Role of mixing between quarkonium and tetraquark on QCD phase diagram

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- Motivation
- A linear sigma model for scalar meson below 2 GeV
- A simple model for quarkonia and tetraquark mixing
- Outlook

- Chiral symmetry is an important ingredient for scalar meson spectrum.
- Lowest scalar meson σ is believed to be the Higgs boson of QCD.
- Chiral phase transition $\rightarrow$ chiral partner become degenerate.
- Scalars are probe of QCD vacuum
- To explain the mass spectrum light scalar meson $\rightarrow$ mixing between quarkonia and tetraquark is necessary.
- Effective model study indicates interesting implication for chiral symmetry restoration.

- identification of the scalar mesons is a long standing puzzle.
- the problem originates from their large decay widths
- one expects non-$\bar{q}q$ scalar objects like glueballs and multiquark states in the mass range below 1800 MeV.
- relevant symmetry:
  $U(3)_L \times U(3)_R \rightarrow SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A$

Iso-Scalar Meson

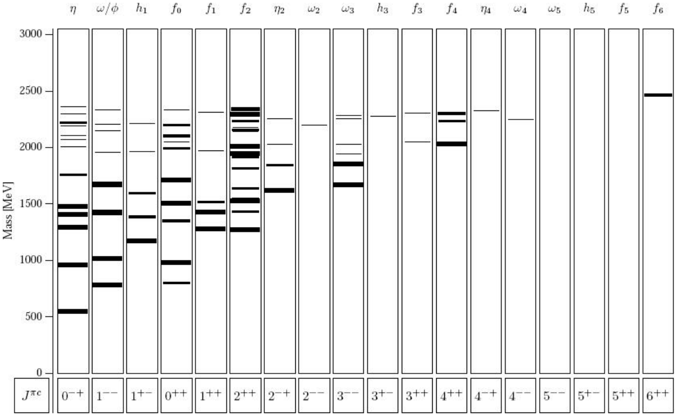

Fig. 7. Experimental light flavoured isoscalar meson spectrum. Data are from [1]. Mean values of resonance positions are indicated by thick lines, less established resonances are represented by medium thick lines, 'further states' by very thin lines.

Unusual Spectroscopy

Vector Mesons:

I = 1:	m[$\rho(776)$] $\approx 776 MeV$	$n\bar{n}$
I = 0:	m[$\omega(783)$] $\approx 783 MeV$	$n\bar{n}$
I = $\frac{1}{2}$:	m[$K^\star(892)$] $\approx 892 MeV$	$n\bar{s}$
I = 0:	m[$\phi(1020)$] $\approx 1020 MeV$	$s\bar{s}$

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I = 0:	m[$f_0(600)$] $\approx 500MeV$	$\sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$
I = $\frac{1}{2}$:	m[κ] $\approx 800MeV$	$\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d$
I = 0:	m[$f_0(980)$] $\approx 980MeV$	$\bar{s}s$
I = 1:	m[$f_0(980)$] $\approx 980MeV$	$\bar{u}d, \bar{d}u, \sqrt{\frac{1}{2}}(\bar{u}u - \bar{d}d)$

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Light Scalars are tetraquark state: Jaffe (Phys. Rev. D 15 (1977))
The States above consecutively can be represented as:
$nn\bar{n}\bar{n}, nn\bar{n}\bar{s}, ns\bar{n}\bar{s}, ns\bar{n}\bar{s}$

Sigma-mesonic mode

Sigma meson $\rightarrow$ Quantum fluctuation of $\bar{\psi}\psi$.

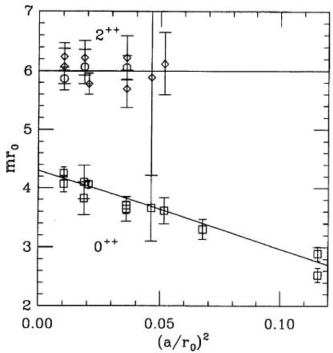
Higgs Particle in QCD $\rightarrow$ Existence in real world is still unclear.

