Hadron spectroscopy from lattice QCD

Mike Peardon
School of Mathematics, Trinity College Dublin, Ireland

33rd International School of Nuclear Physics

Erice, 22nd September 2011

- Motivation
- Lattice techniques for spectroscopy
  - Variational method
  - Smearing and distillation
  - Spin
- Results
  - Isovector mesons
  - Isoscalar mesons
  - Charmonium
  - Baryons
- Scattering and resonances

With evolving techniques, lattice QCD should shed light on questions such as:

- Last decade saw proliferation of new states above open-charm threshold. What are they?
- Are there intrinsic gluonic excitations in hadrons?
- Do glueballs exists as observable resonances?
- Do molecules form? Are there tetraquark states?
- Why is the observed baryon spectrum not reproduced by the quark model?

Techniques for excited-state spectroscopy

Field theory on a Euclidean lattice


- Monte Carlo simulations are only practical using **importance sampling**
- Need a non-negative weight for each field configuration on the lattice

Minkowski $\longrightarrow$ Euclidean

- **Problem:** direct information on scattering is lost and must be inferred indirectly.
- **Benefit:** can isolate lightest states in the spectrum.
- For excitations and resonances, must use a **variational method.**

Variational method in Euclidean QFT

- Ground-state energies found from $t \to \infty$ limit of:

Euclidean-time correlation function

$$
\begin{aligned}
c(t) &= \langle 0|\, \Phi(t) \quad \Phi^\dagger(0)\, |0\rangle \\
&= \sum_{k,k'} \langle 0|\, \Phi|k\rangle\langle k|e^{-\hat{H}t}|k'\rangle\langle k'|\Phi^\dagger\, |0\rangle \\
&= \sum_{k} |\langle 0|\, \Phi|k\rangle|^2 \quad e^{-E_k t}
\end{aligned}
$$

- So $\lim_{t\to\infty} c(t) = Ze^{-E_0 t}$
- Variational idea: find operator Φ to maximise $c(t)/c(t_0)$ from sum of basis operators $\Phi = \sum_a c_a \phi_a$

[C. Michael and I. Teasdale. NPB215 (1983) 433]
[M. Lüscher and U. Wolff. NPB339 (1990) 222]

Variational method

If we can measure $C_{ab}(t) = \langle 0|\phi_a(t)\phi_b^\dagger(0)|0\rangle$ for all a, b and solve generalised eigenvalue problem:

$$\mathbf{C}(t)\,\underline{v} = \lambda \mathbf{C}(t_0)\,\underline{v}$$

then

$$\lim_{t-t_0\to\infty} \lambda_k = e^{-E_k t}$$

For this to be practical, we need:

- a 'good' basis set that **resembles the states** of interest
- **all elements** of this correlation matrix measured

Quarks on the computer

- Most computer time spent handling quark dynamics
- Calculation of two-point correlator between isovector quark bilinears:

$$C(t) = \frac{\int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \; \bar{\psi}_u\Gamma^a\psi_d(t)\;\bar{\psi}_d\Gamma^b\psi_u(0)\; e^{-S_G[U]+\bar{\psi}_f M_f[U]\psi_f}}{\int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \; e^{-S_G[U]+\bar{\psi}_f M_f[U]\psi_f}}$$

$$= \frac{\int \mathcal{D}U \;\mathrm{Tr}\,\Gamma^a M_d^{-1}(t,0)\Gamma^b M_u^{-1}(0,t)\;\det M^2[U]\; e^{-S_G[U]}}{\int \mathcal{D}U\;\det M^2[U]\; e^{-S_G[U]}}$$

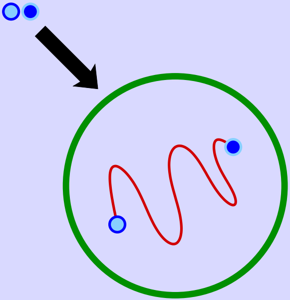
- Quarks in lagrangian → **determinant**
- Quarks in measurement → **propagators**

Both present their own specific problems

Improving measurements with quarks

- M^{-1} is too large to manipulate directly ...
- ... but can solve the linear system $M\underline{x} = \underline{y}$: **point propagator** method

- Smearing: construct good creation operators from extended objects

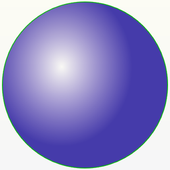


Distillation

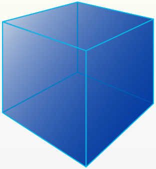
New technique

- smearing extracts relevant modes
- much smaller propagation problem
- freedom to build operators ...
- ... which enables variational method, isoscalars, 2 hadrons, ...

