Confinement, chiral symmetry in hadrons and in a dense matter

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Contents of the Talk

- Key questions to QCD
- Chiral symmetry and origin of hadron mass
- The quark condensate and the Dirac operator
- Extraction of the physical states on the lattice
- Hadrons after unbreaking of the chiral symmetry
- Confined but chirally symmetric dense, cold matter?

Key questions to QCD.

Key question to QCD: How is the hadron mass generated in the light quark sector?

- How important is the chiral symmetry breaking for the hadron mass?
- Are confinement and chiral symmetry breaking directly interrelated?
- Is there parity doubling and does chiral symmetry get effectively restored in high-lying hadrons?
- Is there some other symmetry?

L.Ya.G., C.B. Lang, M. Schröck, PRD 86 (2012) 014507

What is the hadron mass origin in QCD?

Gell-Mann - Levy sigma model, Nambu - Jona-Lasinio mechanism, many "Bag-like" and microscopical models to QCD:

Chiral symmetry breaking in a vacuum is the source of the hadron mass in the light quark sector.

A typical implication: In a dense medium upon smooth chiral restoration the hadron (ρ,...) mass should drop off (the Brown-Rho scaling).

Is it true?

Is chiral symmetry breaking in QCD and confinement are uniquely interconnected? (A key question for the QCD phase diagram).

The quark condensate and the Dirac operator

Banks-Casher: A density of the lowest quasi-zero eigenmodes of the Dirac operator represents the quark condensate of the vacuum:

$$< 0|\bar{q}q|0 >= -\pi\rho(0).$$

Sequence of limits: $V \to \infty; m_q \to 0$.

The lattice volume is finite and the spectrum is descrete. We remove an increasing number of the lowest Dirac modes from the valence quark propagators and study the effects of the remaining chiral symmetry breaking on the masses of hadrons.

$$S(k) = S - \sum_{i \leq k} \mu^{-1} |v_i >< v_i| \gamma_5,$$

S - standard quark propagator in a given gauge configuration;
μ_i are the real eigenvalues of the Hermitian $D_5 = \gamma_5 D$ Dirac operator;
$|v_i >$ - eigenvectors;
k number of the removed lowest eigenmodes.

Extraction of the physical states on the lattice

Assume we have hadrons (states) with energies $n = 1, 2, 3, ...$ with fixed quantum numbers.

$$C(t)_{ij} = \langle \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \rangle = \sum_n a_i^{(n)} a_j^{(n)*} \mathrm{e}^{-E^{(n)} t} \tag{1}$$

where

$$a_i^{(n)} = \langle 0 | \mathcal{O}_i | n \rangle \ .$$

The generalized eigenvalue problem:

$$\widehat{C}(t)_{ij} u_j^{(n)} = \lambda^{(n)}(t, t_0) \widehat{C}(t_0)_{ij} u_j^{(n)}. \tag{2}$$

Each eigenvalue and eigenvector corresponds to a given state. If a basis $\mathcal{O}_i$ is complete enough, one extracts energies and "wave functions" of all states.

$$\frac{C(t)_{ij} u_j^{(n)}}{C(t)_{kj} u_j^{(n)}} = \frac{a_i^{(n)}}{a_k^{(n)}} \ . \tag{3}$$

Extraction of the physical states on the lattice

E.g., we want to study $I = 1, 1^{--}$ states $\rho = \rho(770)$ and its excitations.
Then a basis of interpolators:

$$\mathcal{O}_V = \bar{q}(x)\tau\gamma^i q(x);$$

$$\mathcal{O}_T = \bar{q}(x)\tau\sigma^{0i} q(x);$$

$$\mathcal{O}_\partial = \bar{q}(x)\tau\partial^i q(x); \ldots$$

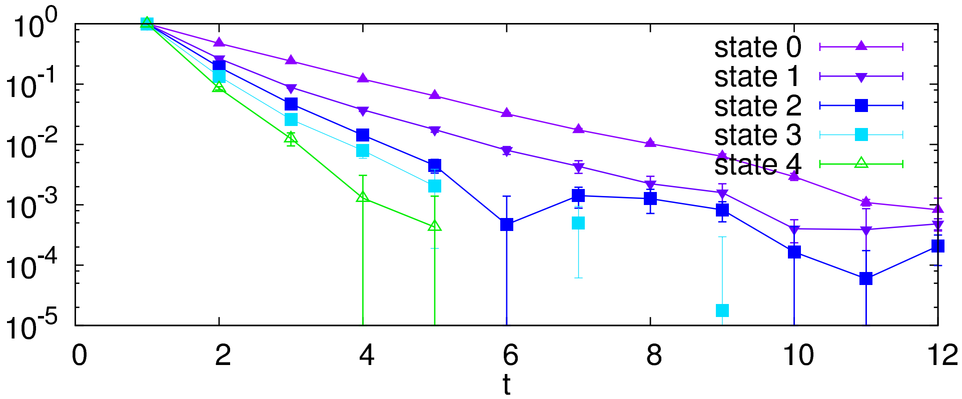
plus interpolators with a Gaussian smearing of the quark fields in spatial directions in the source and sink.

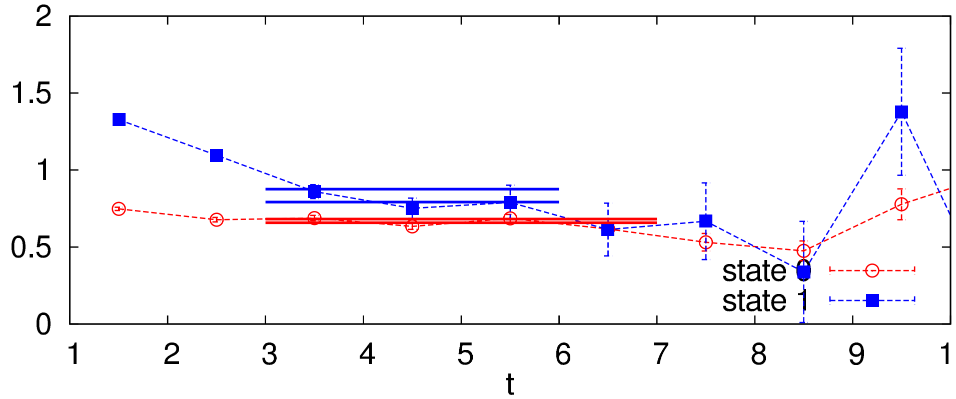
Some lattice details:

- Unquenched QCD with 2 dynamical flavors.
- $L = 2.4$ fm; $a = 0.144$ fm
- $m_\pi = 322$ MeV ($m_{u,d} \sim 15$ MeV)
- Chirally improved fermions

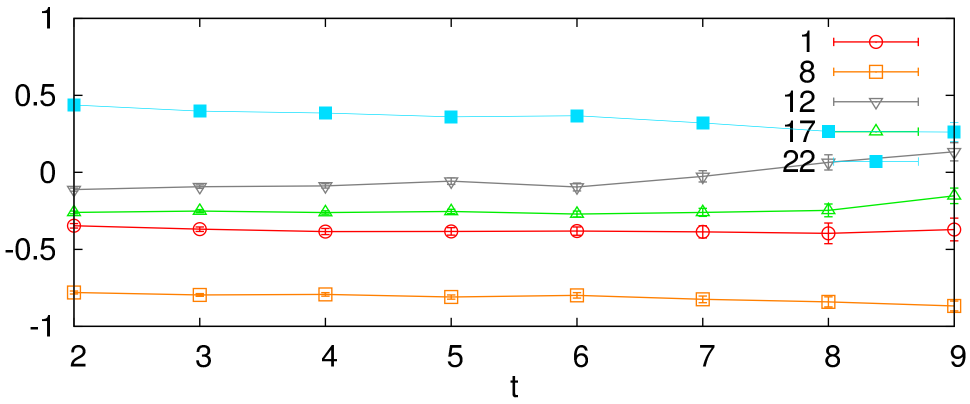
We subtract the low-lying chiral modes from the valence quarks.

