Chiral crossover effects on the shear viscosity of a pion gas

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Collaborators

- based on
  - K. Heckmann, M.B., J. Wambach, work in progress
Motivation

▶ flow data at RHIC

![Graph showing flow data with different values of \( \eta/s \).

\[ v_2 \text{ (percent)} \]

\[ p_T \text{ [GeV]} \]

\[ \eta/s = 0.03 \]

\[ \eta/s = 0.08 \]

\[ \eta/s = 0.16 \]

[STAR]

[P. & U. Romatschke, PRL (2007)]

▶ conventional interpretation: QGP = “nearly perfect fluid” \((\eta/s \sim \frac{1}{4\pi})\)
Motivation

flow data at RHIC

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[H. Niemi et al., PRL (2011)]

- conventional interpretation: QGP = “nearly perfect fluid” ($\eta/s \sim \frac{1}{4\pi}$)
- more recent: hydrodynamics more sensitive to hadronic phase
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► aim: microscopic understanding of the shear viscosity in the hadronic phase
Motivation

- flow data at RHIC

![Graph showing flow data at RHIC](image)

[Romatschke & U. (2007)]

- conventional interpretation: QGP = “nearly perfect fluid” \( \frac{\eta}{s} \sim \frac{1}{4\pi} \)

- more recent: hydrodynamics more sensitive to hadronic phase

- aim: microscopic understanding of the shear viscosity in the hadronic phase

- here: BUU approach to \( \pi\pi \)-scattering in the NJL model

  - correct low-temperature limit

  - imprints of the chiral crossover and the compositeness of the pions

[H. Niemi et al., PRL (2011)]
Viscous relativistic hydrodynamics

- basic ingredients and conservation laws:
  - fluid 4-velocity $u^\mu(x)$, $u^\mu(x)u_\mu(x) = 1$
  - energy-momentum tensor $T^{\mu\nu}(x)$, $\partial_\mu T^{\mu\nu}(x) = 0$
  - particle current $J^\mu(x) = n(x)u^\mu(x)$, $\partial_\mu J^\mu(x) = 0$

- additional assumption:
  EoS + local thermal equilibrium $\Rightarrow \epsilon(x) = \epsilon(p(x))$
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- additional assumption:
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- gradient expansion:
  $T^{\mu\nu} = T^{(0)\mu\nu} + T^{(1)\mu\nu} + \ldots$ (and similar for $J^\mu$)
  - ideal fluid:
    $T^{(0)\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$
  - 1st-order viscous correction:
    $T^{(1)\mu\nu} = \eta \left( \partial^\mu u^\nu + \partial^\nu u^\mu + u^\mu u^\lambda \partial_\lambda u^\nu + u^\nu u^\lambda \partial_\lambda u^\mu \right)$
    $+ (\zeta - \frac{2}{3} \eta) (g^{\mu\nu} - u^\mu u^\nu) \partial_\lambda u^\lambda$
underlying assumption: mean free path $\lambda \gg$ interaction range $r$
Quantum relativistic kinetic theory

- underlying assumption: mean free path $\lambda \gg$ interaction range $r$
- particle phase-space distribution function: $f_a(\vec{x}, \vec{p}, t)$

$$T^{\mu\nu}(x) = \sum_a g_a \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p} f_a(\vec{x}, \vec{p}, t)$$
Quantum relativistic kinetic theory

- **underlying assumption:** mean free path $\lambda \gg$ interaction range $r$

- **particle phase-space distribution function:** $f_a(\vec{x}, \vec{p}, t)$

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- **Boltzmann-Uehling-Uhlenbeck (BUU) equation** $\text{(2} \rightarrow \text{2)}$

\[ \frac{df_a(\vec{x}, \vec{p}, t)}{dt} = \sum_b \frac{g_b}{1+\delta_{ab}} \int \frac{d^3p'}{(2\pi)^3} \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p'_1}{(2\pi)^3} \left\{ |\mathcal{M}_{ab}|^2 \frac{(2\pi)^4 \delta^4(p+p_1-p'-p'_1)}{16EE_1E'_1} \right. \]

\[ \times \left[ f'_a f'_{1b}(1 + f_a)(1 + f_{1b}) - f_a f_{1b}(1 + f'_a)(1 + f'_{1b}) \right] \]
Quantum relativistic kinetic theory