Spin on the lattice



$O(3)$



O_h

- Lattice regulator breaks $SO(3) \to O$
- Spin no longer a good quantum number
- States classified according to irreps of O_h not J^P

- O has 5 irreps: $\{A_1, A_2, E, T_1, T_2\}$
- To continuum: subduce reps $O(3) \to O_h$

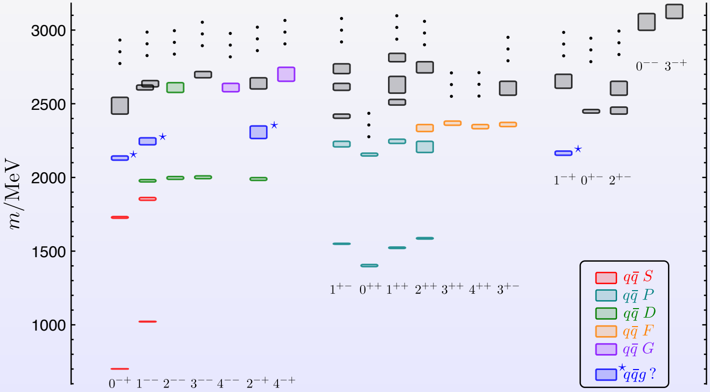
	A_1	A_2	E	T_1	T_2
$J=0$	1				
$J=1$				1	
$J=2$			1		1
$J=3$		1		1	1
$J=4$	1		1	1	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- Enough to search for degeneracy patterns in the spectrum? $4 \equiv 0 \oplus 1 \oplus 2!$
- Start with continuum operators built from derivatives
- Find patterns in operator overlaps

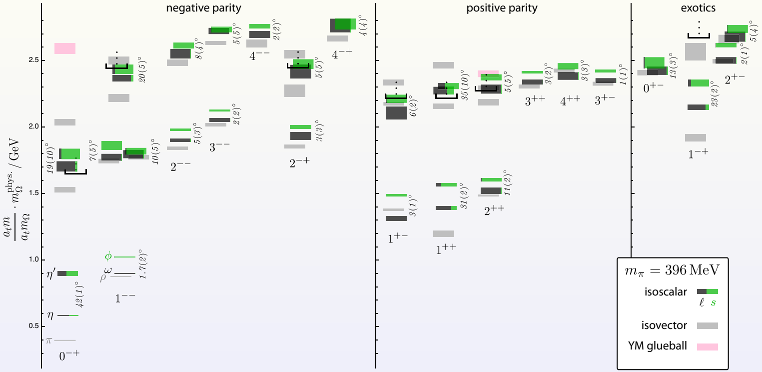
Results

Isovector meson spectrum ($m_\pi = 700$ MeV)

- Below $2GeV$, data resembles quark model
- First identification of the hybrid singlet/triplet?
- Still at unphysical m_π (and not in continuum limit)



Isoscalar meson spectrum

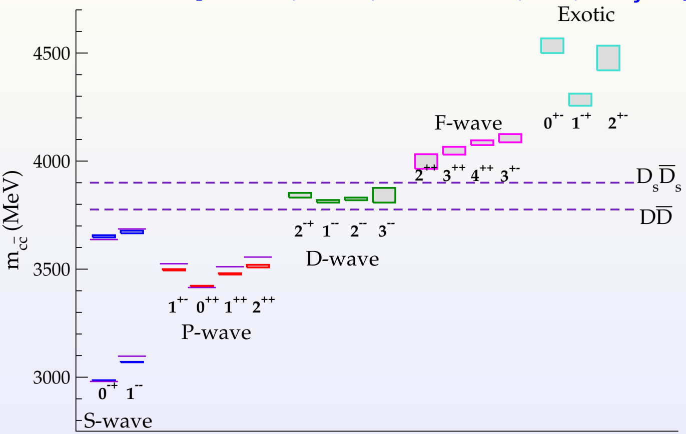


- $V = 16^3$ using GPUs to compute all-t propagators
- Percent-level statistical precision possible
- light-strange mixing computed
- **BUT** - 0^{++} not shown here!

