Constructing and validating a chiral effective model for dilepton production in NN and AA collisions

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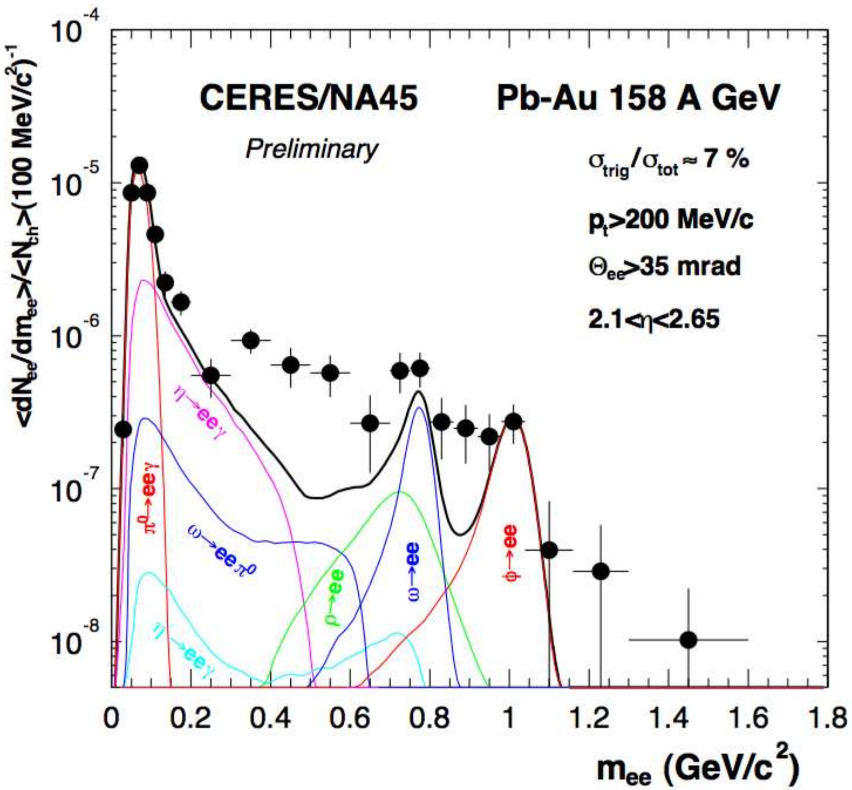
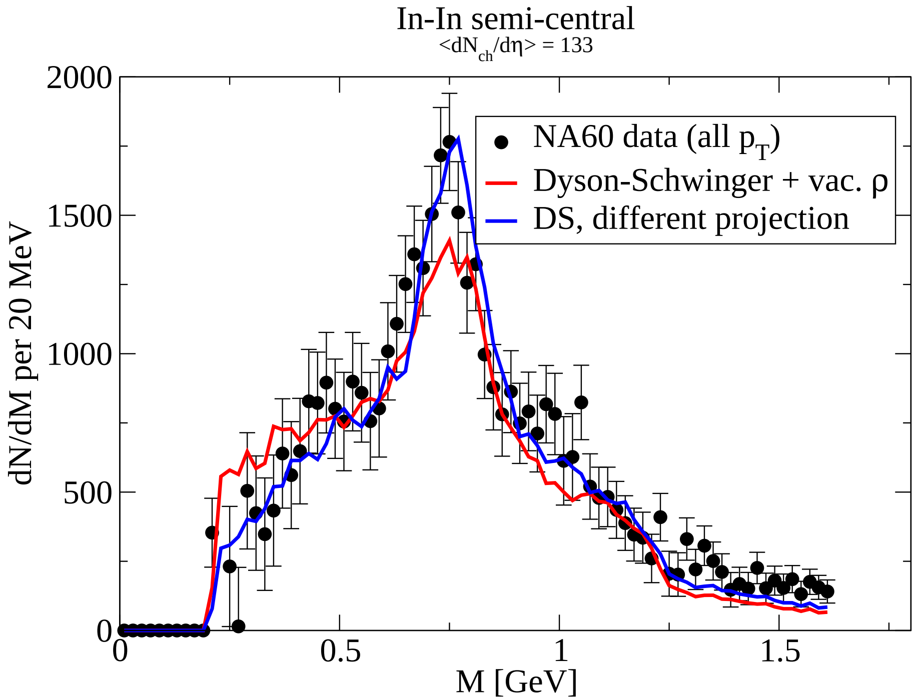
Motivation (I)

Dileptons carry information from hot and dense stages of heavy-ion collisions:



CERES/NA45 collaboration



NA60 collaboration

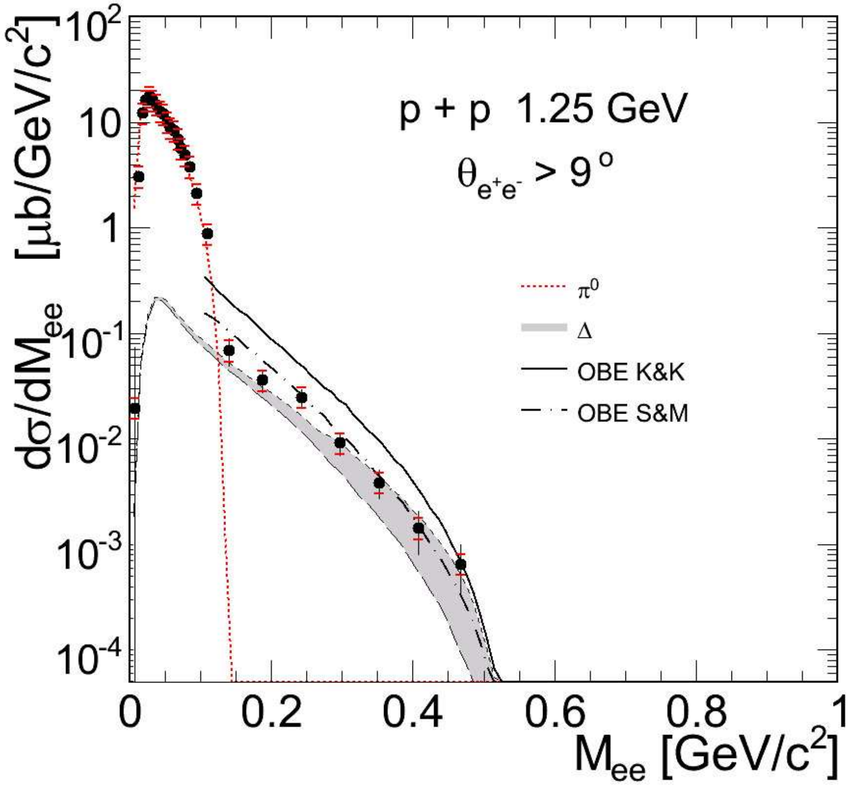
(fig. courtesy of Thorsten Renk)

