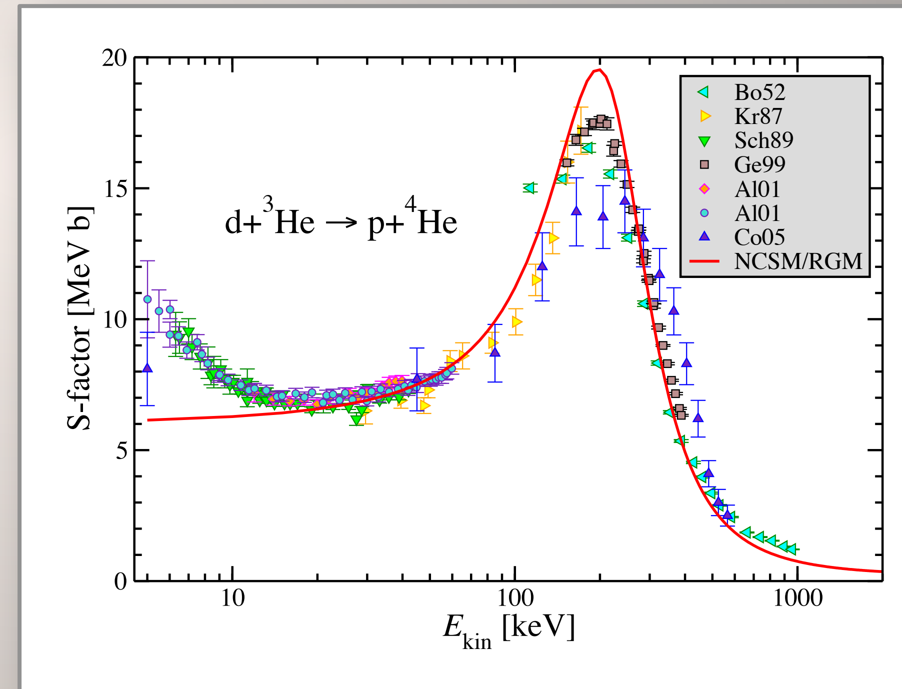
NCSM/RGM calculations of transfer reactions

$$\int dr\, r^2 \begin{pmatrix} \left\langle {}^{r'}_{n}\ \alpha \right| \hat{A}_1 (H-E) \hat{A}_1 \left| \alpha\ {}^{r}_{n} \right\rangle & \boxed{\left\langle {}^{r'}_{n}\ \alpha \right| \hat{A}_1 (H-E) \hat{A}_2 \left| {}^{3}\mathrm{H}\ {}^{r}_{d} \right\rangle} \\ \boxed{\left\langle {}^{r'}_{d}\ {}^{3}\mathrm{H} \right| \hat{A}_2 (H-E) \hat{A}_1 \left| \alpha\ {}^{r}_{n} \right\rangle} & \left\langle {}^{r'}_{d}\ {}^{3}\mathrm{H} \right| \hat{A}_2 (H-E) \hat{A}_2 \left| {}^{3}\mathrm{H}\ {}^{r}_{d} \right\rangle \end{pmatrix} \begin{pmatrix} \frac{g_1(r)}{r} \\ \frac{g_2(r)}{r} \end{pmatrix} = 0$$

Straightforward to couple different mass partitions in the NCSM/RGM formalism

Applications to (d,p) and (d,n) reactions
Example: **^{3}He(d,p)^{4}He**

Work in progress:
^{7}Li(d,p)^{8}Li & ^{8}Li(d,p)^{9}Li
Technical issue: Calculation of kernels with three-body densities for systems with A>5

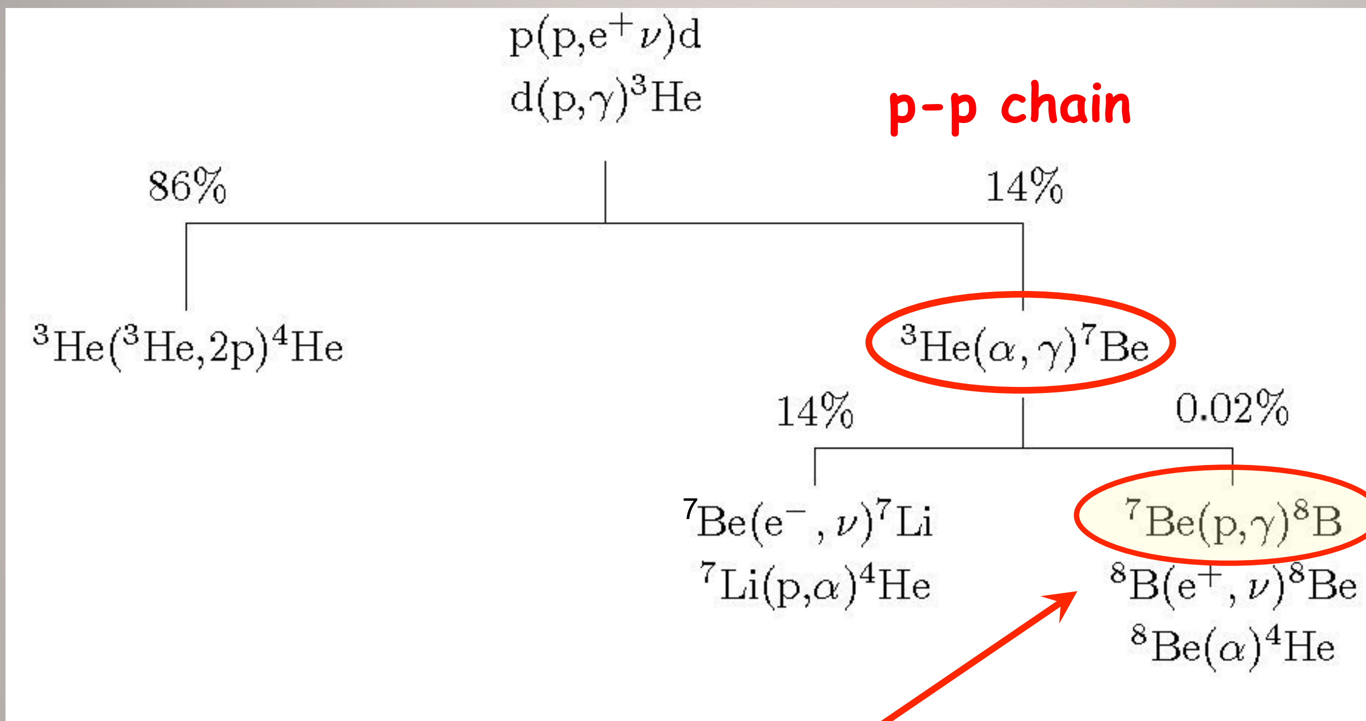


PRL **108,** 042503 (2012) PHYSICAL REVIEW LETTERS week ending 27 JANUARY 2012

***Ab Initio* Many-Body Calculations of the ^{3}H(d, n)^{4}He and ^{3}He(d, p)^{4}He Fusion Reactions**

Petr Navrátil[1,2] and Sofia Quaglioni[2]

