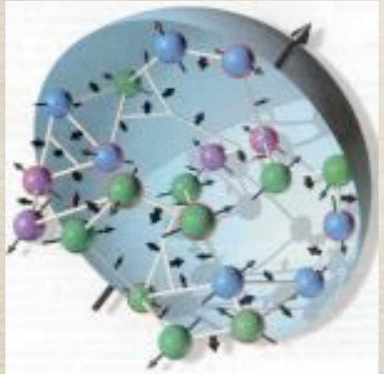
Proton form factors in SL and TL regions at large momentum transfer

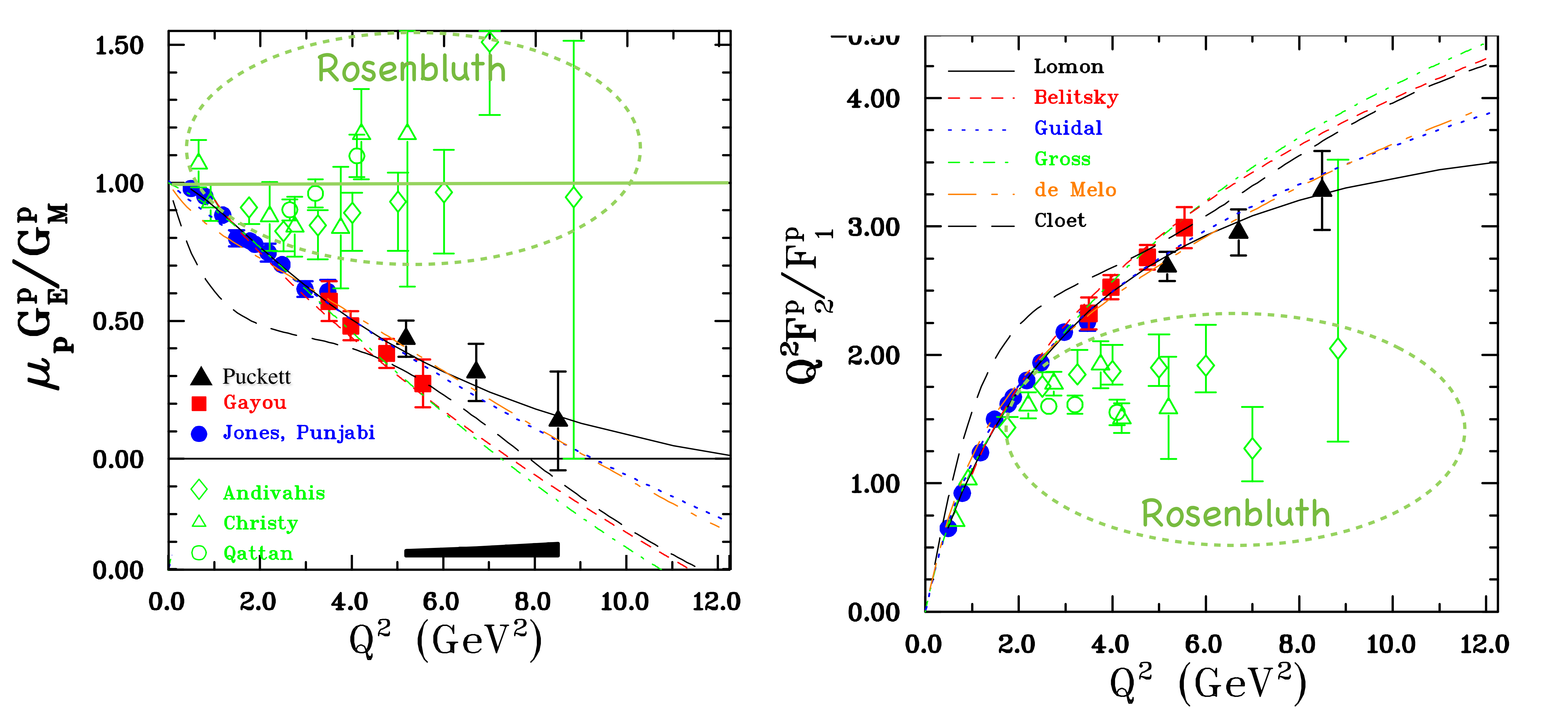
Nikolay Kivel

Helmholtz Institute Mainz, Germany



Ratio GE/GM

$$G_M = F_1 + F_2 \qquad G_E = F_1 - \frac{Q^2}{4m_N^2} F_2$$



JLab recoil pol. experiments

GEp(I) 1999, PRL84

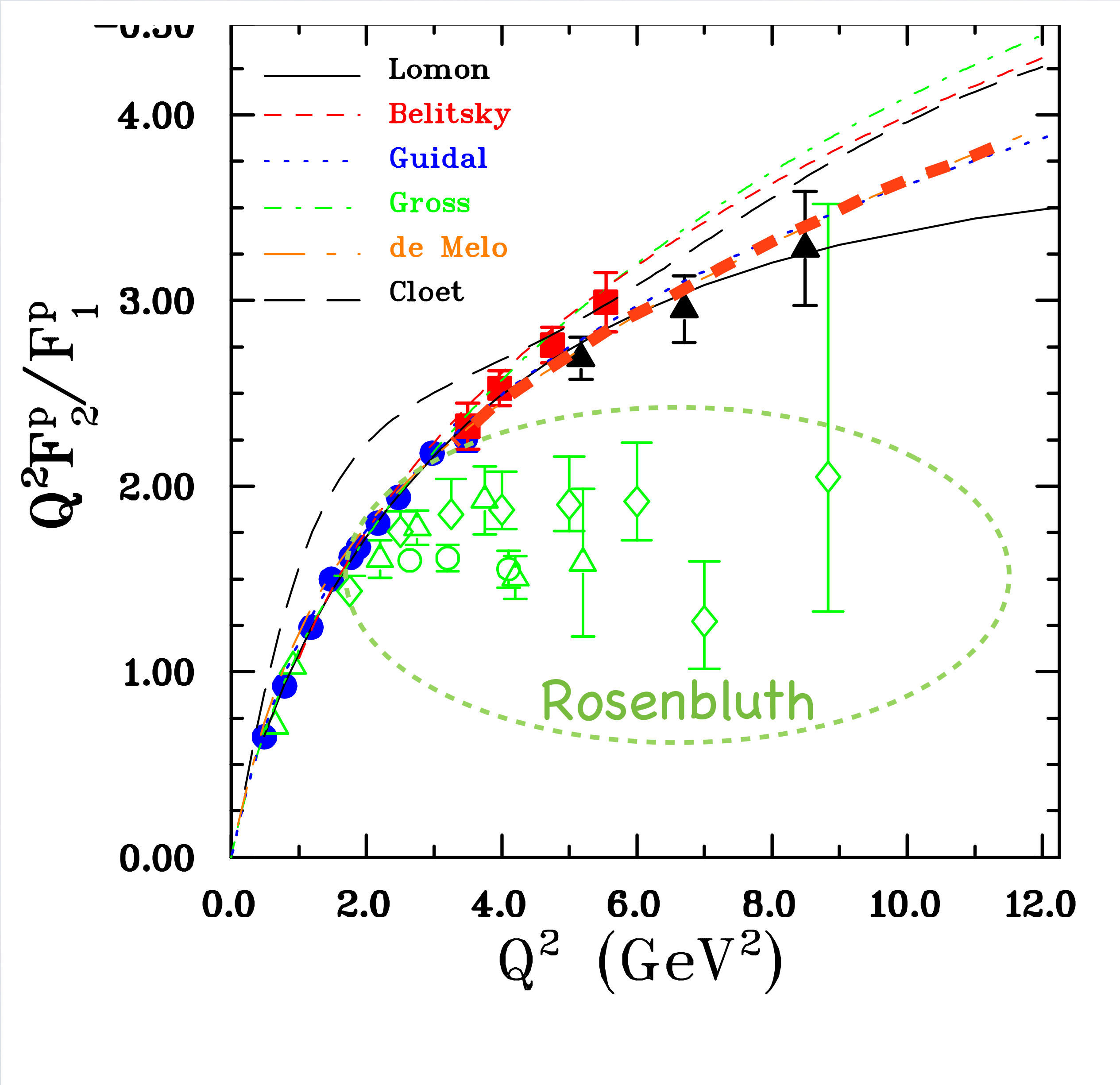
GEp(II) 2001, PRL88

GEp(III) 2010, PRL104

Large Q^2 limit: test of QCD predictions

QCD predictions: $F_1 \sim 1/Q^4$ $\Rightarrow$ $Q^2 F_2/F_1 \sim const$

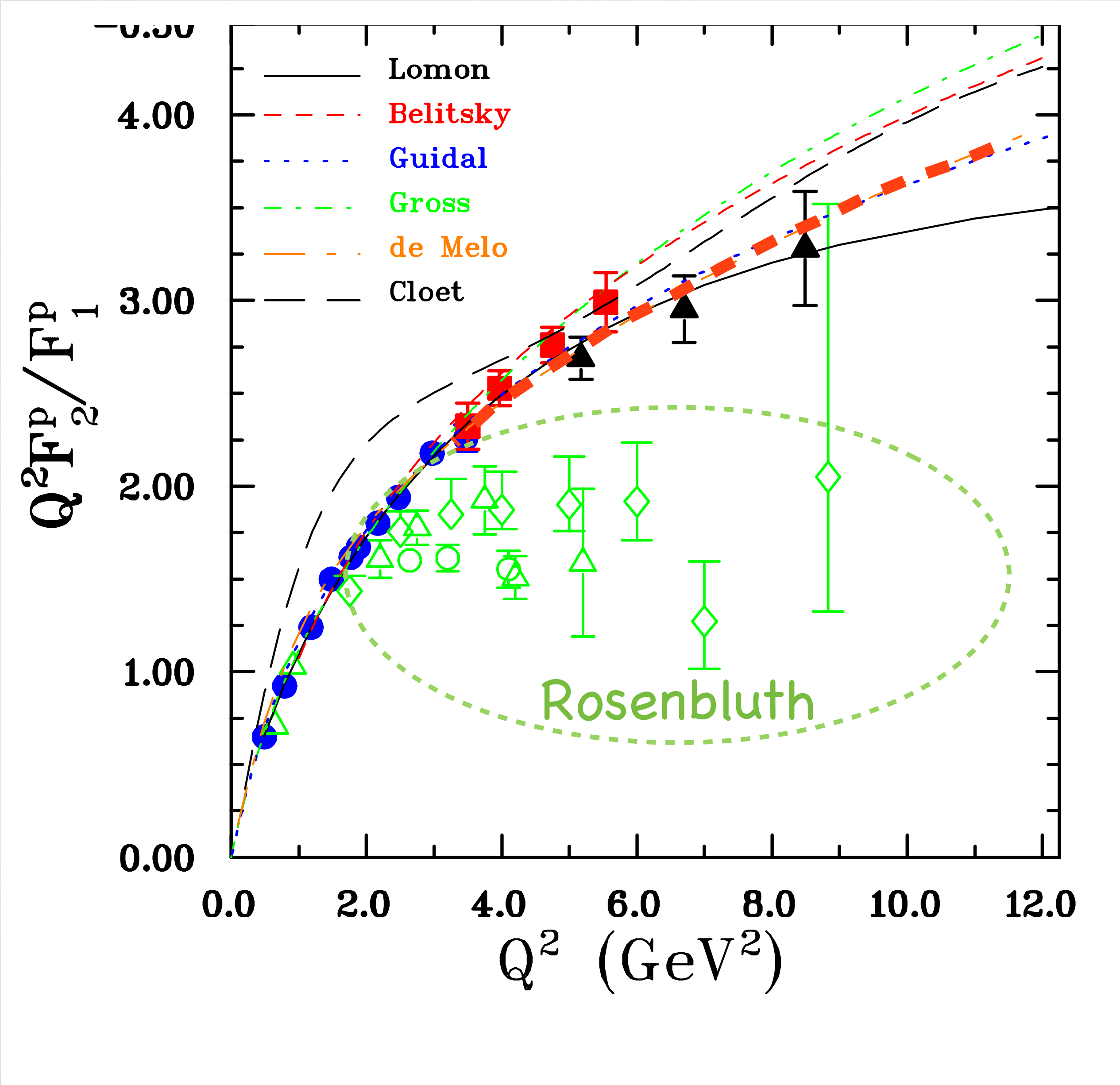
