The theory of double beta decay
and
determination of neutrino mass

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D. Frekers (Muenster)
OUTLINE

- $0\nu\beta\beta$
- $0\nu\epsilon\epsilon$
- $0\nu\epsilon\beta$
- $m_{\beta\beta}$
- $0\nu\beta\beta$ NMEs
- CP-phases
- $\nu$ mass scale
- Nuclear structure
Neutrinoless Double-Beta Decay

\[(A,Z) \rightarrow (A,Z+2) + e^- + e^-\]

Study of the $0\nu\beta\beta$-decay is one of the highest priority issues in particle and nuclear physics.
What is the nature of neutrinos?

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

Only the $0\nu\beta\beta$-decay can answer this fundamental question

Analogy with kaons: $K_0$ and $\bar{K}_0$

Could we have both? (light Dirac and heavy Majorana)

Analogy with $\pi_0$
The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei.

\[
\left( T_{1/2}^{0\nu} \right)^{-1} = \left| \frac{m_{\beta\beta}}{m_e} \right|^2 g_A^4 |M_{0\nu}|^2 G_{0\nu}
\]

| transition | \(G_{01}(E_0, Z) \times 10^{14}\) | \(Q_{\beta\beta}\) [MeV] | Abund. (%) | \(|M_{0\nu}|^2\) |
|------------|-----------------|-----------------|-----------|----------------|
| \(^{150}\text{Nd} \rightarrow ^{150}\text{Sm}\) | 26.9 | 3.667 | 6 | ? |
| \(^{48}\text{Ca} \rightarrow ^{48}\text{Ti}\) | 8.04 | 4.271 | 0.2 | ? |
| \(^{96}\text{Zr} \rightarrow ^{96}\text{Mo}\) | 7.37 | 3.350 | 3 | ? |
| \(^{116}\text{Cd} \rightarrow ^{116}\text{Sn}\) | 6.24 | 2.802 | 7 | ? |
| \(^{136}\text{Xe} \rightarrow ^{136}\text{Ba}\) | 5.92 | 2.479 | 9 | ? |
| \(^{100}\text{Mo} \rightarrow ^{100}\text{Ru}\) | 5.74 | 3.034 | 10 | ? |
| \(^{130}\text{Te} \rightarrow ^{130}\text{Xe}\) | 5.55 | 2.533 | 34 | ? |
| \(^{82}\text{Se} \rightarrow ^{82}\text{Kr}\) | 3.53 | 2.995 | 9 | ? |
| \(^{76}\text{Ge} \rightarrow ^{76}\text{Se}\) | 0.79 | 2.040 | 8 | ? |

The NMEs for \(0\nu\beta\beta\)-decay must be evaluated using tools of nuclear theory.
Neutrinoless double beta decay of $^{110}\text{Pd}$

With its high natural abundance, the new results reveal $^{110}\text{Pd}$ to be an excellent candidate for double-$\beta$ decay studies.

Q-Value and Half-Lives for the Double-Beta-Decay Nuclide $^{110}\text{Pd}$

D. Fink, et al.


<table>
<thead>
<tr>
<th></th>
<th>$^{82}\text{Se}$</th>
<th>$^{110}\text{Pd}$</th>
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<tbody>
<tr>
<td>$Z$</td>
<td>34</td>
<td>46</td>
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<tr>
<td>Abund. (%)</td>
<td>8.73</td>
<td>11.72</td>
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<tr>
<td>$Q$ [keV]</td>
<td>2 995</td>
<td>2 017.8</td>
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<tr>
<td>$G^{0\nu}$ [$10^{-15}$ yr$^{-1}$]</td>
<td>10.16</td>
<td>4.815</td>
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<tr>
<td>$0\nu\beta\beta$ NME</td>
<td>4.64</td>
<td>5.76</td>
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<tr>
<td>$T^{2\nu}_{1/2}$ [yr]</td>
<td>$0.92 \times 10^{20}$</td>
<td>$1.5(6) \times 10^{20}$ (SSD)</td>
</tr>
</tbody>
</table>
Effective Majorana neutrino mass

\[
\left( T_{1/2}^{0\nu} \right)^{-1} = \left| \frac{m_{\beta\beta}}{m_e} \right|^2 g_A^4 \left| M_{\nu}^{0\nu} \right|^2 G^{0\nu}
\]
An accurate knowledge of the nuclear matrix elements, which is not available at present, is however a pre-requisite for exploring neutrino properties.
Neutrinos mass spectrum

mass differences:
$|\Delta m^2_{\text{sol}}| = 7.65 \times 10^{-5} \text{ eV}^2$
$\Delta m^2_{\text{atm}} = 2.43 \times 10^{-3} \text{ eV}^2$

Tritium decay

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2}$$

Cosmology

$$\sum_{i=1}^{3} m_i$$
Daya Bay: $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005 \quad \text{(March 2012)}$

$$|m_{\beta\beta}^{(3 \nu)}| = |c_{12}^2 c_{13}^2 e^{2i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{2i\alpha_2} m_2 + s_{13}^2 m_3|$$

**Issue: Lightest neutrino mass $m_0$**

- Claim for evidence
- IS
- NS
- disfavored by cosmology

**GUT’s**

(Rodejohann pres.)
On the possibility of measuring CP Majorana phases in the $0\nu\beta\beta$-decay

**Majorana phases**

\[ P = \text{diag}(e^{-i\alpha_1/2}, e^{-i\alpha_2/2}, e^{-i\alpha_3/2}) \]
\[ \alpha_3/2 = \delta \]

\[ |m_{\beta\beta}| = |c_{12}^2 c_{13}^2 e^{i\alpha_1} m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3| \]