Solar *p-p* chain

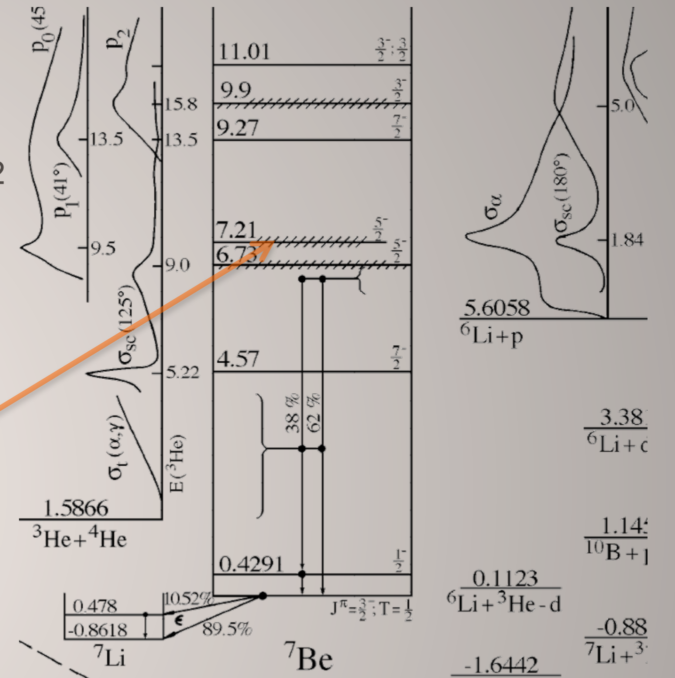


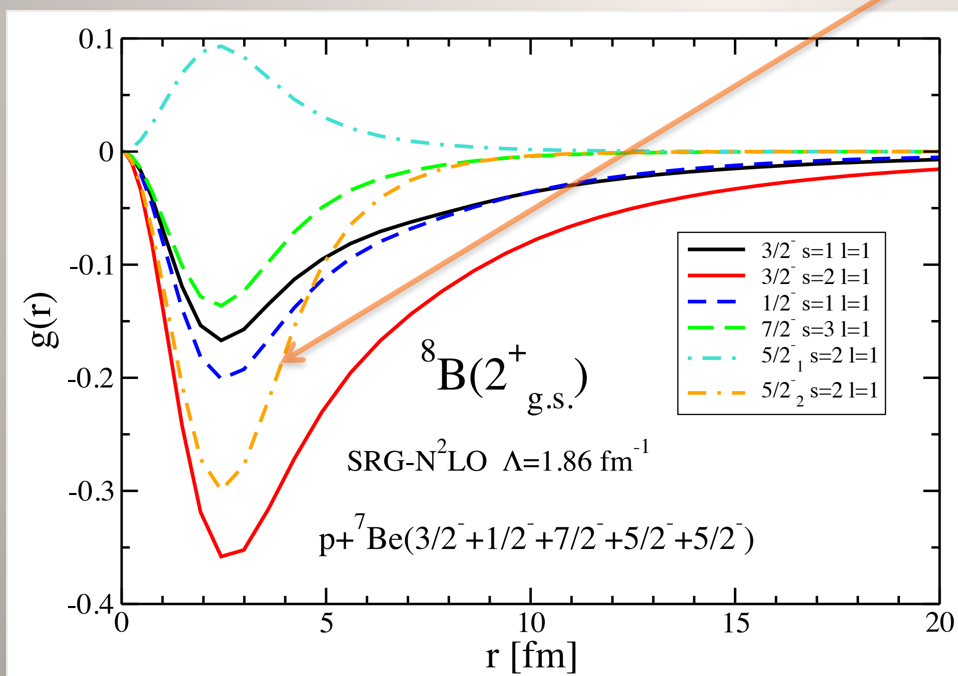
Solar neutrinos

E_ν < 15 MeV

Structure of the ^{8}B ground state

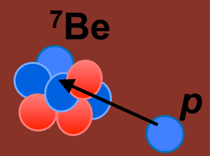
- NCSM/RGM *p*-^{7}Be calculation
  - five lowest ^{7}Be states: $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
  - Soft NN SRG-N^{3}LO with λ = 1.86 fm^{-1}
- ^{8}B 2^+ g.s. bound by 136 keV (Expt 137 keV)
  - Large *P*-wave $5/2^-_2$ component





$5/2^-_2$ state of ^{7}Be should be included in ^{7}Be(p, γ)^{8}B calculations

TRIUMF



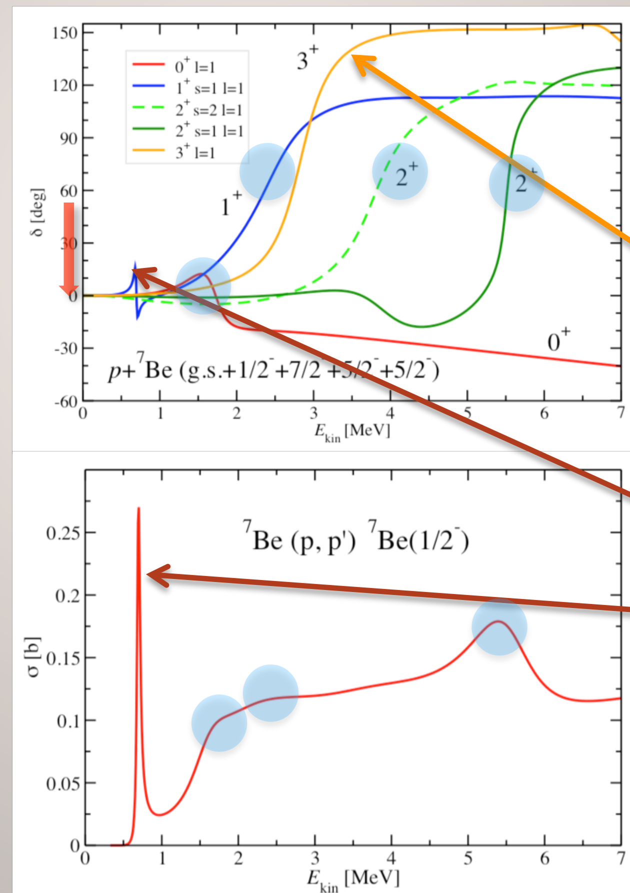
p-^{7}Be scattering

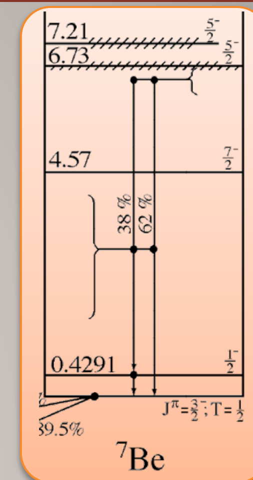
- NCSM/RGM calculation of p-^{7}Be scattering
  - ^{7}Be states $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
  - Soft NN potential (SRG-N^{3}LO with λ = 1.86 fm^{-1})

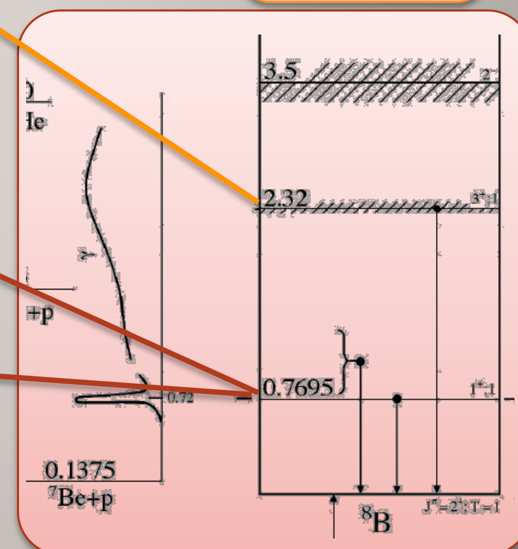
^{8}B 2^+ g.s. **bound** by 136 keV
(expt. bound by 137 keV)

New 0^+, 1^+, and two 2^+ resonances **predicted**

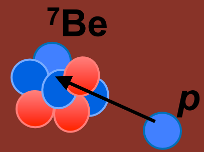
$s=1$ $l=1$ 2^+ clearly visible in (p,p') cross sections





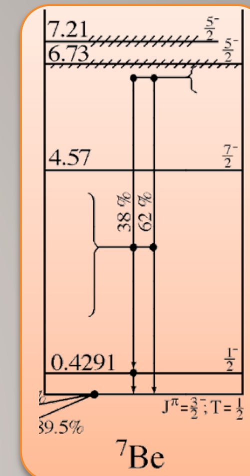


PRC **82**, 034609 (2010)
PLB **704** (2011) 379



^{7}Be(p,γ)^{8}B radiative capture

- NCSM/RGM calculation
  - ^{7}Be states $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-_1$, $5/2^-_2$
  - Soft NN potential (chiral SRG-N^{3}LO with λ = 1.86 fm^{-1})