$F_2 \sim 1/Q^6$



Large Q^2 limit: test of QCD predictions

QCD predictions: $F_1 \sim 1/Q^4$ $\Rightarrow$ $Q^2 F_2/F_1 \sim const$

$F_2 \sim 1/Q^6$



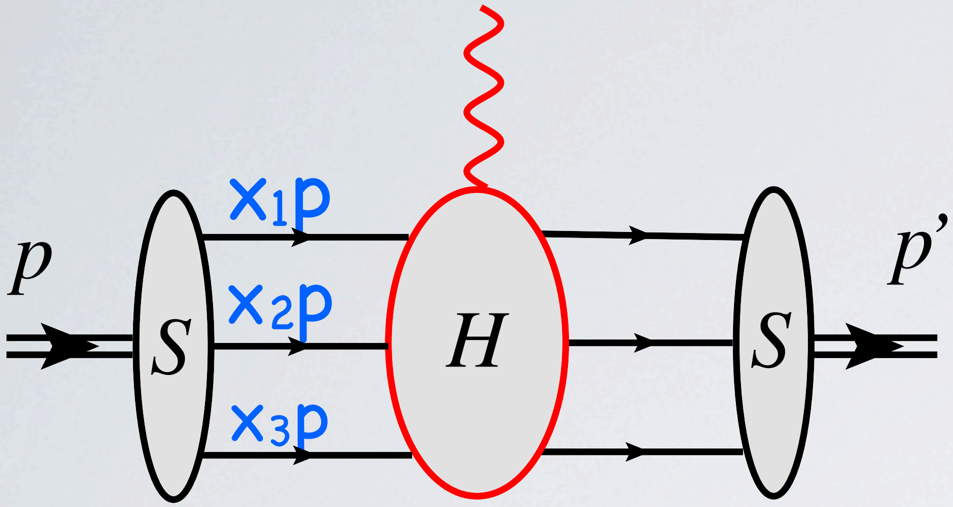
Belitsky, Ji, Yuan 2003

Log^2 empirical observation

$$Q^2 F_2/F_1 \sim \ln^2 Q^2/\Lambda^2$$

$$\Lambda = 300\text{MeV}$$

Proton FFs at large Q^2: hard scattering picture: Dirac FF F_1



Chernyak, Zhitnitsky 1977
Brodsky, Lepage 1979
Efremov, Radyushkin 1980

$$F_1^{(h)} = \frac{\Lambda^4}{Q^4} \sum_i C_i \left\{ \ln[\Lambda^2/Q^2]^{\gamma_i - 2} + \mathcal{O}(1/\ln[Q/\Lambda]) \right\}$$

