

Time-like Electromagnetic form factors at PANDA

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Nucleon form factor related talks from Erice2011 program

- Diego Bettoni (Ferrara)
Antiproton physics (timelike processes at PANDA)
- Nikolay Kivel (Mainz)
Nucleon FF in space- and time-like regions
- Dmitry Khanefit (Mainz)
Feasibility Study on the extraction of the time-like form factors via
the process $p\bar{p} \rightarrow e^+e^-$ with PANDA-Experiment at FAIRT using
the PandaRoot frame work.

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Outline

Theoretical preparation

- Definition & interpretation of form factors (FFs)
- Time-like FFs & spin observables

Experimental aspect

- $p\bar{p} \rightarrow e^+ e^-$
- eppi0 with TDA simulation
- Polarized target R&D

Definition of form factors

- Dirac equation with external field

$$(\gamma_\mu p_\mu - m)\psi = 0 \quad \gamma_\mu p_\mu \rightarrow \gamma_\mu p_\mu + e\gamma_\mu A_\mu$$

- Pauli equation (non-relativistic limit of Dirac equation)

$$\left[\frac{1}{2m} \left(\vec{p} - \frac{e}{c} \vec{A} \right)^2 + e\phi + \mu_B \hat{\sigma} \cdot \vec{B} \right] \psi = i\hbar \frac{\partial \psi}{\partial t}$$

- W. Pauli Rev. Mod. Phys. 13, 203 (1941)

$$-i\kappa \gamma_\mu \gamma_\nu \left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \right)$$

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*Pauli term
contributes
to
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moment*

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Extension

- Relativistic covariance & gauge invariance
- Linear in electromagnetic potential
- Doesn't vanish in static case
- L.I. Foldy, Phys. Rev. 87 688 (1952)

$e\gamma_\mu A_\mu$	$e\gamma_\mu \square^n A_\mu$	$e\gamma_\mu \sum_{n=0}^{\infty} \square^n A_\mu$
$-i\kappa\gamma_\mu\gamma_\nu \left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \right)$	$-i\kappa\gamma_\nu\gamma_\mu \square^n \left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \right)$	$-i\kappa\gamma_\nu\gamma_\mu \sum_{n=0}^{\infty} \square^n \left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \right)$

Scattering matrix

$$S_{fi} = -i \int dx e^{-iqx} \bar{u}_2 (-i) \left(F_{Dirac} \gamma_\mu A_\mu + \frac{1}{2} \kappa F_{Pauli} \gamma_\mu \gamma_\nu \left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \right) \right) u_1$$

$$S_{fi} = -i \int dx e^{-iqx} \bar{u}_2 (-i) \left(F_{Dirac} \gamma_\mu + \frac{1}{2} i \kappa F_{Pauli} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) q^\nu \right) u_1 A_\mu(x)$$

- By inserting the summation of Dirac and Pauli term, each D'Alembert operator contributes a q^2
- Form factors as a function of q^2 instead of constant

Scattering matrix

Dirac term

Pauli term

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Dirac & Pauli FFs

- By inserting the summation of Dirac and Pauli term, each D'Alembert operator contributes a q^2
- Form factors as a function of q^2 instead of constant

Sachs form factor

- Physics interpretation of F_{Dirac} and F_{Pauli} :
 - F_{Dirac} containing both charge and magnetic terms
 - F_{Pauli} only for anomalous magnetic momentum
 - interference expression in cross section
- Non-relativistic limit
J.D. Walecka Nuovo Cimento 11 821 (1959)
- Breit frame (one value for each q)
R.G. Sachs, Phys. Rev. 126, 2256(1962)

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$$\bar{u}\left(\frac{1}{2}\vec{q}\right)\vec{F}(\vec{q}, 0)u\left(-\frac{1}{2}\vec{q}\right) \propto (\vec{\sigma} \times \vec{q})G_M(\vec{q}^2)$$

$$G_M(q^2) = F_1(q^2) + \kappa F_2(q^2)$$

$$\bar{u}\left(\frac{1}{2}\vec{q}\right)F_4(\vec{q}, 0)u\left(-\frac{1}{2}\vec{q}\right) \propto G_E(\vec{q}^2)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2}\kappa F_2(q^2)$$

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- Non-relativistic limit

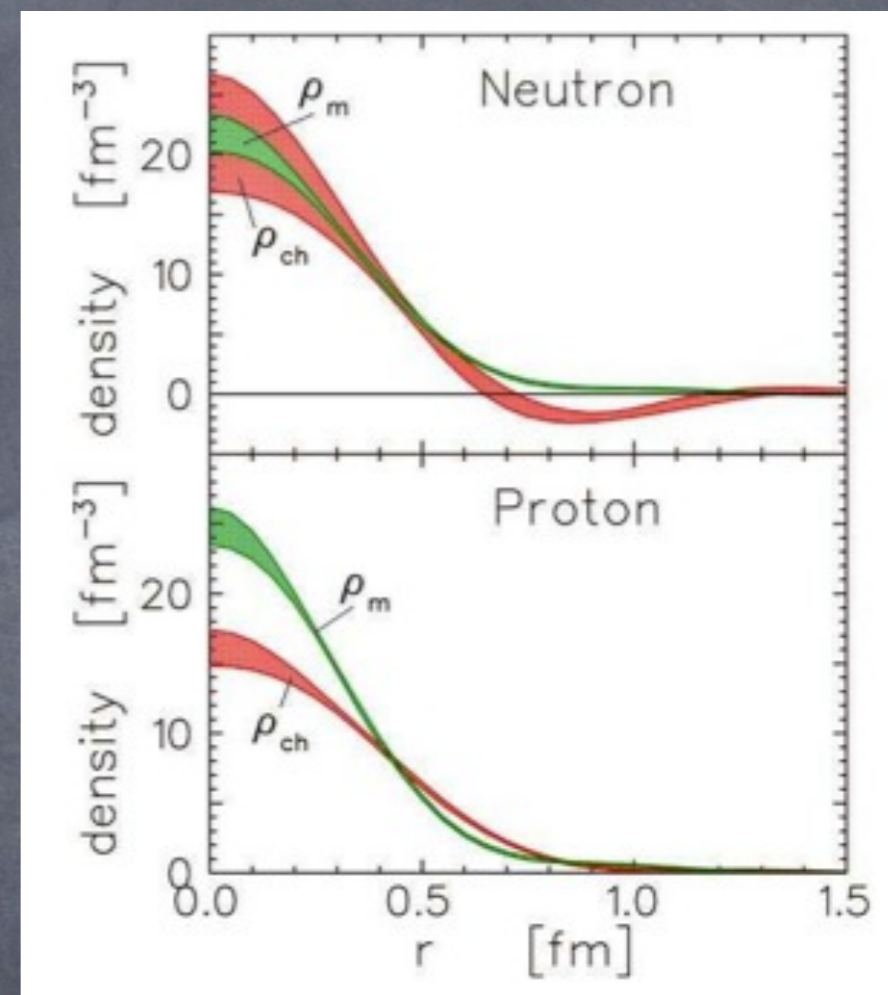
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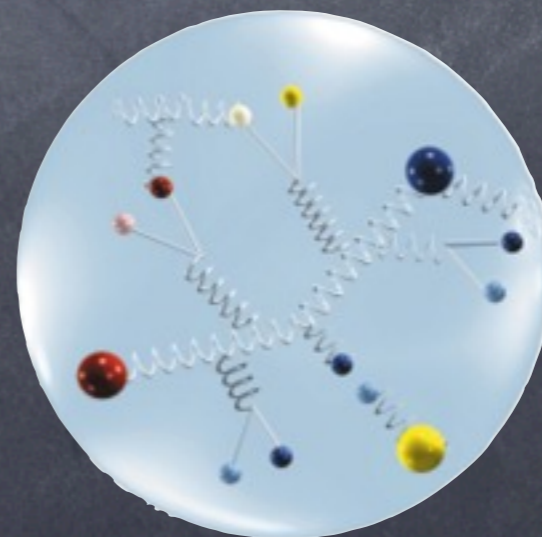
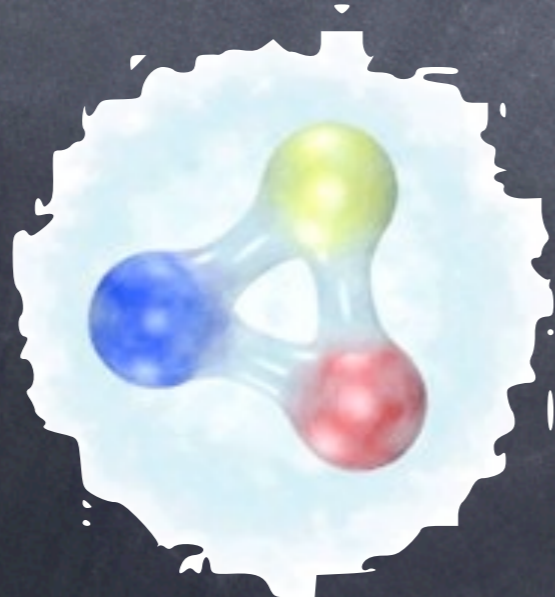
J.J. Kelly Phys. Rev. C 66 065203 (2002)