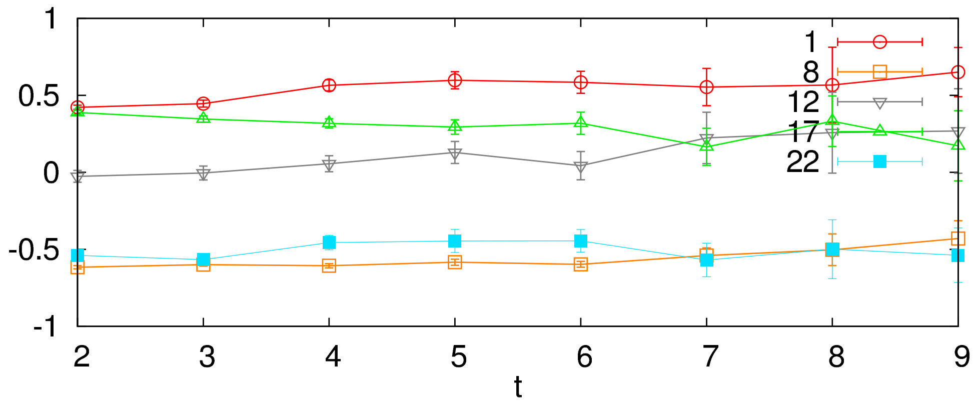
$\rho(I=1,1^{--})$ with 12 eigenmodes subtracted





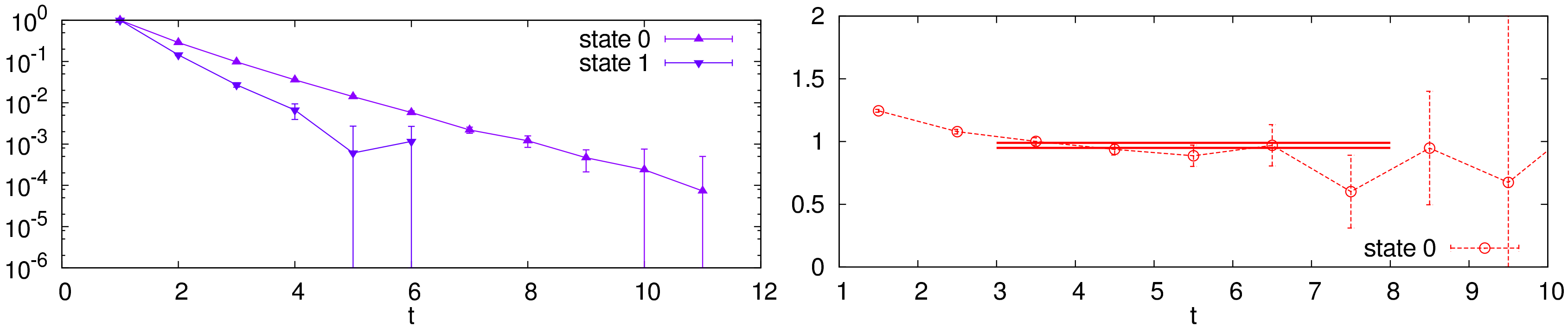
The correlators $\lambda_n(t) \sim \exp(-E_n t)$ for all eigenstates (left) and the effective mass plot $E_n(t) = \log(\lambda_n(t)/\lambda_n(t+1))$ for the two lowest states (right).



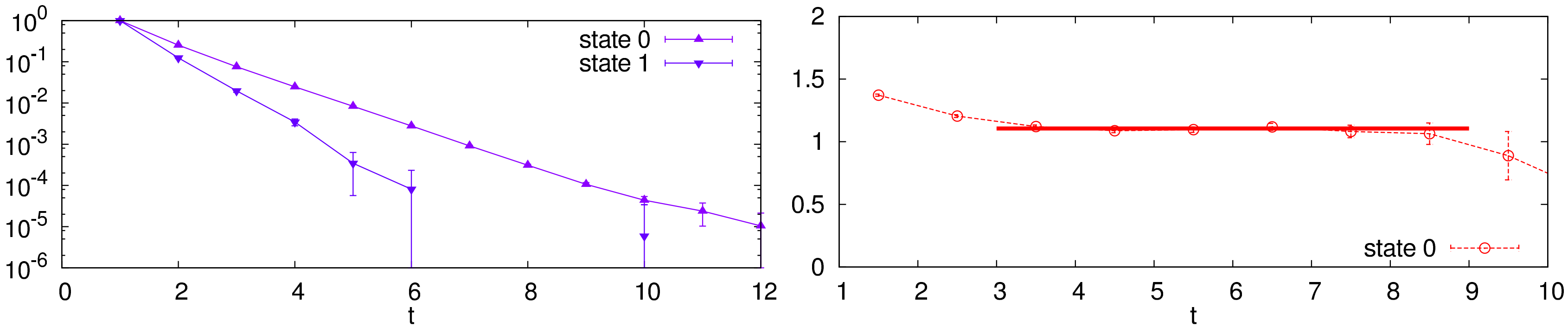


Eigenvectors corresponding to the ground state (left) and 1st excited state (right)