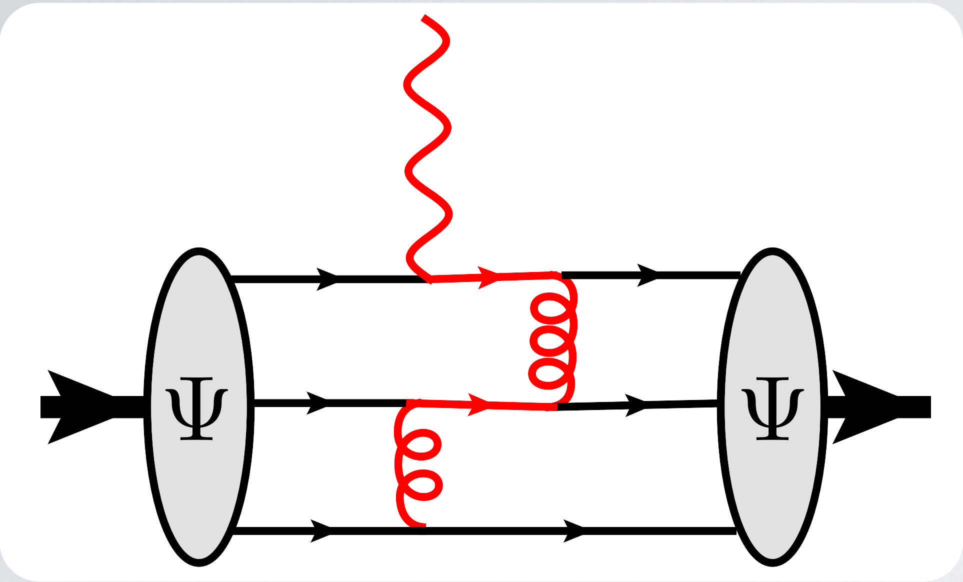
leading log's

- UNIVERSAL coefficients C_i are defined by non-perturbative physics: $C_i = \langle p | O_i | 0 \rangle$

Interpretation: describe how the long. momentum is shared between the constituents

- scaling behavior is model independent QCD prediction (test of QCD!)
- Logarithmic corrections can be computed systematically in pQCD

Proton FFs at large Q^2: hard scattering picture: Dirac FF F_1



Chernyak, Zhitnitsky 1977
Brodsky, Lepage 1979
Efremov, Radyushkin 1980

$$F_1^{(h)} = \frac{\Lambda^4}{Q^4} \sum_i C_i \left\{ \ln[\Lambda^2/Q^2]^{\gamma_i - 2} + \mathcal{O}(1/\ln[Q/\Lambda]) \right\}$$

leading log's

- UNIVERSAL coefficients C_i are defined by non-perturbative physics: $C_i = \langle p | O_i | 0 \rangle$

Interpretation: describe how the long. momentum is shared between the constituents

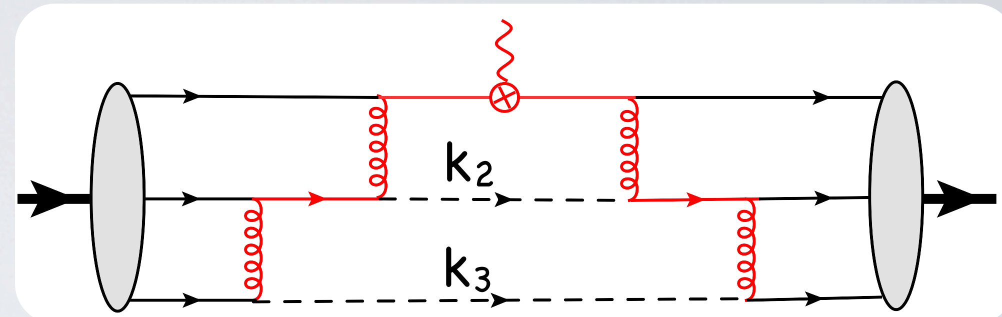
- scaling behavior is model independent QCD prediction (test of QCD!)
- Logarithmic corrections can be computed systematically in pQCD

Proton FFs at large Q^2 Dirac FF F_1

Duncan, Mueller 1980

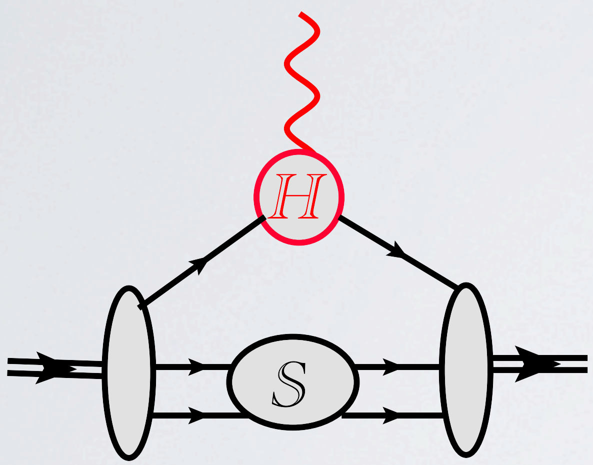
soft spectator scattering
is also relevant!

"visible" in pQCD
from 2-loops



Milshtein, Fadin 1981/82

complete 2- and 3- loop calc



large "nonstandard"
logarithm:

$$\sim \alpha^4 \ln[Q/\Lambda]/Q^4$$

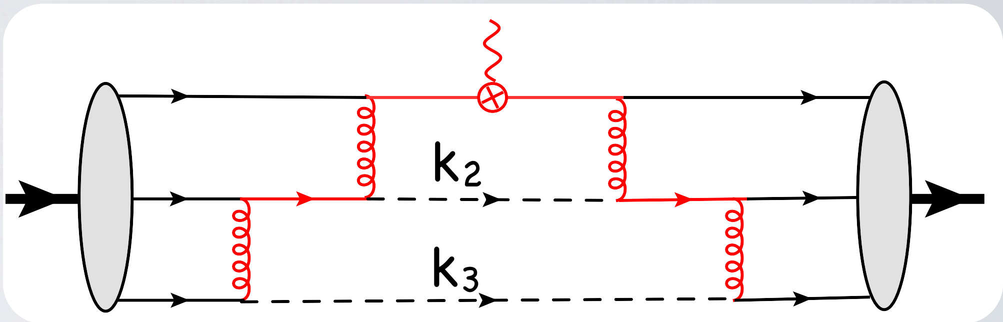
same power behavior
as in hard rescattering

Proton FFs at large Q^2 Dirac FF F_1

Duncan, Mueller 1980

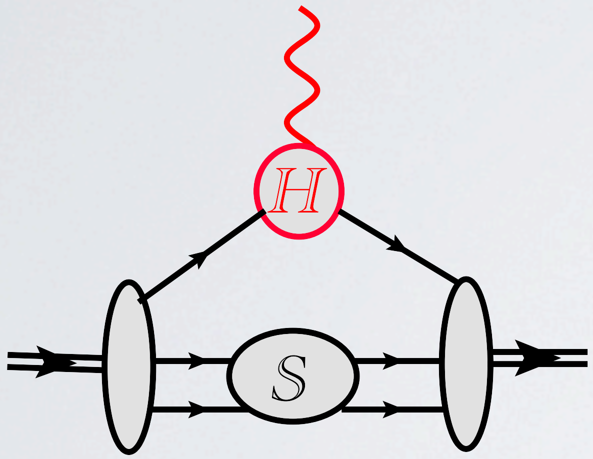
soft spectator scattering
is also relevant!