$$G_M(q^2) = F_1(q^2) + \kappa F_2(q^2)$$

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Dirac equation vs. nucleon

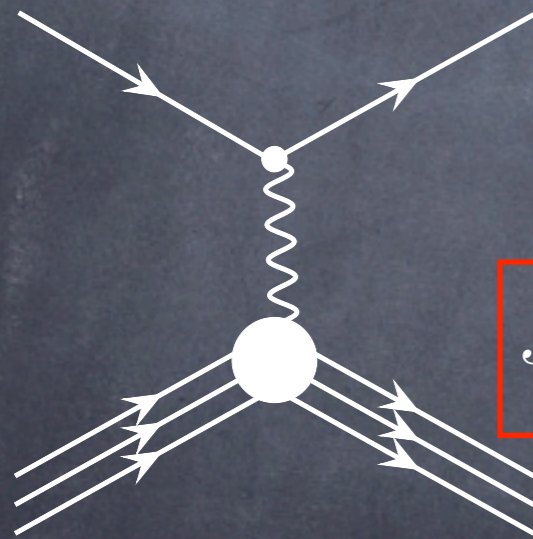
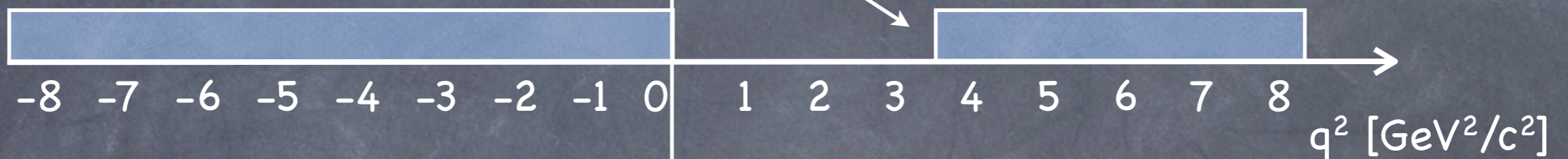
- Anomalous magnetic moment occurs
- Form factors as a function of q^2 instead of constant
- Accommodate complicated meson clouds/quarks within Dirac equation
- By requiring T&P invariance, $2s+1$ form factors for spin s particles



Time-like vs. space-like

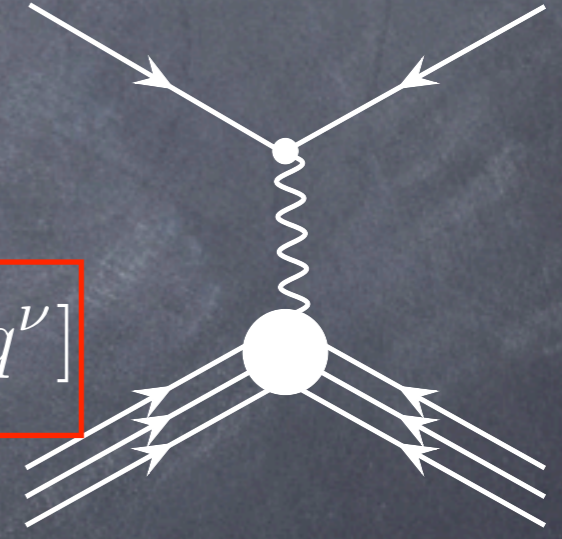
$q^2 < 0$
space-like region

$q^2 > 0$
time-like region



$$j_\mu = ie\gamma_\mu$$

$$J_\mu = ie\left[F_1(q^2)\gamma_\mu + \frac{\kappa}{2M}F_2(q^2)i\sigma_{\mu\nu}q^\nu\right]$$



crossing symmetry

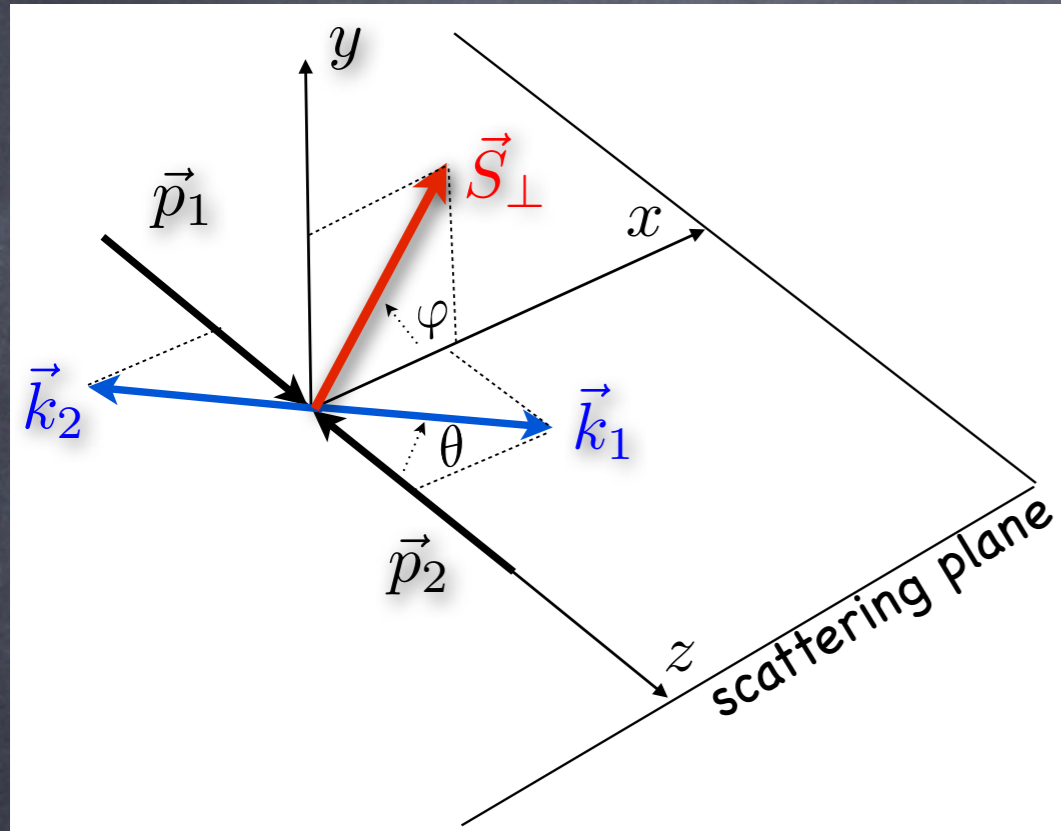
$$\langle \bar{u}_{p'} | J_\mu | u_p \rangle \frac{1}{q^2} \langle \bar{u}_{e'} | j_\mu | u_e \rangle \longleftrightarrow \langle \bar{u}_{p'} u_p | J_\mu | 0 \rangle \frac{1}{q^2} \langle 0 | j_\mu | \bar{u}_{e'} u_e \rangle$$