$b_1(I=1,1^{+-})$ states



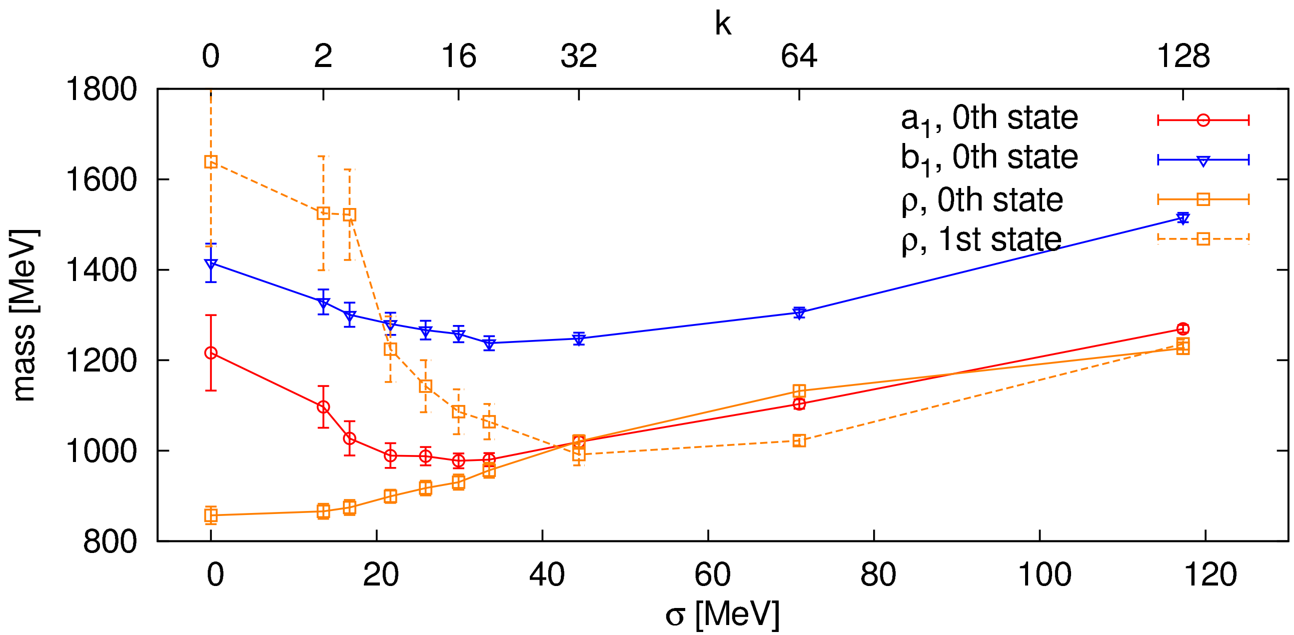
The correlators $\lambda_n(t) \sim \exp\left(-E_n t\right)$ for all eigenstates with 2 eigenmodes subtracted and the effective mass plot $E_n(t) = \log(\lambda_n(t)/\lambda_n(t+1))$ for the lowest state.



The same with 128 eigenmodes subtracted.

The quality of the exponential decay essentially improves with increasing the number of removed eigenmodes for ALL hadrons. By unbreaking the chiral symmetry we remove from the hadron its pion cloud and subtract all higher Fock components like πN, $\pi\Delta$, $\pi\pi$,...

What do meson degeneracies and splittings tell us?



The $SU(2)_L \times SU(2)_R \times C_i$ (chiral-parity) multiplets for $J = 1$ mesons:

$$(0,0) \quad : \quad \omega(0,1^{--}) \quad f_1(0,1^{++})$$

$$(\tfrac{1}{2},\tfrac{1}{2})_a \quad : \quad h_1(0,1^{+-}) \quad \rho(1,1^{--})$$

$$(\tfrac{1}{2},\tfrac{1}{2})_b \quad : \quad \omega(0,1^{--}) \quad b_1(1,1^{+-})$$

$$(0,1)+(1,0) \quad : \quad a_1(1,1^{++}) \quad \rho(1,1^{--})$$

The h_1, ρ, ω and b_1 states would form an irreducible multiplet of the $SU(2)_L \times SU(2)_R \times U(1)_A$ group.