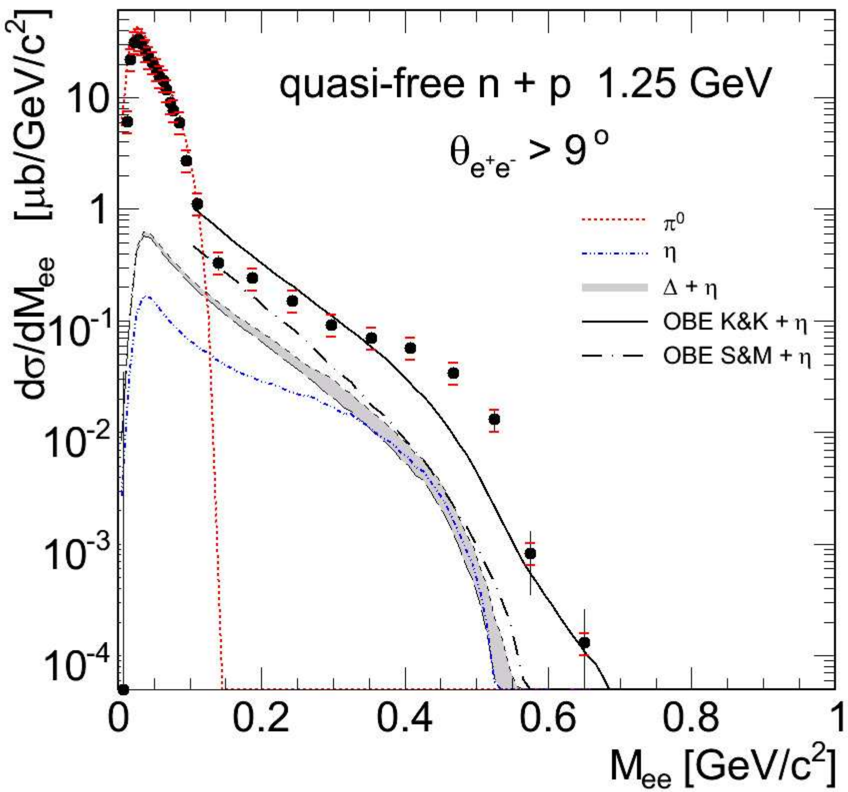
$\Longrightarrow$ **learn about chiral symmetry restoration in hot and dense hadronic matter!**

see R. Rapp, J. Wambach, Adv. Nucl. Phys. 25 (2000) 1

Motivation (II)

Prior to describing AA: understand dilepton production in NN collisions!





HADES collaboration, Acta Phys.Polon. B41 (2010) 365

The chiral effective model

Chiral symmetry of QCD: global $U(N_f)_r \times U(N_f)_\ell$ symmetry (classically)

$\Longrightarrow$ spontaneously broken in vacuum by nonzero quark condensate $\langle \bar{q}q \rangle \neq 0$

$\Longrightarrow$ restored at nonzero temperature T and chemical potential μ

$\Longrightarrow$ degeneracy of hadronic chiral partners in the chirally restored phase

$\Longrightarrow$ for this application: chiral symmetry must be linearly realized

$\Longrightarrow$ Linear sigma model

Disclaimer: No attempt to fit precision data for hadron vacuum phenomenology!

(No attempt to compete with chiral perturbation theory)

Nevertheless: achieve reasonable description of hadron vacuum phenomenology!

Moreover: strong statement on the nature of the scalar mesons!

scalar-meson puzzle: too many scalar states to fit into a $q\bar{q}$ meson nonet

$$f_0(600),\ f_0(980),\ f_0(1370),\ f_0(1500),\ f_0(1710)$$

$\Longrightarrow$ Jaffe's conjecture: R.L. Jaffe, PRD 15 (1977) 267, 281

two scalar $[qq][\bar{q}\bar{q}]$ tetraquark states mix with two scalar $q\bar{q}$ meson states

$\Longrightarrow$ fifth scalar meson could be due to mixing with glueball

Scalar and pseudoscalar mesons

$$\mathcal{L}_S = \text{Tr}\left(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2\, \Phi^\dagger \Phi\right) - \lambda_1 \left[\text{Tr}\left(\Phi^\dagger \Phi\right)\right]^2 - \lambda_2 \text{Tr}\left(\Phi^\dagger \Phi\right)^2 + c\left(\det\Phi - \det\Phi^\dagger\right)^2 + \text{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right]$$

$\Phi \in (N_f^*, N_f) \implies \Phi \equiv \phi_a T_a$, T_a generators of $U(N_f)$, $\phi_a \equiv \sigma_a + i\pi_a$, $H \equiv h_a T_a$

$h_a = c = 0$, $m^2 > 0$: $U(N_f)_r \times U(N_f)_\ell$ symmetry

$h_a = c = 0$, $m^2 < 0$: v.e.v. $\langle \Phi \rangle = \phi\, N_f\, T_0$, $\phi \equiv \langle \sigma_0 \rangle > 0$

Spontaneous symmetry breaking (SSB):

$U(N_f)_r \times U(N_f)_\ell \longrightarrow U(N_f)_V \quad (V \equiv \ell + r)$

$h_a = 0$, $c \neq 0$: $U(1)_A$ anomaly $(A \equiv \ell - r)$

Explicit symmetry breaking (ESB):

$U(N_f)_r \times U(N_f)_\ell \longrightarrow SU(N_f)_r \times SU(N_f)_\ell \times U(1)_V$

$m^2 < 0$: SSB: $SU(N_f)_r \times SU(N_f)_\ell \longrightarrow SU(N_f)_V$

$\dim[SU(N_f)_r \times SU(N_f)_\ell / SU(N_f)_V] = 2\,(N_f^2 - 1) - (N_f^2 - 1) = N_f^2 - 1$

$\implies$ $N_f^2 - 1$ Goldstone bosons $\implies$ pseudoscalar mesons!

h_a, $c \neq 0$, $m^2 < 0$: ESB $\implies$ $N_f^2 - 1$ pseudo - Goldstone bosons

Vector and axial-vector mesons

$$\begin{aligned}\mathcal{L}_V = & -\frac{1}{4}\,\mathrm{Tr}(\mathcal{L}^0_{\mu\nu}\mathcal{L}_0^{\mu\nu} + \mathcal{R}^0_{\mu\nu}\mathcal{R}_0^{\mu\nu}) + \frac{1}{2}\,\mathrm{Tr}\left[\left(m_1^2 + 2\hat{\delta}\right)\,(\mathcal{L}_\mu\mathcal{L}^\mu + \mathcal{R}_\mu\mathcal{R}^\mu)\right] \\ & + i\,\frac{g_2}{2}\,\mathrm{Tr}\left\{\mathcal{L}^0_{\mu\nu}[\mathcal{L}^\mu,\mathcal{L}^\nu] + \mathcal{R}^0_{\mu\nu}[\mathcal{R}^\mu,\mathcal{R}^\nu]\right\} \\ & + g_3\,\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}^\nu\mathcal{L}_\mu\mathcal{L}_\nu + \mathcal{R}^\mu\mathcal{R}^\nu\mathcal{R}_\mu\mathcal{R}_\nu) - g_4\,\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}_\mu\mathcal{L}^\nu\mathcal{L}_\nu + \mathcal{R}^\mu\mathcal{R}_\mu\mathcal{R}^\nu\mathcal{R}_\nu) \\ & + g_5\,\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}_\mu)\;\mathrm{Tr}\,(\mathcal{R}^\nu\mathcal{R}_\nu) \\ & + g_6\,\left[\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}_\mu)\;\mathrm{Tr}\,(\mathcal{L}^\nu\mathcal{L}_\nu) + \mathrm{Tr}\,(\mathcal{R}^\mu\mathcal{R}_\mu)\;\mathrm{Tr}\,(\mathcal{R}^\nu\mathcal{R}_\nu)\right]\end{aligned}$$

$$\mathcal{L}^0_{\mu\nu} \equiv \partial_\mu\mathcal{L}_\nu - \partial_\nu\mathcal{L}_\mu,\;\; \mathcal{R}^0_{\mu\nu} \equiv \partial_\mu\mathcal{R}_\nu - \partial_\nu\mathcal{R}_\mu,\;\; \mathcal{L}_\mu \equiv L^a_\mu T_a,\;\; \mathcal{R}_\mu \equiv R^a_\mu T_a$$

vector mesons: $V^a_\mu \equiv \frac{1}{2}\left(L^\mu_a + R^a_\mu\right)$, axial-vector mesons: $A^a_\mu \equiv \frac{1}{2}\left(L^\mu_a - R^a_\mu\right)$