Dispersion relation

Dispersion relation

- Causality: effect cannot exceed cause (Titchmarsh theorem)
- analyticity of G_E and G_M over complex q^2 plane (application of Cauchy integral with multiple cuts)
- Causality vs. analyticity
- c.f. Simone Pacetti recent talk

Polarization: complete measurement of time-like FFs



P_x : perpendicular to beam
(inside scattering plane)

P_y : normal to scattering plane

P_z : beam direction

$$P_y \propto \sin(2\theta) \text{Im} G_E^* G_M,$$

perpendicular to scattering plane,
either target or outgoing baryon

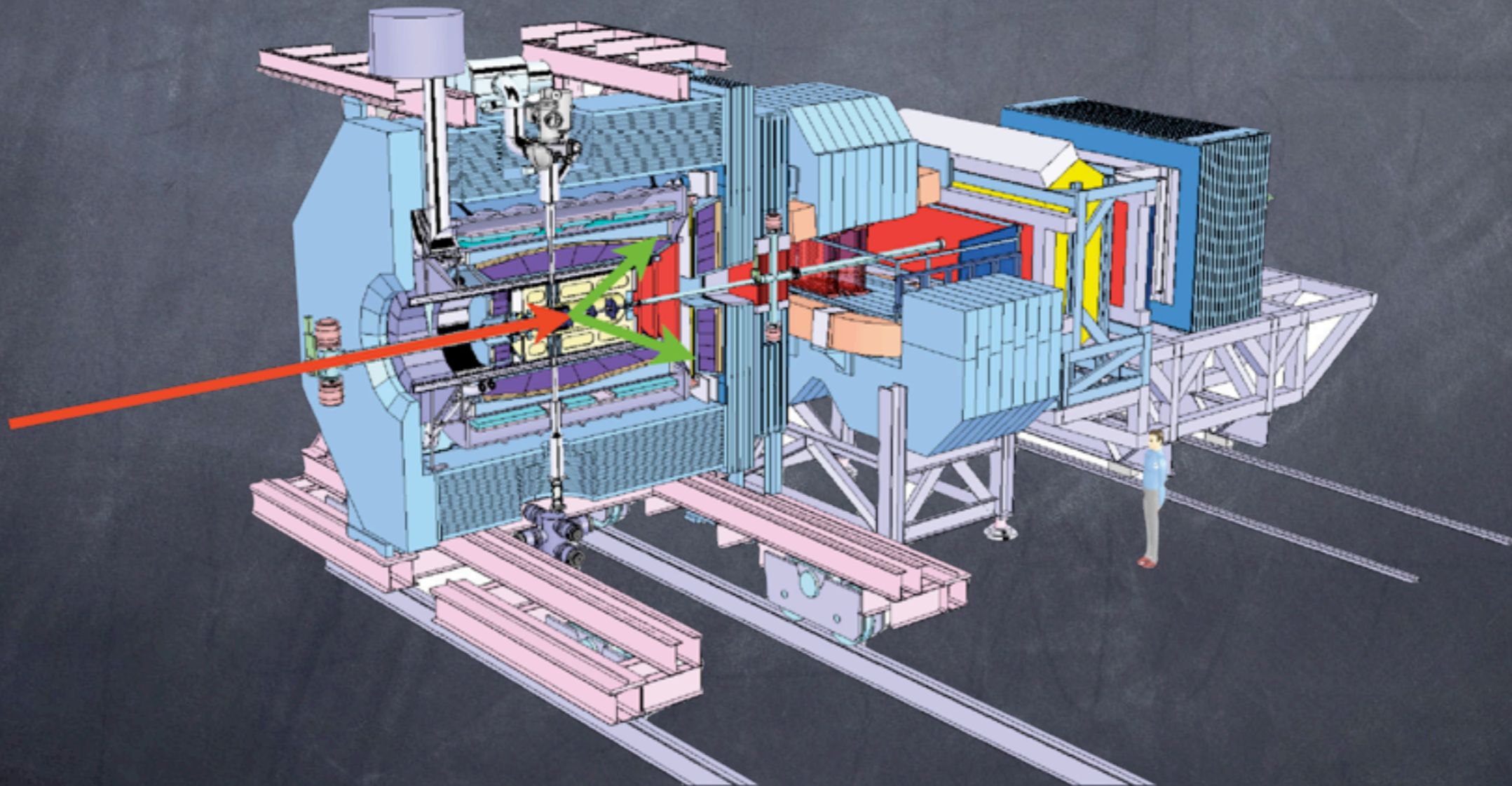
$$P_{zx} = P_{xz} \propto \frac{1}{\sqrt{\tau}} \sin 2\theta \text{Re} G_E G_M^*$$

Sensitive to the real part of $G_E G_M$;
Together with P_y , a **complete**
measurement of G_E and G_M in time
like region can be made.

E. Tomasi-Gustafsson, et al. Eur. Phys. J. A 24, 419–430 (2005)

Experimental aspects

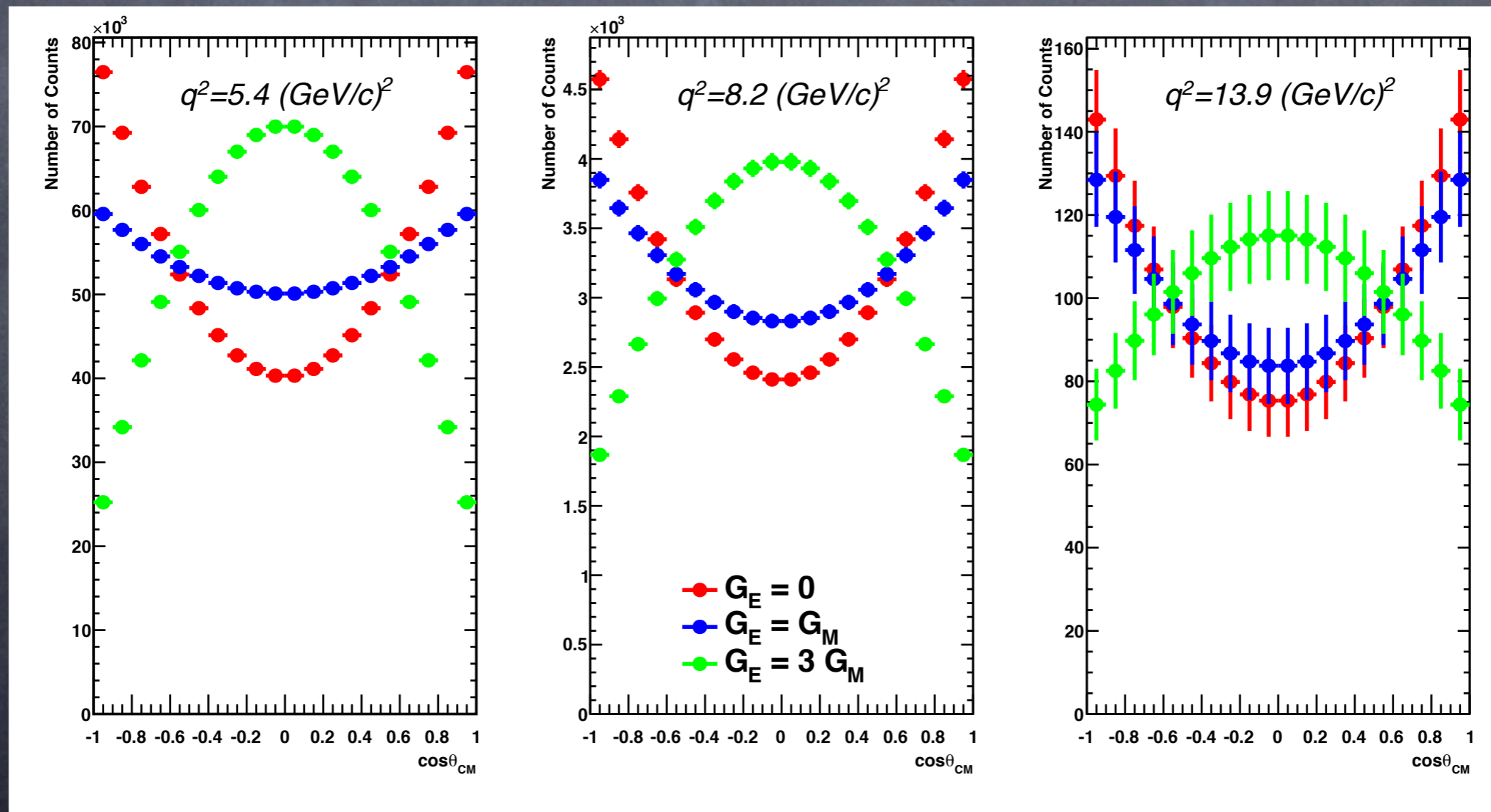
- Good tracking capability;
- High luminosity $L=1.6 \times 10^{32} \text{cm}^{-2}\text{s}^{-1}$;
- Wide momentum range: $1.5 \text{ GeV}/c \sim 15 \text{ GeV}/c$