- underlying assumption: mean free path $\lambda \gg$ interaction range $r$
- particle phase-space distribution function: $f_a(\vec{x}, \vec{p}, t)$

$$T^{\mu\nu}(x) = \sum_a g_a \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p} f_a(\vec{x}, \vec{p}, t)$$

Boltzmann-Uehling-Uhlenbeck (BUU) equation ($2 \rightarrow 2$)

$$\frac{d}{dt} f_a(\vec{x}, \vec{p}, t) = \sum_b g_b \frac{1+\delta_{ab}}{1+\delta_{ab}} \int \frac{d^3 p'}{(2\pi)^3} \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p'_1}{(2\pi)^3} \left\{ |\mathcal{M}_{ab}|^2 \frac{(2\pi)^4 \delta^4(p+p_1-p'-p'_1)}{16 E E_1 E'_1} \right. \frac{1}{E} \left. \times \left[ f'_a f'_1 b (1 + f_a)(1 + f_{1b}) - f_a f_{1b} (1 + f'_a)(1 + f'_{1b}) \right] \right\}$$

- linearization ($2^{nd}$-order Chapman-Enskog expansion)

$$f_a = f_a^{(0)} + f_a^{(1)} + \ldots$$

local equilibrium: $f_a^{(0)}(x, p) = \frac{1}{\exp\left[\left(\mu^\mu u_{\mu}(x) - \mu_\pi(x)\right)/T(x) - 1\right]}$
Quantum relativistic kinetic theory

- **underlying assumption:** mean free path $\lambda \gg$ interaction range $r$

- **particle phase-space distribution function:** $f_a(\vec{x}, \vec{p}, t)$

\[
T^\mu\nu(x) = \sum_a g_a \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p} f_a(\vec{x}, \vec{p}, t)
\]

- **Boltzmann-Uehling-Uhlenbeck (BUU) equation** ($2 \rightarrow 2$)

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\frac{d}{dt} f_a(\vec{x}, \vec{p}, t) = \sum_b \frac{g_b}{1+\delta_{ab}} \int \frac{d^3p'}{(2\pi)^3} \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p'_1}{(2\pi)^3} \left\{|\mathcal{M}_{ab}|^2 \frac{(2\pi)^4 \delta^4(p+p_1-p'-p'_1)}{16EE_1E'_1} \times \left[f'_af'_b(1+f_a)(1+f_{1b}) - f_af_{1b}(1+f'_a)(1+f'_{1b})\right]\right\}
\]

- **linearization** ($2^{nd}$-order Chapman-Enskog expansion)

\[
f_a = f_a^{(0)} + f_a^{(1)} + \ldots, \quad \text{local equilibrium: } f_a^{(0)}(x, \rho) = \frac{1}{\exp\left[(p^\mu u_\mu(x) - \mu_\pi(x))/T(x) - 1\right]}
\]

- **linear integral equation for $\eta$**
Quantum relativistic kinetic theory

- **underlying assumption:** mean free path $\lambda \gg$ interaction range $r$

- **particle phase-space distribution function:** $f_a(\vec{x}, \vec{p}, t)$

\[ T^{\mu\nu}(x) = \sum_a g_a \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p} f_a(\vec{x}, \vec{p}, t) \]

- **Boltzmann-Uehling-Uhlenbeck (BUU) equation** ($2 \rightarrow 2$)

\[ \frac{d}{dt} f_a(\vec{x}, \vec{p}, t) = \sum_b \frac{g_b}{1+\delta_{ab}} \int \frac{d^3p'}{(2\pi)^3} \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p'_1}{(2\pi)^3} \left\{ |\mathcal{M}_{ab}|^2 \frac{(2\pi)^4 \delta^4(p+p_1-p'-p'_1)}{16EE_1E'E'_1} \cdot \left[ f'_a f'_1(1 + f_a)(1 + f_{1b}) - f_a f_{1b}(1 + f'_a)(1 + f'_{1b}) \right] \right\} \]

- **linearization** ($2^{nd}$-order Chapman-Enskog expansion)

\[ f_a = f_a^{(0)} + f_a^{(1)} + \ldots, \quad \text{local equilibrium: } f_a^{(0)}(x, p) = \exp\left[ \frac{1}{(p^\mu u^\mu(x) - \mu_\pi(x)) / T(x) - 1} \right] \]

- **linear integral equation for $\eta$**

- **physics input:** scattering matrix element $\mathcal{M}_{ab}$ **here:** $\pi\pi \rightarrow \pi\pi$ in NJL
Mesons in the NJL model