$f_0(600)$or σ $\rightarrow$ Mass (400-1200) MeV

$\rightarrow$ Full Width (600-1000) MeV

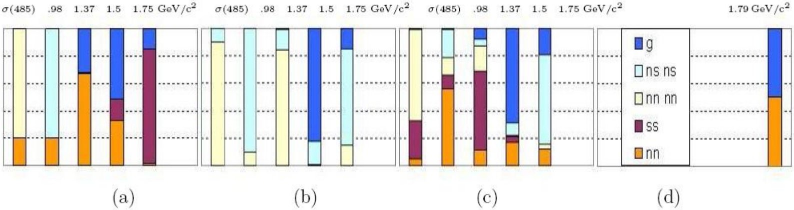
Composition: quark bilinear state or tetraquark state?

Glueball



$r_0 = 0.5\text{fm}$
extrapolation to $a = 0$ gives:
$m = 1611 \pm 30 \pm 160\text{MeV}.$

Model Prediction



Decomposition of scalar isoscalar states into different components.

Ref.: Eur. Phys. J. C21, 531, 2001, Eur. Phys. J. C21, 531, 2001, Phys. Rev. D74, 054030, hep-ph/0603018

Figure Ref.: Phys. Rept. 454:1202,2007.

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- Basic Lagrangian:

$$\mathcal{L} = \mathrm{Tr}\ (\partial_\mu \Phi\ \partial^\mu \Phi^\dagger) + \mathrm{Tr}\ (\partial_\mu \Phi'\ \partial^\mu \Phi^{\dagger\prime}) + \partial_\mu Y \partial^\mu Y^\star - V_0 - V_{SB}$$

Some Remarks

- Tetraquark field:
  a) molecular type :

$$M^b{}_a = (q_{bA})^\dagger \gamma_4 \frac{1+\gamma_5}{2} q_{aA}; \Phi^b{}_a = \epsilon_{acd}\epsilon^{bef}(M^\dagger)^c{}_e(M^\dagger)^d{}_f$$

  b) scalar di-quark + anti-diquark :

$$\phi_i = \sqrt{\frac{1}{2}}\epsilon_{ijk} q^\dagger{}_j C\gamma^5 q_k; \Phi_{ij} = \phi^\dagger{}_i \phi_j$$

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- Glueball field:
  We interpret the spurion field as effective glueball field. To accommodate realistic glueball field it is widely used practice to introduce a flavor singlet complex field to the linear/non-linear sigma model.[Phys Rev. D 21, 3393 (1980), Nucl. Phys. B175, 477 (1980), Prog. Theor. Phys. 66, 1789 (1981), Phys. Rev. D 80, 014014 (2009)].

Lagrangian

$$\mathcal{L}_{\mathcal{S}} = Tr(\partial_\mu \Phi \partial^\mu \Phi^\dagger) + Tr(\partial_\mu \Phi' \partial^\mu \Phi^{\dagger\prime}) + \partial_\mu Y \partial^\mu Y^\star - {m_\Phi}^2 Tr(\Phi^\dagger \Phi)$$
$$- {m_{\Phi'}}^2 Tr(\Phi^{\dagger\prime}\Phi') - {m_Y}^2 YY^\star - \lambda_1 Tr(\Phi^\dagger \Phi \Phi^\dagger \Phi) - {\lambda_1}' Tr(\Phi^{\dagger\prime}\Phi'\Phi^{\dagger\prime}\Phi')$$
$$- \lambda_2 Tr(\Phi^\dagger \Phi \Phi^{\dagger\prime}\Phi') - \lambda_Y (YY^\star)^2 - [\lambda_3 \epsilon_{abc}\epsilon^{def} {\Phi_d}^a {\Phi_e}^b {\Phi_f}'^c + h.c.]$$
$$+ [kYDet(\Phi) + h.c.] \quad (1)$$

$$\mathcal{L}_{\mathcal{SB}} = [Tr(B.\Phi) + h.c.] + [Tr(B'.\Phi') + h.c.] + (D.Y + h.c.)$$
$$- [\lambda_m Tr(\Phi\Phi^{\dagger\prime}) + h.c.] \quad (2)$$

Mixing and Parameter Fixing:

Isospin	$I=1$	$I=\frac{1}{2}$	$I=0$
PseudoScalars(P=-1)	$\{\pi,\pi'\}$	$\{K,\ K'\}, \{K^*, K^{*\prime}\}$	$\{\eta_1,\eta_2,\eta_3,\eta_4,\eta_5\ \}$
Scalars(P=1)	$\{a,\ a'\ \}$,	$\{\kappa,\ \kappa'\}, \{\kappa^*,\ \kappa^{*\prime}\}$,	$\{f_1,f_2,f_3,f_4,f_5\}$

- For I $=1/2$, 1 states: Two and four quarks states mixed with each other.

