Quark number susceptibilities from resummed perturbative QCD

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In collaboration with Jens O. Andersen (Trondheim), Nan Su and Aleksi Vuorinen (Bielefeld)
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2 Hard-Thermal-Loop perturbation theory

3 Dimensional Reduction

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Why finite $T$ and $\mu$ QCD via analytical methods?

- Alternative to Lattice field theory methods at finite density
  - Heavy ion collisions
  - Neutron star physics
  - Early universe
- Different approach, deeper understanding of the nature of QGP
- Connection with the $T \to \infty$ case

What are the challenges (at least some of)?

- Lower the range of $T$ for which predictions are reliable
- Improve the apparent convergence of the perturbative series
- Set up a self consistent and systematic framework
- Generalization to finite chemical potential, and far from $\frac{\mu}{T} \ll 1$

What do we intend to do here?

- Studying at which extent one can understand high precision lattice data, using weakly coupled quasiparticle picture
Imaginary time formalism: \( P_0 \equiv (2n + 1) \pi T - i\mu \)

Dimensional regularization in the momentum space:

\[
\sum_{\{k\}} \sim T \sum_{n=-\infty}^{n=+\infty} \int \frac{d^{3-2\epsilon} p}{(2\pi)^{3-2\epsilon}}
\]

Lagrangian density of QCD:

\[
L_{\text{QCD}} = -\frac{1}{2} \text{Tr} (F_{\mu\nu} F^{\mu\nu}) + i \bar{\psi} \gamma^\mu D_\mu \psi
+L_{\text{gf}} + L_{\text{ghost}} + \Delta L_{\text{QCD}}
\]

Relation between the thermodynamic potential \( \Omega \) and the diagonal quark number susceptibility (QNS) \( \chi \):

\[
\chi(T) \equiv - \frac{\partial^2 \Omega(T, \mu)}{\partial \mu^2} \bigg|_{\mu=0}
\]

\( p \) the pressure: \( p(T, \mu) \equiv -\Omega(T, \mu) \)
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Reorganizing the perturbative series of thermal QCD:

\[ \mathcal{L} = (\mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{HTL}}) \bigg|_{g \to \sqrt{\delta} g} + \Delta \mathcal{L}_{\text{HTL}} \]

With the HTL improvement term:

\[ \mathcal{L}_{\text{HTL}} = -\frac{1}{2} (1 - \delta) m_D^2 \text{Tr} \left( F_{\mu \alpha} \left\langle \frac{y^\alpha y^\beta}{(y \cdot D)^2} \right\rangle_y F^\mu_{\beta} \right) \]

\[ + (1 - \delta) i m_q^2 \gamma^\mu \left\langle \frac{y^\mu}{y \cdot D} \right\rangle_y \psi \]

\[ \delta: \text{ formal expansion parameter, set to 1 in the end,} \]

\[ \# \text{ of HTL dressed loops} \]

\[ \Delta \mathcal{L}_{\text{HTL}}: \text{HTL counterterm(s); } y^\mu: \text{ lightlike 4-vector} \]

\[ m_D/m_q: \text{Debye/quark thermal mass parameters} \]

Adding \( \mathcal{L}_{\text{HTL}} \) “SHIFTS” the expansion to an ideal gas of thermal quasiparticles...
Leading order, 1-loop sum-integrals with dressed propagators:

Typical integral: $\sum_{\{P\}} \log \left[ \frac{A_S^2 - A_0^2}{P^2} \right]$ 

where $A_0$ and $A_S$ are tricky functions of $P_0, p, m_q$ and $T_P$ with:

$$(T_P)^m = \left( \frac{\Gamma \left( \frac{3}{2} - \epsilon \right)}{\Gamma \left( \frac{3}{2} \right) \Gamma (1 - \epsilon)} \right)^m (P_0^2)^m$$

$$\int_0^1 dc_1 \ldots \int_0^1 dc_m \left\{ \frac{(1 - c_1^2)^{-\epsilon}}{(P_0^2 + p^2 c_1^2)} \ldots \frac{(1 - c_m^2)^{-\epsilon}}{(P_0^2 + p^2 c_m^2)} \right\}$$

Here, result for the High-T truncation (no branch cuts):