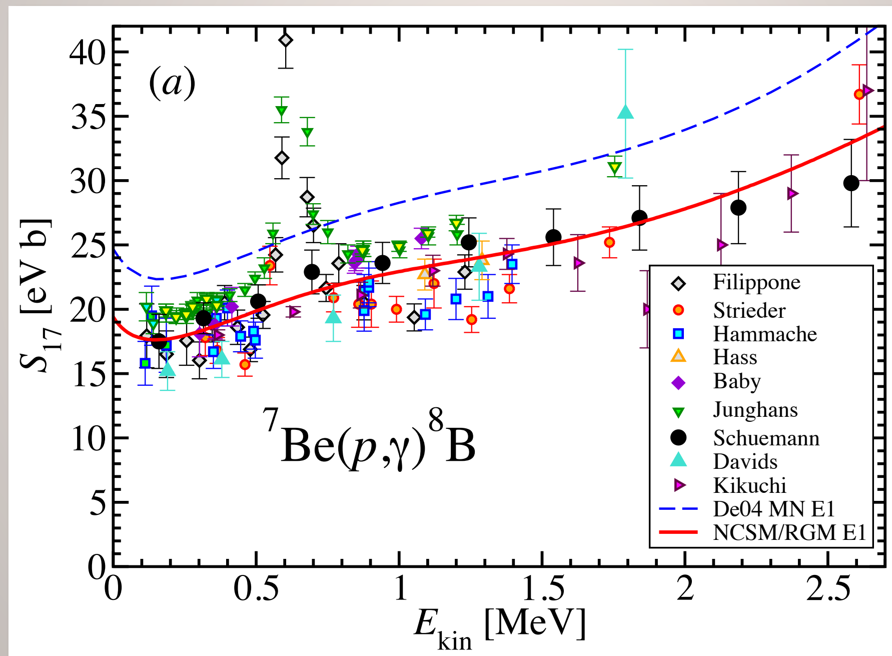
^{8}B 2$^+$ g.s. **bound** by 136 keV

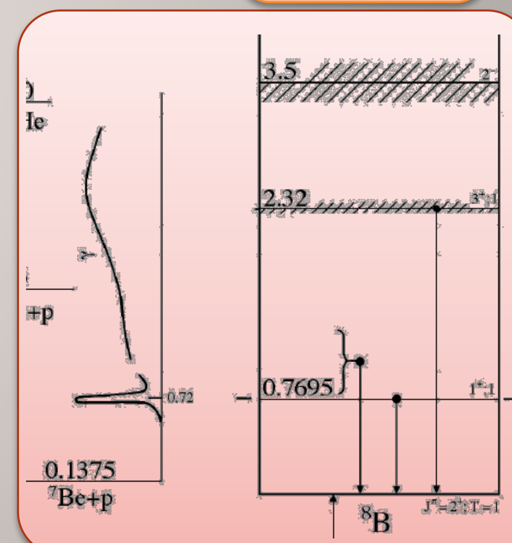
(expt. 137 keV)

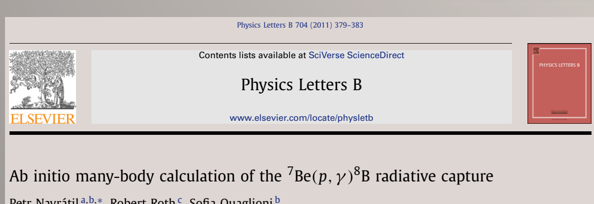
S(0) ~ 19.4(0.7) eV b

Data evaluation:

S(0)=20.8(2.1) eV b






Physics Letters B 704 (2011) 379–383

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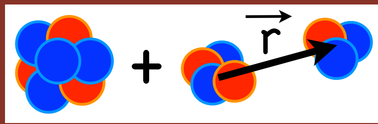
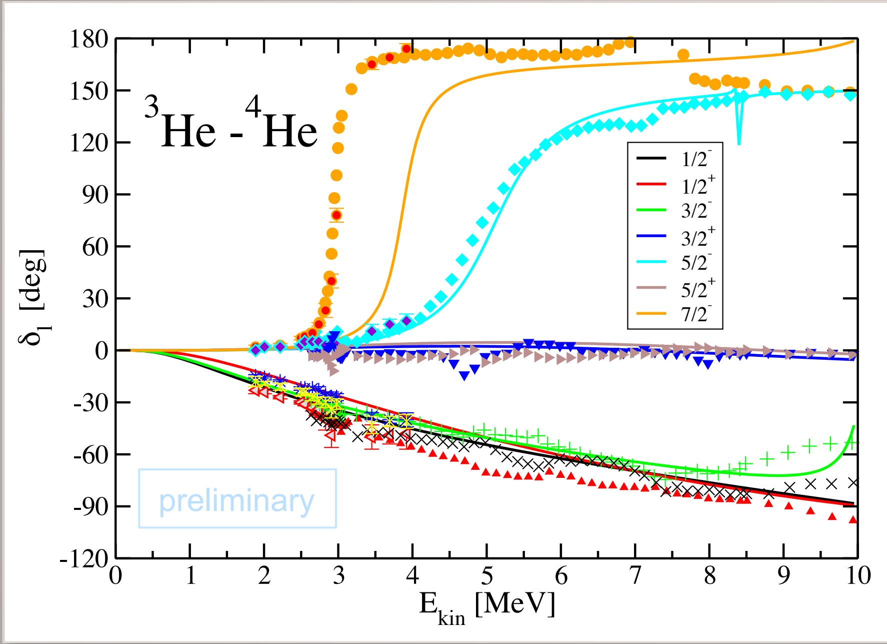
Ab initio many-body calculation of the ^{7}Be(p, γ)^{8}B radiative capture

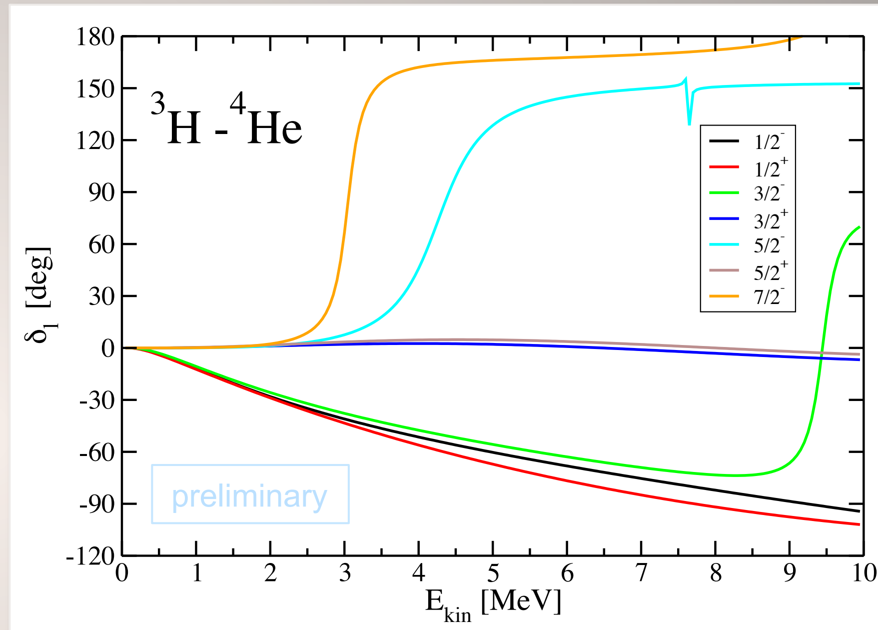
Petr Navrátil [a,b,*], Robert Roth [c], Sofia Quaglioni [b]