**Measured quantity**

\[ |m_{\beta\beta}|^2 = c_{12}^4 c_{13}^4 m_1^2 + s_{12}^4 c_{13}^4 m_2^2 + s_{13}^4 m_3^2 \]
\[ + 2c_{12}^2 s_{12}^2 c_{13}^2 m_1 m_2 \cos (\alpha_1 - \alpha_2) \]
\[ + 2c_{12}^2 c_{13}^2 s_{13}^2 m_1 m_3 \cos \alpha_1 + 2s_{12}^2 c_{13}^2 s_{13}^2 m_2 m_3 \cos \alpha_2. \]

**Normal hierarchy**

\[ m_1 \ll \sqrt{\Delta m_{\text{SUN}}^2} \]
\[ m_2 \approx \sqrt{\Delta m_{\text{SUN}}^2} \]
\[ m_3 \approx \sqrt{\Delta m_{\text{ATM}}^2} \]

**Inverted hierarchy**

\[ m_3 \ll \sqrt{\Delta m_{\text{ATM}}^2} \]
\[ m_1 \approx m_2 \approx \sqrt{\Delta m_{\text{ATM}}^2} \]

**Assuming lightest neutrino mass to be zero**

\[ \cos \alpha_2 \approx \frac{|m_{\beta\beta}|^2 - s_{12}^4 c_{13}^4 \Delta m_{\text{SUN}}^2 - s_{13}^4 \Delta m_{\text{ATM}}^2}{2s_{12}^2 c_{13}^2 s_{13}^2 \sqrt{\Delta m_{\text{SUN}}^2 \Delta m_{\text{ATM}}^2}} \]

\[ \cos \alpha_{12} = \frac{|m_{\beta\beta}|^2 - c_{13}^4 (1 - 2s_{12}^2 c_{12}^2) \Delta m_{\text{ATM}}^2}{2c_{12}^2 s_{12}^2 c_{13}^4 \Delta m_{\text{ATM}}^2} \]
\[ |m_{\beta\beta}| = \frac{1}{\sqrt{T_{1/2}^0 G^{0\nu}(Q_{\beta\beta}, Z)|M^{0\nu}|}} \]

\[ \frac{\sigma_{\beta\beta}}{|m_{\beta\beta}|_{\text{obs}}} = \sqrt{\frac{1}{4} \left( \frac{\sigma_{\text{exp}}}{T_{1/2}^{0\nu-\text{obs}}} \right)^2 + \left( \frac{\sigma_{\text{th}}}{|M^{0\nu}|} \right)^2} \]

0%, 15%, 25%

Averaged over second phase and \( m_0 \leq 10 \text{ meV} \)
Phase-space factor

\[ (T_{1/2}^{0\nu})^{-1} = \left| \frac{m_{\beta\beta}}{m_e} \right|^2 g_A^4 |M_{0\nu}^0|^2 G^{0\nu} \]
Dirac equations:

\[ \psi_{\epsilon \kappa \mu}(r) = \left( \frac{g_\kappa(\epsilon, r) \chi^\mu_\kappa}{if_\kappa(\epsilon, r) \chi^-_\kappa}, \right) \]

Finite nuclear size

WF1: approx. sol. inside nucl. (M. Rose approach)

WF2: numerical solution of Dirac eq.

WF3: numerical solution of Dirac eq. with consideration of electron screening

\[ \begin{align*}
\frac{dg_\kappa(\epsilon, r)}{dr} &= -\frac{\kappa}{r} g_\kappa(\epsilon, r) + \frac{\epsilon - V + m_e c^2}{\hbar} f_\kappa(\epsilon, r) \\
\frac{df_\kappa(\epsilon, r)}{dr} &= -\frac{\epsilon - V - m_e c^2}{\hbar} g_\kappa(\epsilon, r) + \frac{\kappa}{r} f_\kappa(\epsilon, r)
\end{align*} \]
Nucleus is considered to be spherical
Nuclear Matrix Elements (NMEs)

\[
(T_{1/2}^{0\nu})^{-1} = \left| \frac{m_{\beta\beta}}{m_e} \right|^2 g_A^4 \left| M_{0\nu}^{0\nu} \right|^2 G^{0\nu}
\]
In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited ($0^+$, $2^+$) states of the final nucleus.

It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them.

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge directly the quality of the result.
Many-body Hamiltonian

- Start with the many-body Hamiltonian
  \[ H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{NN}(r_i - r_j) \]

- Introduce a mean-field \( U \) to yield basis
  \[ H = \sum_i \left( \frac{p_i^2}{2m} + U(r_i) \right) + \sum_{i<j} V_{NN}(r_i - r_j) - \sum_i U(r_i) \]

  Residual interaction

The success of any nuclear structure calculation depends on the choice of the mean-field basis and the residual interaction!

- The mean field determines the shell structure
- In effect, nuclear-structure calculations rely on perturbation theory
Goeppert-Mayer and Haxel, Jensen, and Suess proposed the independent-particle shell model to explain the magic numbers 2, 8, 20, 28, 50, 82, 126, 184.

Harmonic oscillator with spin-orbit is a reasonable approximation to the nuclear mean field.

Nuclear structure approaches

In QRPA a large valence space is used, but only a class of configurations is included. Describe collective states, but not details of dominantly few particle states. Relative simple, thus more 0nbb-decay calculations.

In ISM a limited valence space is used but all configurations of valence nucleons are included. Describes well properties of low-lying nuclear states.

IBM: The low-lying states of the nucleus are modeled in terms of $L=0,2$ bosons. They interact through one-body and two-body forces giving rise to bosonic wave functions.