$\hat{\delta}$: matrix which accounts for difference in quark masses

g_3, g_4, g_5, g_6: not determined by global fit to masses and decay widths

Scalar – vector interactions

$$\mathcal{L}_{SV} = i\, g_1 \,\mathrm{Tr}\left[\partial_\mu \Phi \left(\Phi^\dagger \mathcal{L}^\mu - \mathcal{R}^\mu \Phi^\dagger\right) - \partial_\mu \Phi^\dagger \left(\mathcal{L}^\mu \Phi - \Phi \mathcal{R}^\mu\right)\right] + \frac{h_1}{2}\,\mathrm{Tr}\left(\Phi^\dagger \Phi\right) \mathrm{Tr}\left(\mathcal{L}_\mu \mathcal{L}^\mu + \mathcal{R}_\mu \mathcal{R}^\mu\right) + (g_1^2 + h_2)\,\mathrm{Tr}\left(\Phi^\dagger \Phi \mathcal{R}_\mu \mathcal{R}^\mu + \Phi \Phi^\dagger \mathcal{L}_\mu \mathcal{L}^\mu\right) - 2(g_1^2 - h_3)\,\mathrm{Tr}\left(\Phi^\dagger \mathcal{L}_\mu \Phi \mathcal{R}^\mu\right)$$

SSB:

- induces mass splitting $m_A^2 - m_V^2 = (g_1^2 - h_3)\phi^2$
- induces bilinear term $\sim g_1 \phi\, A_a^\mu\, \partial_\mu \pi_a$:

$\Longrightarrow$ eliminate by shift $A_a^\mu \longrightarrow A_a^\mu + w(\phi)\, \partial^\mu \pi_a$, $w(\phi) \equiv \dfrac{g_1 \phi}{m_A^2}$

$\Longrightarrow$ wave function renormalization of pseudoscalar fields

$\pi_a \longrightarrow Z\, \pi_a$, $Z^2 \equiv \left(1 - \frac{g_1^2 \phi^2}{m_A^2}\right)^{-1}$ (KSFR : $Z \equiv \sqrt{2}$)

$\Longrightarrow$ v.e.v. $\phi \equiv Z\, f_\pi$

$\Longrightarrow$ complete meson Lagrangian $\boxed{\mathcal{L}_M = \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_{SV}}$

Vacuum phenomenology: Global fit for $N_f = 3$ (I)

$N_f = 3 \implies$ two scalar-isoscalar mesons f_0^L, f_0^H (combinations of $\bar{q}q$ and $\bar{s}s$)

$\implies$ all (pseudo-)scalar masses and decay widths except those of f_0^L, f_0^H determined by linear combination of m^2, λ_1 and of m_1^2, h_1

Since nature of scalar-isoscalar mesons (quarkonium, glueball, or tetraquark?) is unclear

$\implies$ at first omit scalar-isoscalar mesons from the fit

$\implies$ perform χ^2–fit of $m^2, \lambda_2, c, h_0, h_8, m_1^2, \delta, g_1, g_2, , h_2, h_3$ (11 parameters) to 21 experimental quantities

D. Parganlija, F. Giacosa, P. Kovacs, Gy. Wolf, DHR, arXiv:1208.0585[hep-ph]

Constraints: (i) no isospin violation

$\implies$ experimental error $= \max(\text{PDG error}\,, 5\%)$

(ii) $m^2 < 0$ (SSB)

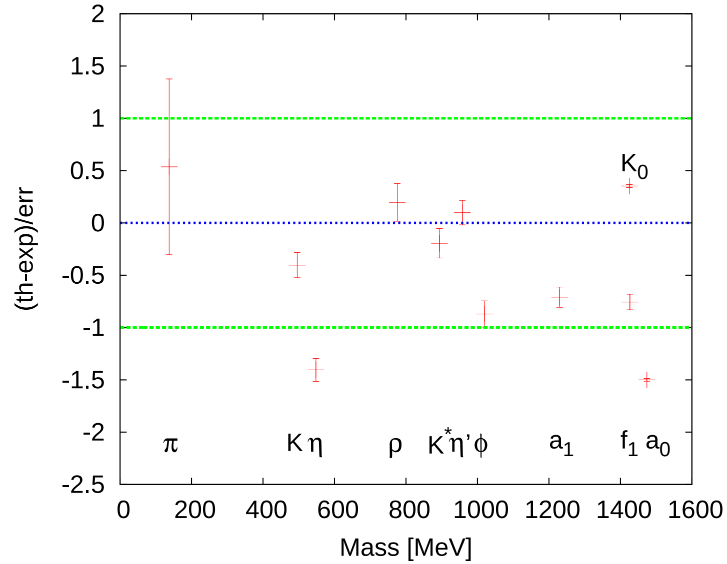
(iii) $\lambda_2 > 0\,,\ \lambda_1 > -\lambda_2/2$ (boundedness of potential)

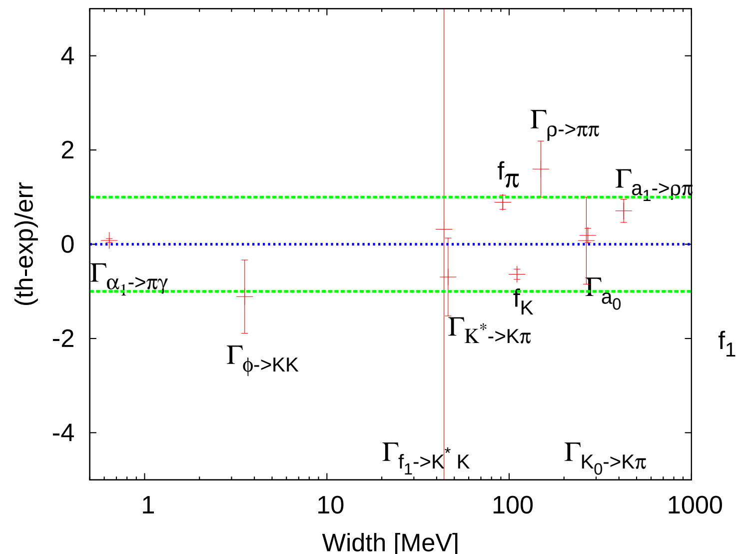
(iv) $m_1 \geq 0$ (boundedness of potential)

(v) $m_1 \leq m_\rho$ (SSB increases mass of vector mesons)

Vacuum phenomenology: Global fit for $N_f = 3$ (II)

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80





accuracy of fit: $\chi^2/\text{d.o.f.} \simeq 1.23$

Vacuum phenomenology: Global fit for $N_f = 3$ (III)

large–N_c suppressed parameters $\lambda_1 = h_1 \equiv 0$:

$\Longrightarrow$ prediction for the masses of the isoscalar-scalar states:

$m_{f_0^L} = 1362.7$ MeV, $m_{f_0^H} = 1531.7$ MeV

$\Longrightarrow$ masses are in the range of the heavy scalar states:

$m_{f_0(1370)} = (1350 \pm 150)$ MeV, $m_{f_0(1500)} = (1505 \pm 75)$ MeV,

$m_{f_0(1710)} = 1720 \pm 86$ MeV

$\Longrightarrow$ mass of f_0^L close to mass of $f_0(1370)$

$\Longrightarrow$ mass of f_0^H close to $f_0(1500)$, but decay pattern similar to that of $f_0(1710)$

$\Longrightarrow$ include mixing with glueball state

$\Longrightarrow$ (most likely) $f_0(1500)$ (predominantly) glueball

$\Longrightarrow$ $f_0(1370)$, $f_0(1710)$ appear to be (predominantly) $\bar{q}q$–states

$\Longrightarrow$ chiral partners of π, η'!