TRIUMF



^{3}He-^{4}He and ^{3}H-^{4}He scattering

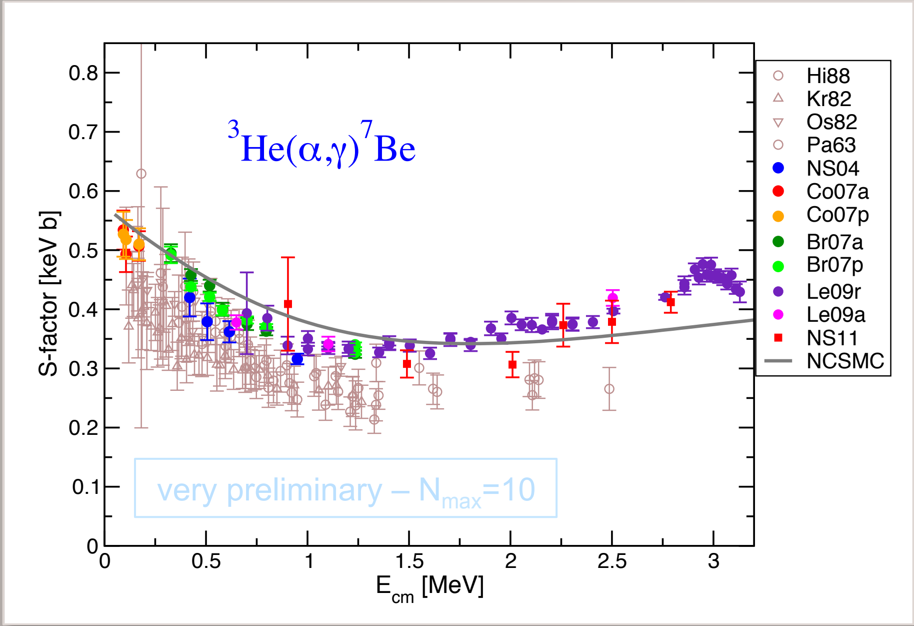


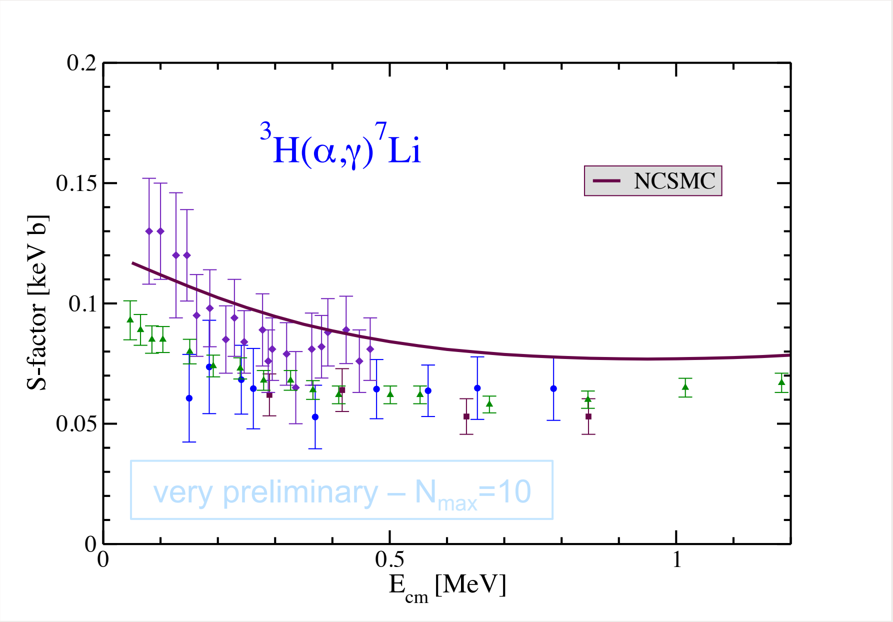


NCSMC calculations with chiral SRG-N^{3}LO *NN* potential (λ=2.1 fm^{-1})
^{3}He, ^{3}H, ^{4}He ground state, 8(π-) + 8(π+) eigenstates of ^{7}Be and ^{7}Li
Preliminary: N_{max}=10, hΩ=20 MeV
E_{th}(^{7}Be)=-1.32 MeV (Expt. -1.59 MeV)
E_{th}(^{7}Li) = -2.20 MeV (Expt. -2.47 MeV)

Goal: Calculations of **^{3}He(^{4}He,γ)^{7}Be & ^{3}H(^{4}He,γ)^{7}Li** capture

^{3}He-^{4}He and ^{3}H-^{4}He scattering





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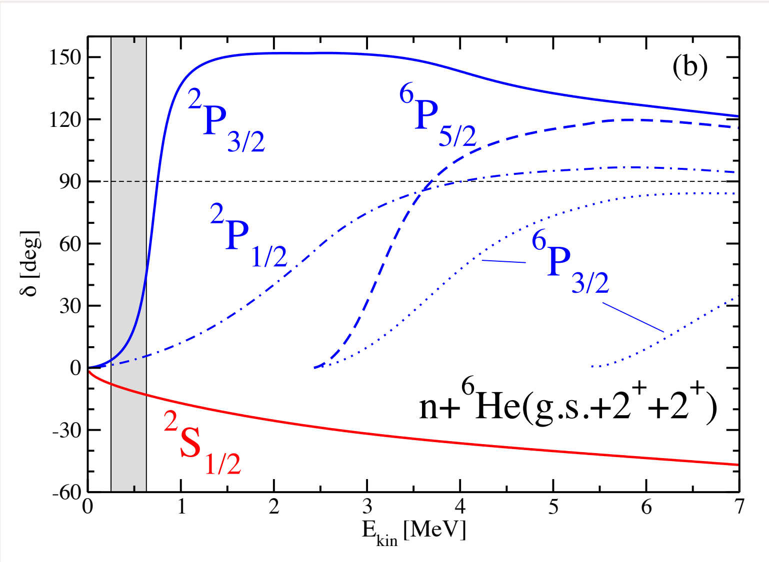
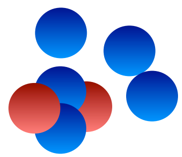
Goal: Calculations of **^{3}He(^{4}He,γ)^{7}Be & ^{3}H(^{4}He,γ)^{7}Li** capture

NCSM with continuum: ^{7}He $\leftrightarrow$ ^{6}He+n



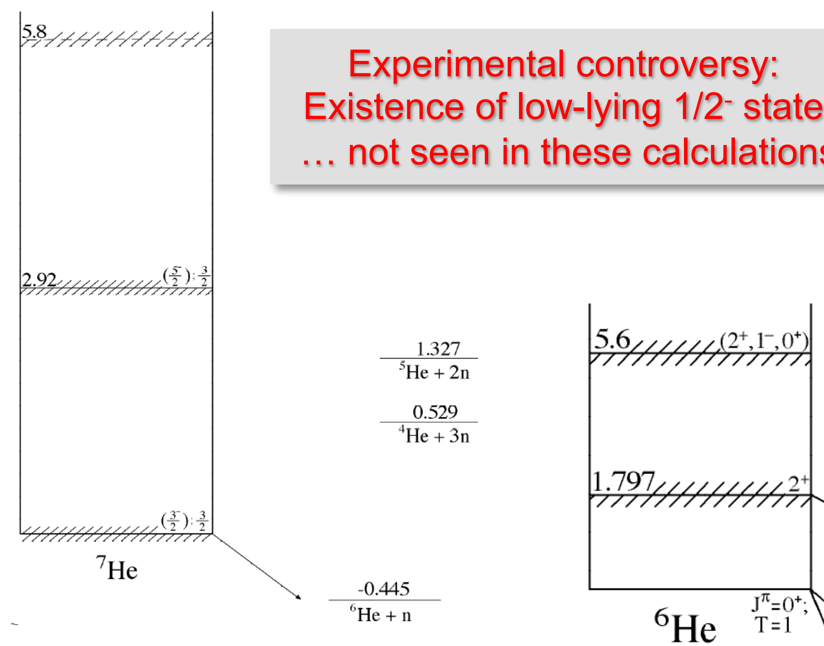
^{7}He



unbound

J^{π}	experiment			NCSMC	
	E_R	Γ	Ref.	E_R	Γ
$3/2^-$	0.430(3)	0.182(5)	[2]	0.71	0.30
$5/2^-$	3.35(10)	1.99(17)	[40]	3.13	1.07
$1/2^-$	3.03(10)	2	[11]	2.39	2.89
	3.53	10	[15]		
	1.0(1)	0.75(8)	[5]		

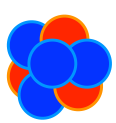
[11] A. H. Wuosmaa *et al.*, Phys. Rev. C **72**, 061301 (2005).