PHFB: Mean field approach based on the Bogoliubov-Hartree-Fock method and Projection. States have good angular momentum due projection.

EDF: Beyond mean field effects are included within generating coordinate method with particle and angular projections.

\[ ^76\text{Ge} \rightarrow ^76\text{Se} \]
0νββ-decay matrix elements

\[
M^{0\nu} = \frac{4\pi R}{g_A^2} \int \left( \frac{1}{(2\pi)^3} \int \frac{e^{-i\vec{q} \cdot (\vec{x}_1 - \vec{x}_2)}}{|q|} \right) \times \sum_m \frac{<0_f^+ | J_{\alpha}^\dagger (\vec{x}_1) | m > < m | J_{\alpha}^\dagger (\vec{x}_2) | 0_i^+ >}{E_m - (E_i + E_f)/2 + |q|} dq d\vec{x}_1 d\vec{x}_2
\]

Weak hadron current

\[
j^{\rho\dagger} = \bar{\Psi} \tau^+ \left[ g_V(q^2) \gamma^\rho + ig_M(q^2) \frac{\sigma^{\rho\nu}}{2m_p} q_\nu - g_A(q^2) \gamma^\rho \gamma_5 - g_P(q^2) q^\rho \gamma_5 \right] \Psi,
\]

Formfactor

\[
g_V(\vec{q}^2) = \frac{g_V}{(1 + \vec{q}^2/M_V^2)^2}
\]

\[
g_A(\vec{q}^2) = \frac{g_A}{(1 + \vec{q}^2/M_A^2)^2}
\]

Weak hadron current in a Breit frame

\[
J^{\rho\dagger}(\vec{x}) = \sum_{n=1}^A \tau_{n}^+ [g^{\rho 0} J^0(\vec{q}^2) + \sum_k g^{\rho k} J_n^k(\vec{q}^2)] \delta(\vec{x} - \vec{r}_n)
\]

\[
J^0(\vec{q}^2) = g_V(q^2)
\]

\[
\vec{J}_n(\vec{q}^2) = g_M(\vec{q}^2) i \frac{\vec{\sigma}_n \times \vec{q}}{2m_p} + g_A(\vec{q}^2) \vec{\sigma} - g_P(\vec{q}^2) \frac{\vec{q} \cdot \vec{\sigma}_n}{2m_p} \frac{\vec{q}}{2m_p}
\]
One two-body operators

\[ \langle p|O(1)|n\rangle \langle p'|O(2)|n'\rangle = \langle p, p'|O'(1, 2)|n, n'\rangle \]

Integration over angular part of \(\nu\) momentum

\[
\int e^{i\vec{q} \cdot \vec{r} - \vec{r}_2} d\Omega_q = \int e^{i\vec{q} \cdot \vec{r}} d\Omega_q = \\
\sqrt{4\pi} \ 4\pi \sum_{l} l! j_l(qr) Y_{lm}(\Omega_r) \int Y_{lm}^*(\Omega_q) Y_{00}(\Omega_q) d\Omega_q = 4\pi j_0(qr)
\]

Neutrino potential

\[
O_F(r_{12}, E_{J\pi}^k) = \tau^+(1)\tau^+(2) H_F(r_{12}, E_{J\pi}^k), \\
O_{GT}(r_{12}, E_{J\pi}^k) = \tau^+(1)\tau^+(2) H_{GT}(r_{12}, E_{J\pi}^k)\sigma_{12}, \\
O_T(r_{12}, E_{J\pi}^k) = \tau^+(1)\tau^+(2) H_T(r_{12}, E_{J\pi}^k)S_{12}
\]

\[
H_K(r_{12}, E_{J\pi}^k) = \\
\frac{2}{\pi g_A^2 R} \int_0^\infty f_K(qr_{12}) \frac{h_K(q^2)q dq}{q + E_{J\pi}^k - (E_i + E_f)/2}
\]

Neutrino potential

\[
\sigma_{12} = \vec{\sigma}_1 \cdot \vec{\sigma}_2, \\
S_{12} = 3(\vec{\sigma}_1 \cdot \hat{r}_{12})(\vec{\sigma}_2 \cdot \hat{r}_{12}) - \sigma_{12}
\]

Nuclear matrix element

\[
M^{0\nu} = -\frac{M_F}{g_A^2} + M_{GT} - M_T
\]

\[
M_K = \sum_{J^+, k, k_f, J, \text{pnp'n'}} \sum (-1)^{j_n+j_{p'}+J+J} \times \\
\sqrt{2J + 1} \begin{pmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & J \end{pmatrix} \times \\
\langle p(1), p'(2); J || \bar{f}(r_{12})O_K f(r_{12}) || n(1), n'(2); J \rangle \times \\
\langle 0^+_f || [c_{p'}^+ \tilde{c}_n']_J || J^\pi k_f \rangle \langle J^\pi k_f || J^\pi k_i \rangle \langle J^\pi k_i || 0^+_i \rangle
\]

9/18/2013
Calculation of two-body matrix elements

From j-j to LS coupling

\[ \mathcal{M}^{2\text{body}} = \langle a(1), b(2); J'|O(1, 2)|c(1)d(2); J' \rangle \]

\[ |n_c l_c j_c, n_d l_d j_d; J' M' \rangle = \sum_{SL} \hat{S}^2 \hat{L}^2 \hat{j}_c \hat{j}_d \begin{bmatrix} 1/2 & l_c & j_c \\ 1/2 & l_d & j_d \\ S & L & J' \end{bmatrix} |n_c l_c, n_d l_d, SL; J' M' \rangle \]

