First Steps Towards QCD Under External Magnetic Fields

- from a Dyson–Schwinger Perspective-

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Outline

1. Introduction
2. The Challenges
3. The Devil In The Details
4. My Status Quo
5. Down The Road

Why are magnetic fields interesting?

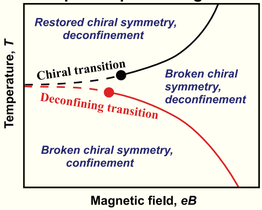
- Heavy Ion Collisions
- Cosmological electroweak phase transition
- Neutron Stars
- Condensed Matter Systems
- (Color-) Superconductors

(first three items:) $\sim 10^{14-16}$ Tesla

Open Issues

- Modification of the (QCD) vacuum structure ?!
- Chiral Magnetic Effect ?
- Magnetic catalysis ?

Expected phase diagram



Mizher, Chernodub, Fraga: PRD 82,105016 (2010)

1. Introduction

A First Step: Magnetic Catalysis

Enhancement of chiral symmetry breaking due to an external magnetic field

Gusynin, Miransky, Shovkovy: PRL 73,26 (1994)

Issues...

- Realization of magnetic field ?
- Mechanism driving enhancement ?
- Effect of approximations/truncations ?
- Origin of discrepancies between lattice and effective model results ?

Aim ...

$\Rightarrow$ Obtain complementary information by non-perturbative finite volume study in a Dyson–Schwinger framework

Reminder: Particles in magnetic fields..

- Constant magnetic field in z-direction: $\vec{\mathcal{B}} = (0, 0, \mathcal{B})^{\mathsf{T}}$

Classical Particle

Lorentz force:

$$F(t) = -e\vec{v} \times \vec{\mathcal{B}}$$

- $v = \omega r$
- $\omega = \frac{e\mathcal{B}}{m}$
- No restriction in z-direction
- Circular orbit $\perp \vec{\mathcal{B}}$

QM Particle

Schrödinger equation:

$$-\frac{1}{2m}[\partial_x^2 + (\partial_y + \mathrm{i}e\mathcal{B}x)^2]\psi = E\psi$$

- $E = \omega(n + \frac{1}{2})$
- Shifted harmonic oscillator *eigen*states
- Infinite degeneracy wrt p_y

Implementing a constant magnetic background

- Abelian field in z-direction

$$A_\mu = (0, \mathcal{B} z, 0, 0)^\mathsf{T}$$

$$\Rightarrow \mathcal{F}_{\mu\nu,ab} = \mathcal{F}_{\mu\nu,ab} + f_{\mu\nu} \cdot \mathbb{1}_{ab}$$

- Principle of minimal coupling

$$\mathrm{D}_\mu = \partial_\mu - ieA_\mu \qquad \mathcal{D}_\mu = \mathrm{D}_\mu + \mathrm{i}g\, t^a \mathcal{A}_{\mu,a}$$

- Leads to the underlying Lagrangian

$$\mathcal{L} = \bar{\psi}\,(i\not{\mathcal{D}} - m)\psi + \mathcal{F}_{\mu\nu}\,\mathcal{F}^{\mu\nu}$$

Deriving the equations

- Work in the Dyson–Schwinger approach
- Propagator from Green's function identity:

$$(\mathrm{i}\not\Pi - m)\, S(x, x') = \delta(x - x')$$

with $\Pi_\mu = \partial_\mu - eA_\mu$

- 'Standard' approach: expand in plane wave functions find diagonal propagator in momentum space
- Challenge: $[\Pi_\mu, p_\nu] \neq 0$
$\rightarrow$'standard' not applicable

Following Ritus' method

...to obtain the (inverse) propagator in momentum space

V.I. Ritus: Annals of Phys. 69,555 (1972)

Ritus Method: The Idea

- Observation I:
  S can only depend on scalar structures built from γ^μ contracted with $\Pi_\mu, F_{\mu\nu}, ...$
- Observation II:
  $[(\not\Pi)^2, S(x,x')] = 0$