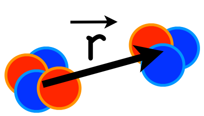
Experimental controversy:
Existence of low-lying 1/2$^-$ state
… not seen in these calculations

NCSMC
with three ^{6}He states
and ten ^{7}He eigenstates
More **7-nucleon correlations**
Fewer ^{6}He-core states needed

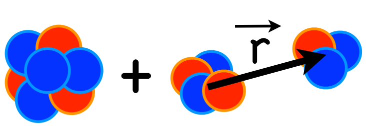
NCSM



NCSM/RGM

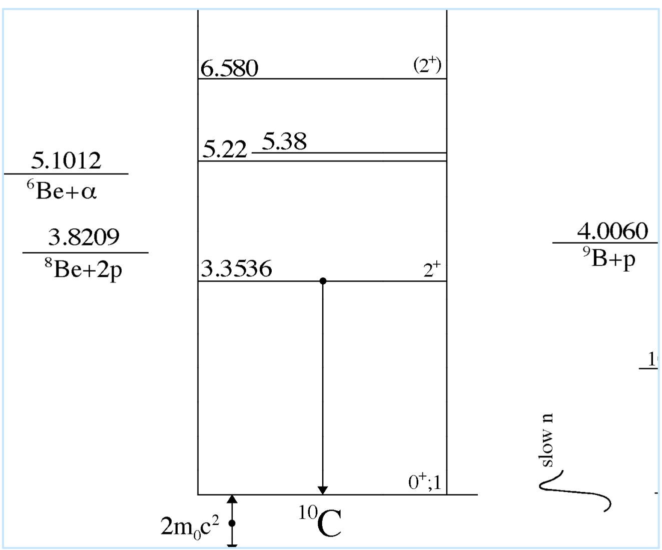


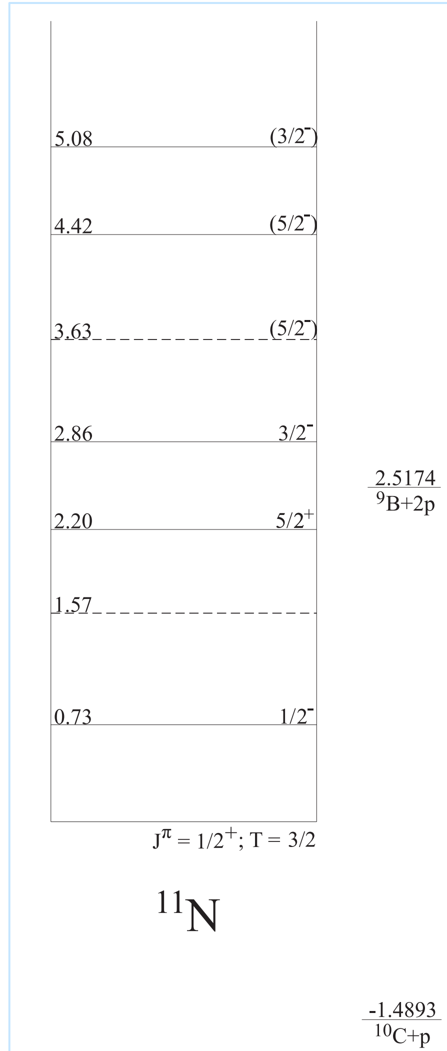
NCSMC

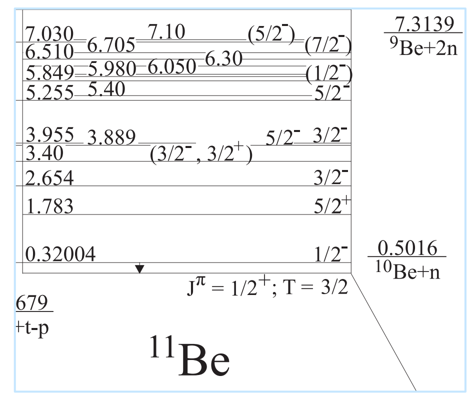


p+^{10}C scattering: structure of ^{11}N resonances

- Limited information about the structure of proton rich ^{11}N – mirror nucleus of ^{11}Be halo nucleus
- Incomplete knowledge of ^{10}C unbound excited states
- Importance of 3N force effects and continuum

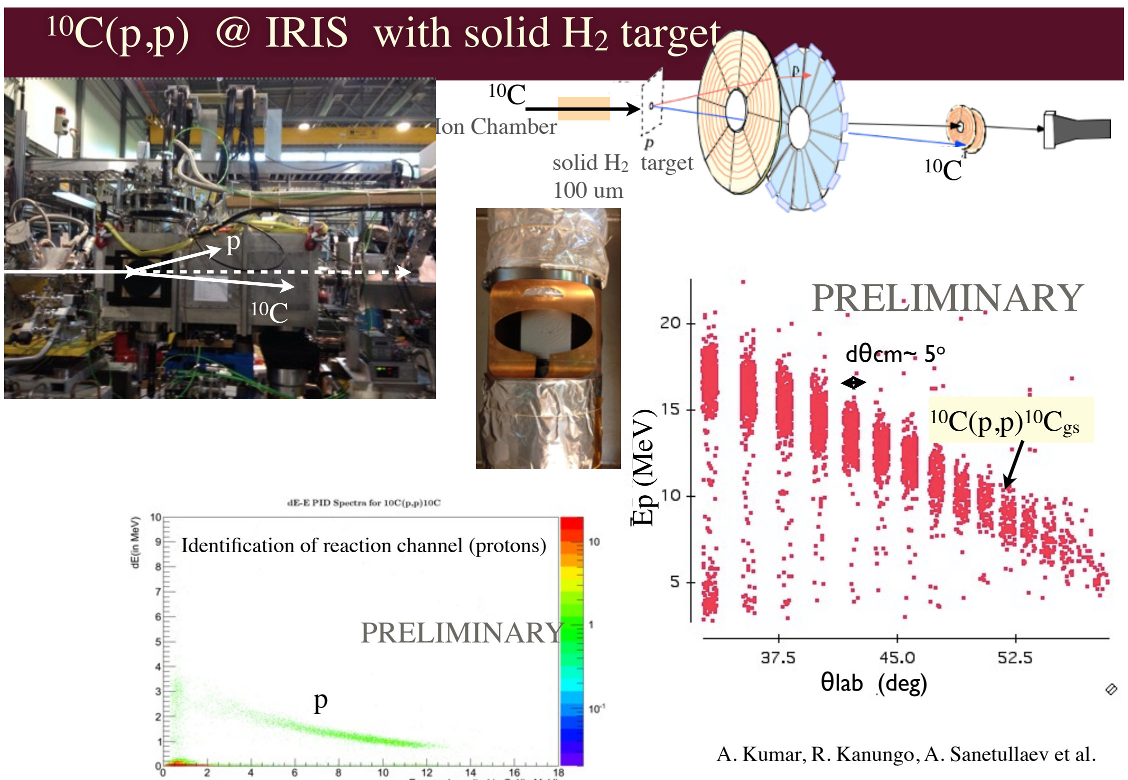






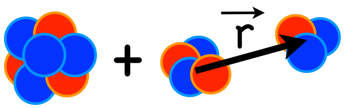
$^{10}C(p,p)$ @ IRIS with solid H_2 target

- New experiment at ISAC TRIUMF with reaccelerated ^{10}C
  - The first ever ^{10}C beam at TRIUMF
  - Angular distributions measured at E_{CM} ~ 4.1 MeV and 4.4 MeV
  - Data analysis under way

