Coarse Graining in Nuclear Interactions

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Motivation

- No unique determination of the nucleon-nucleon interaction
- Effective coarse graining
  - Energy - Oscillator Shell Model
  - Configuration space - Euclidean Lattice EFT
  - Momentum - $V_{\rm lowk}$ interaction
- Effects bellow a characteristic distance ignored ($b \sim a \sim 1/\Lambda \sim 0.5 - 1.0 fm$)
- Different phenomenological NN potentials
  - Parameters fitted to experimental np and pp scattering data
  - High accuracy $\chi^2/\text{d.o.f.} \leq 1$
  - OPE as a long range interaction
  - ~ 40 parameters for the short and intermediate range
  - Repulsive core for most of them
    - Short range correlations
- Nuclear structure calculations become complicated

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Motivation
Nyquist Theorem

- Optimal sampling for a signal with finite bandwidth in Fourier space
- For our case

$$\Delta r \Delta k \sim 1 \tag{1}$$

- de Broglie wavelength of the most energetic particle
- Sampling resolution determined
- Coarse graining in configuration space

Delta Shell Potential

- A sum of delta functions

$$V(r) = \sum_i \frac{\lambda_i}{2\mu} \delta(r - r_i) \tag{2}$$

[Aviles Phys.Rev. C6 (1972) 1467]

- Optimal and minimal sampling of the nuclear interaction
- Pion production threshold $\Delta k = 2$ fm^{-1}
- Optimal sampling around 0.5 fm

Delta Shell Potential

- 3 well defined regions
- Innermost region $r \leq 0.5$ fm
  - Short range interaction
  - No delta shell (No repulsive core)
- Intermediate region $0.5 \leq r \leq 2.0$ fm
  - Unknown interaction
  - λ_i parameters fitted to experimental data
- Outermost region $r \geq 2.0$ fm
  - Long range interaction
  - Described by OPE
  - Sampled with delta shells as well

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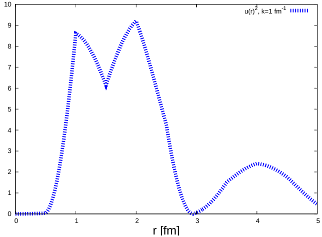
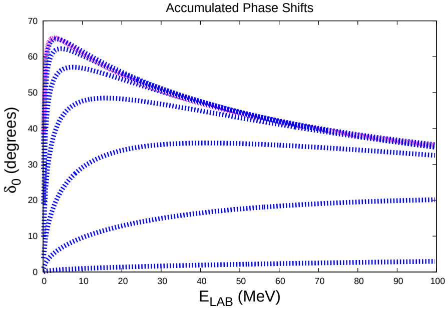
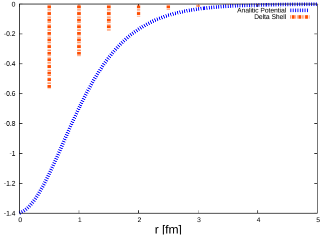
Comparison with known potential

$$V(r) \sim \sum_{i=1}^{n} V(r_i)\delta(r - r_i)\Delta r \qquad (3)$$

$$k \cot \delta = -\frac{1}{\alpha_0} + \frac{1}{2} r_{\text{eff}} k^2 \qquad (4)$$