- Lagrangian: \[ \mathcal{L} = \bar{\psi} (i \not{\partial} - m) \psi + g \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2 \right] \]

- gap equation: \[ \Delta = \Delta + \frac{\Delta^*}{2} \quad \Rightarrow \quad \text{dynamical quark masses} \]

- mesons (RPA): \[ \begin{array}{c}
\includegraphics[width=0.5\textwidth]{mesons_RPA.png}
\end{array} = \begin{array}{c}
\includegraphics[width=0.2\textwidth]{mesons_RPA_1.png}
+ \begin{array}{c}
\includegraphics[width=0.2\textwidth]{mesons_RPA_2.png}
+ \ldots
= \begin{array}{c}
\includegraphics[width=0.2\textwidth]{mesons_RPA_3.png}
+ \begin{array}{c}
\includegraphics[width=0.2\textwidth]{mesons_RPA_4.png}
\end{array}
\end{array}
\end{array} \]
Mesons in the NJL model

- **Lagrangian:**
  \[ \mathcal{L} = \bar{\psi} (i \slashed{\partial} - m) \psi + g \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2 \right] \]

- **Gap equation:**
  \[ \to \text{dynamical quark masses} \]

- **Mesons (RPA):**
  \[ \to \text{mesons (RPA)} \]

- **In-medium masses:**

![Graph showing mass vs. temperature for different particles (q, π, σ)]
Mesons in the NJL model

- **Lagrangian:** \[ \mathcal{L} = \bar{\psi} (i\not\! \partial - m) \psi + g \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} \gamma_5 \vec{\tau} \psi)^2 \right] \]

- **gap equation:** \[ \begin{array}{c}
\begin{array}{c}
\text{dynamical quark masses}
\end{array}
\end{array} \]

- **mesons (RPA):** \[ \begin{array}{c}
\begin{array}{c}
\text{masses}
\end{array}
\end{array} \]

- **in-medium masses:**

- **characteristic temperatures:**
  
  [Quack et al., PLB (1995)]

  - **σ-dissociation temperature:**
    \[ m_\sigma (T_{\text{diss}}) = 2 m_\pi (T_{\text{diss}}) \]
    here: \[ T_{\text{diss}} = 180 \text{ MeV} \]
  
  - **Mott temperature:**
    \[ m_\pi (T_{\text{Mott}}) = 2 m_q (T_{\text{Mott}}) \]
    here: \[ T_{\text{Mott}} = 199 \text{ MeV} \]
In-medium $\pi\pi$-scattering

- **scattering amplitude**
  
  [Bernard et al., PLB (1991), Schulze, JPG (1995)]

- leading order $1/N_c$

- to be taken in $s$-, $t$-, and $u$-channel

- consistent with chiral low-energy theorems
In-medium $\pi\pi$-scattering

- scattering amplitude
  [Bernard et al., PLB (1991), Schulze, JPG (1995)]

- leading order $1/N_c$
- to be taken in $s$-, $t$-, and $u$-channel
- consistent with chiral low-energy theorems

- scattering length
  
  \[ a^l = \frac{1}{32\pi m_\pi} \mathcal{M}^l_{\pi\pi} (s = 4m_\pi^2, t = u = 0) \]

- chiral expansion [Weinberg, PRL (1966)]

  \[ a^0_W = \frac{7m_\pi}{32\pi f_\pi^2}, \quad a^2_W = -\frac{2m_\pi}{32\pi f_\pi^2} \]
In-medium $\pi\pi$-scattering

- scattering amplitude
  [Bernard et al., PLB (1991), Schulze, JPG (1995)]

  ![Diagram of scattering amplitude]

  - leading order $1/N_c$
  - to be taken in $s$-, $t$-, and $u$-channel
  - consistent with chiral low-energy theorems

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  a^l = \frac{1}{32\pi m_\pi^2} \mathcal{M}^l_{\pi\pi} (s = 4m_\pi^2, t = u = 0)
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  \[
  a^0_W = \frac{7m_\pi}{32\pi f_\pi^2}, \quad a^2_W = -\frac{2m_\pi}{32\pi f_\pi^2}
  \]

- numerical results
  cf. [Quack et al. PLB (1995)]

  ![Graph of $a$ vs. $T$]

  - Weinberg values at low $T$
  - “Feshbach resonances” at $T_{\text{diss}}$ and $T_{\text{Mott}}$
In-medium cross section