Moshinsky transformation to relative coordinates

\[ |n_c l_c n_d l_d; LM_L \rangle = \sum_{n_l N L} \langle n l, N L, L|n_c l_c, n_d l_d, L \rangle |n l, N L; LM_L \rangle \]

Two-body m.e.

\[ \mathcal{M}_{F, GT}^{2\text{body}} = \hat{J}' \sum_{SL} \hat{S} \hat{L} \hat{j}_a \hat{j}_b \hat{j}_c \hat{j}_d \begin{bmatrix} 1/2 & l_c & j_c \\ 1/2 & l_d & j_d \\ S & L & J' \end{bmatrix} \begin{bmatrix} 1/2 & l_a & j_a \\ 1/2 & l_b & j_b \\ S & L & J' \end{bmatrix} \]

\times \sum_{n_l N L} \sum_{n_l' N L'} \langle n l, N L, L|n_c l_c, n_d l_d, L \rangle \langle n l', N L', L|n_a l_a, n_b l_b, L \rangle \langle n l', N L'|j_0(q|r_i,j)|nl, N L; L \rangle \langle s_a s_b; S|| \frac{1}{\vec{\sigma}_1 \cdot \vec{\sigma}_2}||s_c s_d; S \rangle \]

\[ \langle n l', N L'|j_0(q|r_i,j)|nl, N L; L \rangle = \delta_{l' l} \delta_{N N'} \delta_{L L'} \langle n l'|j_0(q|r_i,j)|nl \rangle \]

Fermi:

\[ \langle s_a s_b; S||\vec{\sigma}_1 \cdot \vec{\sigma}_2||s_c s_d; S \rangle = \hat{S}(\delta_{S1} - 3\delta_{S0}) \]

Gamow-Teller:

\[ \langle s_a s_b; S||1||s_c s_d; S \rangle = \hat{S}(\delta_{S1} + \delta_{S0}) \]
QRPA and isospin symmetry restoration
F.Š., V. Rodin, A. Faessler, and P. Vogel
PRC 87, 045501 (2013)
$2\nu\beta\beta$-decay NMEs

\[
\frac{1}{T^{2\nu-\text{exp}}_{1/2}} = G^{2\nu}(E_0, Z) \, g_A^4 \, |M_{GT}^{2\nu}|^2
\]

$g_{pp}$ adjusted to $2\nu\beta\beta$-decay half-life
Separation of $g_{pp}$ into $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$

Close values $\text{red}$ and $\text{blue}$ $\Rightarrow$ no new parameter
$M^{2\nu}_{F}$ depends strongly on $g_{pp}^{T=1}$

$M^{2\nu}_{GT}$ depend very weakly on $g_{pp}^{T=1}$
\[ M_{\text{GT-cl}}^{2\nu} = \int_0^\infty C_{\text{GT-cl}}^{2\nu}(r) dr \]

\( C_{\text{GT-cl}}^{2\nu} \) scaled by factor 1/3
Multipole decomposition

$M_{0v}^{0\nu}(\pi)$

old par.
new par.
\[ M^{0\nu} = M_{GT}^{0\nu} \left( 1 + \frac{1}{2} \frac{M_F^{0\nu}}{g_A M_{GT}^{0\nu}} + \frac{M_T^{0\nu}}{M_{GT}^{0\nu}} \right) \]
Differences: mean field; residual int.; size of the m.s.; many-body appr.

**ISM:** Menendez et al. NPA 818 (2009) 139

**EDF:** Rodriguez, Martinez-Pinedo, PRL (2010) 105

**IBM:** Barea, Kotila, Iachello, PRC (2013) 014315

**PHFB:** K. Rath et al., PRC 85 (2012) 014308
QRPA uncertainties and their correlations in the analysis of $0\nu\beta\beta$ decay

A. Faessler, G.L. Fogli, E. Lisi, V. Rodin, A. M. Rotunno, F.Š.,
PRD 87, 053002 (2013)

Effects of isospin restoration not included yet.

For each nucleus $2 \times 2 \times 2 \times 3 = 24$ NMEs
Range of half-lives preferred at 90% C.L. by the $0\nu\beta\beta$ claim of evidence compared with the 90% exclusion limits placed by other experiments.

The comparison involves the NME and their errors as well as their correlations.
Range of $m_{\beta\beta}$ allowed by the $0\nu\beta\beta$ claim of evidence compared with the limits placed by other experiments (All at 90% C.L.).

Before GERDA data have appeared:
Theoretical and experimental constraints in the plane charted by the $0\nu\beta\beta$ half-lives of $^{76}\text{Ge}$ and $^{136}\text{Xe}$.

Horizontal band: range preferred by claim. Slanted band: constraint placed by our QRPA estimates. The combination provides the shaded ellipse, whose projection on the abscissa gives the range preferred at 90% C.L. for the $^{136}\text{Xe}$ half-life.
Allowed regions (ellipses) as derived from Klapdor’s claim in the plane charted by the half-lives of \(^{136}\text{Xe}\) and each of six nuclei.

A large fraction of each ellipse is excluded by the combined EXO & KL-Zen results. (All bounds are at 90% C.L. on one variable.)
Some notes about the $0\nu\beta\beta$-decay NMEs
The $0\nu\beta\beta$-decay NMEs (Status: 2013)

Nobody is perfect:

\[ g_A = 1.25(7), \text{CCm or UCOM s.r.c., } r_0 = 1.20 \text{ fm} \]

Differences:
1) mean field;  
2) residual int.;  
3) size of the m.s.;  
4) many-body appr.