$\bar{p} p \rightarrow e^+ e^-$

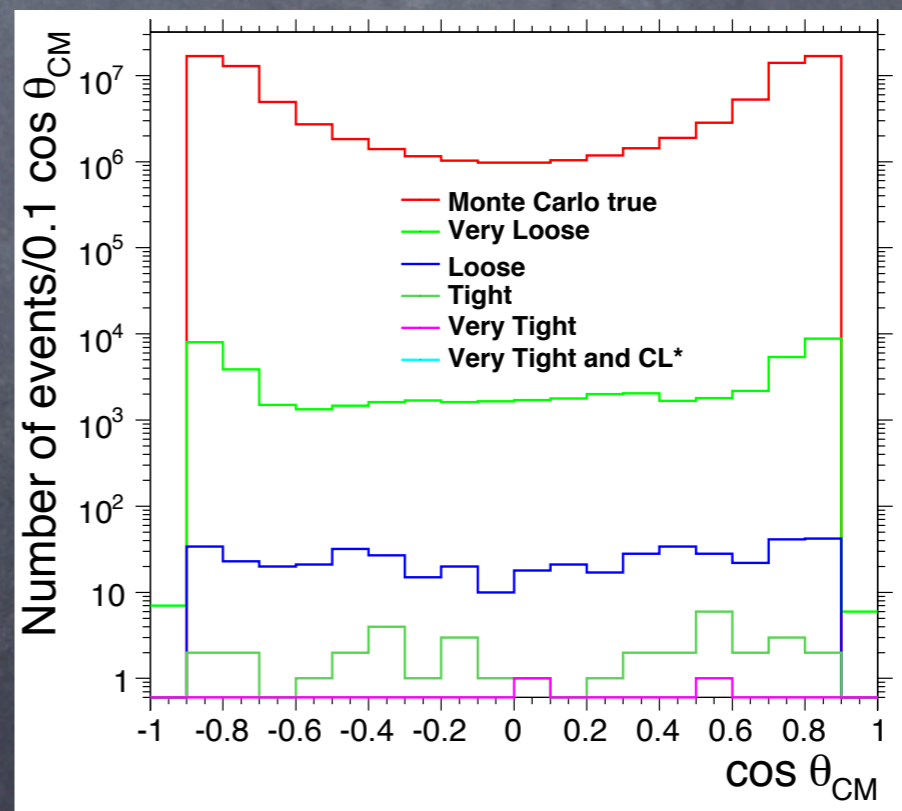
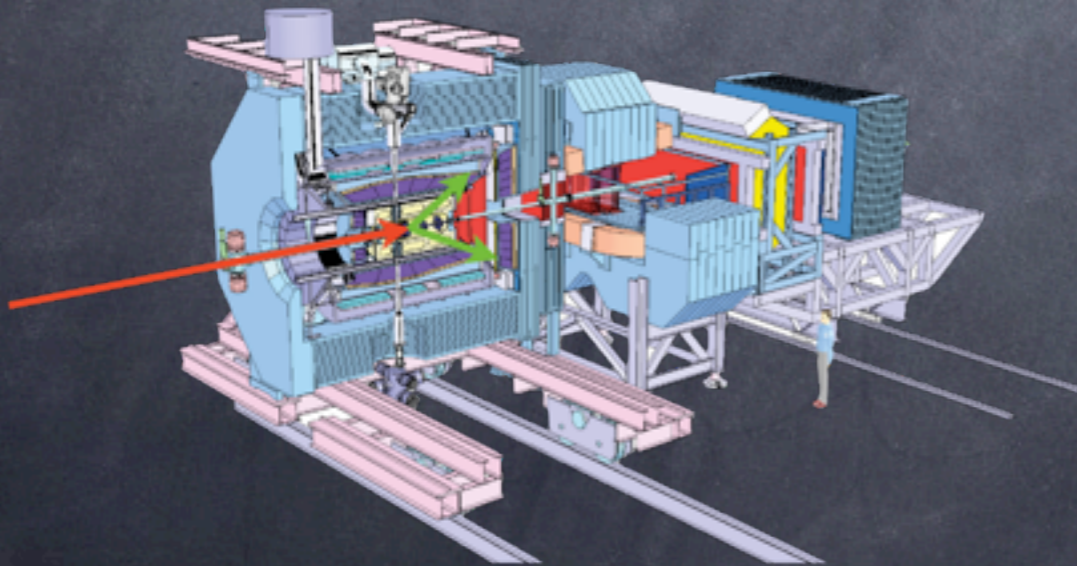
Rosenbluth cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{8M^2\sqrt{\tau(\tau-1)}} \left[|G_M|^2 (1 + \cos^2\theta) + \frac{|G_E|^2}{\tau} (1 - \cos^2\theta) \right], \quad \tau = \frac{-q^2}{4M^2}$$