- isospin averaged cross section:
  \[
  \left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{1}{9} \sum_{l=0}^{2} (2l + 1) \left| \mathcal{M}_{\pi\pi}^{I} \right|^2 \frac{64\pi^2 s}{4\pi}\]

- \( i\mathcal{M}_{\pi\pi} = \) \[\text{diagram}\] + \[\text{diagram}\]
In-medium cross section

- isospin averaged cross section: \[
\left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{1}{9} \sum_{l=0}^{2}(2l + 1) \left| \frac{M_{\pi\pi}^l}{64\pi^2 s} \right|^2
\]

- \[iM_{\pi\pi} = \begin{array}{c}
\text{triangle} \\
\text{square}
\end{array}\]

- approximations:
  1. Weinberg amplitude
     \[M_{\pi\pi}^l = 32\pi m_\pi a_W^l\]
     \((T \text{ and momentum independent})\)
  2. evaluate \(M\) at threshold
     \[M_{\pi\pi}^l = 32\pi m_\pi a^l(T)\]
In-medium cross section

- isospin averaged cross section: 
  \[ \left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{1}{9} \sum_{l=0}^{2} (2l + 1) \left| \frac{M_{\pi\pi}^l}{64\pi^2 s} \right|^2 \]

- \( iM_{\pi\pi} = \begin{pmatrix} \sigma \end{pmatrix} + \begin{pmatrix} \end{pmatrix} \)

- approximations:
  1. Weinberg amplitude
     \[ M_{\pi\pi}^l = 32\pi m_\pi a_W^l \]
     \((T\text{ and momentum independent})\)
  2. evaluate \( M \) at threshold
     \[ M_{\pi\pi}^l = 32\pi m_\pi a^l(T) \]

---

**total cross section** \((T = 0)\)

![Graph showing total cross section vs. \( s^{1/2} \) in MeV. The graph indicates a decrease in cross section as \( s^{1/2} \) increases.](image-url)
In-medium cross section

- **isospin averaged cross section:**
  \[(\frac{d\sigma}{d\Omega})_{cm} = \frac{1}{9} \sum_{l=0}^{2} (2l + 1) \frac{|M_{\pi\pi}^l|^2}{64\pi^2s}\]

- **\(iM_{\pi\pi}\)**
  \[= \begin{array}{c}
  \alpha \\
  + \\
  \end{array} + \begin{array}{c}
  \square
  \end{array}\]

- **approximations:**
  1. Weinberg amplitude
     \[M_{\pi\pi}^l = 32\pi m_\pi a_W^l\]
     \((T\text{ and momentum independent})\)
  2. evaluate \(M\) at threshold
     \[M_{\pi\pi}^l = 32\pi m_\pi a^l(T)\]

**total cross section** \((T = 150\text{ MeV})\)
**In-medium cross section**

- Isospin averaged cross section:
  \[
  \left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{1}{9} \sum_{l=0}^{2} (2l + 1) \left| \mathcal{M}_{\pi\pi}^l \right|^2 \frac{1}{64 \pi^2 s}
  \]

- \( i \mathcal{M}_{\pi\pi} = \)

- Approximations:
  1. Weinberg amplitude
     \[ \mathcal{M}_{\pi\pi}^l = 32\pi \ m_\pi \ a_W^l \]
     \( (T \text{ and momentum independent}) \)
  2. Evaluate \( \mathcal{M} \) at threshold
     \[ \mathcal{M}_{\pi\pi}^l = 32\pi \ m_\pi \ a^l(T) \]

**Total cross section** \( (T = 177 \text{ MeV}) \)
In-medium cross section

- isospin averaged cross section: \[
\left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{1}{9} \sum_{l=0}^{2} (2l + 1) \left| \mathcal{M}_{\pi\pi}^l \right|^2 \frac{64\pi^2}{s}
\]

- \[i\mathcal{M}_{\pi\pi} = \begin{array}{c}
\sigma \\
+ \\
\end{array} + \begin{array}{c}
\end{array}
\]

- approximations:
  1. Weinberg amplitude
     \[\mathcal{M}_{\pi\pi}^l = 32\pi m_\pi a_W^l\]
     \((T\text{ and momentum independent})\)
  2. evaluate \(\mathcal{M}\) at threshold
     \[\mathcal{M}_{\pi\pi}^l = 32\pi m_\pi a^l(T)\]