"visible" in pQCD
from 2-loops



Milshtein, Fadin 1981/82

complete 2- and 3- loop calc



large "nonstandard"
logarithm:

$$\sim \alpha^4 \ln[Q/\Lambda]/Q^4$$

same power behavior
as in hard rescattering

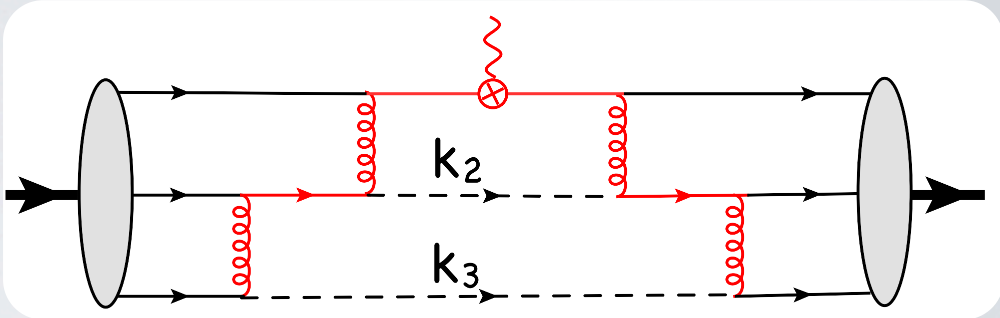
☛ QCD factorization is more complicate:
both, hard and soft rescattering are relevant!

Proton FFs at large Q^2 Dirac FF F_1

Duncan, Mueller 1980

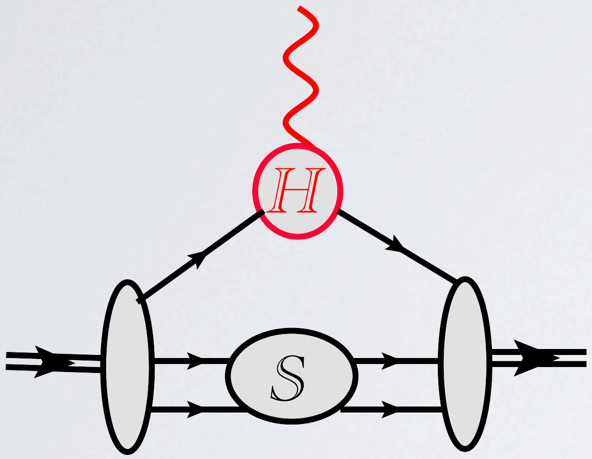
soft spectator scattering
is also relevant!

"visible" in pQCD
from 2-loops



Milshtein, Fadin 1981/82

complete 2- and 3- loop calc



large "nonstandard"
logarithm:

$$\sim \alpha^4 \ln[Q/\Lambda]/Q^4$$

same power behavior
as in hard rescattering

☛ QCD factorization is more complicate:
both, hard and soft rescattering are relevant!

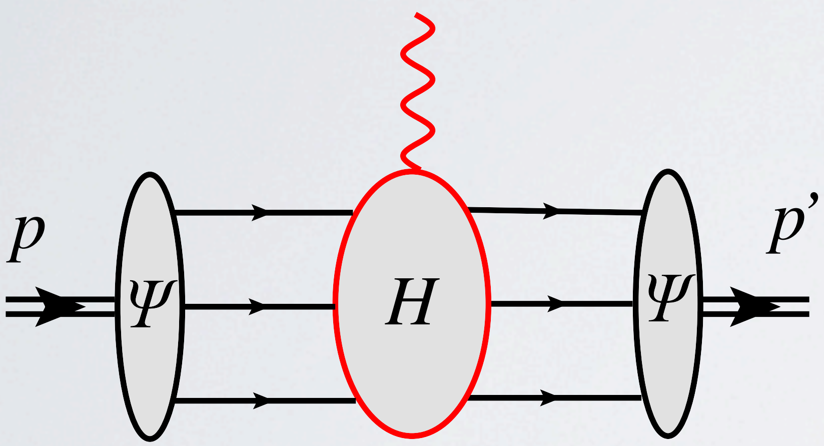
☛ Soft rescattering could be more enhanced in the barion sector comparing to meson ones

Proton FFs at large Q^2 Pauli FF F_2

Large Q behavior is more complicate!

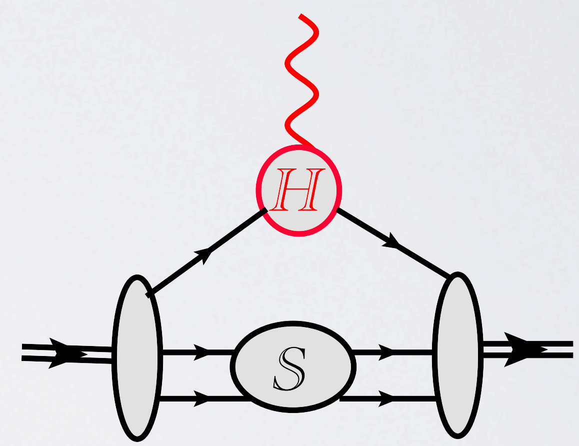
$$F_2 \sim \frac{\Lambda^6}{Q^6}\,\alpha_s(Q^2)\ln^2[\Lambda/Q] \sim \frac{\Lambda^6}{Q^6}$$

two regions overlap



hard rescattering

Log's can not be computed systematically
= NO FACTORIZATION



soft rescattering
= soft-overlap
= Feynman mechanism

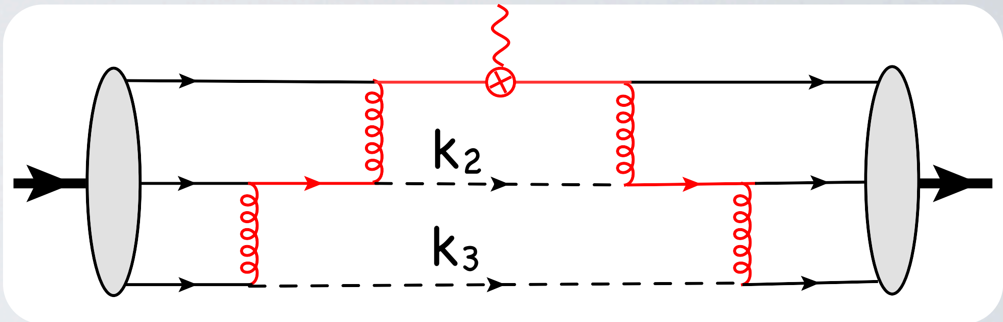
Resume:

$$Q^2 F_2/F_1 \sim \ln^2 Q^2/\Lambda^2$$

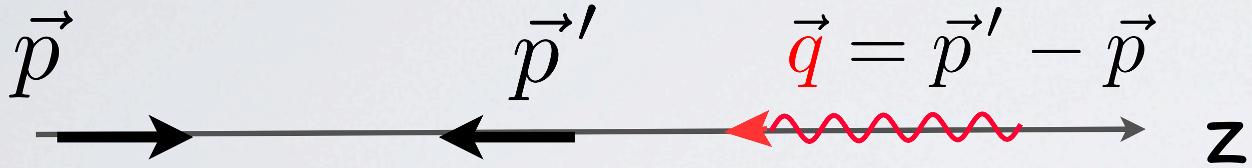
Soft spectator scattering at large Q^2

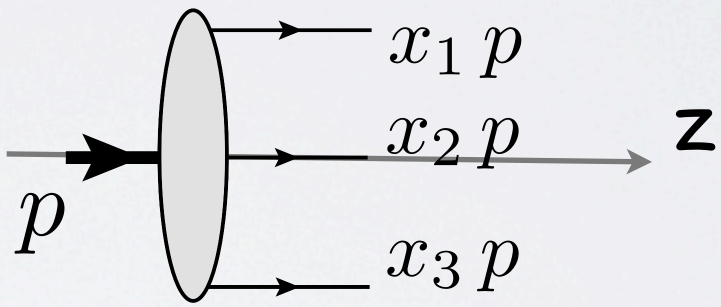
$$p = \frac{1}{2}(Q, 0, 0, Q) + \mathcal{O}(m^2/Q^2)$$

$$p' = \frac{1}{2}(Q, 0, 0, -Q) + \mathcal{O}(m^2/Q^2)$$



$$\sim \alpha^4 \ln[Q/\Lambda]/Q^4$$





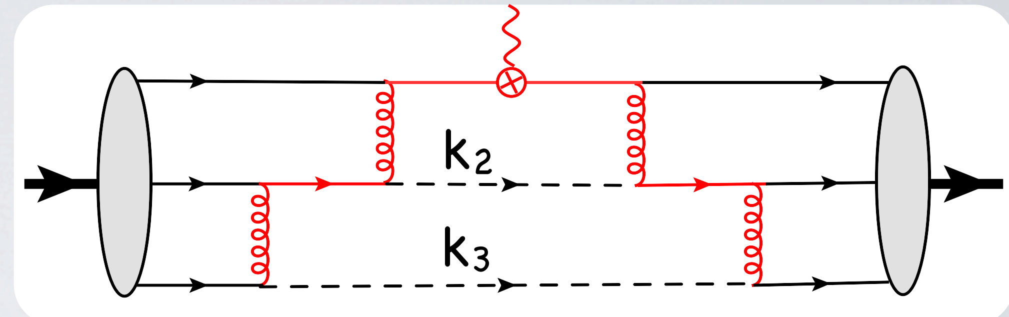
$$\varphi_N(x_1, x_2, x_3) \sim \int_{k_{i\perp} < \mu^2} dk_{i\perp} \Psi_P(x_1, x_2, x_3, k_{i\perp})$$