$p\bar{p} \rightarrow e^+ e^-$

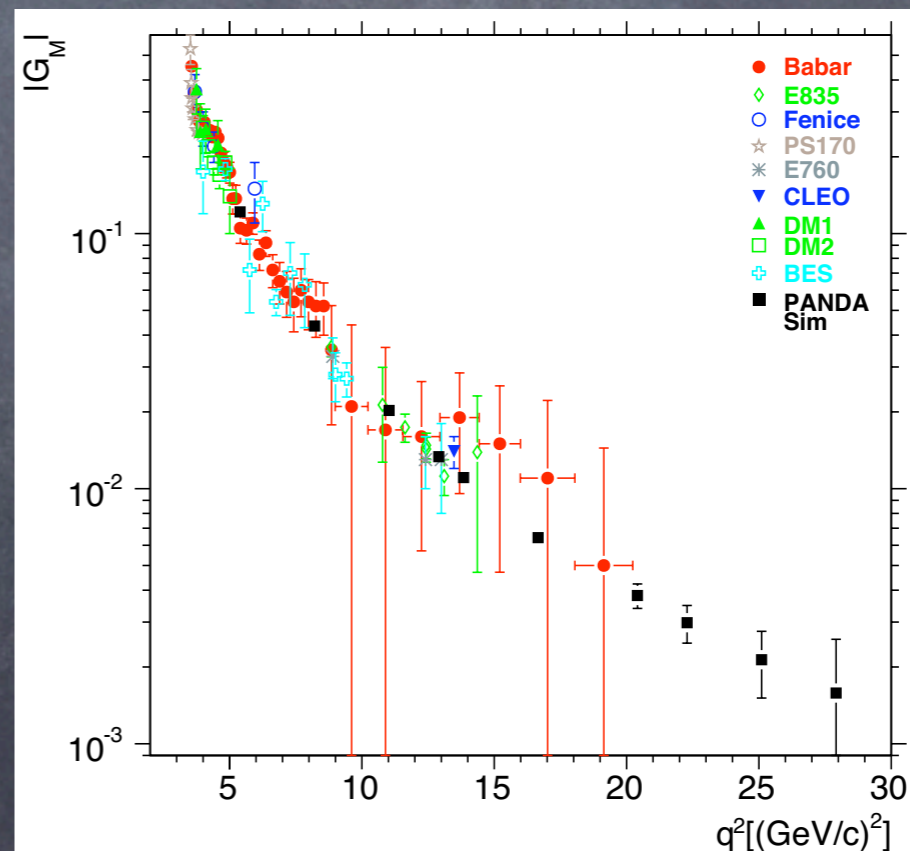
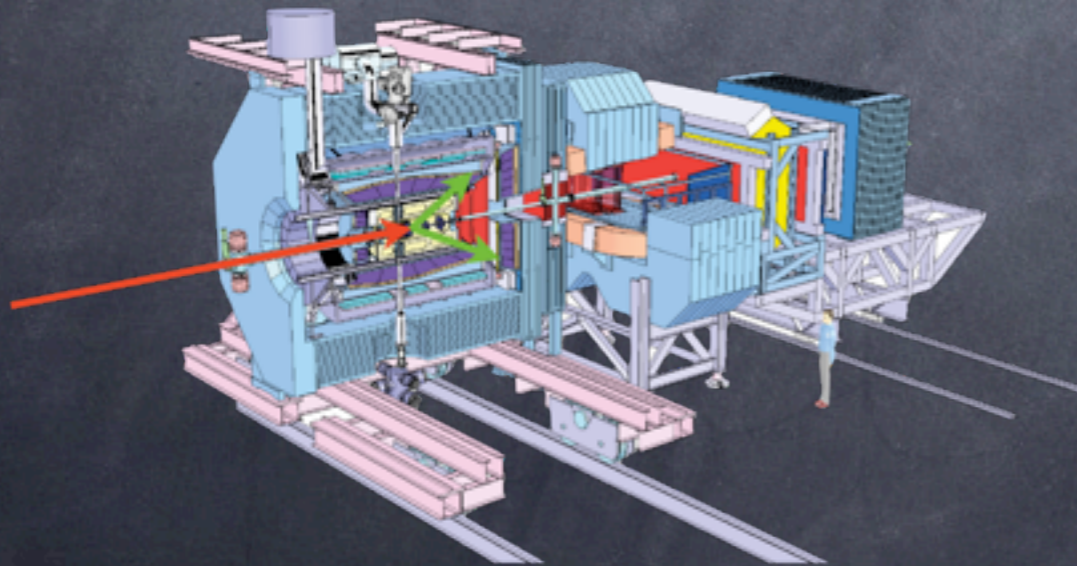
- Simulation done by Mainz and Orsay groups:
 - 100 CPUs in Orsay,
 - 300 CPUs Lyon
 - 200 CPUs at GSI
- event generator (M. Zambrana)
- $\pi^+\pi^-$ background suppression (D. Khanefit)



