What Nuclear Multifragmentation Reactions Imply for Equation of State of Stellar Matter

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Thermal multifragmentation of nuclei:
Production of hot fragments at
T ≈ 3-8 MeV
ρ ≈ 0.1 ρ₀
Nuclear density: ρ₀ = 0.15 fm⁻³ = 2.5 × 10¹⁴ g cm⁻³
Interpretation: liquid-gas type phase transition in finite nuclei. A chance to investigate properties of hot fragments in dense environment of other nuclei and nucleons, which can be different from their ground state properties.

Collapse of massive stars leading to Supernova II explosions:
We expect production of hot fragments in nuclear matter at
T ≈ 1-10 MeV
ρ ≲ 0.3 ρ₀
Characteristic times of the processes are very large (milliseconds), nuclear equilibrium is expected. Properties of hot fragments influence processes during the collapse and explosions.
Statistical ensemble

with \( \text{fix} \ T, \ \rho_B, \ Y_{L(e)} \)

\[
\mu_i = B_i \mu_B + Q_i \mu_Q + L_i \mu_L
\]

For nuclear species \((A, Z)\) :

\[
\mu_{AZ} = A \mu_B + Z \mu_Q
\]

electrons \( e^- : \mu_{e^-} = -\mu_Q + \mu_L = -\mu_{e^+} \)

neutrinos \( \nu : \mu_\nu = \mu_L = -\mu_\bar{\nu} \)

Baryon number conservation :

\[
\rho_B = \frac{B}{V} = \sum_{(A,Z)} A \langle n_{AZ} \rangle \text{ fixed} \quad \rightarrow \quad \mu_B
\]

Electric neutrality

\[
\rho_Q = \frac{Q}{V} = \sum_{(A,Z)} Z \langle n_{AZ} \rangle - n_e = 0 \quad \rightarrow \quad \mu_Q
\]

Lepton number conservation

\[
Y_L = \frac{L}{B} = \frac{n_e + n_\nu}{\rho_B} \quad \text{(trapped} \ \nu) \quad \text{or} \quad Y_e = \frac{n_e}{\rho_B} \quad \text{(free} \ \nu)
\]
**Statistical Model for Supernova Matter: SMSM**

Grand canonic density of fragments with mass $A$ and charge $Z$ in nuclear matter:

$$\langle \rho_{AZ} \rangle = g_{AZ} (1 - \frac{\rho}{\rho_0}) A^{3/2} \exp \left[ - \frac{1}{T} (F_{AZ} - \mu_A - \xi Z) \right]$$

Total density $\rho = M/V = \sum \langle \rho_{AZ} \rangle$, $M$ is number of nucleons and $V$ is volume of the system. $g_{AZ}$ is the degeneracy of fragments, the nucleon thermal wavelengt is

$$\lambda_T = (2\pi h^2 / m_N T)^{1/2}$$

$\mu$ and $\xi$ are the chemical potentials for the nucleon number and charge conservation in the system. Free energy of fragments:

$$F_{AZ}(T, \rho) = F_{AZ}^B + F_{AZ}^S + F_{AZ}^C + F_{AZ}^{\text{sym}}$$

**Bulk energy:**

$$F_{AZ}^B (T) = - \omega_0 \left( \frac{T^2}{\varepsilon_0} \right) A, \quad \omega_0 = 16 \text{MeV}$$

**Surface energy:**

$$F_{AZ}^S (T) = \beta_0 \left( \frac{T_c^2 - T^2}{T_c^2 + T^2} \right) A^{2/3}, \quad \beta_0 = 18 \text{MeV}$$

**Coulomb energy:**

$$F_{AZ}^C (\rho) = \frac{3}{5} c(\rho) \frac{(eZ)^2}{r_0 A^{1/3}}$$

$$c(\rho) = \left[ 1 - \frac{3}{2} \left( \frac{\rho_e}{\rho_p} \right)^{1/3} + \frac{1}{2} \left( \frac{\rho_e}{\rho_p} \right) \right]$$

**Symmetry energy:**

$$F_{AZ}^{\text{sym}} = \gamma \frac{(A - 2Z)^2}{A}, \quad \gamma = 25 \text{MeV}$$

**Reactions with leptons, in equilibrium:**

$$e^- + p \rightarrow n + \nu_e$$

$$e^+ + n \rightarrow p + \bar{\nu}_e$$

(and inverse reactions, also with all nuclei.)