What do meson degeneracies and splittings tell us?

- Chiral symmetry is restored but confinement is still there !

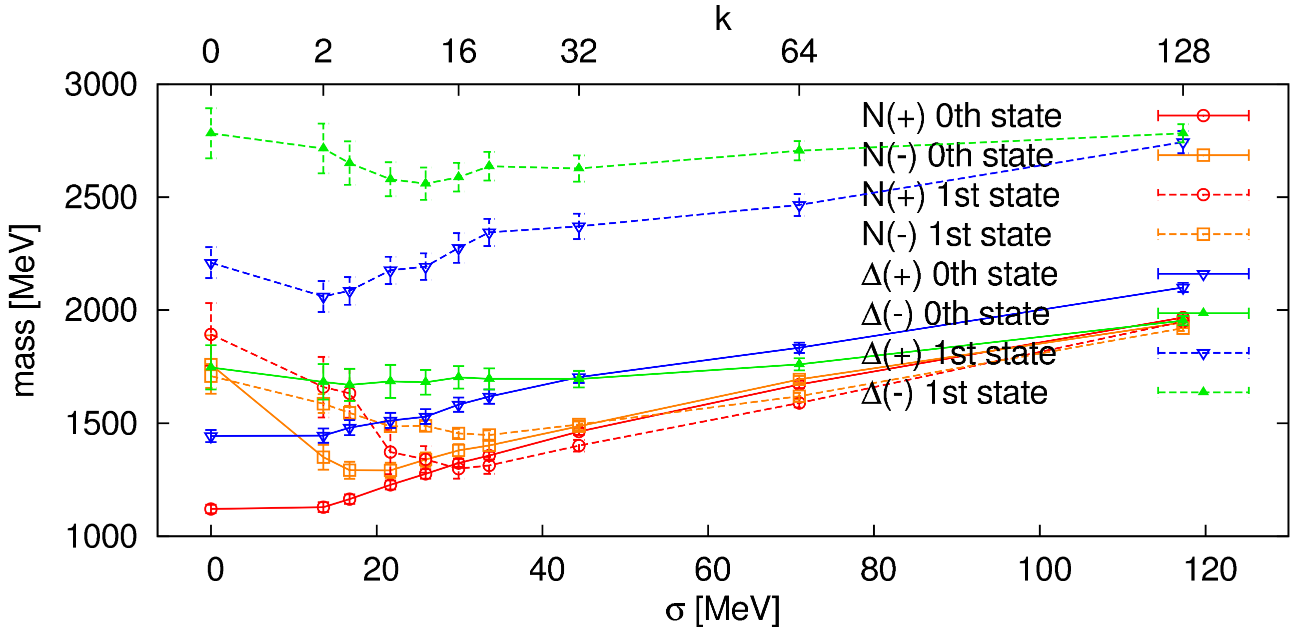
- Hadrons get their large chirally symmetric mass!

- The $SU(2)_L \times SU(2)_R$ gets restored while the $U(1)_A$ is still broken!

- The $U(1)_A$ explicit breaking comes not (not only) from the low-lying modes as the $SU(2)_L \times SU(2)_R$!

- $\rho - \rho'$ degeneracy indicates higher symmetry that includes $SU(2)_L \times SU(2)_R$ as a subgroup. What is this symmetry!? Is this symmetry related with the symmetry of the high-lying mesons?

Baryons



Three possible $SU(2)_L \times SU(2)_R \times C_i$ (chiral-parity) multiplets for any spin

$$(1/2,0)+(0,1/2); \quad (3/2,0)+(0,3/2); \quad (1/2,1)+(1,1/2)$$

Our interpolators have $J = 1/2$ for N and $J = 3/2$ for Δ, i.e. we cannot see $(1/2,1)+(1,1/2)$ quartets.

- Chiral symmetry is restored (all baryons are in doublets), while confinement is still there.
- Baryons have large CHIRALLY SYMMETRIC mass.
- Two $J = 1/2$ N doublets get degenerate - clear sign for a higher symmetry. No this higher symmetry for Δ's.

Confined but chirally symmetric cold, dense matter ?

At some critical density the standard quark-antiquark condensate of the vacuum should vanish because of Pauli blocking.

Above the chiral restoration point: confined matter with vanishing quark-antiquark condensate, i.e. built with confined but chirally symmetric hadrons?