describes how the long. momentum is shared between the constituents

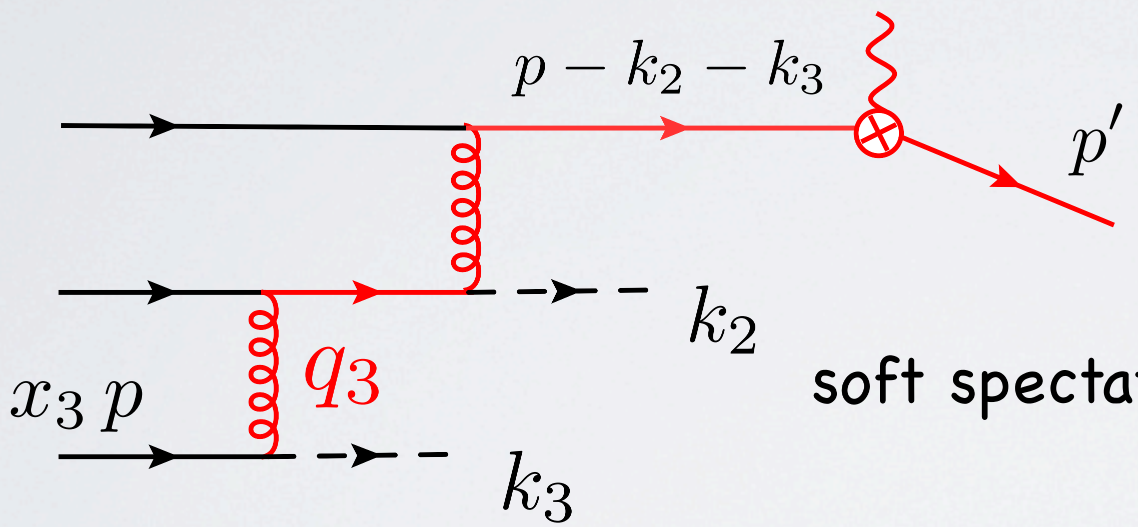
Soft spectator scattering at large Q^2

$$p = \frac{1}{2}(Q, 0, 0, Q) + \mathcal{O}(m^2/Q^2)$$

$$p' = \frac{1}{2}(Q, 0, 0, -Q) + \mathcal{O}(m^2/Q^2)$$



$$\sim \ \alpha^4 \ln[Q/\Lambda]/Q^4$$

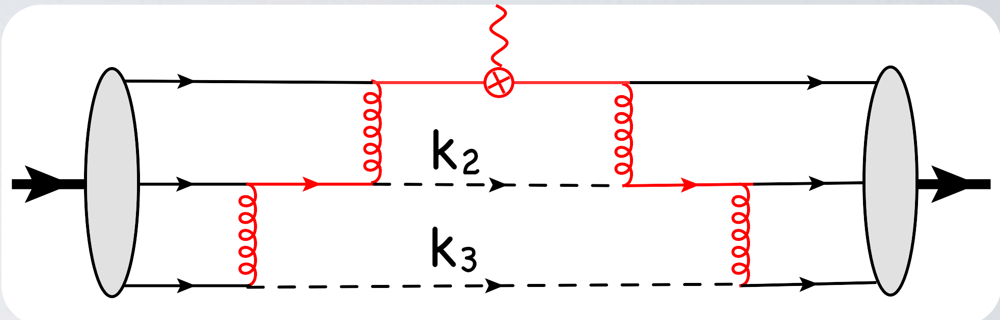


soft spectators $k_{2\mu} \sim k_{3\mu} \sim \Lambda$

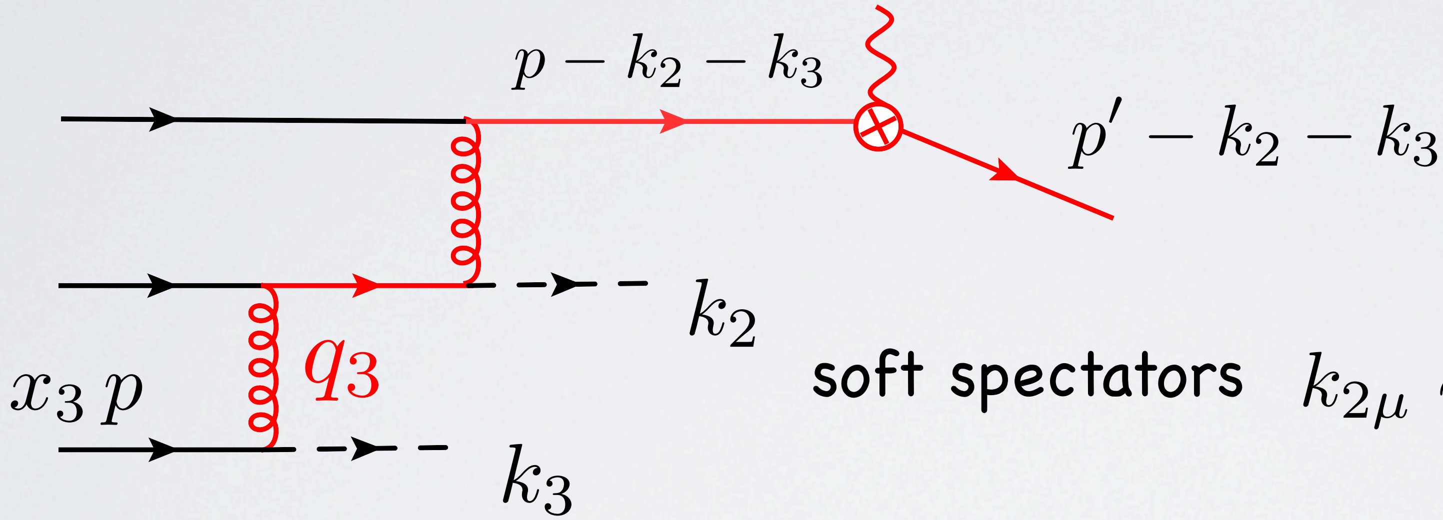
Soft spectator scattering at large Q^2

$$p = \frac{1}{2}(Q, 0, 0, Q) + \mathcal{O}(m^2/Q^2)$$

$$p' = \frac{1}{2}(Q, 0, 0, -Q) + \mathcal{O}(m^2/Q^2)$$



$$\sim \ \alpha^4 \ln[Q/\Lambda]/Q^4$$



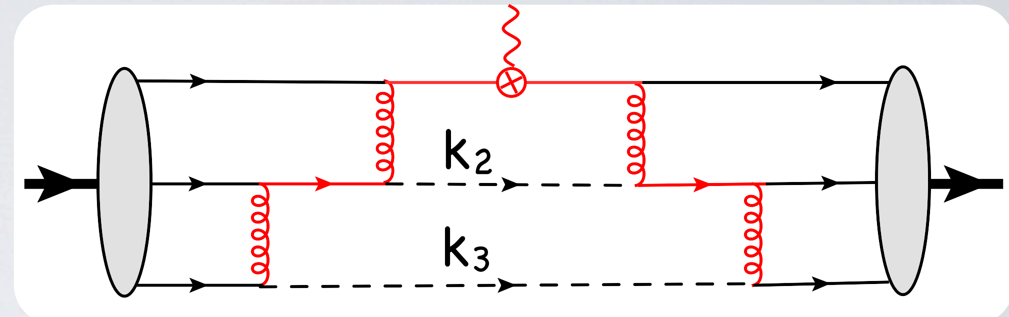
soft spectators $k_{2\mu} \sim k_{3\mu} \sim \Lambda$

$q_3 = x_3 p - k_3$ $\qquad q_3^2 \sim (p \cdot k_3) \sim Q\Lambda \ll Q^2$

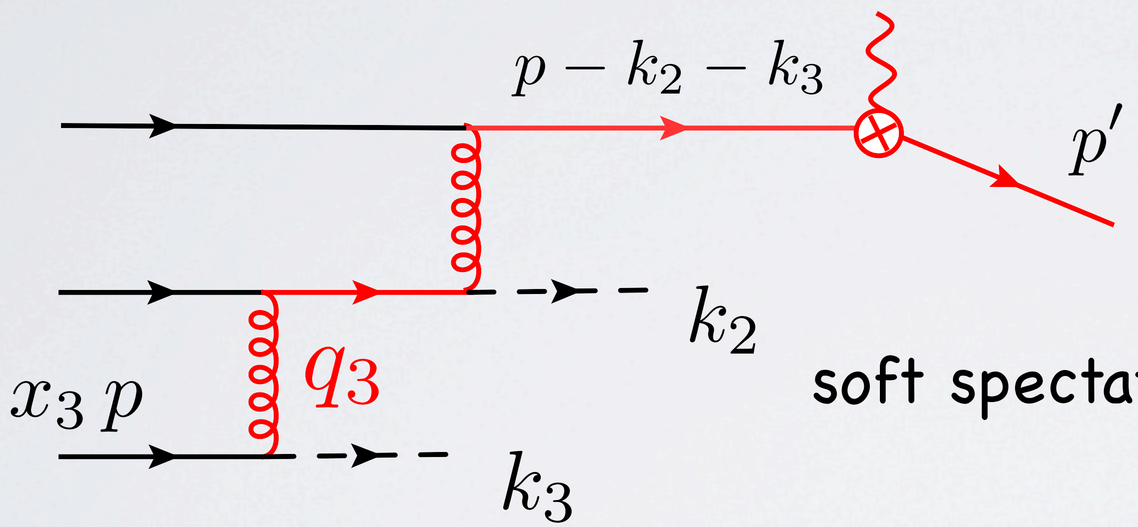
Soft spectator scattering at large Q^2

$$p = \frac{1}{2}(Q, 0, 0, Q) + \mathcal{O}(m^2/Q^2)$$

$$p' = \frac{1}{2}(Q, 0, 0, -Q) + \mathcal{O}(m^2/Q^2)$$



$$\sim \ \alpha^4 \ln[Q/\Lambda]/Q^4$$



soft spectators $k_{2\mu} \sim k_{3\mu} \sim \Lambda$

$q_3 = x_3 p - k_3$ $\qquad q_3^2 \sim (p \cdot k_3) \sim Q\Lambda \ll Q^2$

all red lines can be described as hard-collinear

$q_{hc} \sim (Q, \mathbf{0}, \pm Q) + k \qquad q_{hc}^2 \sim Q\Lambda$

with small residual momenta $k \sim \mathcal{O}(\Lambda)$

Soft spectator scattering at large Q^2

☛ soft spectator scattering involves 3 different scales associated with the virtualities of the scattering particles

hard: $q_h^2 \sim Q^2$ (hard subprocess)

hard-collinear: $q_{hc}^2 \sim Q\Lambda$ (hard-collinear subprocess)

soft: $q_s^2 \sim \Lambda^2$ (soft nonperturbative content)

1. Factorization of hard modes

$$F(Q^2, Q\Lambda, \Lambda^2) = (H_0 + \alpha_s(Q^2)H_1 + \cdots) * f(Q\Lambda, \Lambda^2) + \mathcal{O}(1/Q)$$

2. Factorization of hard-collinear modes

$$f(Q\Lambda, \Lambda^2) = (\alpha_s^n(Q\Lambda)h_n + \alpha_s^{n+1}(Q\Lambda)h_{n+1} + \cdots) * \mathcal{S}(\Lambda^2) + \mathcal{O}(1/Q\Lambda)$$

$$Q\Lambda \gg m_N^2$$

Soft spectator scattering at large Q^2

1. Factorization of hard modes

$$F(Q^2, Q\Lambda, \Lambda^2) = \sum_{n\geq 0} \alpha_s^n(Q^2) H_n * f(Q\Lambda, \Lambda^2) + \mathcal{O}(1/Q)$$