$\Longrightarrow$ light scalar states $f_0(600)$, $f_0(980)$ could be (predominantly) $[qq][\bar{q}\bar{q}]$–states

(see, however, W. Heupel, G. Eichmann, C.S. Fischer, arXiv:1206.5129[hep-ph]

$\Longrightarrow$ light scalars have a dominant meson-molecule component!)

Incorporating the scalar glueball (I)

Another confirmation of the (predominantly) $q\bar{q}$ assignment for the heavy scalar mesons: $\Longrightarrow$ coupling to the glueball/dilaton field! (so far only $N_f = 2$)

S. Janowski, D. Parganlija, F. Giacosa, DHR, PRD 84 (2011) 054007

- dilatation symmetry $\Longrightarrow$ dynamical generation of tree-level meson mass parameters through glueball field G: $m^2 \to m^2 \left(\frac{G^2}{G_0^2}\right)$, $m_1^2 \to m_1^2 \left(\frac{G^2}{G_0^2}\right)$
- add glueball Lagrangian: $$\mathcal{L}_G = \frac{1}{2}\,(\partial_\mu G)^2 - \frac{1}{4}\,\frac{m_G^2}{\Lambda^2}\,G^4 \left(\ln\left|\frac{G}{\Lambda}\right| - \frac{1}{4}\right)$$

  $\Lambda \sim$ gluon condensate $\langle G^a_{\mu\nu} G_a^{\mu\nu}\rangle$

  $\Longrightarrow\ \mathcal{L}_M \longrightarrow \mathcal{L}_M + \mathcal{L}_G$
- shift σ and G by their v.e.v.'s, $\sigma \to \sigma + \phi$, $G \to G + G_0$

  $\Longrightarrow$ v.e.v. G_0 given by $-\frac{m^2}{m_G^2}\,\phi^2\,\Lambda^2 = G_0^4 \,\ln\left|\frac{G_0}{\Lambda}\right|$

  $\Longrightarrow$ glueball mass given by $M_G^2 = m^2\,\frac{\phi^2}{G_0^2} + m_G^2\,\frac{G_0^2}{\Lambda^2}\left(1 + 3\ln\left|\frac{G_0}{\Lambda}\right|\right)$

  $\Longrightarrow$ bilinear term $\sim \sigma G$ $\Longrightarrow$ eliminate by $O(2)$ transformation

$$\begin{pmatrix} \sigma' \\ G' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma \\ G \end{pmatrix}$$

Incorporating the scalar glueball (II)

$\Longrightarrow$ χ^2 fit of Λ, M_σ, m_G^2, m_1^2 to the following experimental quantities:

Quantity	Our Value [MeV]	Experiment [MeV]
$M_{\sigma'}$	1191 ± 26	1350 ± 150
$M_{G'}$	1505 ± 6	1505 ± 6
$G' \to \pi\pi$	38 ± 5	38.04 ± 4.95
$G' \to \eta\eta$	5.3 ± 1.3	5.56 ± 1.34
$G' \to K\bar{K}$	9.3 ± 1.7	9.37 ± 1.69

$\chi^2/\text{d.o.f.} = 0.29$

$\Longrightarrow$ $\theta = (29.7 \pm 3.6)^o$ $\Longrightarrow$ $f_0(1500)$ is 76% glueball!

$\Longrightarrow$ predict the following quantities:

Quantity	Our Value [MeV]	Experiment [MeV]
$G' \to \rho\rho \to 4\pi$	30	54.0 ± 7.1
$G' \to \eta\eta'$	0.6	2.1 ± 1.0
$\sigma' \to \pi\pi$	284 ± 43	325
$\sigma' \to \eta\eta$	72 ± 6	61.8 ± 22.8

$\Longrightarrow$ reasonable description of experimental data!

Predictions for a pseudoscalar glueball

Consider decay of pseudoscalar glueball into scalar and pseudoscalar mesons

$$\mathcal{L}_{\tilde{G}\Phi} = i\, c_{\tilde{G}\Phi}\, \tilde{G}\, \left(\det\Phi - \det\Phi^{\dagger}\right)$$

$\Longrightarrow$ **predict branching ratios for decays into scalar and pseudoscalar mesons**

$\Longrightarrow$ **could be measured in PANDA!**

BR	$M_{\tilde{G}} = 2.6$ GeV	$M_{\tilde{G}} = 2.37$ GeV
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma^{tot}_{\tilde{G}}$	0.049	0.042
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.019	0.011
$\Gamma_{\tilde{G}\to\eta\eta\eta}/\Gamma^{tot}_{\tilde{G}}$	0.016	0.013
$\Gamma_{\tilde{G}\to\eta\eta\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.0017	0.00080
$\Gamma_{\tilde{G}\to\eta\eta'\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.00013	0
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma^{tot}_{\tilde{G}}$	0.46	0.46
$\Gamma_{\tilde{G}\to\eta\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.16	0.16
$\Gamma_{\tilde{G}\to\eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.094	0.088

BR	$M_{\tilde{G}} = 2.6$ GeV	$M_{\tilde{G}} = 2.37$ GeV
$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}}$	0.059	0.069
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}}$	0.082	0.10
$\Gamma_{\tilde{G}\to\eta\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.028	0.033
$\Gamma_{\tilde{G}\to\eta\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0.012	0.0093
$\Gamma_{\tilde{G}\to\eta'\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.019	0.013

W.I. Eshraim, S. Janowski, F. Giacosa, DHR, arXiv:1208.6474[hep-ph]