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- For I = 1/2, 1 states: Two and four quarks states mixed with each other.
- For I = 0 scalar and pseudoscalar states two, four quarks as well as glueball states mixed with each other.

- Input Parameters:
  Mixing angles for π and K within the range $\{-\frac{\pi}{4}, \frac{\pi}{4}\}$ along with their decay constants.
  Two condensates (below 2 GeV)
- Symmetry Breaking Parameters:
  $\frac{B_s}{B_{u,d}} = \frac{m_s}{m_{u,d}} = \frac{{B_s}'}{{B_{u,d}}'}$

Parameter Fixing contd..

- Vacuum Stability Conditions: $\frac{\partial V}{\partial < v_i >} = 0.$
- Physical Input Mass: $(R^{-1}) M^2{}_{bare}(R) = M^2{}_{phys}$

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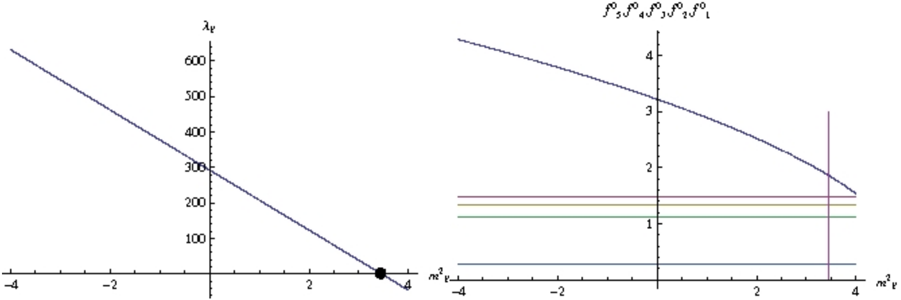
- Vacuum Stability Conditions: $\frac{\partial V}{\partial < v_i >} = 0.$
- Physical Input Mass: $(R^{-1}) {M^2}_{bare} (R) = {M^2}_{phys}$
- Parameters related to Glueball sector:

$$Tr[{M_\eta}^2]_{Model} = Tr[{M_\eta}^2]_{Exp} \ , \tag{3}$$

$$Det[{M_\eta}^2]_{Model} = Det[{M_\eta}^2]_{Exp} \ . \tag{4}$$

Bounded potential constraint $\lambda_Y > 0$



Dependence of λ_Y and scalar meson masses on the scanning parameter ${m_Y}^2$

π' Mass (GeV)	Field	Our Value (GeV)	quarkonia (%)	tetraquark (%)	Experimental Value (GeV)
1.2	a	1.055	38.14	61.8 6	0.98
	a'	1.417	61.86	38.14	1.47
	κ	1.13	62.14	37.86	0.80
	κ'	1.186	37.86	62.14	1.43

Table: Mass spectra and components for the triplet and doublet sector based on our fit are demonstrated where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

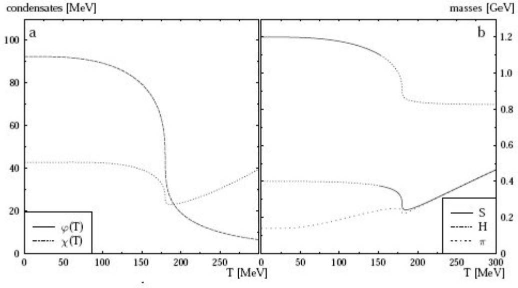
π' Mass (GeV)	$J^{PC} = 0^{-+}$	Our Value (GeV)	quarkonia (%)	tetraquark (%)	glueball (%)	Experimental Value (GeV)
1.2	η_5	1.858	0.037	0.001	99.962	1.756 ± 0.009
	η_4	1.380	75.803	24.167	0.03	1.476 ± 0.004
	η_3	1.291	26.700	73.294	0.006	1.294 ± 0.004
	η_2	0.907	15.852	84.145	0.003	0.95766 ± 0.00024
	η_1	0.595	81.607	18.393	0.0	0.547853 ± 0.000024