In order to proceed we need an assumption.

At large density and small temperatures the matter could be a Fermi liquid or a crystal. Depends on fine details of the microscopic dynamics, that is not under control.

However.

Nuclear matter at $N_c = 3$ is a liquid. Color-superconductor is also a liquid.

Then at $N_c = 3$ the phase between the two is most naturally also a liquid.
If it is a Fermi liquid, then, by assumption, there are both rotational and translational invariancies.

Confined but chirally symmetric cold,dense matter

We cannot solve QCD. To address the issue we need a solvable model that is

(i) manifestly chirally symmetric

(ii) manifestly confining

(iii) provides spontaneous breaking of chiral symmetry

L.Ya.G., R. F. Wagenbrunn, PRD 77 (2008) 054027
L.Ya.G., PRD 79 (2009) 037504
L.Ya.G., PRD 80 (2009) 037701
L.Ya.G., V. Sazonov, R. F. Wagenbrunn, PRD 84 (2011) 095009

A schematic confining and chirally symmetric model. The only "gluonic" interaction is instantaneous Lorentz-vector linear potential (a generalization of the 't Hooft model).

Generalized 't Hooft model in a vacuum

The gap equation:

$$i\Sigma(\vec{p}) = \hbar \int \frac{d^4k}{(2\pi)^4} V_{CONF}(\vec{p}-\vec{k})\gamma_0 \frac{1}{S_0^{-1}(k_0,\vec{k}) - \Sigma(\vec{k})}\gamma_0 .$$

Infrared regularization is required.

$$\frac{\vec{\lambda}_i \cdot \vec{\lambda}_j}{4} V(r_{ij}) = \sigma r_{ij}; \qquad V(p) = \frac{8\pi\sigma}{(p^2+\mu_{IR}^2)^2}$$

The self-energy

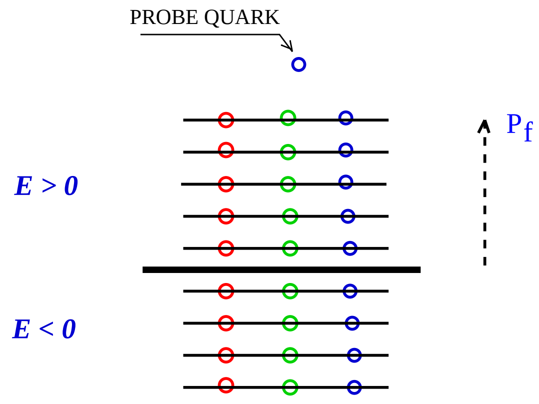
$$\Sigma(\vec{p}) = A_p + (\vec{\gamma}\hat{\vec{p}})[B_p - p].$$

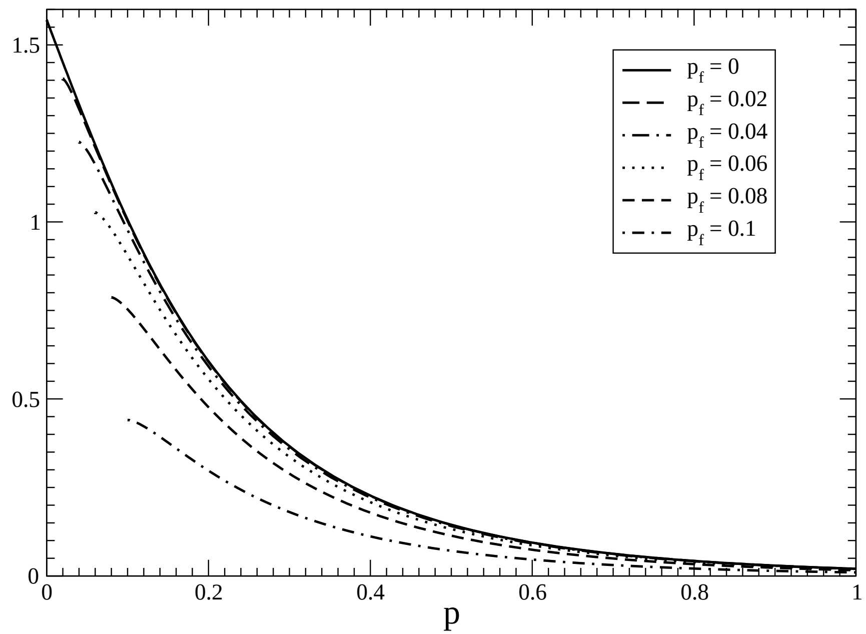
$$A_p = \frac{\sigma}{2\mu_{IR}} sin\varphi_p + A_p^f$$