M. Sudol, et al. Eur. Phys. J. A **44**, 373–384 (2010)

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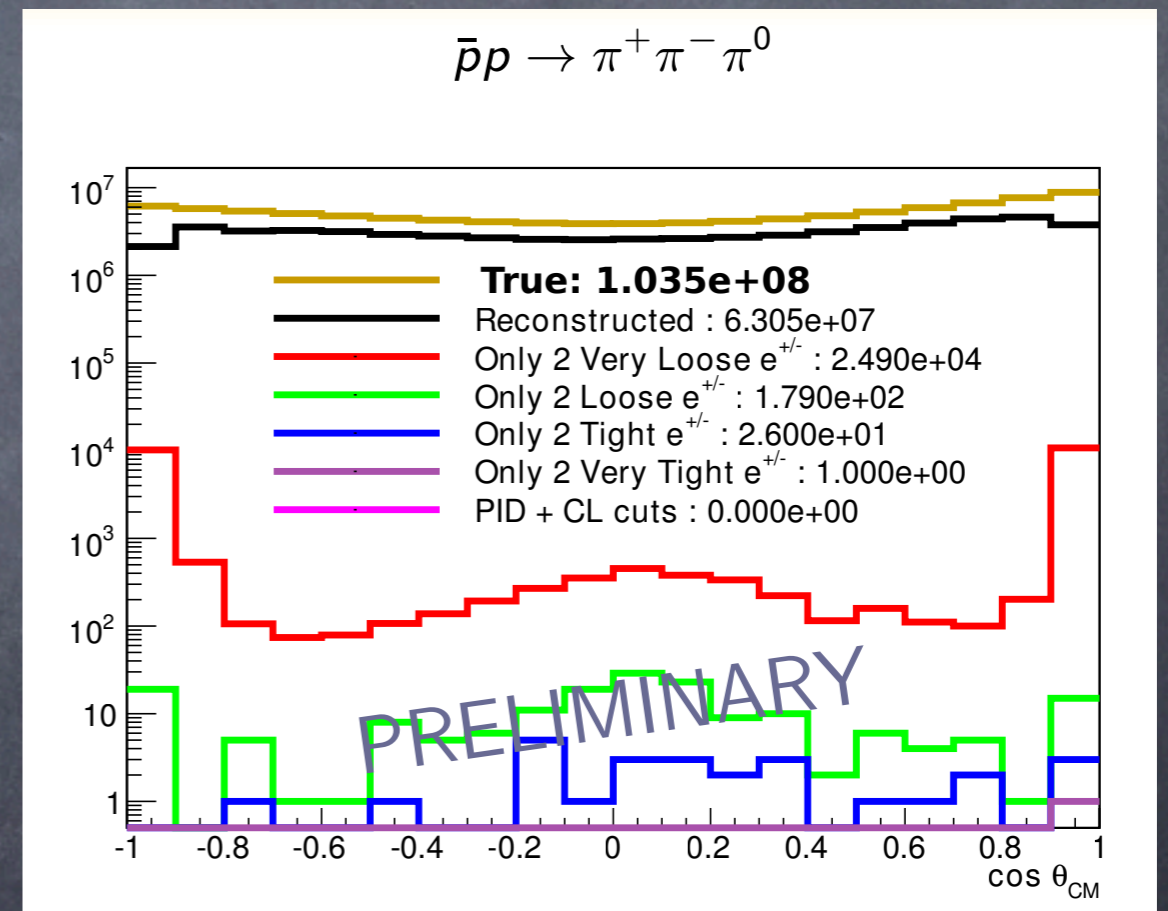
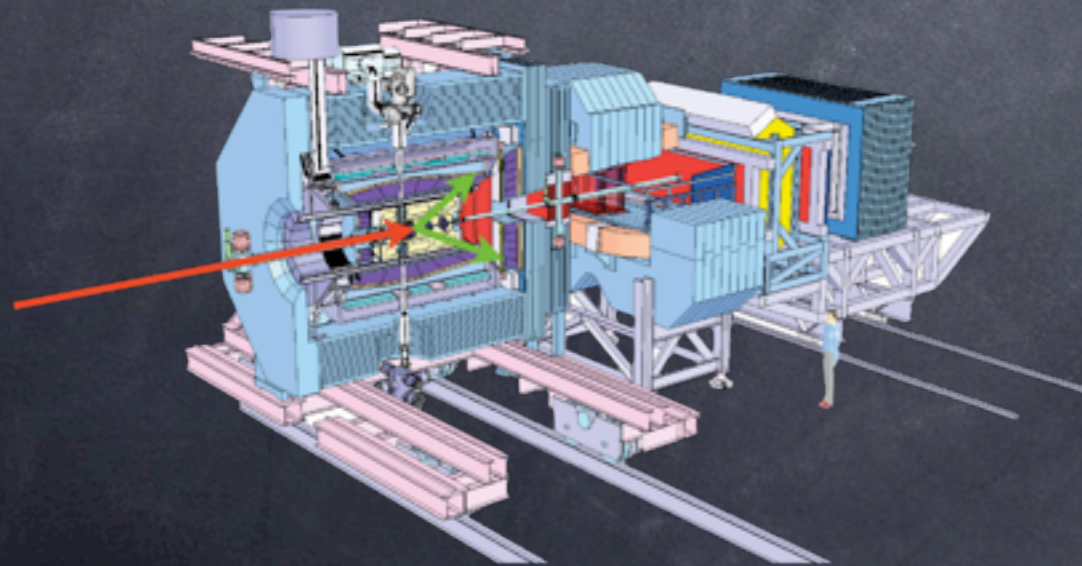
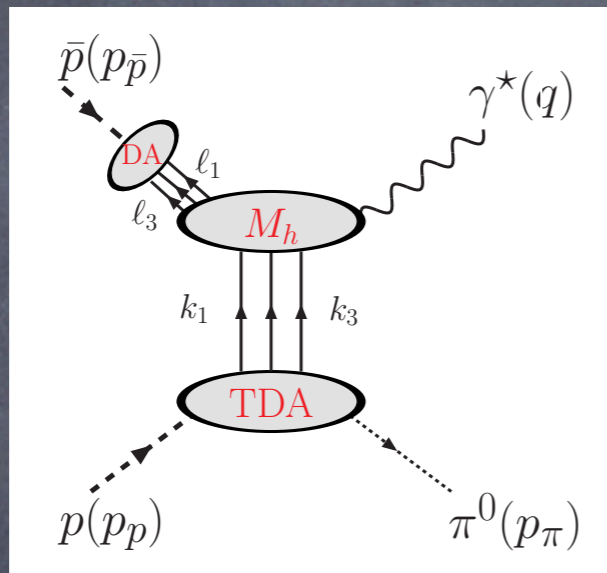
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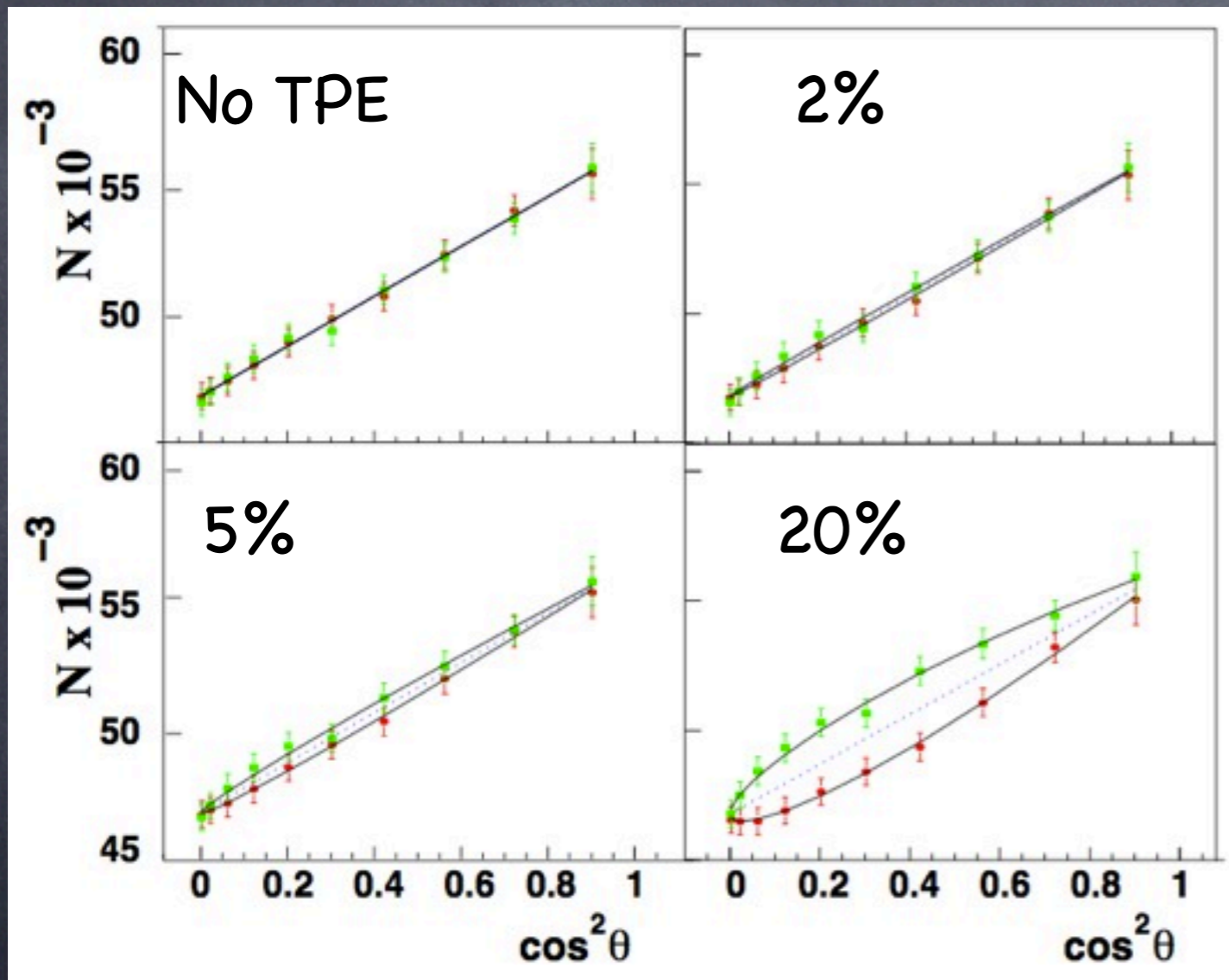
Transition Distribution Amplitude

- Feasibility study by M. Mora Espi (PhD candidate)



PANDA experiment at FAIR:

simulation: PANDA vs. TPE



$$\frac{d\sigma}{d\cos\theta} = \sigma_0(1 + A\cos^2\theta)$$

A: asymmetry due to TPE interference

$$q^2 = 5.4 \text{ (GeV/c)}^2$$

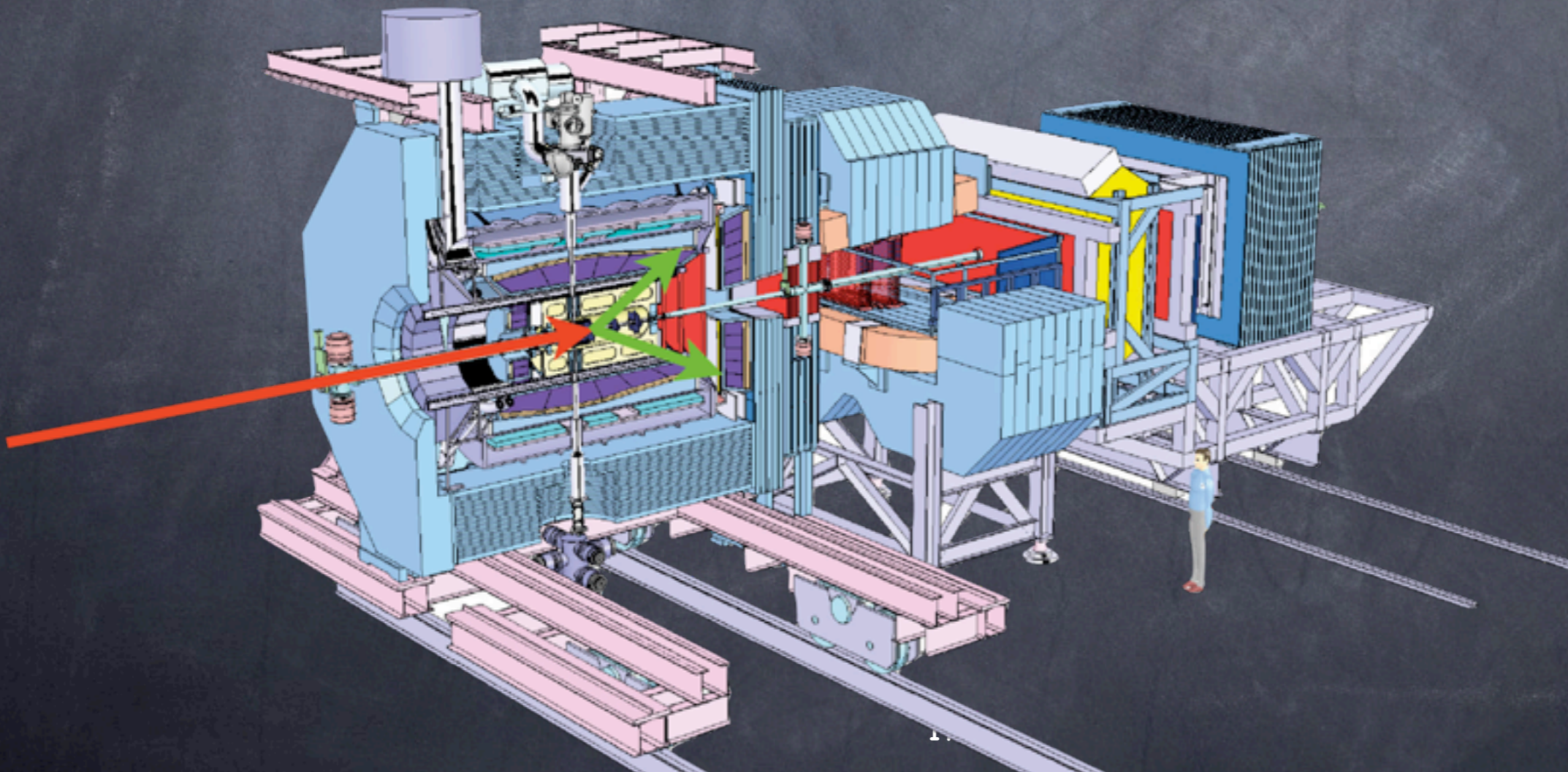
forward lepton
backward lepton

M. Sudol, et al. Eur. Phys. J. A 44, 373-384 (2010)

Is PANDA polarizable?

$$P_y \propto \sin(2\theta) \operatorname{Im} G_E^* G_M,$$

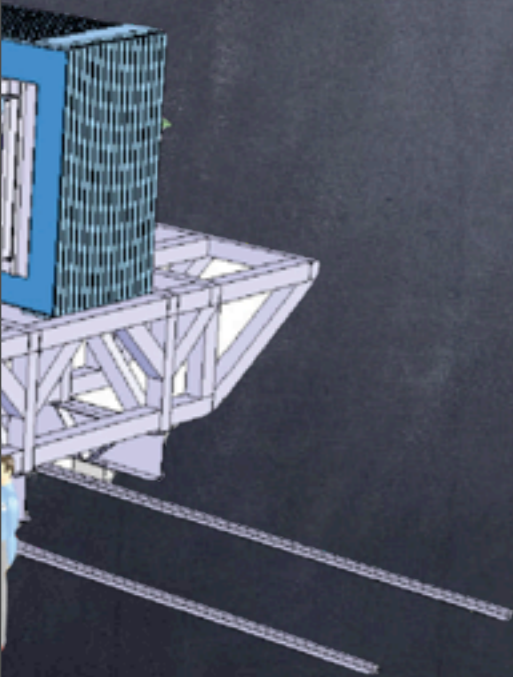
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- Close collaboration with experts from Mainz and IHEP.