[J. Dudek *et.al.* PRD83:111502 (2011)]

Charmonium

PRELIMINARY [G.Moir, L.Liu, P.Vilaseca, MP, S.Ryan]

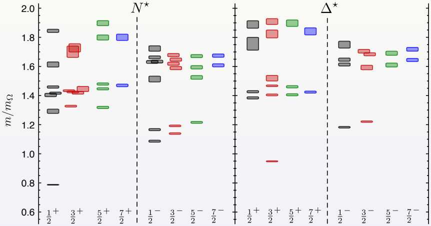


- Distilled charm quarks - good statistical precision again
- Statistical error on 1^{-+} hybrid ≈ 17 MeV

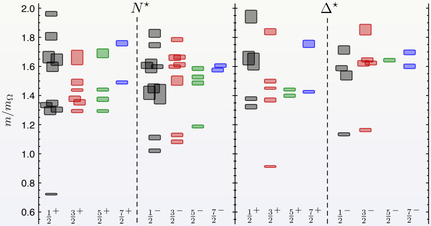
N and Δ excitations

[Edwards *et.al.*: arXiv:1104.5152]

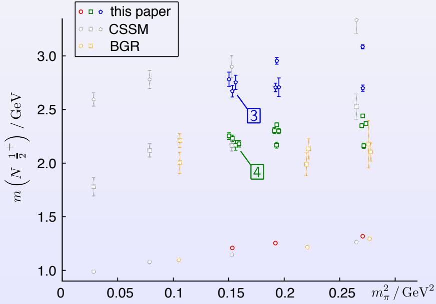
$m_\pi = 524\text{MeV}$



$m_\pi = 396\text{MeV}$



- Large operator basis, inspired by quark model
- With bigger operator basis, new states emerge
- More data closer to physical m_π required to understand the Roper



Scattering on the Euclidean lattice

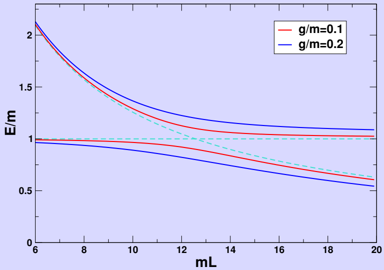
Hadrons in a finite box: scattering

- On a finite lattice with periodic b.c., hadrons have quantised momenta; $\underline{p} = \frac{2\pi}{L}\{n_x, n_y, n_z\}$
- Two hadrons with total $P = 0$ have a discrete spectrum
- These states can have same quantum numbers as those created by $\bar{q}\Gamma q$ operators and QCD can mix these

- This leads to shifts in the spectrum in finite volume
- In an experiment, this is the same physics that makes resonances
- Lüscher's method - relate elastic scattering to energy shifts

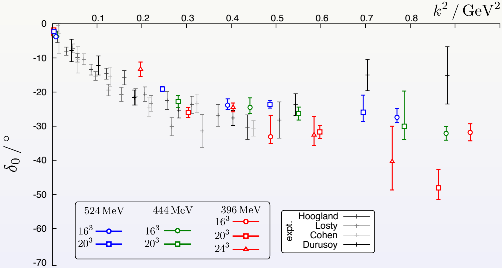
Toy model

$$H = \begin{pmatrix} m & g \\ g & \frac{4\pi}{L} \end{pmatrix}$$



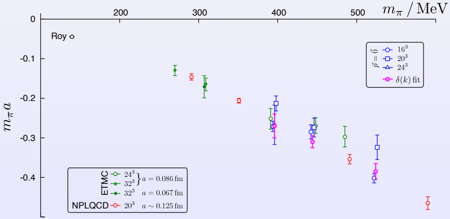
$I = 2$ $\pi\pi$ scattering

[Dudek *et.al.*: PRD83 071504 (2011)]