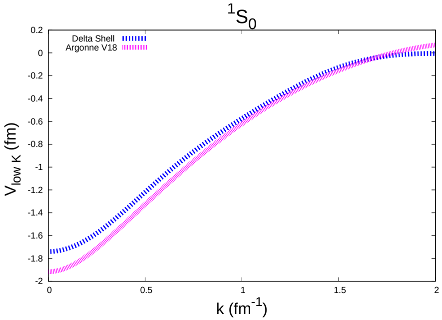


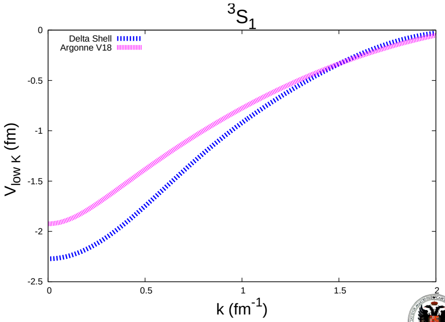
Comparison with V_{lowk}

- Fourier transform of delta shell potential

$$\langle k|V|k\rangle = \sum_i \lambda_i r_i^2 j_l(kr_i)^2 \tag{5}$$

- Coarse graining the interaction





Fitting np Phase shifts

- Several high quality potentials reproduce the np phase shifts
- NN-OnLine Data base
  - Nijm I
  - Nijm II
  - Reid93
  - ESC96
- Fit for every partial wave with $j \leq 4$
- Long range interaction sampled from OPE and Chiral N2LO TPE potentials
- Up to 3 interaction points inside the intermediate range part
- Strength coefficients as fit parameters and fixed concentration radii

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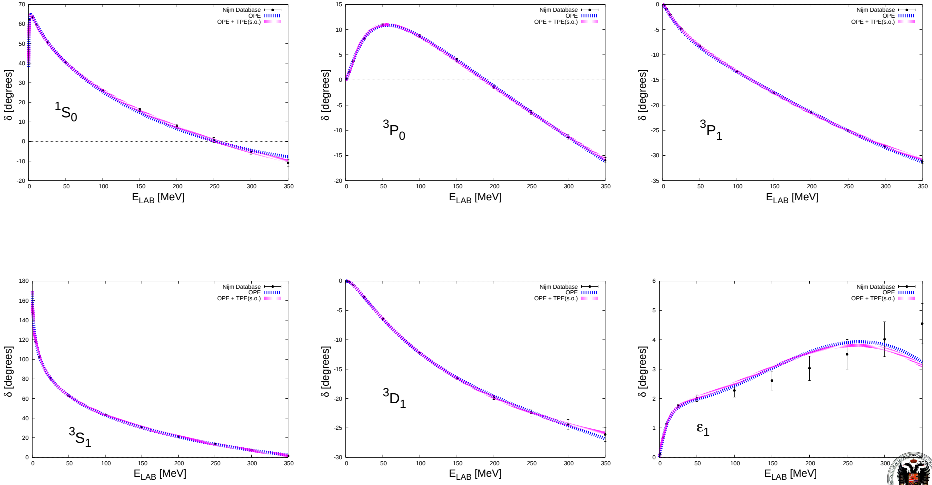
Fitting np Phaseshifts

Fitting Parameters

Wave	Potential	Number of parameters	χ^2/D.o.F
$^1S_0$	OPE	3	2.23
	TPE	3	0.12
$^3P_0$	OPE	2	0.18
	TPE	2	0.49
$^3P_1$	OPE	2	1.04
	TPE	2	0.71
$^3S_1$	OPE	3	
	TPE	3	
$^3D_1$	OPE	2	0.97
	TPE	2	0.54
ε_1	OPE	2	
	TPE	2	

Fitting np Phaseshifts

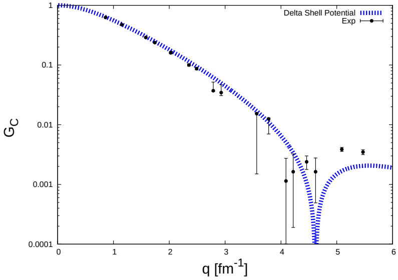


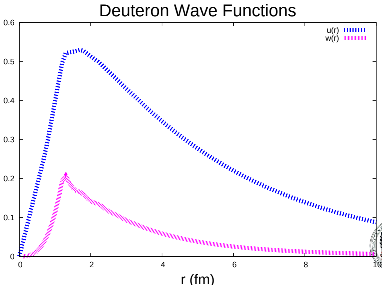
Deuteron Properties

- $^3S_1 -^3 D_1$ Coupled channels
- Schrödinger equation integrated for bound states
- Deuteron wave γ number determined by normalizable wave functions
- Wave functions can be calculated
    - Matter radius
    - Quadrupole moment
    - D-state probability
    - Inverse moment
    - Charge form factor