$$B_p = \frac{\sigma}{2\mu_{IR}} cos\varphi_p + B_p^f$$

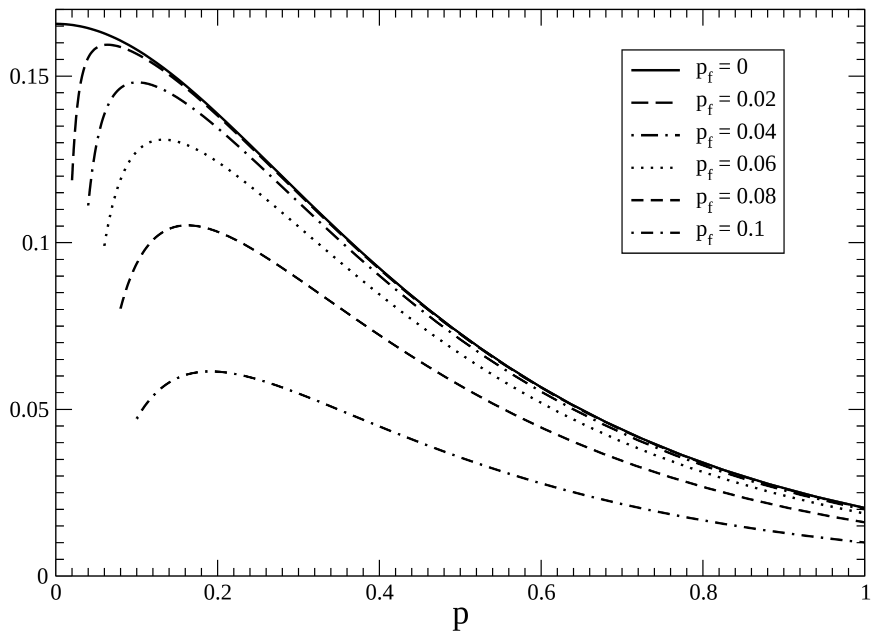
Inclusion of a finite chemical potential

We have to remove from the gap equation all occupied levels below P_f - Pauli blocking.