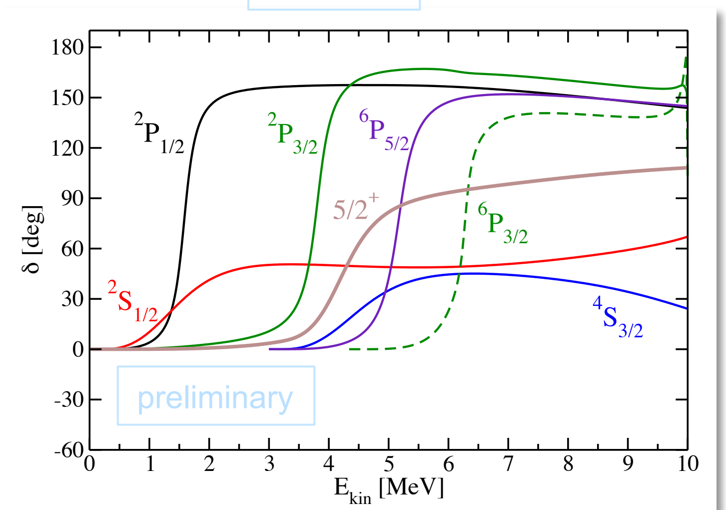


p+^{10}C scattering: structure of ^{11}N resonances

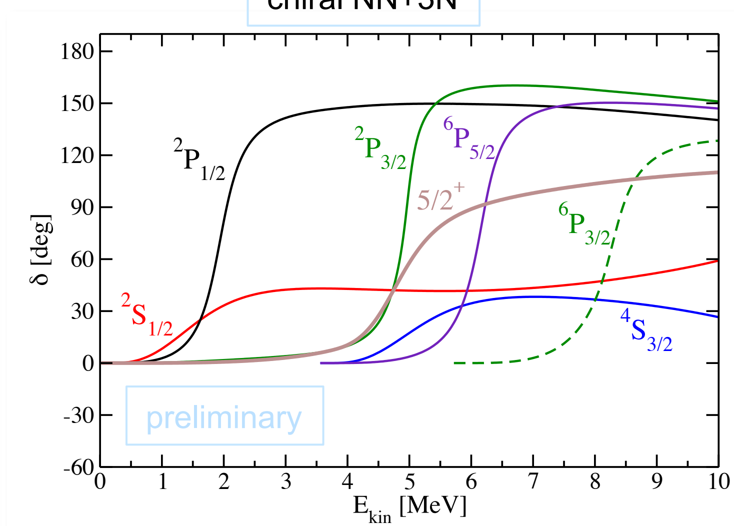
- NCSMC calculations **including chiral 3N**
  - p-^{10}C + ^{11}N
    - ^{10}C: 0^+, 2^+, 2^+ NCSM eigenstates
    - ^{11}N: 6 π = -1 and 3 π = +1 NCSM eigenstates
    - N_{max}= 7, N_{max}=9 under way

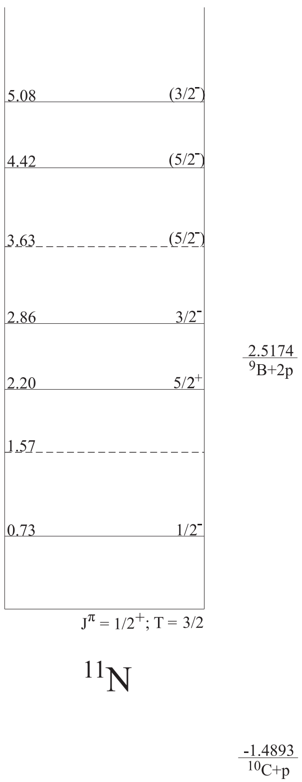


chiral NN



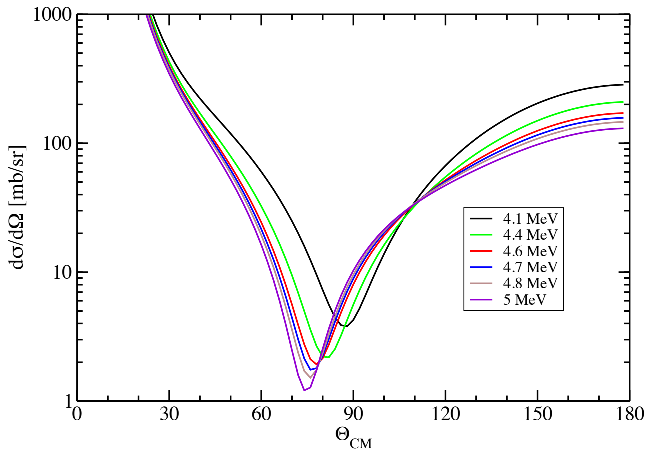
chiral NN+3N

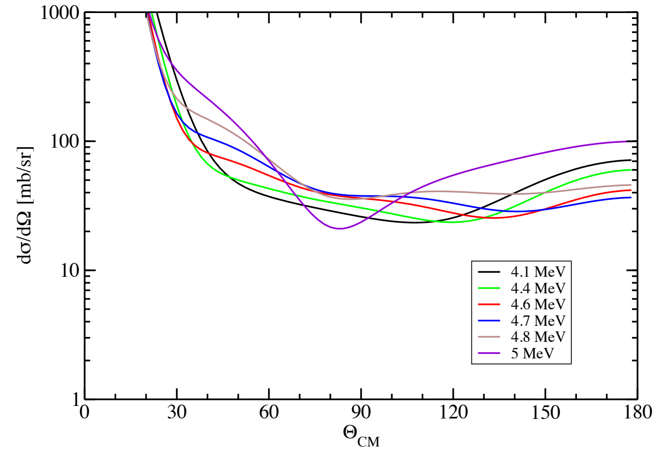




With the 3N the $^2P_{1/2}$ and $^2P_{3/2}$ resonances broader and shifted to higher energy in a better agreement with experiment

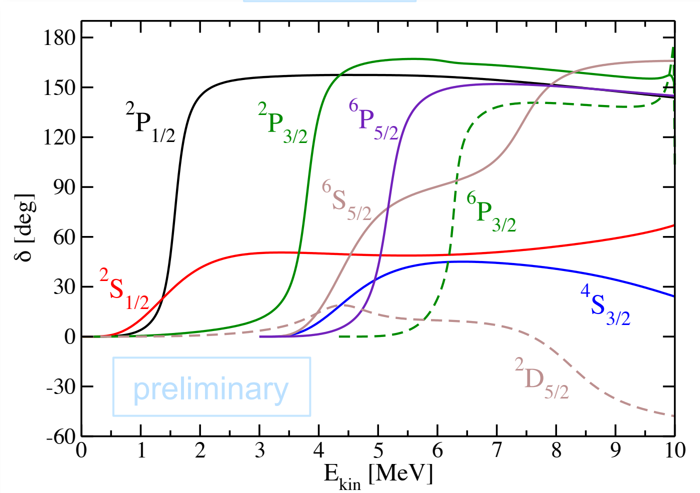
p+^{10}C scattering: Elastic differential cross section

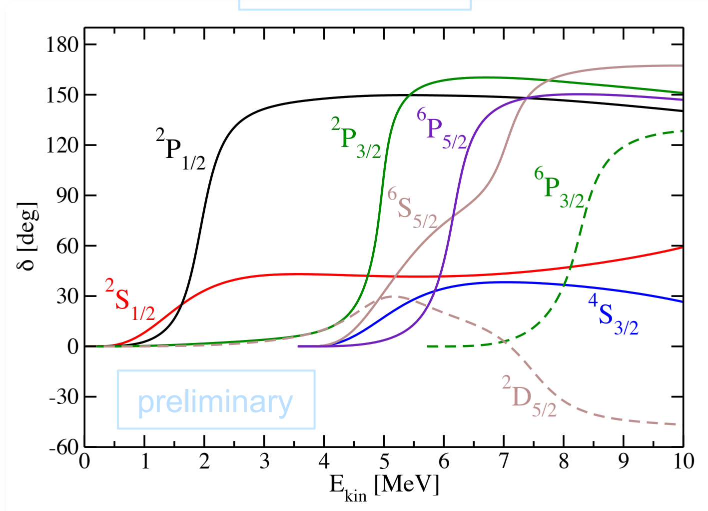


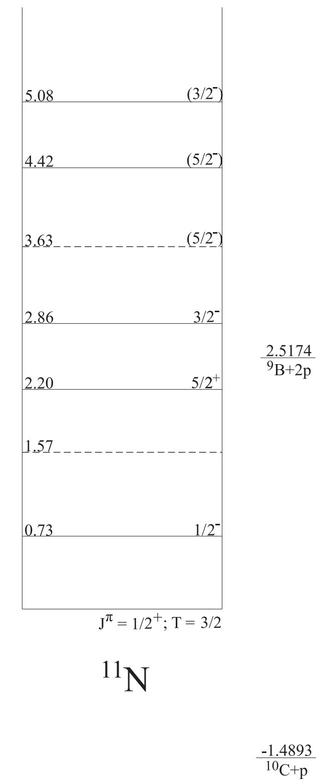


chiral NN

chiral NN+3N







Significant difference in angular distributions in the experimentally explored energy range due to the shift of the $^2P_{3/2}$ resonance