Deuteron Properties

	Delta Shell	Empirical	Nijm I	Nijm II	Reid93	Argonne v_{18}	Units
γ	0.230348	0.231605	Input	Input	Input	Input	fm^{-1}
η	0.02488	0.0256(5)	0.02534	0.02521	0.02514	0.0250	
A_S	0.8768	0.8781(44)	0.8841	0.8845	0.8853	0.8850	$fm^{1/2}$
r_m	1.9676	1.953(3)	1.9666	1.9675	1.9686	1.967	fm
Q_D	0.2693	0.2859(3)	0.2719	0.2707	0.2703	0.270	fm^2
P_D	5.498	5.67(4)	5.664	5.635	5.699	5.76	%
$\langle r^{-1} \rangle$	0.4596			0.4502	0.4515		fm^{-1}





Nuclear Structure

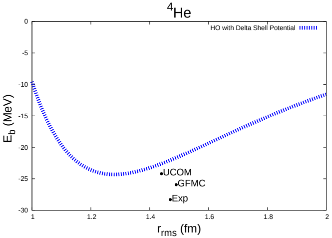
- Harmonic Oscillator Shell Model
- Anti-symmetrized product of single particle wave functions
- Relative coordinates representation - Talmi-Moshinsky coefficients
- Binding Energy for closed shell nuclei
    - ^{4}He, $(1s)^4$
    - ^{16}O, $(1s)^4(1p)^{12}$
    - ^{40}Ca, $(1s)^4(1p)^{12}(2s)^4(1d)^{20}$
- Variational approach with oscillator length parameter

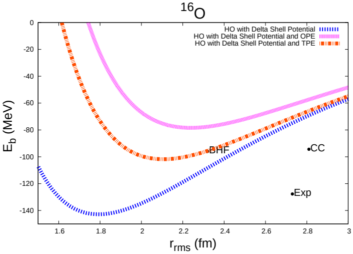
Nuclear Structure

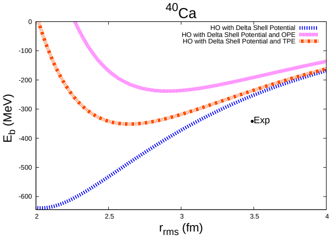
- ^{4}He occupies the 1s shell
- Potential energy expectation value

$$\langle V \rangle = 3\langle 1s | V_{^1S_0} | 1s \rangle + 3\langle 1s | V_{^3S_1} | 1s \rangle$$

- Similar for ^{16}O and ^{40}Ca

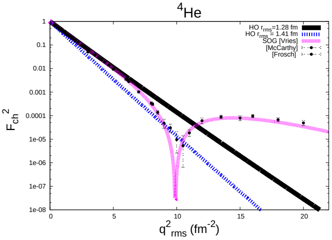


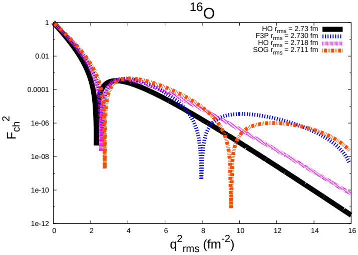


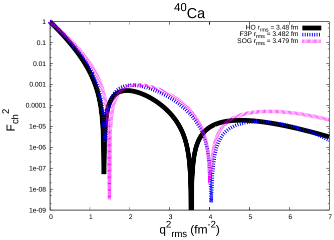


Nuclear Structure

- Charge form factors for closed shell nuclei
- Comparison with experimental data (^{4}He) and other parametrizations







Summary

- Sampling of the NN interaction by a delta shell potential
- Sampling resolution determined by the deBroglie wavelength of the most energetic particle in our scheme
- 3 well defined regions
- Coarse graining in configuration space, analogous to $V_{\rm lowk}$
- Fit to the np phase shifts in the Nijmegen data base
    - Every partial wave with $j \leq 4$
    - 40 fitting parameters
    - χ^2/d.o.f. $\lesssim 2$ (less than 1 in some waves)
- Nuclear structure calculations are analytic and simple! (Android App)
- Deuteron wave functions and properties
- Good agreement with empirical values and other calculations
- Variational approach to nuclei binding energy
- Charge Form factors of closed shell nuclei