2. Factorization of hard-collinear modes

$$f(Q\Lambda, \Lambda^2) = \sum_{n\geq 0} \alpha_s^n(Q\Lambda) h_n * \mathcal{S}(\Lambda^2) + \mathcal{O}(1/Q\Lambda)$$

$$Q\Lambda \gg m_N^2$$

☛ moderate values of Q^2: $Q\Lambda \sim m_N^2$ hard-collinear scale is not large

$$Q^2 = 9 - 25\text{GeV}^2$$

$$\Lambda \simeq 0.3\text{GeV}$$

⇒ $Q\Lambda \simeq 0.9 - 1.5\text{GeV}^2$

Soft spectator scattering at large Q^2: factorization

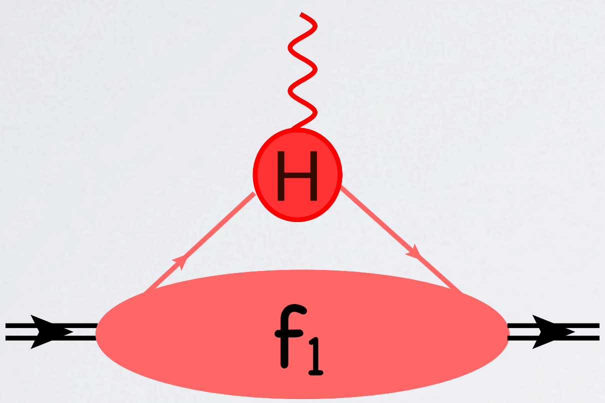
Soft Collinear Effective Theory

QCD: hard
hard-collinear
collinear & soft

factorizing
hard modes

SCET-I
hard-collinear
collinear & soft

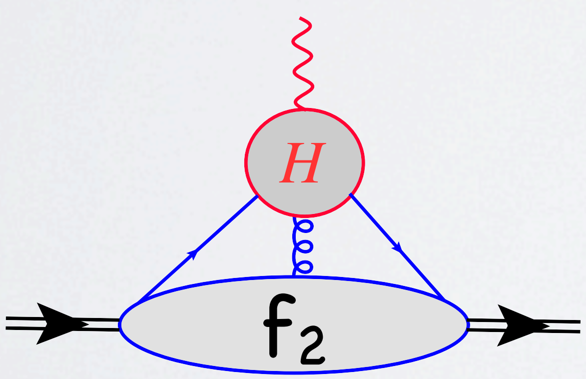
NK, Vanderhaeghen 2010

$F_1(Q) =$ 

$\simeq e^{-S(Q)} U_1(Q)\, f_1(Q\Lambda)$

LLog's

SCET-I FFs

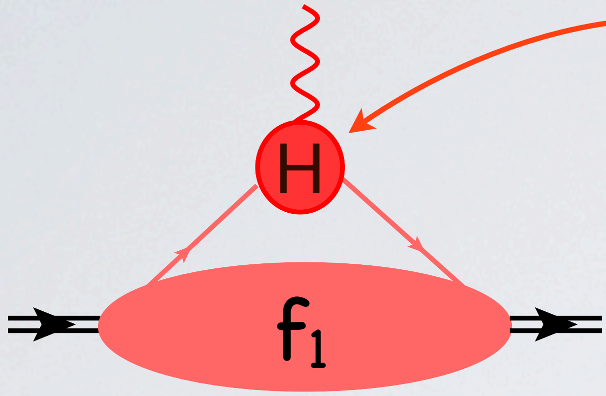
$F_2(Q) =$ 

$\simeq e^{-S(Q)} U_2\,(Q)\ \dfrac{m_N^2}{Q^2} f_2(Q\Lambda)$

Soft spectator scattering at large Q^2: SCET form factors

NK, Vanderhaeghen 2010

$F_1(Q) =$



$\simeq e^{-S(Q)} U_1(Q)\, f_1(Q\Lambda)$

$$p' \simeq \frac{1}{2}(Q,0,0,-Q) = \frac{1}{2}Qn \qquad p \simeq \frac{1}{2}(Q,0,0,Q) = \frac{1}{2}Q\bar{n}$$

SCET FF

$$\langle p'|\bar{\chi}_n(0)\gamma_{\perp\mu}\chi_{\bar{n}}(0)|p\rangle_{SCET} = \bar{N}(p')\,\gamma_{\perp\mu}\,N(p)\,f_1(Q\Lambda)$$

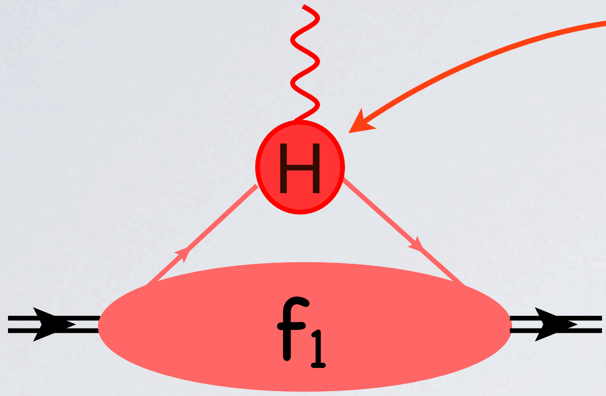
quark-gluon "jet"

$$\chi_{\bar{n}}(0) = \bar{\mathrm{P}}\exp\left\{-ig\int_{-\infty}^{0} ds\, n\cdot A(sn)\right\}\frac{\not{\bar{n}}\not{n}}{4}\psi(0)$$

- process independent = universal
  - constructed from the different collinear fields $\bar{\chi}_n$ and $\chi_{\bar{n}}$
    - fact. scale dependence: f_1 obeys well defined RG equation (allows to sum large logarithms)

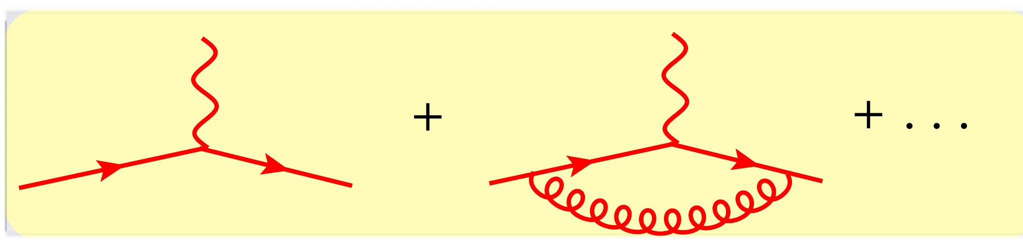
Soft spectator scattering at large Q^2: coefficient function

NK, Vanderhaeghen 2010

$F_1(Q) =$ 

$\simeq e^{-S(Q)} U_1(Q)\, f_1(Q\Lambda)$

Coefficient function



$$= 1 + \alpha_s \ln^2 Q/\Lambda + \alpha_s \ln Q/\Lambda + \ldots$$

large log's

- large logarithms can be resummed by RG equation

Sudakov Log's:

$$S(Q) = \frac{\alpha_s(Q^2)}{4\pi} C_F \ln^2 \frac{Q}{\Lambda} + \ldots$$

"standard" Log's:

$$U_1(Q) = \left(\frac{\alpha_S(Q^2)}{\alpha_S(Q\Lambda)} \right)^{\gamma_1/\beta_0}$$

Soft spectator scattering at large Q^2: leading log results

moderate Q^2, Leading Log approximation NK, Vanderhaeghen 2010

$$F_1(Q) \simeq e^{-S(Q)} U_1(Q)\, f_1(Q\Lambda) \qquad F_2(Q) \simeq e^{-S(Q)} U_2\,(Q)\; \frac{m_N^2}{Q^2} f_2(Q\Lambda)$$

Soft spectator scattering at large Q^2: leading log results

moderate Q^2, Leading Log approximation NK, Vanderhaeghen 2010

$$F_1(Q) \simeq e^{-S(Q)} U_1(Q)\, f_1(Q\Lambda) \qquad F_2(Q) \simeq e^{-S(Q)} U_2(Q)\, \frac{m_N^2}{Q^2} f_2(Q\Lambda)$$