### Including electrons

**Density of electrons:**

$$\rho_e = \rho_{e^-} - \rho_{e^+}, \quad \sum \rho_{AZ} Z = \rho_e$$

**Charge conservation** [electro-neutrality]

$$Y_e = \rho_e / \rho$$

**Equilibrium**

$$\mu_e = -\xi$$

**Relativistic degenerate electron gas**

$$\rho_e = \frac{g_e}{6\pi^2} \left[ \mu_e^3 + \mu_e \left( \pi^2 T^2 - \frac{3}{2} m_e^2 \right) \right]$$

### Including electron neutrinos

**Density of neutrinos**

$$\rho_{\nu_e} = \rho_{\nu_e} - \rho_{\bar{\nu}_e}$$

**Conservation of leptons**

$$Y_{\text{lept}} = (\rho_e + \rho_\nu) / \rho = \text{const}$$

**Equilibrium**

$$\rho_{\nu} = \frac{1}{6\pi^2} \left( \frac{\mu_\nu}{\hbar c} \right)^3 \left[ 1 + \mu_\nu^{-2} \pi^2 T^2 \right]$$

**Self-consistent calculation of all densities** $\rho_{AZ}, \rho_e$ and $\rho_{\nu}$

+ including photons
Production of nuclear fragments in multifragmentation and supernova explosions

**Multifragmentation Reactions**
- $A_s=197$, $Z_s=79$
- $3A\text{MeV}$
- $5A\text{MeV}$
- $8A\text{MeV}$

**Supernova Explosions**
- $Y_e=0.2$, $\rho=0.1\rho_0$
- $T=4\text{MeV}$
- $T=5\text{MeV}$
- $T=6\text{MeV}$

Variation of mass distribution with $T$ and $E^*$
- U-shape
- Power law
- Exponential

SMM
- SMSM
Investigation of the symmetry energy term

The symmetry energy coefficient $\gamma$ was investigated in several independent experiments, which use both the isoscaling phenomenon and isotope distributions of fragments.

$$F_{AZ}^{\text{sym}} = \gamma (A - 2Z)^2 / A,$$

A.S. Botvina et al., PRC72(2005)048801

G. Souliotis et al., PRC75(2007)011601

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**SMM hot fragments**

**cold fragments**

1 stable nuclei

2 EPAX

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**FRS experiment**

$\Delta^{238}U + Pb$

$^{238}U + Ti$

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Influence of the symmetry energy in nuclear reactions


Ye=0.4
No considerable difference for γ=25 and 14 MeV.

Ye=0.2
Considerable difference for γ=25 and 14 MeV.

Influence of the symmetry energy term on $Y_n$ and $Y_{\text{heavy}}$ in stellar matter

Considerable difference for $\gamma=25$ and 14 MeV.
Comparison of SMSM and SHEN EOS


One can see that two models predict similar mass fractions of neutrons, protons, alpha particles and heavy nuclei for different temperature and Ye.
Properties of hot fragments: the surface energy term $B_0$.

$Z^{-\tau}$ analysis of IMF yields

We obtain an evolution of the surface energy of hot fragments toward region of full multifragmentation projectiles with different isospin.

ALADIN

A.S.Botvina et al., PRC74(2006)044609
Influence of the surface energy term on mass distributions

nuclear reactions  stellar matter

A.S. Botvina et al., PRC74(2006)044609

Even small variations of the surface energy coefficient B0 lead to considered changing in fragment production similarity!
Influence of the surface energy term on $Y_n$ and $Y_{\text{heavy}}$ in stellar matter

Variations of the surface energy coefficient $B_0$ (from 16 to 20 MeV) lead to practically no effect.

Even small variations of the surface energy coefficient $B_0$ (from 18 to 20 MeV) lead to considering changes in fragment production at $T=4$ MeV.
Influence of the surface energy term on $Y_p$ and $Y_\alpha$ in stellar matter
Important Results: connection of nuclear multifragmentation with astrophysics

1. Similar conditions of nuclear matter are reached in multifragmentation reactions and during the collapse and explosion of massive stars.

2. The statistical models successfully applied for nuclear multifragmentation can be generalized for astrophysical conditions. Nuclear parameters of the models, in particular, the symmetry energy, can be extracted from multifragmentation experiments.

3. Broad variety of nuclear species including, exotic and neutron rich nuclei, are produced in stellar matter. Modification of the symmetry energy and surface energy of nuclei in dense hot medium is important for rates of electro-weak reactions, and for synthesis of heavy elements.