LSSM (small m.s., negative parity states)  
PHFB (GT force neglected)  
IBM (Hamiltonian truncated)  
(R)QRPA (g.s. correlations not accurate enough)
\[ \chi_F = \frac{M_{0v}^F}{M_{0v}^{GT}} \approx -\frac{1}{3} \]

Fermi:

\[ 1 = \Omega(S=0) + \Omega(S=1) \]

Gamow-Teller:

\[ \sigma \cdot \sigma = -3 \Omega(S=0) + \Omega(S=1) \]
Tensor part of the $0^{\nu}\beta\beta$ NME
(some disagreement)

ISM: effect is small, QRPA(J): negligible; PHFB, EDF: not calculated; QRPA(TBC), IBM: up to 10%
A comparison of QRPA calc.

QRPA(TBC): restor. of isospin, tensor (10%), self. const.-Argonne, \( g_A = 1.269, \) lms

QRPA(JL): Bonn-A, UCOM, \( g_A = 1.25, \) tensor (negligible), sms

WS not adjusted
Deformed QRPA

<table>
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<tr>
<th>Element</th>
<th>this work</th>
<th>Sk3</th>
<th>SG2</th>
<th>Ref. [25]</th>
<th>Ref. [26]</th>
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<td>$^{76}\text{Ge}$</td>
<td>0.184$^a$</td>
<td>0.161</td>
<td>0.157</td>
<td>0.095(30)</td>
<td>0.2623(9)</td>
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<tr>
<td>$^{76}\text{Se}$</td>
<td>-0.018</td>
<td>-0.181</td>
<td>-0.191</td>
<td>0.163(33)</td>
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<tr>
<td>$^{130}\text{Te}$</td>
<td>0.01</td>
<td>-0.076</td>
<td>-0.039</td>
<td>0.035(23)</td>
<td>0.1184(14)</td>
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<tr>
<td>$^{130}\text{Xe}$</td>
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<td>$^{136}\text{Xe}$</td>
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<td>0.001</td>
<td>0.016</td>
<td>-</td>
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<tr>
<td>$^{136}\text{Ba}$</td>
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<td>0.009</td>
<td>0.070</td>
<td>-</td>
<td>0.1258(12)</td>
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<td>$^{150}\text{Nd}$</td>
<td>0.27</td>
<td>0.266</td>
<td>0.271</td>
<td>0.367(86)</td>
<td>0.2853(21)</td>
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<tr>
<td>$^{150}\text{Sm}$</td>
<td>0.22</td>
<td>0.207</td>
<td>0.203</td>
<td>0.230(30)</td>
<td>0.1931(21)</td>
</tr>
</tbody>
</table>

$^{130}\text{Te}$ +0.035(23) 0.1184(14) -0.076 0.169(6) 0.108

**Skyrme int:** Mustonen, Engel, arXiv:1301.6997 [nucl-th]
**Argonne int:** Fang, Faessler, Rodin, F.Š., PRC 83 (2011) 034320
Quenching of $g_A$ and two-body currents

Menendez, Gazit, Schwenk, PRL 107 (2011) 062501; MEDEX13 contribution

The $0\nu\beta\beta$ operator calculated within effective field theory. Corrections appear as 2-body current predicted by EFT. The 2-body current contributions are related to the quenching of Gamow-Teller transitions found in nuclear structure calc. 

$g_A = 1.269$

$g_{\text{eff}}^A = 0.70$

$(1.269)^4 = 2.6$
On the relation between 0νββ-decay and 2νββ-decay (GT) NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

\[ M^{0\nu} = M^{0\nu}_{GT} \left( 1 + \frac{1}{g_A^2} \frac{M^{0\nu}_F}{M^{0\nu}_{GT}} + \frac{M^{0\nu}_T}{M^{0\nu}_{GT}} \right) \]
Going to relative coordinates:

\[ M_{GT-cl}^{2\nu} = \int_0^\infty C_{GT-cl}^{2\nu}(r) dr \]

**Neutrino potential**

\[ M_{GT}^{0\nu} = \int_0^\infty H_{GT}^{0\nu}(r) C_{GT-cl}^{2\nu}(r) dr \]

A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

Neutrino potential prefer short distances
$M^{0\nu}_{GT}$ depends weakly on $g_A/g_{pp}$ and QRPA approach unlike $M^{2\nu}_{GT}$.

$$M^{0\nu}_{GT}(r_0) = \int_0^{r_0} H^{0\nu}_{GT}(r) C^{2\nu}_{GT-cl}(r) dr$$

Different QRPA-like approaches depend on axial-vector coupling.

F.Š., Fedor Simkovic
There is no proportionality between 0νββ-decay and 2νββ-decay NMEs!!!

Frekers et al.
Charge exchange reactions
Heavy $\nu \, 0\nu\beta\beta$-decay NMEs
(type II see-saw)

LHC (scale!?)
and L-R symmetric models

Discrete LR symmetry to parity (U=V)

Presentation of W. Rodejohann

9/18/2013 Fedor Simkovic
Heavy $\nu$: $0\nu\beta\beta$ NMEs - status 2013

**PHFB:** K. Rath et al., PRC 85 (2012) 014308

**IBM:** Barea, Kotila, Iachello, PRC (2013) 014315

**SQRPA:** Vergados, Ejiri, F. Š., RPP 75 (2012) 106301

**ISM:** Menendez, private communications
Co-existence of few mechanisms of the $0\nu\beta\beta$-decay

*It may happen that in year 201? (or 2????) the $0\nu\beta\beta$-decay will be detected for 2-3 or more isotopes ...*