chiral angle

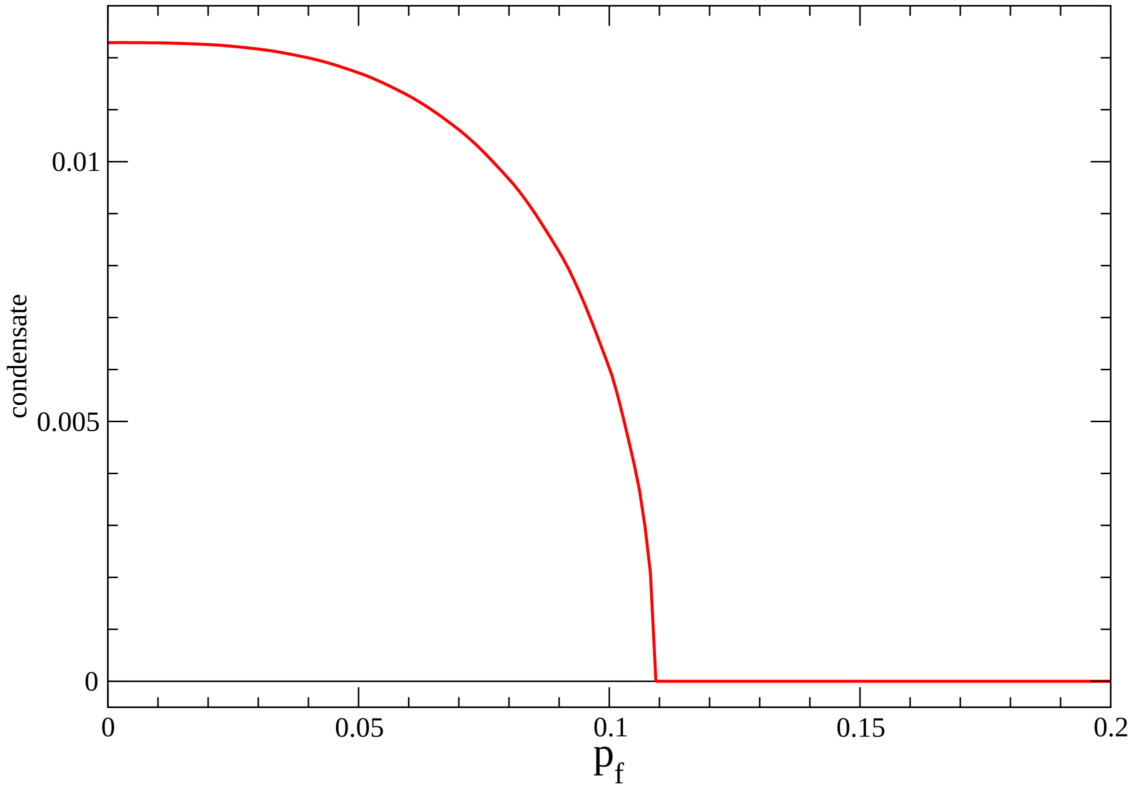


dynamical mass

Chiral symmetry restoration

Above the critical Fermi momentum, $P_f > P_f^{cr}$, there is no nontrivial solution of the gap equation. Chiral symmetry gets restored:

$$\varphi_p = 0; \qquad M(p) = 0; \qquad < \bar{q}q >= 0$$



Chiral symmetry restoration

$$\varphi_p = 0 \longrightarrow M(p) = 0; \qquad < \bar{q}q >= 0$$

Then in the self-energy operator,

$$\Sigma(\vec{p}) = A_p + (\vec{\gamma}\hat{\vec{p}})[B_p - p],$$

$$A_p = 0; \qquad B_p \to infrared \;\; divergent$$

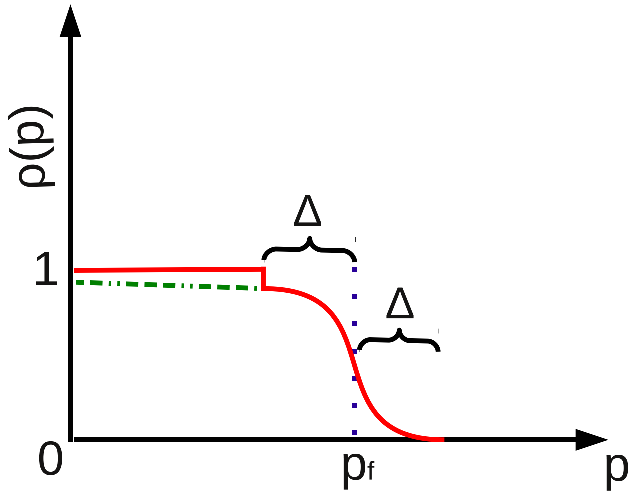
Quarks are still confined, because a single-quark energy is still infrared-divergent:

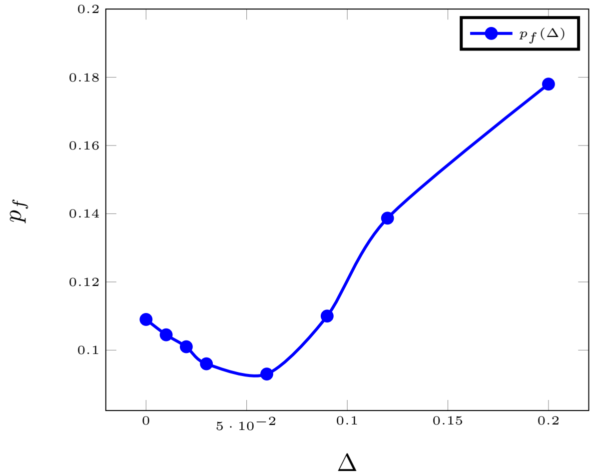
$$E(p) = \sqrt{A_p^2 + B_p^2} = \frac{\sigma}{2\mu_{IR}} + E_{fin}(p)$$

A single quark is removed from the spectrum at any chemical potential.

Not a rigid quark Fermi surface

Around the Fermi surface in the confining mode active degrees of freedom are the color-singlet baryons. Quarks interact inside these baryons. Consequently, there cannot be a rigid quark Fermi sphere. Instead a smooth distribution of quarks. Will the chiral restoration phase transition survive?





Conclusion: A chirally symmetric but confining dense, cold matter is possible, at least within a chirally symmetric and manifestly confining model.