Table: Mass spectra and components for the pseudo-scalar mesons based on our fit are shown where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

π' Mass (GeV)	$J^{PC} = 0^{++}$	Our Value (GeV)	quarkonia (%)	tetraquark (%)	glueball (%)	Experimental Value (GeV)
1.2	${f_5}^0$	2.09	0.01	0.0	99.99	-
	${f_4}^0$	1.487	77.469	22.53	0.001	1.505 ± 0.006
	${f_3}^0$	1.347	22.177	77.82	0.003	1.2-1.5
	${f_2}^0$	1.124	21.561	78.439	0.0	0.980 ± 0.010
	${f_1}^0$	0.274	78.784	21.211	0.005	0.4-1.2

Table: Mass spectra and components for the scalar mesons based on our fit are shown where the best value of $m_{\pi'}$ is found to be $m_{\pi'} = 1.2$ GeV.

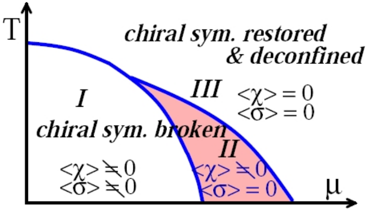
Effect of Mixing



- light tetraquark meson becomes degenerate with pions after chiral symmetry is restored.
- after transition chiral condensate approaches to zero but tetraquark condensate tends to rise. A. Heinz et al., Phys.Rev. D79 037502

Alternate Symmetry breaking

- Alternative breaking of chiral symmetry
  $SU(N_f)_L \times SU(N_f)_R \rightarrow$
  $SU(N_f)_V \times (Z_{N_f})_A \rightarrow SU(N_f)_V$.
  M. Harada et al. arXiv:0908.1361



Fermion-Boson Lagrangian

- Basic degrees of freedom: quarks and diquarks.
- Lagrangian:

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m_0)q + \frac{G_\sigma}{2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] + (\partial_\mu D^\dagger \partial^\mu D - M_s^2 D^\dagger D) + \frac{G_D}{2}(D^\dagger D)^2 + G_I(\bar{q}q)(D^\dagger D)$$

- To construct baryons it is necessary to include axial-vector isovector diquark field with axial vector coupling to quark.

But we don't account for baryons in our present study.

Fermion-Boson Lagrangian

- Effective vacuum Lagrangian (after bosonization):

$$\mathcal{L} = -i\mathrm{Tr}lnS_q^{-1} + \frac{i}{2}\mathrm{Tr}ln\Delta_D^{-1} - \frac{\sigma^2}{2G_\sigma} - \frac{\pi^2}{2G_\sigma} - \frac{\chi^2}{2G_D} - G_I\frac{\sigma\chi}{G_\sigma G_D}$$

- We assume apart from chiral condensate, tetraquark condensate $< D^\dagger D >$ is also present in the vacuum.

- Here, because of mixing, constitutent masses of quark and diquark depend on both the chiral and tetraquark condensates:

$$m_q = m_0 - \sigma - \frac{G_I}{G_D}\chi$$
$$M_D^2 = M_S^2 - \chi - \frac{G_I}{G_\sigma}\sigma$$

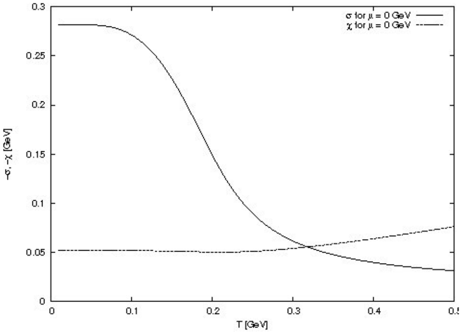
(Note: In our notation, $\sigma = G_\sigma < \bar{q}q >$ and $\chi = G_D < D^\dagger D >$)