The Procedure

- Use *eigen*functions of $\not\Pi^2$ to diagonalize propagator
- End up with 'modified' propagator
  diagonal in momentum space
  depending on special subset of momenta

$$
\begin{aligned}
(\mathrm{i}\not\Pi - \mathrm{m})\, S(x,x') &= \int \mathrm{dp}\, \mathbb{E}_p\, (\bar{\not p} - \mathrm{m}) S(p)\, \bar{\mathbb{E}}_p \\
&\overset{!}{=} \delta(x - x')
\end{aligned}
$$

3. The Devil In The Details

- Diagonalization procedure shows

$$S(\bar{p}) = (\bar{\not{p}} - \mathrm{m})^{-1}$$

- With momenta given by

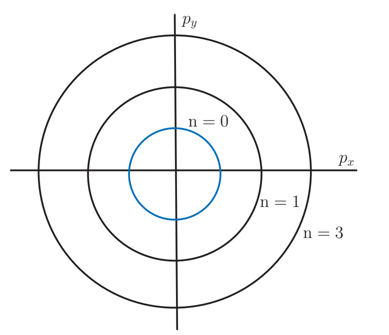
$$\bar{p} = (p_0, 0, \sqrt{k}, p_z)^{\mathsf{T}}$$

- And $\sqrt{k}$ encoding the particles' Landau Levels

$$\sqrt{k} = \sqrt{|e\mathcal{B}|(2\mathrm{n}+1) + \sigma e\mathrm{B}\, sgn(e\mathcal{B})}$$

- States per unit area:
  $\frac{|e\mathcal{B}|}{2\pi}$ for n = 0
  $\frac{|e\mathcal{B}|}{\pi}$ for $\mathrm{n} \geqslant 1$

3. The Devil In The Details

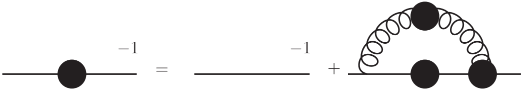


- Lowest Landau level approximation (LLLA) : n = 0 (spin polarized state)
- Dimensional reduction $n = 0 \Rightarrow \sqrt{k} = 0$

$$\bar{p} = (p_0, 0, 0, p_z)^T$$

- NO application of the Mermin–Wagner theorem $\rightarrow$ gluons are 4-dimensional
- Drawback/Limit of LLLA: $\mathcal{B} \rightarrow 0$
- Beyond LLLA: include $n = 1, 2, \ldots$

The Dyson–Schwinger Equations



with the dressed propagator

$$S(\bar{p})^{-1} = B(\bar{p}) + \mathrm{i}\,(A_0(\bar{p})\gamma^0\bar{p}_0 + A_2(\bar{p})\gamma^2\bar{p}_2 + A_3(\bar{p})\gamma^3\bar{p}_3)$$

- $A_1(p)$ and $A_2(p)$ are not accessible in LLL approximation
- Gluonic input from lattice calculations

  → Fischer, Maas, Pawlowski: Annals Phys.324 (2009)
- Landau gauge
- Modified bare vertex approximation
- Solution in a finite volume → (1+1) torus

Magnetic Flux

$$\int dx_\mu A_\mu = \mathcal{B} \cdot \mathrm{F}$$

$$\int dx_\mu A_\mu = \mathcal{B} \cdot (\mathrm{F} - L_x L_y)$$

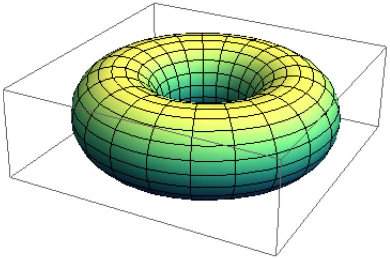
Charged Particles

$$\exp(iq\mathcal{B}\mathrm{F}) \stackrel{!}{=} \exp(iq\mathcal{B}(\mathrm{F} - L_x L_y))$$

$$\Rightarrow q\mathcal{B} = \frac{2\pi}{L_x L_y} b$$

with $b = 0, 1, 2 \ldots$

Magnetic Field In A Finite Volume



Full Quark Propagator

$$S(\bar{p})^{-1} = B(\bar{p}) + \mathrm{i}\, A_\mu(\bar{p})\gamma^\mu \bar{p}_\mu$$