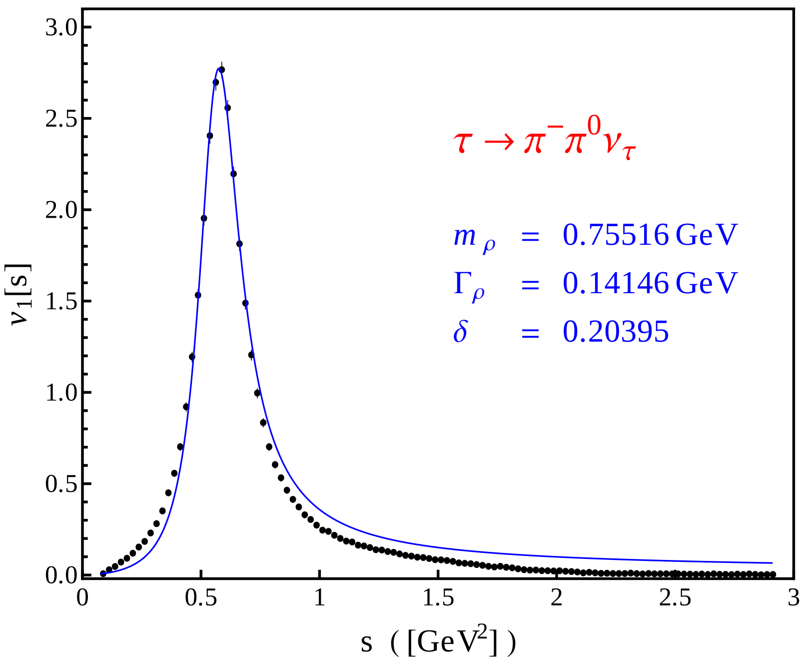
Electroweak interactions

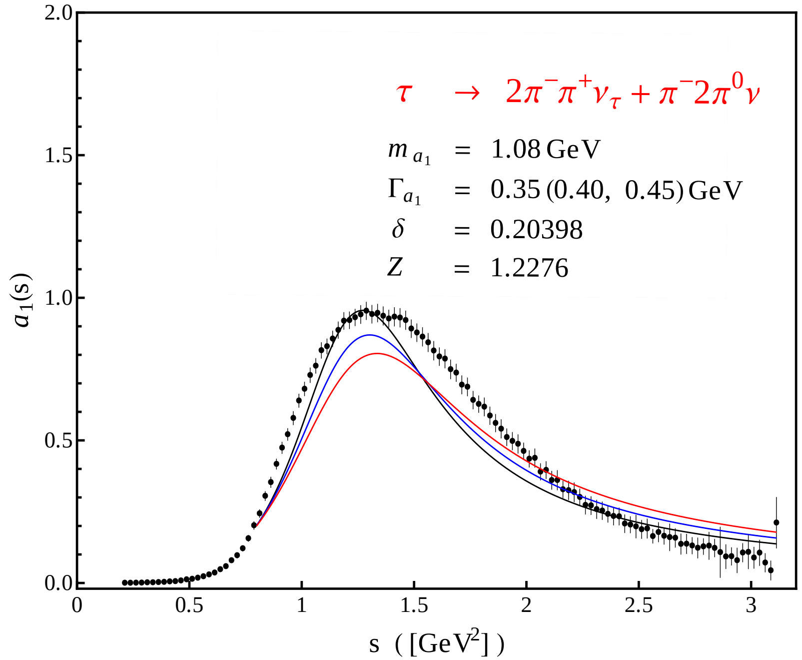
A. Habersetzer, F. Giacosa, DHR, in preparation

$$\partial^\mu \Phi \longrightarrow D^\mu \Phi \equiv \partial^\mu \Phi - i\, e\, A^\mu\, [T_3, \Phi] - i\, g \cos\theta_C\, (W_1^\mu T_1 + W_2^\mu T_2)\Phi$$
$$- i\, g \cos\theta_W\, (Z^\mu T_3 \Phi + \tan^2\theta_W\, \Phi T_3 Z^\mu)$$

$$\mathcal{L}_0^{\mu\nu} \longrightarrow \mathcal{L}^{\mu\nu} \equiv \partial^\mu \mathcal{L}^\nu - i\, e\, A^\mu [T_3, \mathcal{L}^\nu] - i\, g [W_1^\mu T_1 + W_2^\mu T_2, \mathcal{L}^\nu]$$
$$- \partial^\nu \mathcal{L}^\mu + i\, e\, A^\nu [T_3, \mathcal{L}^\mu] + i\, g [W_1^\nu T_1 + W_2^\nu T_2, \mathcal{L}^\mu] \quad \text{(similarly for } R_0^{\mu\nu}\text{)}$$

$$\mathcal{L}_M \longrightarrow \mathcal{L}_M + \frac{\delta}{2}\, g\, \cos\theta_C\, \mathrm{Tr}[W_{\mu\nu} \mathcal{L}^{\mu\nu}] + \frac{\bar{\delta}}{2}\, e\, \mathrm{Tr}[B_{\mu\nu} \mathcal{R}^{\mu\nu}] + \frac{1}{4}\, \mathrm{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2]$$





cf. M. Urban, M. Buballa, J. Wambach, NPA 697 (2002) 338

Baryons and their chiral partners

Inclusion of baryons and their chiral partners:

$\Longrightarrow$ **Mirror assignment:** C. DeTar and T. Kunihiro, PRD 39 (1989) 2805

$$\Psi_{1,r} \longrightarrow U_r \, \Psi_{1,r} \; , \quad \Psi_{1,\ell} \longrightarrow U_\ell \, \Psi_{1,\ell} \; , \quad \text{but: } \Psi_{2,r} \longrightarrow U_\ell \, \Psi_{2,r} \; , \quad \Psi_{2,\ell} \longrightarrow U_r \, \Psi_{2,\ell}$$

$\Longrightarrow$ **new, chirally invariant mass term:**

$$\begin{aligned} \mathcal{L}_B = & \, \bar{\Psi}_{1,\ell} \, i\partial\!\!\!/ \, \Psi_{1,\ell} + \bar{\Psi}_{1,r} \, i\partial\!\!\!/ \, \Psi_{1,r} + \bar{\Psi}_{2,\ell} \, i\partial\!\!\!/ \, \Psi_{2,\ell} + \bar{\Psi}_{2,r} \, i\partial\!\!\!/ \, \Psi_{2,r} \\ & + m_0 \left(\bar{\Psi}_{2,\ell} \, \Psi_{1,r} - \bar{\Psi}_{2,r} \, \Psi_{1,\ell} - \bar{\Psi}_{1,\ell} \, \Psi_{2,r} + \bar{\Psi}_{1,r} \, \Psi_{2,\ell} \right) \end{aligned}$$

Note: chiral symmetry restoration:

chiral partners become degenerate, but not necessarily massless!

$\Longrightarrow$ m_0 **models contribution from gluon condensate to baryon mass**

$\Longrightarrow$ **allows for stable nuclear matter ground state! (see below)**

Vector – baryon interactions

$$\mathcal{L}_{VB} = c_1 \left(\bar{\Psi}_{1,\ell} \, \not{\mathcal{L}} \, \Psi_{1,\ell} + \bar{\Psi}_{1,r} \, \not{\mathcal{R}} \, \Psi_{1,r} \right) + c_2 \left(\bar{\Psi}_{2,\ell} \, \not{\mathcal{R}} \, \Psi_{2,\ell} + \bar{\Psi}_{2,r} \, \not{\mathcal{L}} \, \Psi_{2,r} \right)$$

Note: in general $c_1 \neq c_2$

$\Longrightarrow$ **allows to fit axial coupling constants (see below)!**

Scalar – baryon interactions

Yukawa interaction:

$$\mathcal{L}_{SB} = -\hat{g}_1 \left(\bar{\Psi}_{1,\ell}\, \Phi\, \Psi_{1,r} + \bar{\Psi}_{1,r}\, \Phi^\dagger\, \Psi_{1,\ell} \right) - \hat{g}_2 \left(\bar{\Psi}_{2,r}\, \Phi\, \Psi_{2,\ell} + \bar{\Psi}_{2,\ell}\, \Phi^\dagger\, \Psi_{2,r} \right)$$