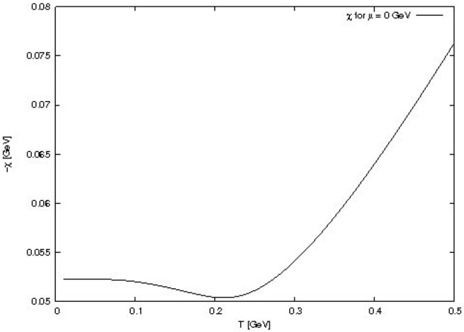
Input Parameters

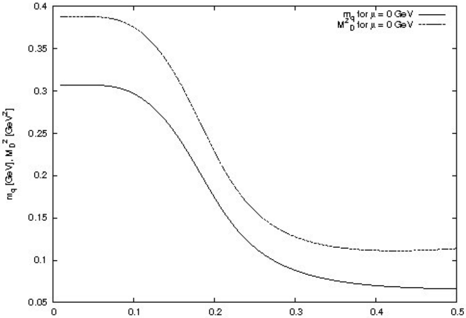
$$< \bar{u}u >= (0.251)^3 GeV^3,\ f_\pi = 92.4 MeV,\ m_\pi = 0.14 GeV,$$
$$f_0(600) = 0.4 - 1.2 GeV,\ f_0(1370) = 1.2 - 1.5 GeV$$

- GMOR relation: $G_\sigma = \frac{m_0 m_q}{m_\pi^2 f_\pi^2}$
- Two scenarios possible:

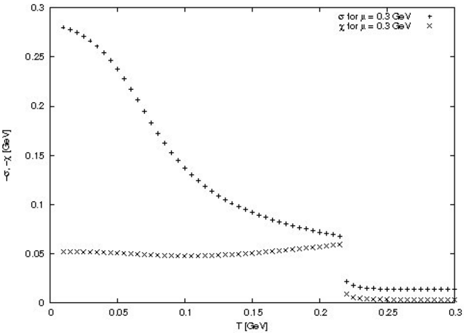
  $f_0(600)$ is a conventional meson whereas $f_0(1370)$ is a tetraquark meson and vice versa.
- We assume: $\chi = \frac{\sigma}{n}$ where n is a positive integer.


σ for μ = 0 GeV
χ for μ = 0 GeV
−σ, −χ [GeV]
T [GeV]


χ for μ = 0 GeV
−χ [GeV]
T [GeV]


m_q for μ = 0 GeV
M^2_D for μ = 0 GeV
m_q [GeV], M_D^2 [GeV^2]

$$\frac{\partial\Omega}{\partial\chi} = \frac{\chi}{G_D} + \frac{G_I}{G_\sigma G_D}\sigma - 12\int \frac{p^2 dp}{2\pi^2}\frac{\partial E_f}{\partial\chi}[1 - \frac{1}{exp[\beta(E_f-\mu)]+1} - \frac{1}{exp[\beta(E_f+\mu)]+1}] - \frac{1}{2}\int \frac{p^2 dp}{2\pi^2}\frac{\partial E_b}{\partial\chi}[1 + \frac{1}{exp[\beta(E_b-\mu)]-1} + \frac{1}{exp[\beta(E_b+\mu)]-1}]$$



Finite density results.

Note: In all these results we have assumed $f_0(600)$ is a conventional $\bar{q}q$ meson and $f_0(1370)$ is a tetraquark meson. The values of the parameters: $m_0 = 5.5$ MeV, $M_s = 0$ MeV, $G_\sigma = 9.26$ GeV^{-2} $G_D = 28.72$, $G_I = 11.04$ GeV^{-1}.

- To understand the vacuum phenomenology we need to incorporate scale anomaly
- we need to couple tetraquark to glueball in our framework
- need to analyze the various decay widths of the mesons.
- To study the effect of mixing between quarkonium and tetraquark on the QCD phase diagram within our model we need to analyze the allowed parameter space to study the consequences
- need to incorporate baryonic degrees of freedom.
- need to study the medium behaviour of the mesons under chiral phase transition in the context of mixing between quarkonium and tetraquark.

THANK YOU!