*(If there will be enough money for enrichment of isotopes!?)*
There exist heavy Majorana neutral leptons $N_i$ (singlet of SU(2)$\times$U(1) group)

$$\mathcal{L} = -\sqrt{2} \sum_{i,l} Y_{li} \overline{L}_{lL} N_{iR} \tilde{H} + \text{h.c.}$$

Effective interaction for processes with virtual $N_i$ at electroweak scale

$$\mathcal{L}_{\text{eff}} = -\frac{1}{\Lambda} \sum_{\nu',l,i} \overline{L}_{\nu'L} \tilde{H} \sum_i (Y_{\nu' i} \frac{\Lambda}{M_i} Y_{li}) C \tilde{H}^T (\overline{L}_{lL})^T + \text{h.c.}$$

After spontaneous violation of the electroweak symmetry the left-handed Majorana mass term is generated

$$M^L = Y \frac{\nu^2}{M} Y^T = U m U^T$$

$$\mathcal{L}^M = -\frac{1}{2} \sum_{\nu,l} \overline{\nu}_{\nu'L} M_{\nu'l}^L (\nu_{lL})^c + \text{h.c.}$$

$$= -\frac{1}{2} \sum_i m_i \bar{\nu}_i \nu_i, \quad \text{for } \nu_i$$
Probing the standard see-saw mechanism

Inverted hierarchy

Heavy ν Mass at GUT scale

Inverted hierarchy
Co-existence of 2, 3 or more interfering mechanisms of $0\nu\beta\beta$-decay

It is well-known that there exist many mechanisms that may contribute to the $0\nu\beta\beta$. Let consider 3 mechanisms: i) light $\nu$-mass mechanism, ii) heavy $\nu$-mass mechanism, iii) R-parity breaking SUSY mechanism with gluino exchange and CP conservation.

\[
\frac{1}{T_{1/2}^{0\nu}} = G^{0\nu}(E_0, Z) \left| \frac{m_{\beta\beta}}{m_e} M_{\nu}^{0\nu} + \eta_N^L M_N^{0\nu} + \eta_{\chi'111} M_{\chi'111}^{0\nu} \right|^2
\]

\[
m_{\beta\beta} = \sum_k \left( U_{ek}^L \right)^2 \xi_k m_k
\]

\[
\eta_N^L = \sum_k U_{ek}^2 \frac{m_p}{M_k}
\]

Claim of evidence:

\[
T_{1/2}^{0\nu}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25} \text{ y}
\]

\[
T_{1/2}^{0\nu}(^{100}Mo) \geq 5.8 \times 10^{23} \text{ y}
\]

\[
T_{1/2}^{0\nu}(^{130}Te) \geq 3.0 \times 10^{24} \text{ y}
\]

We introduce

\[
\xi = \frac{|M_1^{\nu}| \sqrt{T_1 G_1}}{|M_2^{\nu}| \sqrt{T_2 G_2}}^{r_s}
\]

$\xi=0$, non-observation ($T_2 \to \infty$)

$\xi=1$, solution for single active mech. is reproduced

$\xi_{Te} < 1.2$

$\xi_{Mo} < 2.6$
2 active mechanisms of the $0\nu\beta\beta$-decay: 
Light and heavy $\nu$-mass mechanism

Non-observation of the $0\nu\beta\beta$-decay for some isotopes might be in agreement with non-zero $m_{\beta\beta}$

$\pm 1 \frac{1}{\sqrt{T_1 G_1}} = \frac{m_{\beta\beta}}{m_e} M_1^{\nu} + \eta M_1^{\eta}$

$\pm 1 \frac{1}{\sqrt{T_2 G_2}} = \frac{m_{\beta\beta}}{m_e} M_2^{\nu} + \eta M_2^{\eta}$

$|m_{\beta\beta}| = \begin{vmatrix}
\frac{m_e}{M_1^{\nu}} \frac{M_1^{\nu} M_2^{\eta}}{M_1^{\nu} \sqrt{T_1 G_1} (M_1^{\nu} M_2^{\eta} - M_2^{\nu} M_1^{\eta})} \\
\pm \frac{m_e}{M_2^{\nu}} \frac{M_2^{\nu} M_1^{\eta}}{M_2^{\nu} \sqrt{T_2 G_2} (M_1^{\nu} M_2^{\eta} - M_2^{\nu} M_1^{\eta})}
\end{vmatrix}$

F.Š., J.D. Vergados, A. Faessler, PRD 82, 113015 (2010)

Non-observation for $^{130}$Te
Two non-interfering mechanisms of the $0\nu\beta\beta$-decay (light LH and heavy RH neutrino exchange)

\[
T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 1.9 \times 10^{25} \text{y}, \quad T_{1/2}^{0\nu}(^{100}\text{Mo}) \leq 5.8 \times 10^{24} \text{y}, \quad T_{1/2}^{0\nu}(^{130}\text{Te}) \leq 3.0 \times 10^{25} \text{y}
\]

Half-life:

\[
\frac{1}{T_{1/2}^{0\nu}} \cong |\eta_\nu|^2 |M'_{1,1,\nu}|^2 + |\eta_R|^2 |M'_{1,1,N}|^2
\]

\[
\eta_\nu = \frac{m_{\beta\beta}}{m_e}
\]

Set of equations:

\[
\frac{1}{T_1 G_1} = |\eta_\nu|^2 |M'_{1,\nu}|^2 + |\eta_R|^2 |M'_{1,N}|^2
\]

\[
\frac{1}{T_2 G_2} = |\eta_\nu|^2 |M'_{2,\nu}|^2 + |\eta_R|^2 |M'_{2,N}|^2
\]