Quantized B-Field

$$|e\mathcal{B}| = \frac{2\pi}{L_x L_y} b$$

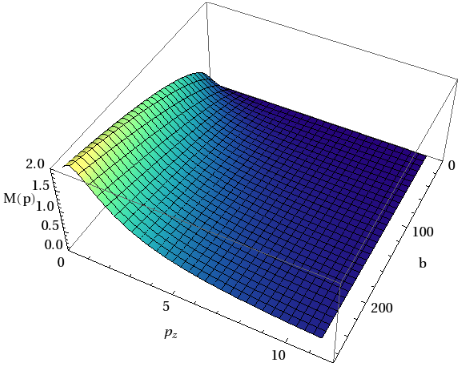
$$b \in [0, L_x \cdot L_y]$$

Typical Tori

- Box Length: 6 fm
- Mom. points: 8×33
- $\mathcal{B}_{max} \sim 1.8\ \mathrm{GeV}^2$

Dynamically Generated Mass

$$M(p) = \frac{B(p)}{A(p)} \qquad [M(p)] = GeV$$



Result

- Lowest Landau Level approximation ⇒
  Enhanced mass generation with increasing magnetic field

4. My Status Quo

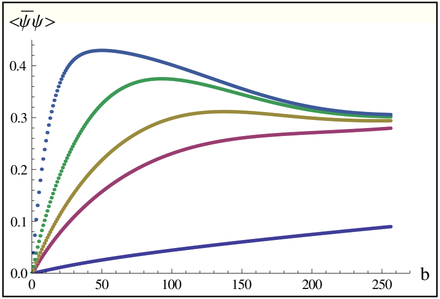
Chiral Condensate

$$\langle\bar{\psi}\psi\rangle_b \sim b \sum_{n_t,n_z} \frac{B_b(p)}{B_b(p)^2+(A_{0b}(p)\,p_0)^2+(A_{2b}(p)\,p_2)^2+(A_{3b}(p)\,p_3)^2}$$

Max. B-field

- $|e\mathcal{B}| = \frac{2\pi}{L_x L_y} b$
- $\mathcal{B}_{max} \sim 1.8\ \text{GeV}^2$

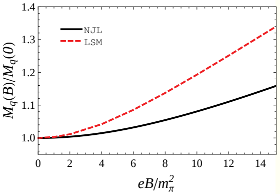
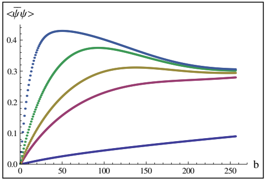
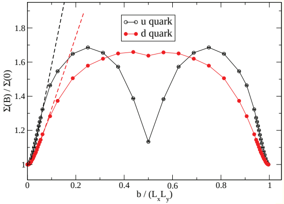
- Limit $\mathcal{B} \to 0$ is not reliable in LLLA



Result

- Including more Landau Level $\Rightarrow$ Non-monotonic chiral condensate with increasing magnetic field

4. My Status Quo

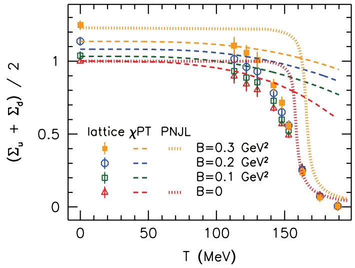
Effective Model Approaches	Dyson–Schwinger Approach	Lattice Gauge Theory
		
Ferrari, Garcia, Pinto: arXiv 1207.3714v2		D'Elia, Negro: PRD 83 (2011)
• Mostly LLLA • Infinite volume • Increasing chiral symmetry breaking with B	• Beyond LLLA • Finite volume • Saturation effects in chiral condensate	• Beyond LLLA • Finite volume • Saturation effects in chiral condensate

Wrap up

- Concept of constant external magnetic fields
- Effects on particles' propagators and momenta
- Discussion of (Lowest) Landau Level approximation

The next steps

- Volume studies
- Finite temperatures
- Finite chemical potential
- ...



Bali, Bruckmann, Enrödi, Fodor, Katz, Schäfer: arXiv 1206.4205

Thank you for your attention!

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