**total cross section \((T = 188\text{ MeV})\)**

![Graph showing total cross section against \(s^{1/2}\) in MeV units.](image)
In-medium cross section

- isospin averaged cross section:
  \[ \left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{1}{9} \sum_{l=0}^{2} (2l + 1) \frac{|M^{ll}_{\pi\pi}|^2}{64\pi^2s} \]

- \[ iM^{\pi\pi} = \begin{cases} \sigma & \text{if } T = 0 \\ \text{box} & \text{if } T = 0 \end{cases} \]

- approximations:
  1. Weinberg amplitude
     \[ M^{ll}_{\pi\pi} = 32\pi \ m_{\pi} \ a_{W}^{l} \]
     \( (T \text{ and momentum independent}) \)
  2. evaluate \( M \) at threshold
     \[ M^{ll}_{\pi\pi} = 32\pi \ m_{\pi} \ a_{I}^{l}(T) \]
  3. keep momentum dependence of the \( \sigma \) exchange
     \( (\text{but still evaluate quark triangles and boxes at threshold}) \)

- total cross section \( (T = 0) \)

\[ \sigma_{tot} [mb] \]

\[ s^{1/2} [MeV] \]
In-medium cross section

- **isospin averaged cross section:**
  \[
  \left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{1}{9} \sum_{l=0}^{2} (2l + 1) \left| \mathcal{M}_{\pi\pi}^l \right|^2 \frac{64\pi^2}{s} 
  \]

- **\( i\mathcal{M}_{\pi\pi} \) = \[ \text{diagram} \] + \[ \text{diagram} \]**

- **approximations:**
  1. **Weinberg amplitude**
     \[
     \mathcal{M}_{\pi\pi}^l = 32\pi \ m_\pi \ a_W^l 
     \]
     \((T \text{ and momentum independent})\)
  2. **evaluate \( \mathcal{M} \) at threshold**
     \[
     \mathcal{M}_{\pi\pi}^l = 32\pi \ m_\pi \ a^l(T) 
     \]
  3. **keep momentum dependence of the \( \sigma \) exchange**
     \((\text{but still evaluate quark triangles and boxes at threshold})\)

**total cross section** \((T = 0)\)
In-medium cross section

- Isospin averaged cross section:
\[
\left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{1}{9} \sum_{l=0}^{2} (2l + 1) \left| \frac{M_{\pi\pi}^l}{64\pi^2 s} \right|^2
\]

- \( iM_{\pi\pi} = \) \( \sigma \) + \( \square \)

- Approximations:
  1. Weinberg amplitude
     \( M_{\pi\pi}^l = 32\pi m_\pi a_W^l \)
     \((T\) and momentum independent\)
  2. Evaluate \( M \) at threshold
     \( M_{\pi\pi}^l = 32\pi m_\pi a^l(T) \)
  3. Keep momentum dependence of the \( \sigma \) exchange
     (but still evaluate quark triangles and boxes at threshold)

Total cross section \((T = 150\text{ MeV})\)
In-medium cross section

- isospin averaged cross section:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{1}{9} \sum_{l=0}^{2} (2l + 1) \left| \mathcal{M}^{l}_{\pi\pi} \right|^2 \frac{64\pi^2 s}{64\pi^2 s}
\]

- \( i\mathcal{M}^{l}_{\pi\pi} = \) \( \sigma \) \( + \)

- approximations:
  1. Weinberg amplitude
     \( \mathcal{M}^{l}_{\pi\pi} = 32\pi m_{\pi} a_{W}^{l} \)
     \( (T\) and momentum independent\)
  2. evaluate \( \mathcal{M} \) at threshold
     \( \mathcal{M}^{l}_{\pi\pi} = 32\pi m_{\pi} a^{l}(T) \)
  3. keep momentum dependence of the \( \sigma \) exchange
     (but still evaluate quark triangles and boxes at threshold)

- total cross section \( (T = 177\text{ MeV}) \)

\[
\sigma_{tot} [\text{mb}]
\]

\[
s^{1/2} [\text{MeV}]
\]
In-medium cross section

- isospin averaged cross section:
  \[
  \left( \frac{d\sigma}{d\Omega} \right)_{cm} = \frac{1}{9} \sum_{l=0}^{2} (2l + 1) \left| \mathcal{M}_{\pi\pi}^l \right|^2 \frac{1}{64\pi^2 s}
  \]