Solutions:

\[
|\eta_\nu|^2 = \frac{|M'_{2,N}|^2 / T_1 G_1 - |M'_{1,N}|^2 / T_2 G_2}{|M'_{1,\nu}|^2 |M'_{2,N}|^2 - |M'_{1,N}|^2 |M'_{2,\nu}|^2}
\]

\[
|\eta_R|^2 = \frac{|M'_{1,\nu}|^2 / T_2 G_2 - |M'_{2,\nu}|^2 / T_1 G_1}{|M'_{1,\nu}|^2 |M'_{2,N}|^2 - |M'_{1,N}|^2 |M'_{2,\nu}|^2}
\]

\[
\eta_N^R = \left( \frac{M_W}{M_{WR}} \right)^4 \sum_{k} V_{ek}^2 \frac{m_p}{M_k}
\]

Two non-interfering mechanisms of the $0
\nu\beta\beta$-decay (light LH and heavy RH neutrino exchange)

The positivity condition:

\[
\frac{T_1 G_1 |M_{1,1}^{0\nu}|^2}{G_2 |M_{2,1}^{0\nu}|^2} \leq T_2 \leq \frac{T_1 G_1 |M_{1,1}^{0\nu}|^2}{G_2 |M_{2,2}^{0\nu}|^2}
\]

Very narrow ranges!

\[
0.15 \leq \frac{T_{1/2}^{0\nu}(^{100}Mo)}{T_{1/2}^{0\nu}(^{76}Ge)} \leq 0.18
\]

\[
0.17 \leq \frac{T_{1/2}^{0\nu}(^{130}Te)}{T_{1/2}^{0\nu}(^{76}Ge)} \leq 0.22
\]

\[
1.14 \leq \frac{T_{1/2}^{0\nu}(^{130}Te)}{T_{1/2}^{0\nu}(^{100}Mo)} \leq 1.24
\]

\[
\eta_N^R = \left( \frac{M_W}{M_{WR}} \right)^4 \sum \text{heavy} V_{ek}^2 \frac{m_p}{M_k}
\]

\[
\eta_\nu = \frac{m_{\beta\beta}}{m_e}
\]
**Resonant Neutrinoless Double-Electron Capture**

\[(A,Z) \rightarrow (A,Z-2)^{**}\]


Additional modes of the 0vECEC-decay:

\[e_b + e_b^+ (A,Z) \rightarrow (A,Z-2) + \gamma + 2\gamma + e^+e^- + M\]
Oscillations of atoms

New $0\nu\bar{\nu}$ transitions with parity violation to ground and excited states of final atom/nucleus were found. Selection rules for the $0\nu\bar{\nu}$ transitions were established. The explicit form of corresponding NMEs was derived.

Available data of atomic masses, as well as nuclear and atomic excitations were used to select the most likely candidates for resonant $0\nu\bar{\nu}$ transitions. Assuming an effective Majorana neutrino mass of 1 eV, some half-lives has been predicted to be as low as $10^{22}$ years in the unitary limit.

More accurate atomic mass measurements in the context of the $0\nu\bar{\nu}$ were initialized, which have been partially accomplished using the modern high-precision ion traps. In addition, new $0\nu\bar{\nu}$ experiments were initialized.
Nuclear matrix elements for $0^{\nu}\beta\beta$

Ground state to ground state nuclear transitions

Suppression of the NME depends not only on the relative deformation but also their absolute values.

| Initial (final) | $\beta_{Q_p}$ | $\beta_{B(E2)}$ | $\langle BCS_i | BCS_f \rangle$ |
|----------------|--------------|----------------|----------------|
| $^{152}$Gd ($^{152}$Sm) | (+0.29) | 0.212 (0.306) | 0.44 |
| $^{164}$Er ($^{164}$Dy) | 0.36 (+0.32) | 0.333 (0.348) | 0.73 |
| $^{180}$W ($^{180}$Hf) | 0.27 (+0.27) | 0.252 (0.273) | 0.75 |

Deformed QRPA

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$M^{2\nu}_{GT}$ [$MeV^{-1}$]</th>
<th>$M^{0\nu}$ sph. QRPA</th>
<th>$M^{0\nu}$ QRPA ($\beta_2 = 0$)</th>
<th>$M^{0\nu}$ QRPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{152}$Gd</td>
<td>0.10</td>
<td>7.59</td>
<td>7.50</td>
<td>3.23</td>
</tr>
<tr>
<td>0.00</td>
<td>7.21</td>
<td></td>
<td>2.67</td>
<td></td>
</tr>
<tr>
<td>$^{164}$Er</td>
<td>0.10</td>
<td>6.12</td>
<td>7.20</td>
<td>2.64</td>
</tr>
<tr>
<td>0.00</td>
<td>5.94</td>
<td></td>
<td>2.27</td>
<td></td>
</tr>
<tr>
<td>$^{180}$W</td>
<td>0.10</td>
<td>5.79</td>
<td>6.22</td>
<td>2.05</td>
</tr>
<tr>
<td>0.00</td>
<td>5.56</td>
<td></td>
<td>1.79</td>
<td></td>
</tr>
</tbody>
</table>

Fang et al., PRC 85, 035503 (2012)

EDF

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$M^{0\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{152}$Gd</td>
<td>0.89, 1.07</td>
</tr>
<tr>
<td>$^{164}$Er</td>
<td>0.64, 0.50</td>
</tr>
<tr>
<td>$^{180}$W</td>
<td>0.58, 0.38</td>
</tr>
</tbody>
</table>