- S-wave phase shift
- No mass dependence observed
- Consistent with experimental data
- D-wave measured too

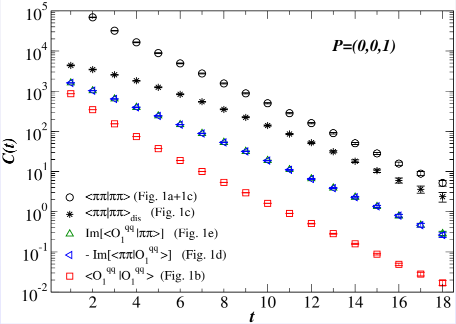
- Scattering length determined
- Compares well with other lattice determinations

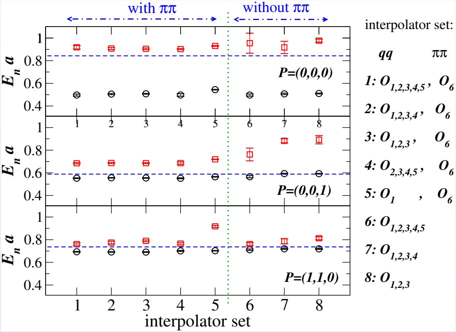


$I = 1$ scattering using distillation

[C.Lang *et.al.* arXiv:1105.5636]

- Number of groups have measured Γ_ρ on the lattice.
- Need non-zero relative momentum of pions in final state (P-wave decay)
- New calculation using distillation

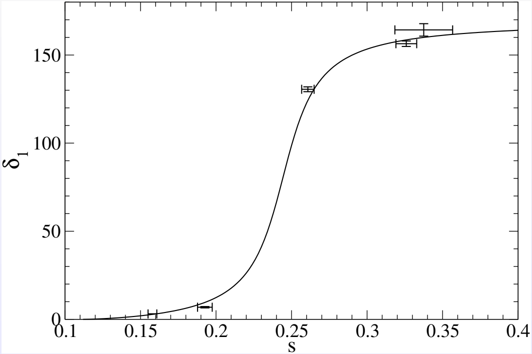




$I = 1$ $\pi\pi$ phase shift

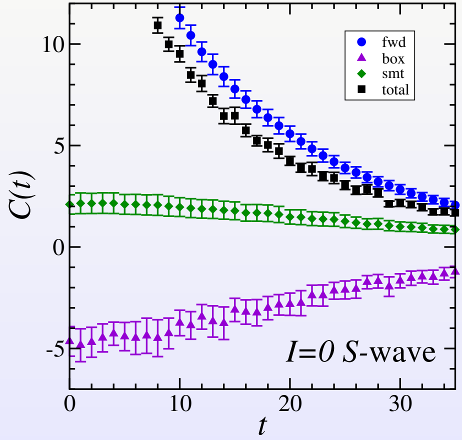
[C.Lang *et.al.* arXiv:1105.5636]

- $m_\pi \approx 266$ MeV
- Better resolution by studying moving ρ as well
- ρ resonance resolved clearly, with $m_\rho = 792(7)(8)$ MeV
- $g_{\rho\pi\pi} = 5.13(20)$



I=0 $\pi - \pi$ scattering - measuring $\langle \pi\pi | \pi\pi \rangle$

- Stochastic insertion into distillation space works well



[C. Morningstar *et.al.*: PRD83:114505 (2011)]

Conclusions

- Current state-of-the-art lattice simulations include quark dynamics and are approaching the physical pion masses
- The **variational method** is well established as the best way of studying excitations, scattering states and resonances
- New techniques enable the variational method to be exploited in more interesting ways
- Good resolution of the excited-state spectra of mesons and baryons seen up to ≈ 2.5 GeV: caveat - up/down quarks still heavy. Method working in charmonium sector
- Good resolution of isoscalar spectrum
- Scattering states in basis are essential
- First simulations of scattering using new methods reported recently - results are promising.