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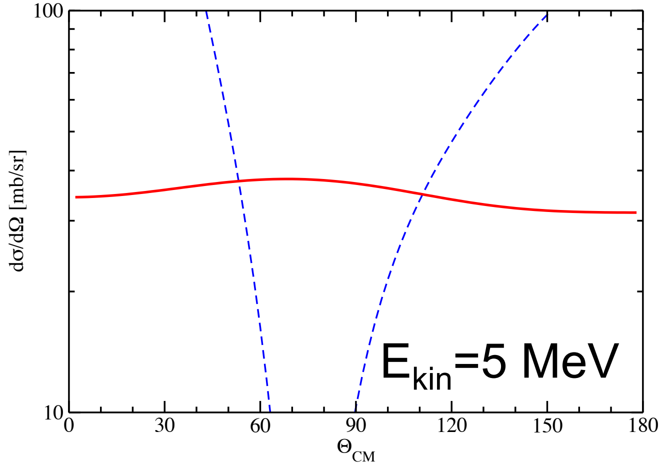
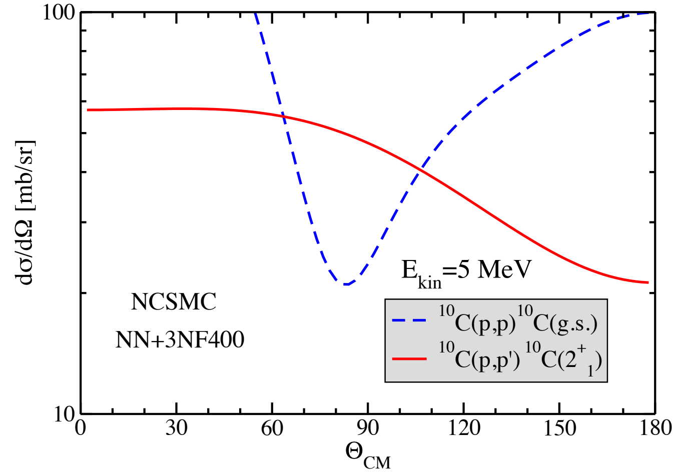
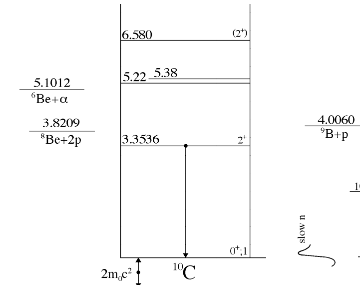
$^{10}C(p,p')^{10}C(2^+_1)$ scattering: Differential cross section

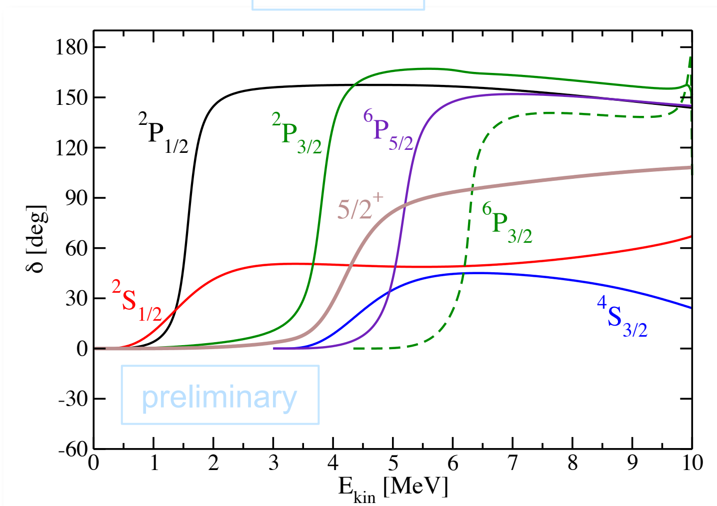


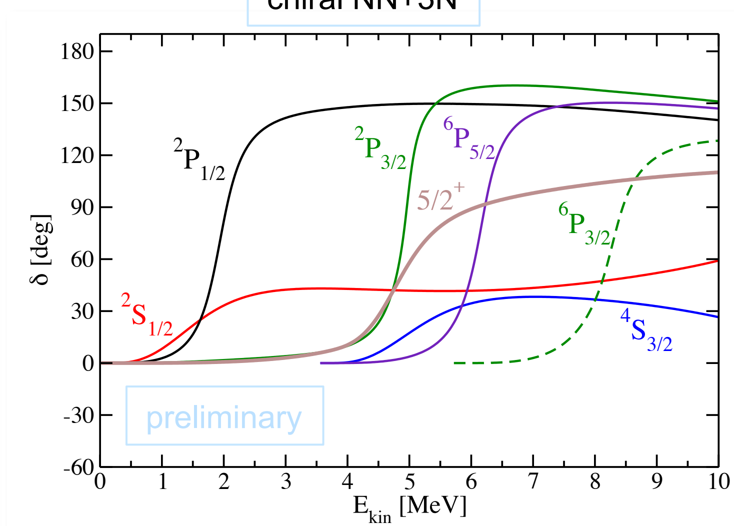
chiral NN

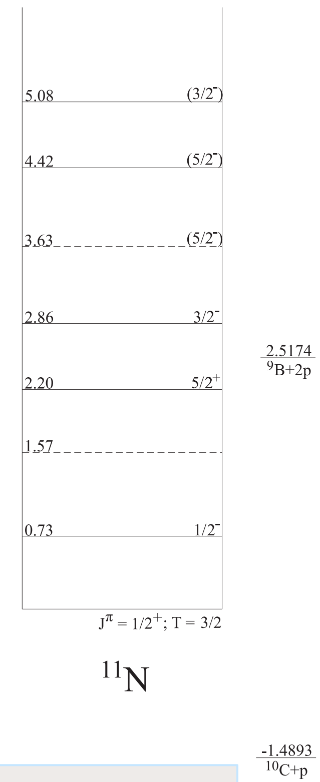


chiral NN+3N



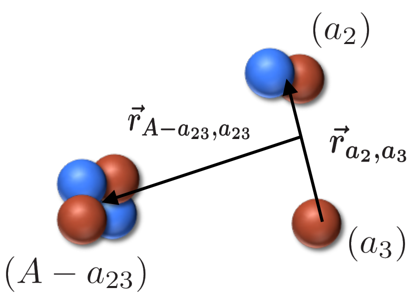






Significant difference in the shape of the inelastic differential g.s. to 2^+_1 cross section around E_{kin} ~ 5 MeV
The shape determined by an interference of $5/2^+$ and $3/2^-$ resonances

NCSM/RGM for three-body clusters: Structure of ^{6}He

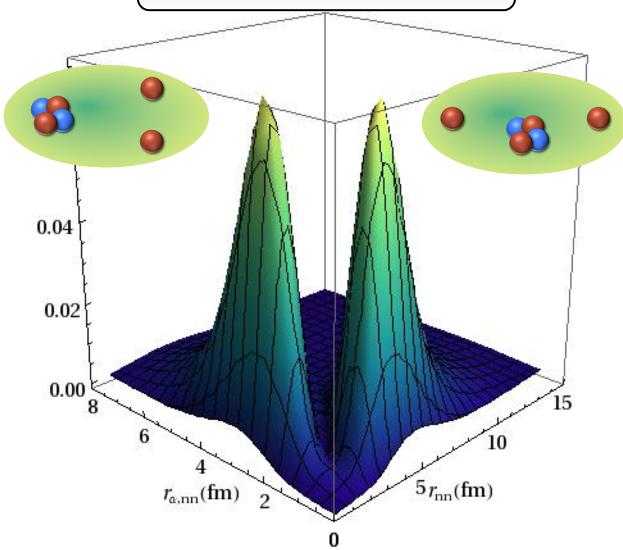


$$|\Phi^{J^\pi T}_{\nu r}\rangle \sim \underbrace{\Psi^{A-a_{23}}_{\alpha_1}\ \Psi^{a_2}_{\alpha_2}\ \Psi^{a_3}_{\alpha_3}}_{\text{NCSM}}$$