$N_f = 2$ mass eigenstates:

$$\begin{pmatrix} N \\ N^* \end{pmatrix} \equiv \begin{pmatrix} N^+ \\ N^- \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5\, e^{-\delta/2} \\ \gamma_5\, e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} , \quad \sinh\delta = \frac{\phi}{4\, m_0} (\hat{g}_1 + \hat{g}_2)$$

$$m_\pm = \sqrt{m_0^2 + \frac{\phi^2}{16}(\hat{g}_1 + \hat{g}_2)^2} \pm \frac{\phi}{4}(\hat{g}_1 - \hat{g}_2) \quad \longrightarrow \quad m_0 \qquad (\phi \to 0)$$

axial coupling constant:

$$g_A = +\tanh\delta \left[1 - \frac{c_1 + c_2}{2\, g_1} \left(1 - \frac{1}{Z^2} \right) \right] - \frac{c_1 - c_2}{2\, g_1} \left(1 - \frac{1}{Z^2} \right)$$

$$g_A^* = -\tanh\delta \left[1 - \frac{c_1 + c_2}{2\, g_1} \left(1 - \frac{1}{Z^2} \right) \right] - \frac{c_1 - c_2}{2\, g_1} \left(1 - \frac{1}{Z^2} \right) \neq -g_A\,!$$

$\Longrightarrow$ for $c_1 \neq c_2$ compatible with $g_A \simeq 1.26\,,\ g_A^* \simeq 0\,!$

T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

T. Maurer, T. Burch, L.Ya. Glozman, C.B. Lang, D. Mohler, A. Schäfer, arXiv:1202.2834[hep-lat]

Vacuum phenomenology: The chiral partner of the nucleon (I)

Baryon sector ($N_f = 2$): S. Gallas, F. Giacosa, DHR, PRD 82 (2010) 014004

Determine $m_0\,,\ c_1\,,\ c_2\,,\ \hat{g}_1\,,\ \hat{g}_2$ **through** χ^2 **fit to**

$$M_N\,,\ M_{N^*}\,,\ g_A = 1.267 \pm 0.004\,,\ g_A^*\,,\ \Gamma(N^* \to N\pi)$$

(i) Scenario A: $N = N(940),\ N^* = N(1535)$

$\Longrightarrow$ $g_A^* = 0.2 \pm 0.3$ T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

$\Gamma(N^* \to N\pi) = (67.5 \pm 23.6)$ MeV

(ii) Scenario B: $N = N(940),\ N^* = N(1650)$

$\Longrightarrow$ $g_A^* = 0.55 \pm 0.2$ T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

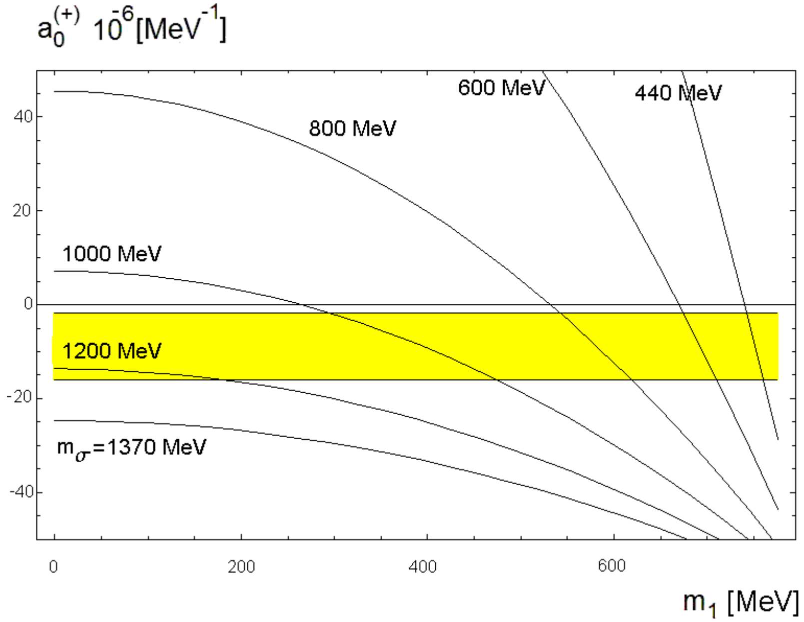
$\Gamma(N^* \to N\pi) = (128 \pm 44)$ MeV

Test validity of the two scenarios through comparison to:

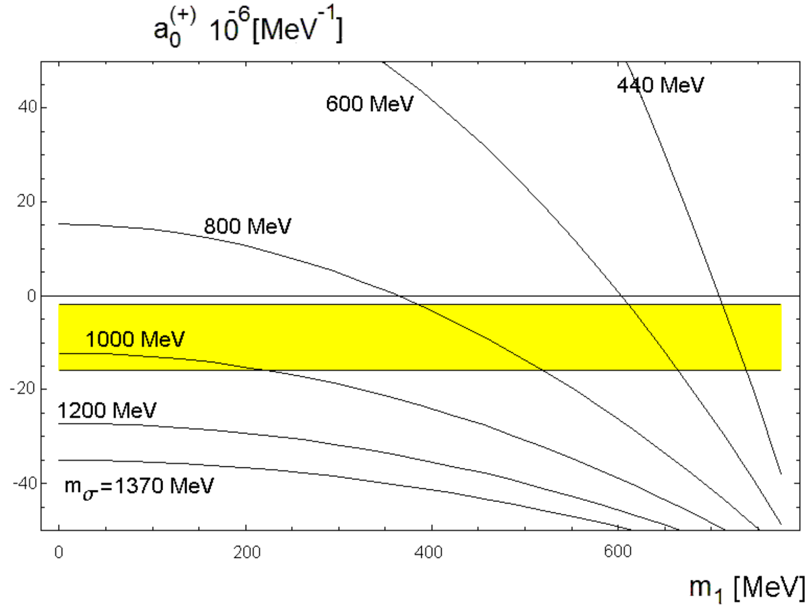
- πN scattering lengths
- decay width $\Gamma(N^* \to N\eta)$

Vacuum phenomenology: The chiral partner of the nucleon (II)

πN scattering lengths $a_0^{(\pm)}$:



m$_{N^*}$ = 1535 MeV



m$_{N^*}$ = 1655 MeV

$$a_0^{(-)} = (6.04 \pm 0.63) \cdot 10^{-4}\ \text{MeV}^{-1} \qquad\qquad a_0^{(-)} = (5.90 \pm 0.46) \cdot 10^{-4}\ \text{MeV}^{-1}$$

for comparison: $a_{0,\text{exp}}^{(-)} = (6.4 \pm 0.1) \cdot 10^{-4}\ \text{MeV}^{-1}$

However: $\Gamma(N^* \to N\eta) = (10.9 \pm 3.8)$ MeV $\qquad$ $\Gamma(N^* \to N\eta) = (18.3 \pm 8.5)$ MeV

$\Gamma_{\text{exp}}(N^* \to N\eta) = (78.7 \pm 24.3)$ MeV! $\qquad$ $\Gamma_{\text{exp}}(N^* \to N\eta) = (10.7 \pm 6.7)$ MeV

$\Longrightarrow$ **Scenario B seems to be favored!**

Vacuum phenomenology: The chiral partner of the nucleon (III)

$\Longrightarrow$ **But then: what is the chiral partner of $N(1535)$?**

Remember L.Ya. Glozman, PRL 99 (2007) 191602:

Heavy chiral partners are closer in mass than lighter ones

$\Longrightarrow$ **Signal of chiral symmetry restoration in the QCD mass spectrum**

$\Longrightarrow$ **Could the partner of $N(1535)$ be $N(1440)$?**

Nuclear matter saturation (I)