- \[ i\mathcal{M}_{\pi\pi} = \begin{array}{c}
\end{array} + \begin{array}{c}
\end{array} \]

- approximations:
  1. Weinberg amplitude
     \[
     \mathcal{M}_{\pi\pi}^l = 32\pi m_\pi a_W^l
     \]
     \( (T \text{ and momentum independent}) \)
  2. evaluate \( \mathcal{M} \) at threshold
     \[
     \mathcal{M}_{\pi\pi}^l = 32\pi m_\pi a^l(T)
     \]
  3. keep momentum dependence of the \( \sigma \) exchange
     (but still evaluate quark triangles and boxes at threshold)

![Graph showing total cross section vs. \( s^{1/2} \) for different values of \( T \)]
Including the sigma-decay width

► physical inconsistency:

\[ \sigma \leftrightarrow \pi \pi \] considered in scattering, but not in RPA sigma propagator

\[ \Rightarrow \text{width strongly underestimated} \]
Including the sigma-decay width

- physical inconsistency:
  \( \sigma \leftrightarrow \pi \pi \) considered in scattering, but not in RPA sigma propagator
  \( \Rightarrow \) width strongly underestimated

- include \( \frac{1}{N_c} \)-correction term
  - but there are many more
  \( \Rightarrow \) inconsistency with chiral symmetry
Including the sigma-decay width

- **physical inconsistency:**
  \[ \sigma \leftrightarrow \pi\pi \] considered in scattering, but not in RPA sigma propagator
  \[ \Rightarrow \] width strongly underestimated

- **include**

  - 1/\(N_c\)-correction term
  - but there are many more
  \[ \Rightarrow \] inconsistency with chiral symmetry

- **include only imaginary part:**

  \[ \Pi_{\text{dressed}} = \Pi^{RPA}_{\sigma} + \text{Im} \Pi^{\pi\pi}_{\sigma} \]
  \[ D^{\text{dressed}}_{\sigma} = \frac{-2g}{1-2g\Pi^{\text{dressed}}_{\sigma}} \]
Including the sigma-decay width

- **physical inconsistency:**
  \[
  \sigma \leftrightarrow \pi\pi \text{ considered in scattering, but not in RPA sigma propagator}
  \]
  \[\Rightarrow \text{ width strongly underestimated}\]

- **include**
  \[\begin{array}{c}
  \square
  \end{array}\]
  - 1/\(N_c\)-correction term
  - but there are many more
    \[\Rightarrow \text{ inconsistency with chiral symmetry}\]

- **include only imaginary part:**
  \[
  \Pi^{dressed}_\sigma = \Pi^{RPA}_\sigma + \text{Im } \Pi^{\pi\pi}_\sigma
  \]
  \[
  D^{dressed}_\sigma = \frac{-2g}{1-2g\Pi^{dressed}_\sigma}
  \]

- **(unnormalized) spectral function**
  \[
  \rho_\sigma(q) = -2 \text{Im } D_\sigma(q)
  \]
  \[T = 0\]
Including the sigma-decay width

- physical inconsistency:
  \( \sigma \leftrightarrow \pi\pi \) considered in scattering, but not in RPA sigma propagator
  \( \Rightarrow \) width strongly underestimated

- include

  - \( 1/N_c \)-correction term
  - but there are many more
  \( \Rightarrow \) inconsistency with chiral symmetry

- include only imaginary part:
  \[
  \Pi^{\text{dressed}} = \Pi^{\text{RPA}} + \text{Im} \Pi^{\pi\pi}
  \]
  \[
  D^{\text{dressed}}_{\sigma} = \frac{-2g}{1-2g\Pi^{\text{dressed}}_{\sigma}}
  \]

- (unnormalized) spectral function
  \[
  \rho_{\sigma}(q) = -2 \text{ Im} \ D_{\sigma}(q)
  \]
  \( T = 150 \text{ MeV} \)
Including the sigma-decay width

- physical inconsistency:
  \( \sigma \leftrightarrow \pi \pi \) considered in scattering, but not in RPA sigma propagator
  \( \Rightarrow \) width strongly underestimated

- include
  
  - \( 1/N_c \)-correction term
  - but there are many more
  \( \Rightarrow \) inconsistency with chiral symmetry

- include only imaginary part:
  \[
  \Pi^{dressed}_\sigma = \Pi^{RPA}_\sigma + \text{Im} \Pi^{\pi \pi}_\sigma
  \]
  \[
  D^{dressed}_\sigma = \frac{-2g}{1-2g\Pi^{dressed}_\sigma}
  \]