Expansion in power of $m_{q,D}/T$, truncation at order $O(m_{q,D}^4)$

Prescription for the mass parameters:

$$m_D^2 \equiv \alpha_S \pi \left( \frac{4N_c}{3} T^2 + 2N_f \frac{\mu^2}{\pi^2} \right); \quad m_q^2 \equiv \alpha_S \pi \frac{N_c^2 - 1}{4N_c} \left( T^2 + \frac{\mu^2}{\pi^2} \right)$$
\[
\Omega_{\text{HTL}}^{\text{LO, high-}T} \equiv -\frac{d_A \pi^2 T^4}{45} \left\{ 1 + \frac{d_F}{d_A} \left( \frac{7}{4} + 30\hat{\mu}^2 + 60\hat{\mu}^4 \right) \right.
\]
\[
- \frac{15}{2} \hat{m}_D^2 - \left[ 30 \frac{d_F}{d_A} (1 + 12\hat{\mu}^2) \right] \hat{m}_q^2
\]
\[
+ 30\hat{m}_D^3 + \left[ 60 \frac{d_F}{d_A} (6 - \pi^2) \right] \hat{m}_q^4
\]
\[
+ \left[ \frac{45}{4} \left( \gamma_E - \frac{7}{2} + \frac{\pi^2}{3} + \log \left( \frac{\Lambda}{4\pi T} \right) \right) \right] \hat{m}_D
\]
\[
+ O(m_D^6, m_q^6) \right\}
\]

where \( \hat{m} \equiv \frac{m}{2\pi T} \), \( d_A \equiv N_c^2 - 1 \), and \( d_F \equiv N_c N_f \). As a preview,...
High-T truncation vs weak coupling up to $\alpha_s^2 \log \alpha_s$
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Separation of the pressure into different contributions:

\[ p_{\text{QCD}} \equiv p_{\text{hard}} + p_{\text{soft}}(m_E, \lambda, g_E) \]

⇒ Typical in-medium momentum scales

- \( p_{\text{hard}} \): From the hard modes \((\propto 2\pi T)\), via strict perturbative expansion in the 4D theory
- \( p_{\text{soft}} \): From the soft modes \((\propto gT)\), via the effective 3D Yang-Mills plus adjoint Higgs theory i.e. EQCD

Lagrangian density of EQCD:

\[
\mathcal{L}_{\text{EQCD}} = \frac{1}{2} \text{Tr} F_{ij}^2 + \text{Tr} [D_i, A_0]^2 + m_E^2 \text{Tr} A_0^2 \\
+ ig^3 \frac{1}{3\pi^2} \sum_f \mu_f \text{Tr} A_0^3 + \delta \mathcal{L}_E
\]

See Aleksi Vuorinen’s talk (on Tuesday)
Already known 4-loop result (at finite $\mu$) [A. Vurinen; 2003]

**BUT**

- Effect of the strange quark mass within 5%
  Ratio of free gas result [M. Laine, Y. Schroder; 2006]:
  \[ p(m_s) \approx \frac{p_{SB}(m_s)}{p_{SB}(m_s=0)} \times p(0) \]

- New resummation scheme applied to the finite $\mu$ case
  Keep the EQCD parameters unexpanded
  [M. Laine, Y. Schroder; 2006]
  ⇒ Resums a certain class of diagrams to all order
  ⇒ Substantially reduces the renormalization scale dependance!

As a preview,...
Effect of the strange quark mass in DR
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QNS from resummed perturbative QCD
■ Running of $\alpha_S$: 1-loop for HTLpt, and 2-loop for DR

■ Renormalization scale varied by a factor of 2 around:
\[ \Lambda_{\text{Ren.}}^{N_f = 2 - 3} \approx (1.29 - 1.44) \times 2\pi T \] for DR (optimal scale)
[K. Kajantie et al.; 1997]
\[ \Lambda_{\text{Ren.}}^{N_f = 2 - 3} \equiv 2\pi T \] for HTLpt

■ QCD scale:
\[ \Lambda_{\text{MS}}^{N_f = 2 - 3} = 180 - 200 \text{ MeV} \] for DR (fitted to HRG pressure)
\[ \Lambda_{\text{MS}}^{N_f = 2 - 3} \approx 140 - 160 \text{ MeV} \] for HTLpt (using lattice $\alpha_S$ value)

■ Comparison to recent high precision lattice data:
[S. Borsanyi et al., A. Bazavov et al.; 2012]
$N_f = 2$ perturbation theory vs Lattice data

Nf=2, P th. vs. lattice data; Andersen, Mogliacci, Su, Vuorinen (in preparation).
$N_f = 2 + 1$ perturbation theory vs Lattice data

Nf=2+1, P th. vs. lattice data; Andersen, Mogliacci, Su, Vuorinen (in preparation).
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Computation of $\chi$ in QCD via 2 different resummation schemes

The results are consistent with recent lattice data down to $T \approx 250 - 300$ MeV!!

HTLpt results are from the high T truncation of the free quasiparticle gas (1-loop)
Seems to account for a next to leading order computation
(full computation underway)
Perturbative pressure and problem of apparent convergence

Figure: Weak coupling expansion of the pressure up to $g^5$. 
HTLpt pressure up to 3-loops

Figure: HTLpt pressure for QCD with $N_f = 3$ up to 3-loops at zero $\mu$. Reference: [J.O. Andersen, L.E. Leganger, M. Strickland, N. Su; 2011].