Rodriguez, Martinez-Pinedo, PRC 85, 044310 (2012)
\[ m_{\beta\beta} = 50 \text{ meV} \]

\[ \beta \beta \text{ half-lives} \]

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>((n2j) _a)</th>
<th>((n2j) _b)</th>
<th>(E_a)</th>
<th>(E_b)</th>
<th>(E_C)</th>
<th>(\Gamma_{ab} \text{ (keV)})</th>
<th>(\Delta \text{ (keV)})</th>
<th>(T_{1/2}^{\text{min}} \text{ (y)})</th>
<th>(T_{1/2}^{\text{max}} \text{ (y)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{152}\text{Gd})</td>
<td>110</td>
<td>210</td>
<td>46.83</td>
<td>7.74</td>
<td>0.34</td>
<td>(2.3 \times 10^{-2})</td>
<td>(-0.83 \pm 0.18)</td>
<td>(4.7 \times 10^{28})</td>
<td>(4.8 \times 10^{29})</td>
</tr>
<tr>
<td>(^{152}\text{Gd})</td>
<td>110</td>
<td>211</td>
<td>46.83</td>
<td>7.31</td>
<td>0.32</td>
<td>(2.3 \times 10^{-2})</td>
<td>(-1.27 \pm 0.18)</td>
<td>(4.2 \times 10^{31})</td>
<td>(1.1 \times 10^{32})</td>
</tr>
<tr>
<td>(^{152}\text{Gd})</td>
<td>110</td>
<td>310</td>
<td>46.83</td>
<td>1.72</td>
<td>0.11</td>
<td>(3.2 \times 10^{-2})</td>
<td>(-7.07 \pm 0.18)</td>
<td>(9.4 \times 10^{31})</td>
<td>(1.1 \times 10^{32})</td>
</tr>
<tr>
<td>(^{164}\text{Er})</td>
<td>210</td>
<td>210</td>
<td>9.05</td>
<td>9.05</td>
<td>0.22</td>
<td>(8.6 \times 10^{-3})</td>
<td>(-6.82 \pm 0.12)</td>
<td>(7.5 \times 10^{32})</td>
<td>(8.4 \times 10^{32})</td>
</tr>
<tr>
<td>(^{164}\text{Er})</td>
<td>210</td>
<td>211</td>
<td>9.05</td>
<td>8.58</td>
<td>0.23</td>
<td>(8.3 \times 10^{-3})</td>
<td>(-7.28 \pm 0.12)</td>
<td>(4.2 \times 10^{34})</td>
<td>(4.6 \times 10^{34})</td>
</tr>
<tr>
<td>(^{164}\text{Er})</td>
<td>210</td>
<td>310</td>
<td>9.05</td>
<td>2.05</td>
<td>0.11</td>
<td>(1.8 \times 10^{-2})</td>
<td>(-13.92 \pm 0.12)</td>
<td>(3.5 \times 10^{33})</td>
<td>(3.9 \times 10^{33})</td>
</tr>
<tr>
<td>(^{180}\text{W})</td>
<td>110</td>
<td>110</td>
<td>63.35</td>
<td>63.35</td>
<td>1.26</td>
<td>(7.2 \times 10^{-2})</td>
<td>(-11.24 \pm 0.27)</td>
<td>(1.3 \times 10^{31})</td>
<td>(1.8 \times 10^{31})</td>
</tr>
</tbody>
</table>
\[ (A,Z) \rightarrow (A,Z+2) + e^- + e^- \]

\[ e^- + e^- + (A,Z) \rightarrow (A,Z-2)^{**} \]

**Perturbation theory**

\[ \frac{1}{T_{1/2}^{0\nu}} = \left( \frac{m_{\beta\beta}}{m_e} \right)^2 G^{01}(E_0, Z) |M^{0\nu}|^2 \]

**Breit-Wigner form**

\[ \Gamma^{0\nu E\gamma \gamma \gamma}(J^\pi) = \frac{|V_{\alpha\beta}(J^\pi)|^2}{(M_i - M_f)^2 + \Gamma_{\alpha\beta}^2/4} \Gamma_{\alpha\beta} \]

- 2νββ-decay background can be a problem
- Uncertainty in NMEs factor ~2, 3
- 0^+ \rightarrow 0^+, 2^+ transitions
- Large Q-value
- \( ^{76}\text{Ge}, ^{82}\text{Se}, ^{100}\text{Mo}, ^{130}\text{Te}, ^{136}\text{Xe} \) …
- Many exp. in construction, potential for observation in the case of inverted hierarchy (2020)

- 2νεε-decay strongly suppressed
- NMEs need to be calculated
- 0^+ \rightarrow 0^+, 0^-, 1^+, 1^- transitions
- Small Q-value
- Q-value needs to be measured at least with 100 eV accuracy
- \( ^{152}\text{Gd} \), looking for additional
- small experiments yet
Universe as a laboratory to study LN violation
Belyaev, Ricci, Simkovic, Truhlik, arXiv: 1212.3155, Truhlik, MEDEX13 presentation

Cooling of strongly magnetized iron White dwarfs

\[ \text{e}^- + ^{56}\text{Fe} \rightarrow ^{56}\text{Cr} + \text{e}^+ \]

\[ \text{e}^- + \text{e}^- + ^{56}\text{Fe} \rightarrow ^{56}\text{Cr} \]
Historically, there are > 100 experimental limits on $T_{1/2}$ of the $0\nu\beta\beta$ decay.

However, during the last decade the complexity and cost of such experiments increased dramatically. The constant slope is no longer maintained.

Actually, when NMEs will be needed to analyze data?
Theory of neutrinoless double-beta decay

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