- (unnormalized) spectral function
  \[
  \rho_\sigma(q) = -2 \text{Im} D_\sigma(q)
  \]
  \( T = 177 \text{ MeV} \)
Including the sigma-decay width

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  \[ \begin{array}{c}
    \hline
    \end{array} \]
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- (unnormalized) spectral function
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\[ T = 188 \text{ MeV} \]
Cross section with sigma-decay width

Total cross section \((T = 0)\)

- \(T = 0\): \(\sigma = \sigma_{\text{Weinberg}}\) at threshold
Cross section with sigma-decay width

\[ \sigma_{\text{tot}}(T) = 150 \text{ MeV} \]

- \( T = 0 \): \( \sigma = \sigma_{\text{Weinberg}} \) at threshold
- small and intermediate \( T \): \( \sigma \gg \sigma(M)_{\text{thresh}} \gg \sigma_{\text{Weinberg}} \)
Cross section with sigma-decay width

Total cross section \( (T = 177 \text{ MeV}) \)

- \( T = 0: \) 
  \[ \sigma = \sigma_{\text{Weinberg}} \text{ at threshold} \]

- Small and intermediate \( T: \) 
  \[ \sigma \gg \sigma(\mathcal{M})_{\text{thresh}} \gg \sigma_{\text{Weinberg}} \]

- \( T \approx T_{\text{diss}}: \) 
  \[ \sigma(\mathcal{M})_{\text{thresh}} \gg \sigma \gg \sigma_{\text{Weinberg}} \]
Cross section with sigma-decay width

\[
\sigma_{\text{tot}} \quad (T = 188 \text{ MeV})
\]

- \(T = 0:\)
  \[\sigma = \sigma_{\text{Weinberg}} \text{ at threshold}\]

- Small and intermediate \(T:\)
  \[\sigma \gg \sigma(\mathcal{M})_{\text{thresh}} \gg \sigma_{\text{Weinberg}}\]

- \(T \approx T_{\text{diss}}:\)
  \[\sigma(\mathcal{M})_{\text{thresh}} \gg \sigma \gg \sigma_{\text{Weinberg}}\]

- \(T > T_{\text{diss}}:\)
  \[\sigma \to \pi\pi \text{ irrelevant}\]

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Shear viscosity: numerical results

Weinberg
Shear viscosity: numerical results

- Weinberg
- $\mathcal{M} = \mathcal{M}_{\text{threshold}}$
Weinberg
\[ \mathcal{M} = \mathcal{M}_{\text{threshold}} \]
RPA $\sigma$-propagator
Shear viscosity: numerical results

- Weinberg
- $\mathcal{M} = \mathcal{M}_{\text{threshold}}$
- RPA $\sigma$-propagator
- dressed $\sigma$-propagator
Shear viscosity: numerical results

- Weinberg
- $\mathcal{M} = \mathcal{M}_{\text{threshold}}$
- RPA $\sigma$-propagator
- dressed $\sigma$-propagator

- validity of the kinetic approach:
  - criterion: $\frac{\lambda}{r} \gg 1$ (dilute gas)
  - $\lambda = \frac{1}{n\sigma}$ mean free path
  - $r = $ interaction range
    (e.g., $1/m_\sigma$, $1/m_\pi$, hard sphere: $\sqrt{\frac{\sigma}{\pi}}$)
Fluidity

- most popular measure: $\eta/s$
  - $\eta$ from our “best model” (dressed $\sigma$-meson)
  - entropy density of an ideal pion gas

- alternative measure: $L_\eta/L_n$
  [Liao & Koch, PRC (2010)]
  - $L_\eta = \frac{\eta}{\hbar c s}$, $L_n = n^{-1/3}$
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Conclusions

summary:

- shear viscosity from $\pi\pi$-scattering in the NJL model in kinetic theory
- agreement with lowest-order $\chi$PT (Weinberg) at low $T$, much lower values when approaching the crossover
- quantitative results very sensitive to details of the model
Conclusions

summary:

- shear viscosity from $\pi\pi$-scattering in the NJL model in kinetic theory
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- quantitative results very sensitive to details of the model

outlook:

- better description of $p$-wave $\pi\pi$ scattering (include $\rho$-meson)
- further scattering channels, e.g., kaons
- ...

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