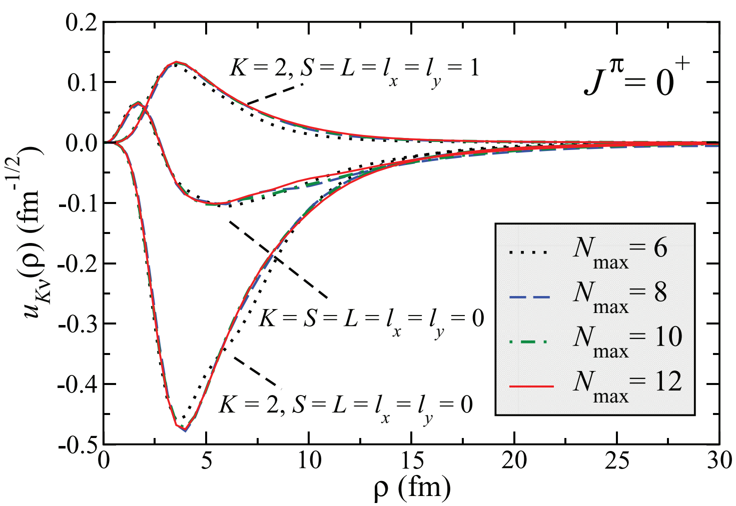
$$\boxed{|\Psi^{J^\pi T}\rangle = \sum_\nu \int dx\, x^2 \int dy\, y^2 G_\nu^{J^\pi T}(x,y)\hat{A}_\nu |\Phi^{J^\pi T}_{\nu x y}\rangle}$$

^{4}He(g.s.)+n+n

$\ell_x = \ell_y = L = S_{nn} = 0$



^{6}He ground state calculation with proper asymptotic conditions



PHYSICAL REVIEW C **88**, 034320 (2013)

Three-cluster dynamics within an *ab initio* framework

Sofia Quaglioni,[1,*] Carolina Romero-Redondo,[2,†] and Petr Navrátil[2,‡]

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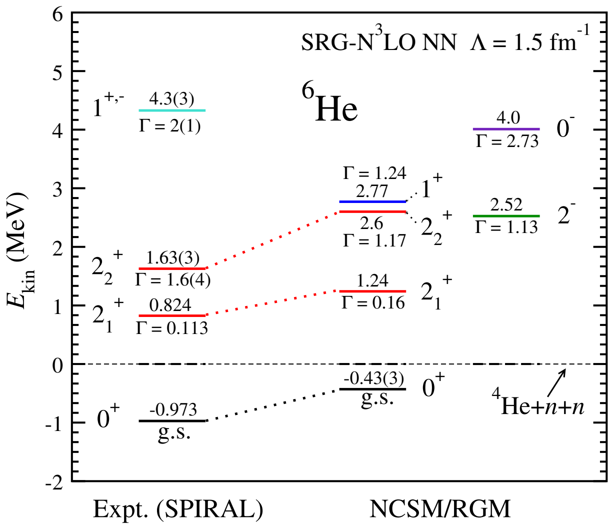
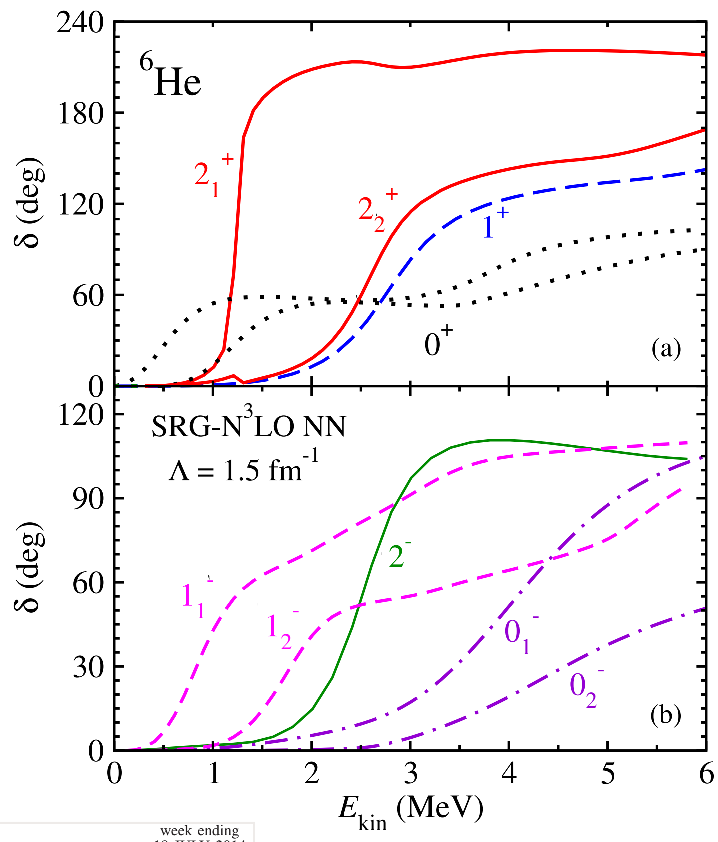
NCSM/RGM for three-body clusters: Structure of ^6He

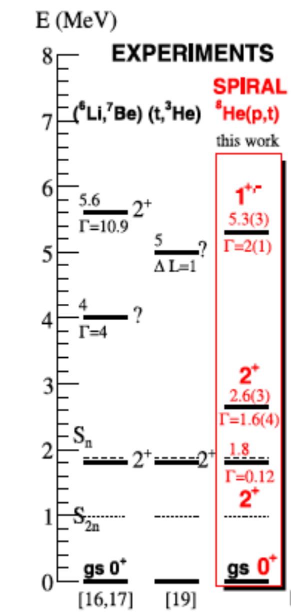
^4He(g.s.)+n+n

Soft SRG-evolved chiral N^{3}LO NN potential, λ=1.5 fm^{-1}

Recent experiment:
PLB 718 (2012) 441







Narrow 2$^+$ resonance

A second low-lying broader 2$^+$ resonance: Found in recent GANIL experiment

2$^-$, 1$^+$ and 0$^-$ resonances

0$^+$ and 1$^-$ very broad

PRL **113**, 032503 (2014) PHYSICAL REVIEW LETTERS week ending 18 JULY 2014

^{4}He + n + n Continuum within an *Ab initio* Framework

Carolina Romero-Redondo,[1,*] Sofia Quaglioni,[2,†] Petr Navrátil,[1,‡] and Guillaume Hupin[2,§]
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Conclusions and Outlook

- *Ab initio* calculations of nuclear structure and reactions is a dynamic field with significant advances

- We developed a new unified approach to nuclear bound and unbound states
  - Merging of the NCSM and the NCSM/RGM = **NCSMC**
  - Inclusion of three-nucleon interactions in reaction calculations for *A*>5 systems
  - Extension to three-body clusters (^{6}He ~ ^{4}He+n+n)
  - Applications to capture reactions important for astrophysics

- Outlook:
  - Extension to composite projectiles (deuteron, ^{3}H, ^{3}He)
  - Transfer reactions
  - Bremsstrahlung
  - Alpha-clustering (^{4}He projectile)
    - ^{12}C and Hoyle state: ^{8}Be+^{4}He
    - ^{16}O: ^{12}C+^{4}He

NCSMC and NCSM/RGM collaborators

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Joachim Langhammer, Robert Roth (TU Darmstadt)

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Guillaume Hupin (Notre Dame)

Simone Baroni (ULB)

Wataru Horiuchi (Hokkaido)