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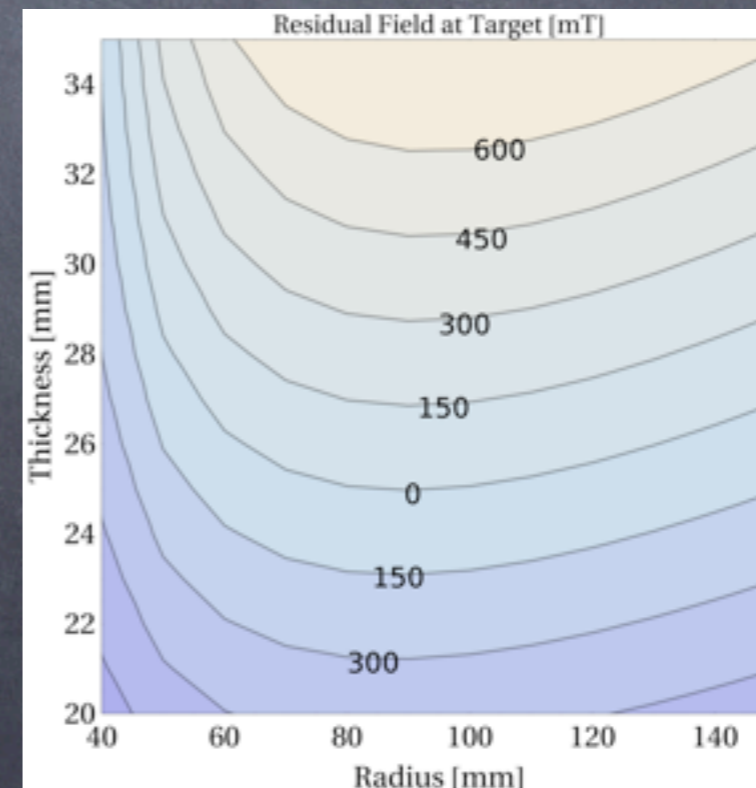
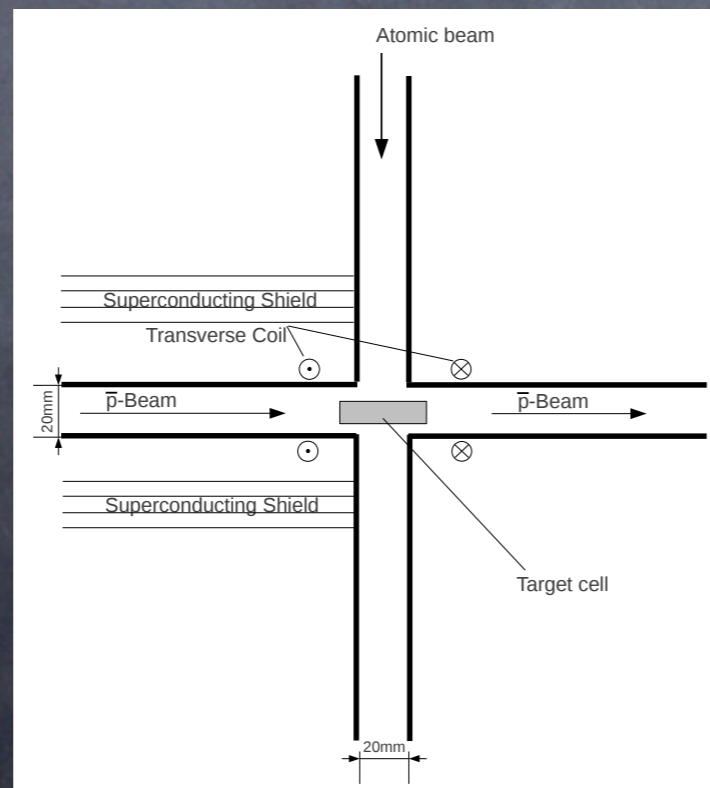
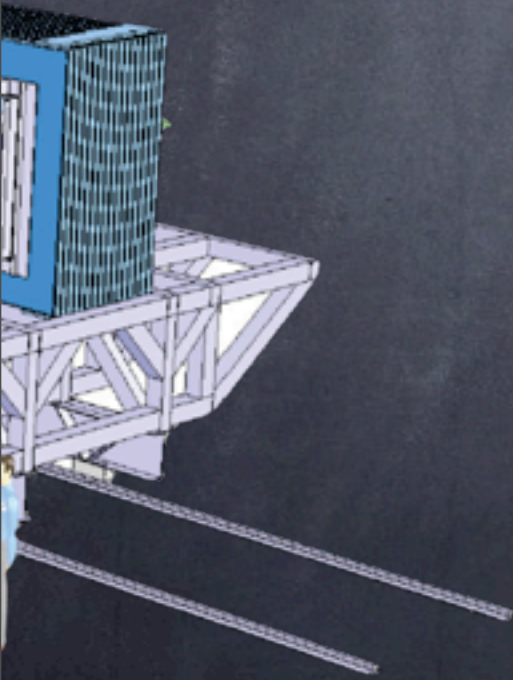
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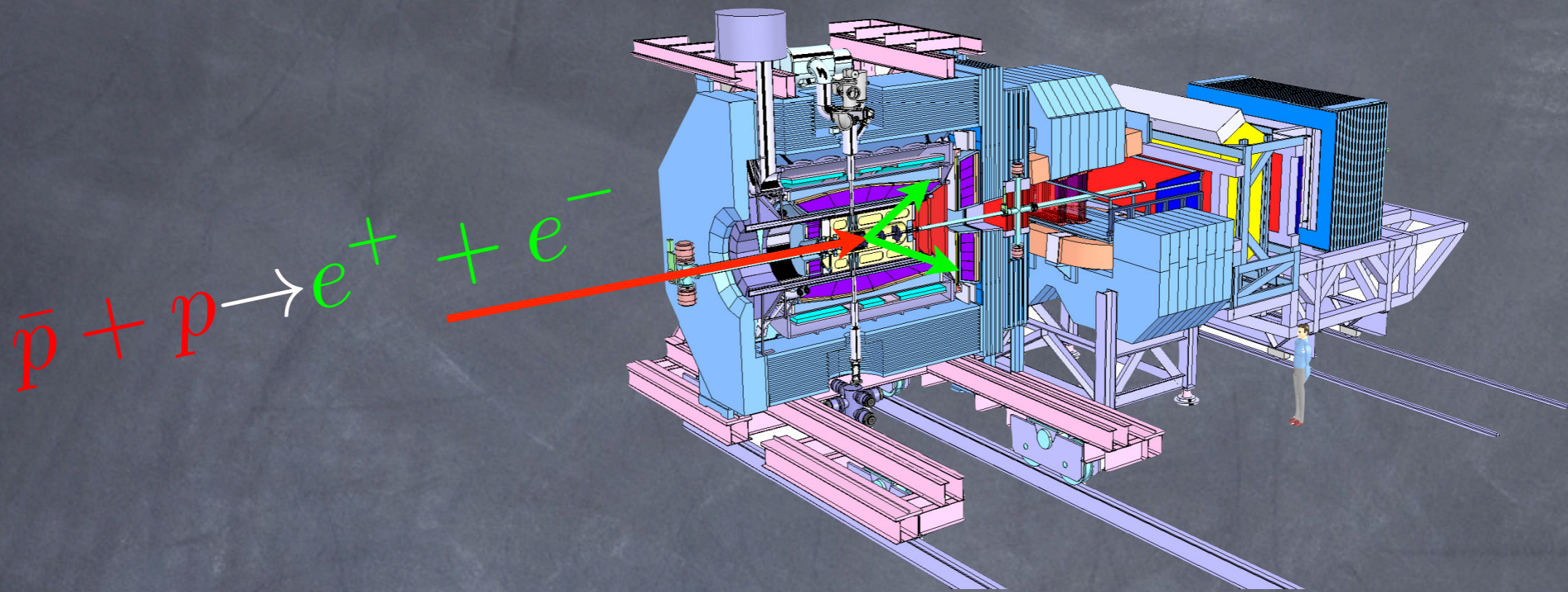


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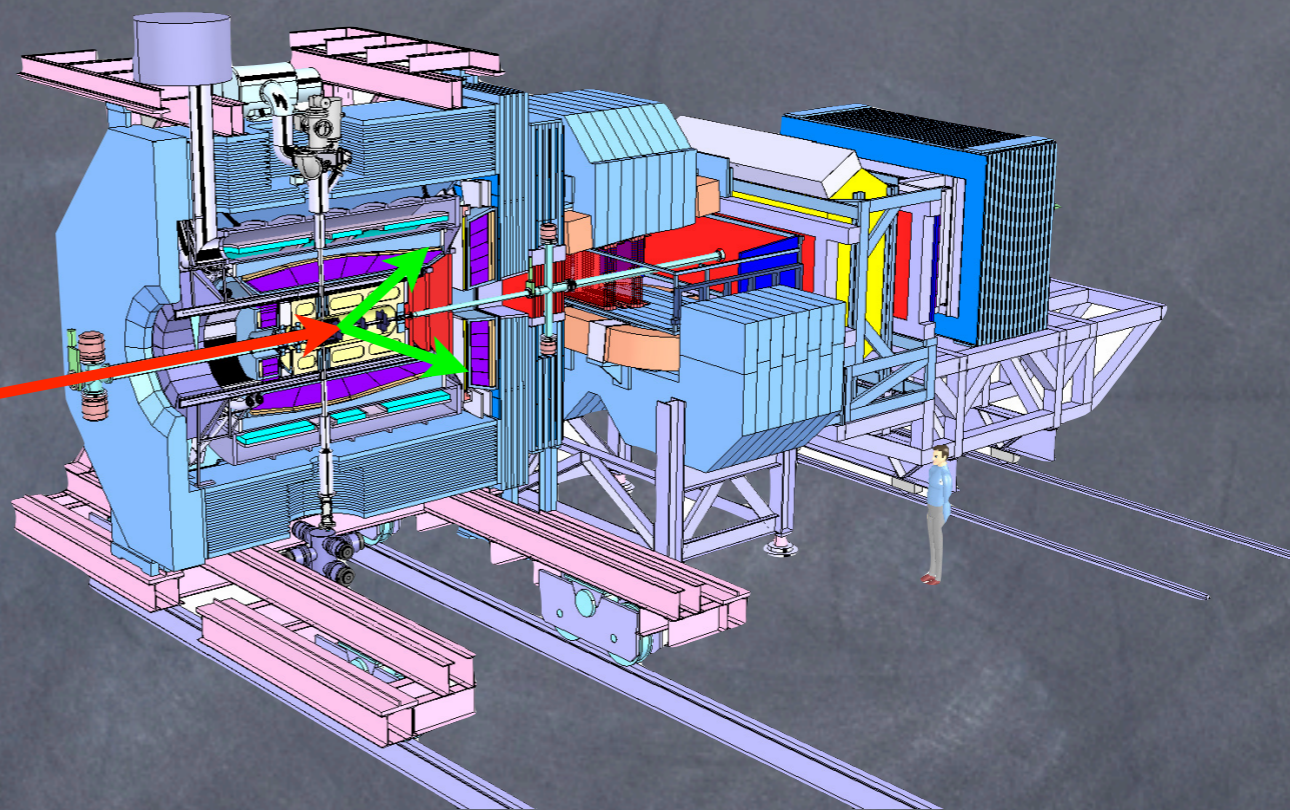
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- Essentially improve data in TL region
- Possibility to measure relative phase (G_E , G_M)
- Determine contribution of TPE
- Other interesting EM processes



The end