D. Zschiesche, L. Tolos, J. Schaffner-Bielich, R.D. Pisarski, PRC75 (2007) 055202

studied cold nuclear matter within the mirror assignment

used effective potential in mean-field approximation:

$$\mathcal{V}_{\rm eff}(\sigma,\omega_0) = \sum_{i=\pm} \frac{d_i}{(2\pi)^3} \int_0^{k_{F,i}} \mathrm{d}^3\vec{k}\; \left[E_i^*(k) - \mu_i^*\right] + \frac{1}{2}\, m^2\, \sigma^2 + \frac{1}{4}\,\lambda\,\sigma^4 - h\sigma - \frac{1}{2}\, m_1^2\,\omega_0^2 - g_4\,\omega_0^4$$

d_i internal degrees of freedom of N, N^*

$k_{F,i} = \sqrt{\mu_i^{*\,2} - m_i^2}$ Fermi momentum

$E_i^*(k) = \sqrt{k^2 + m_i^2}$ single-particle energy

$\mu_i^* = \mu_i - g_\omega\,\omega_0$ effective chemical potential

$$m^2 = \frac{1}{2}\left(3m_\pi - m_\sigma^2\right)\,, \quad \lambda = \frac{m_\sigma^2 - m_\pi^2}{2\sigma}\,, \quad h = f_\pi\, m_\pi^2\,,$$

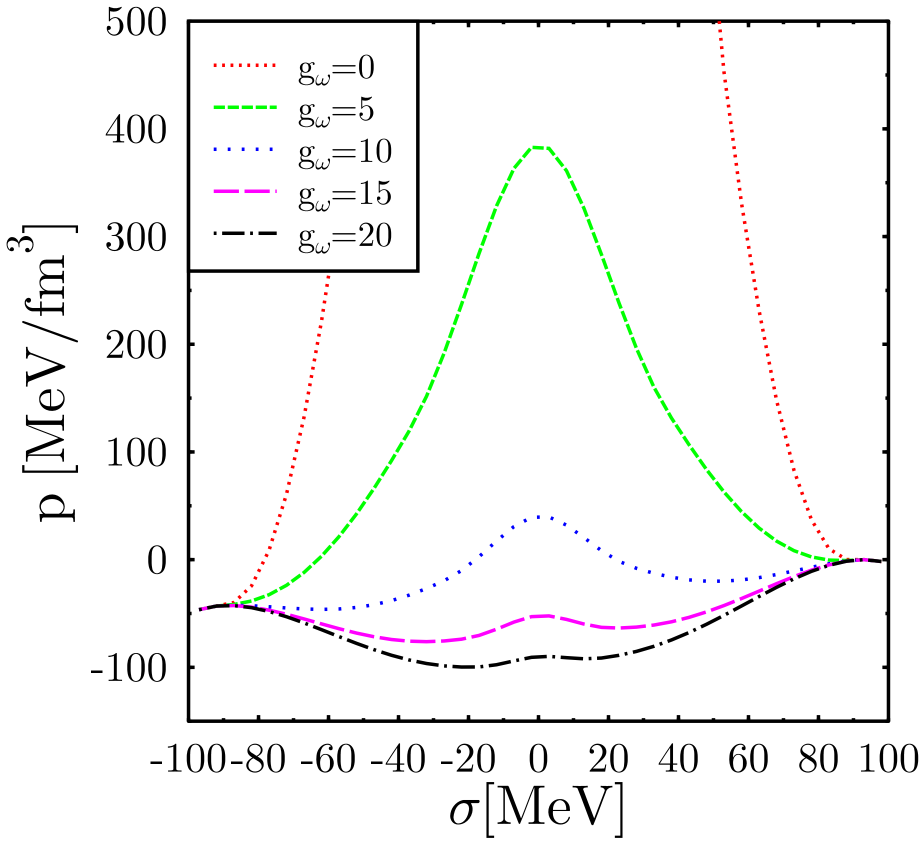
v.e.v.'s $\phi = \langle\sigma\rangle$, $\bar{\omega} = \langle\omega_0\rangle$ determined by

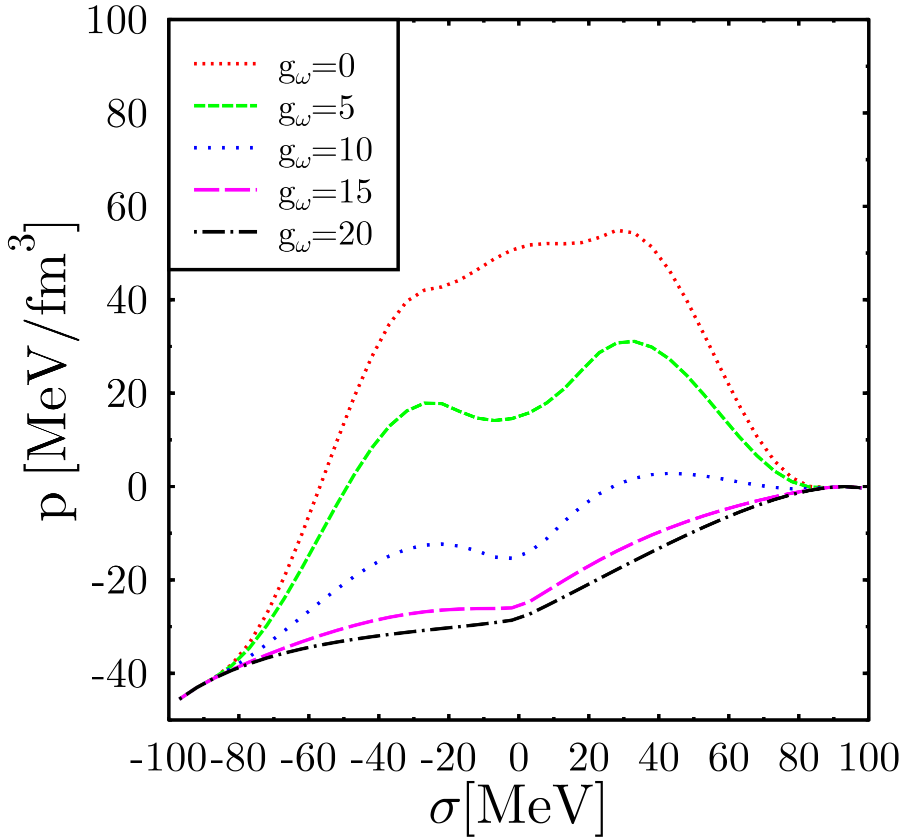
$$\left.\frac{\delta\mathcal{V}_{\rm eff}(\sigma,\omega_0)}{\delta\sigma}\right|_{\phi,\bar{\omega}} = \left.\frac{\delta\mathcal{V}_{\rm eff}(\sigma,\omega_0)}{\delta\omega_0}\right|_{\phi,\bar{\omega}} = 0$$

Nuclear matter saturation (II)

$m_0 = 0$: $\Longrightarrow \nexists\, g_\omega$ **for which nuclear matter saturates**

$m_0 > 0$: $\Longrightarrow \exists\, g_\omega$ **for which nuclear matter saturates**





$\Longrightarrow$ **ground state is either vacuum or chirally restored phase**

(**both figs.:** $\mu_B = 923$ **MeV**, $g_4 = 0$, $m_- = 1.5$ **GeV**
left: $m_\sigma = 1$ **GeV**, **right:** $m_\sigma = 400$ **MeV**)

Nuclear matter saturation (III)

$\exists$ **nuclear matter ground state for:**

m_- [GeV]	m_0 [MeV]	m_σ [MeV]	g_4	$m_+(n_0)/m_+$	$m_-(n_0)/m_-$	K [MeV]
1.5	790	370.63	0	0.84	0.73	510.57
1.5	790	346.59	3.8	0.83	0.72	440.51
1.2	790	318.56	0	0.86	0.79	436.41
1.2	790	302.01	3.8	0.86	0.78	374.75

$\Longrightarrow$ **scalar meson too light, compressibility too large!**

S. Gallas, F. Giacosa, G. Pagliara, NPA 872 (2011) 13

inclusion of tetraquark d.o.f. χ : m_0 **dynamically generated,** $m_0 = a\,\chi$

$$\Longrightarrow \quad \mathcal{V}_{\rm eff}(\sigma,\omega_0,\chi) = \mathcal{V}_{\rm eff}(\sigma,\omega_0) - g\,\chi\,\sigma^2 + \tfrac{1}{2}\,m_\chi^2\,\chi^2$$

v.e.v. $\bar{\chi} = \langle \chi \rangle$ **determined by** $\left.\dfrac{\delta \mathcal{V}_{\rm eff}(\sigma,\omega_0,\chi)}{\delta\chi}\right|_{\phi,\bar{\omega},\bar{\chi}} = 0$

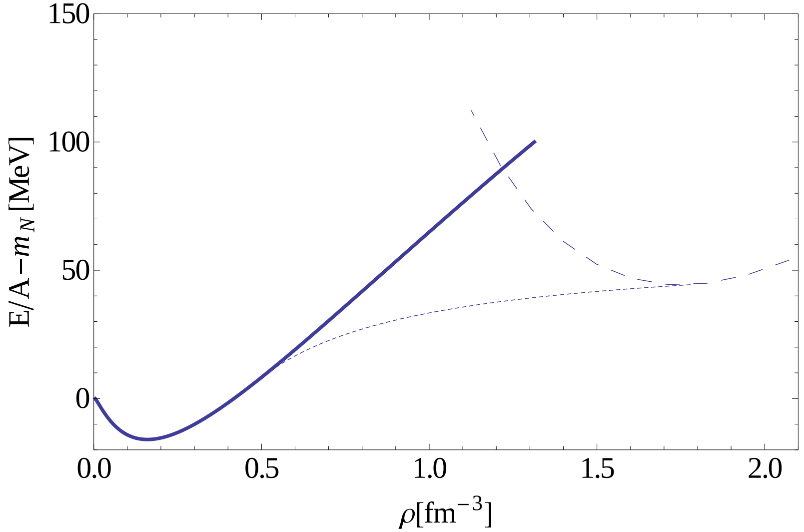
$\Longrightarrow$ **nuclear matter ground state:**

m_- [GeV]	m_0 [MeV]	m_σ [GeV]	g_4	m_χ [MeV]	K [MeV]
1.535	500	1.294	0	612	194

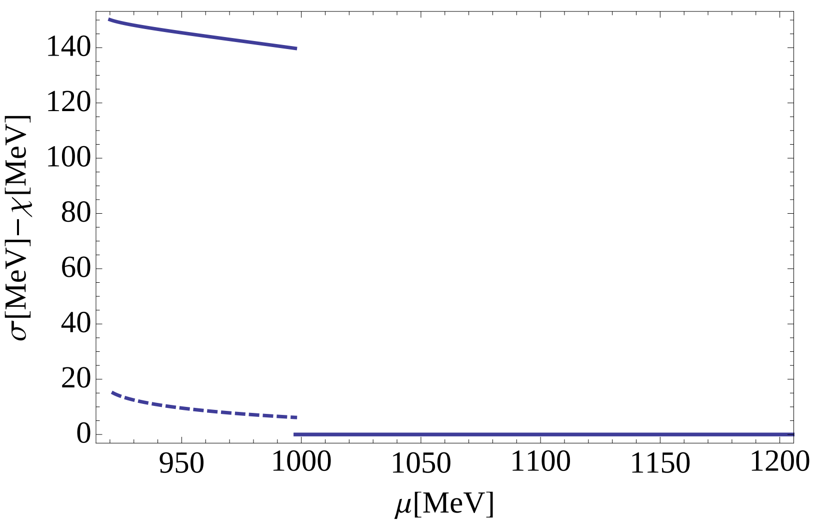
Note: fit to vacuum properties requires $m_0 = 460 \pm 130$ **MeV**

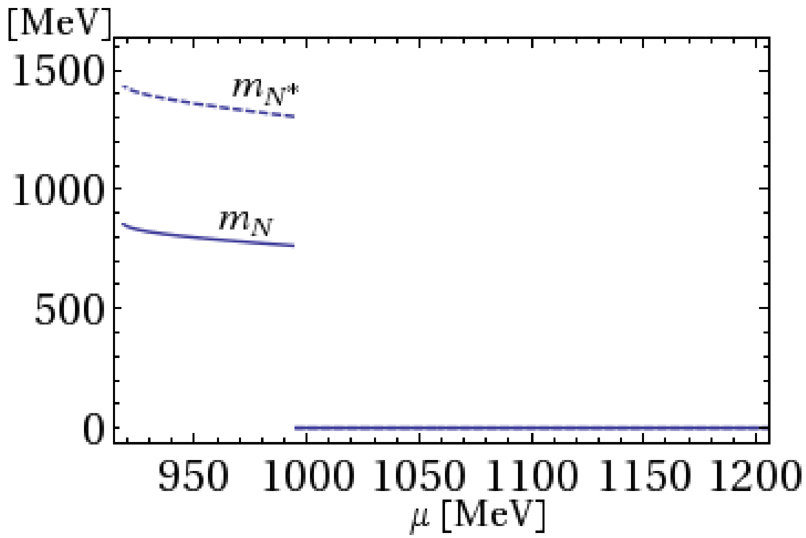
Nuclear matter at large densities

⟹ 1st order phase transition to chirally restored phase:



S. Gallas, F. Giacosa, G. Pagliara,
NPA 872 (2011) 13





Conclusions

I. Linear σ model with $U(N_f)_r \times U(N_f)_\ell$ symmetry with scalar, vector mesons, baryons and their chiral partners

II. Vacuum phenomenology:

1. **Excellent fit of mesonic vacuum properties for $N_f = 3$**
2. **The scalar meson puzzle: evidence for tetraquark assignment for the light scalar mesons $f_0(600)$, $f_0(980)$, glueball is most likely (predominantly) $f_0(1500)$**
3. **The chiral partner of the nucleon: is it $N(1650)$ instead of $N(1535)$?**

III. Nonzero densities:

1. **Nuclear matter ground state: correctly described by chiral effective model with mirror assignment for chiral partner of N**

Outlook: Further studies

1. **Vacuum:**
   - (i) Extension to $N_f = 4$ W. Eshraim
   - (ii) Full scalar mixing scenario including $q\bar{q}$, tetraquark, and glueball states
     S. Janowski
     cf. also T. Mukherjee, M. Huang, Q.-S. Yan, arXiv:1203.5717[hep-ph], 1209.1191[hep-ph]
   - (iii) electroweak interactions, τ decay A. Habersetzer, F. Giacosa, DHR
   - (iv) Δ resonance S. Gallas
   - (v) NN scattering W. Deinet
   - (vi) Exclusive hadron, dilepton production in elementary NN collisions
     K. Teilab
2. **Nonzero T, μ:**
   - (i) $q\bar{q}$−tetraquark mixing
     A. Heinz, S. Strüber, F. Giacosa, DHR, PRD 79 (2009) 037502
   - (ii) Inhomogeneous phases A. Heinz, M. Wagner
   - (iii) Hadron properties, signals for chiral symmetry restoration