- Two gluon hard exchange is subleading effect

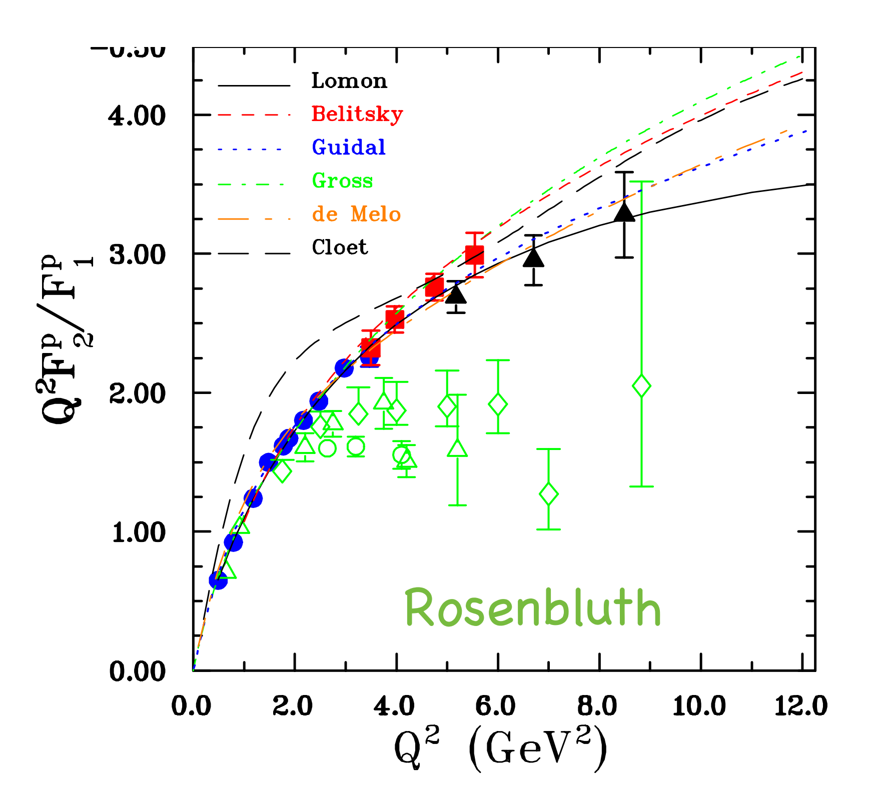
Soft spectator scattering at large Q^2: leading log results

moderate Q^2, Leading Log approximation NK, Vanderhaeghen 2010

$$F_1(Q) \simeq e^{-S(Q)} U_1(Q)\, f_1(Q\Lambda) \qquad F_2(Q) \simeq e^{-S(Q)} U_2(Q)\, \frac{m_N^2}{Q^2} f_2(Q\Lambda)$$

- Two gluon hard exchange is subleading effect
- FF ratio moderate Q^2

$$\frac{Q^2 F_2}{F_1} \simeq 0.96\; m_N^2 \frac{f_2(Q\Lambda)}{f_1(Q\Lambda)}$$



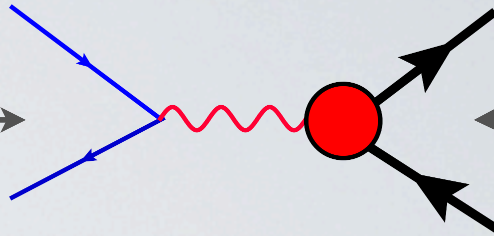
hard contribution: Sudakov log's cancel, very weak Q-dependence

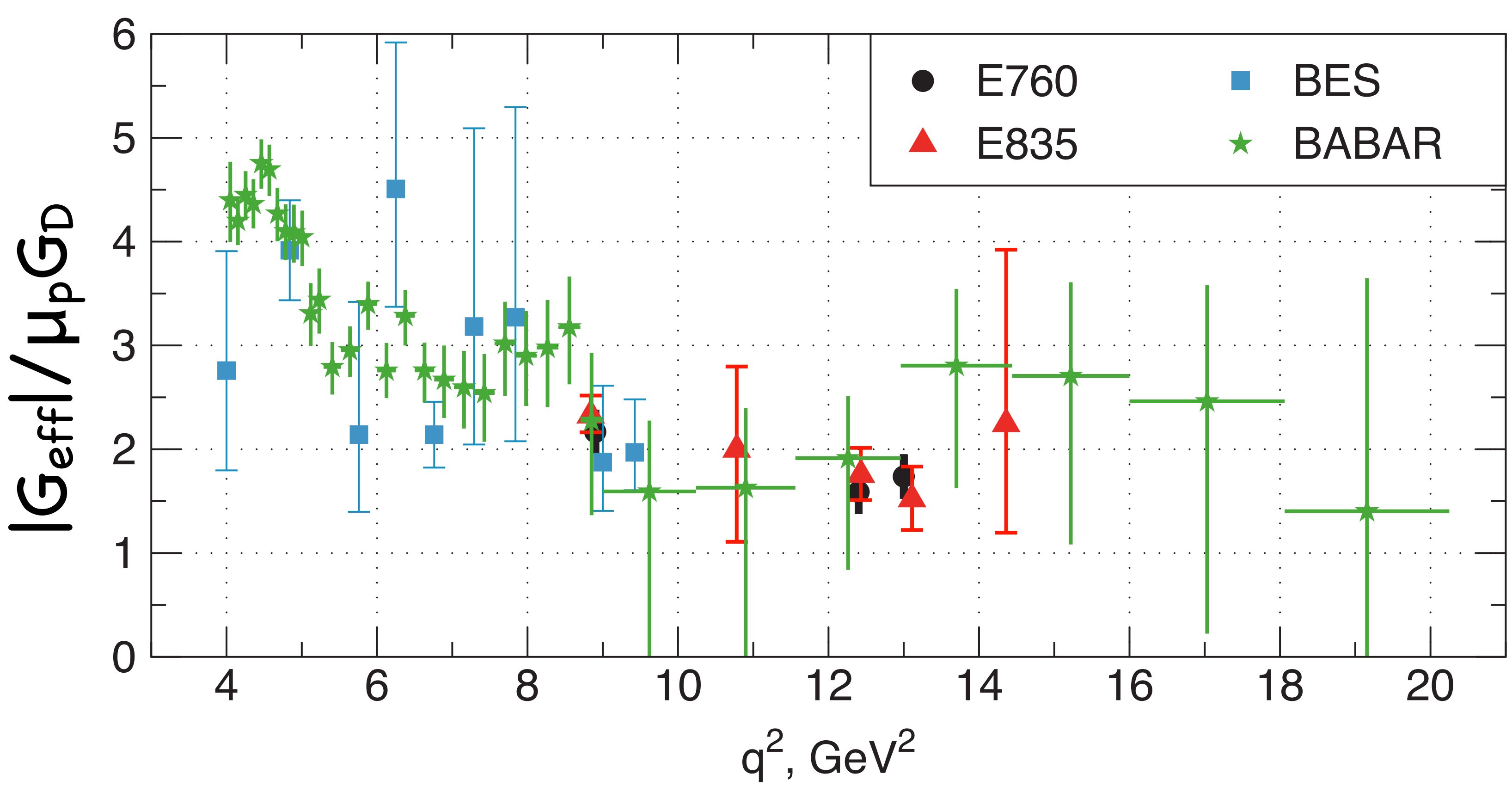
Dominant contribution due to SCET FFs

$\Rightarrow \quad Q\Lambda < 1.5\mathrm{GeV}$ hard-coll scale defines main contribution

Nucleon FFs in the timelike region $q^2>0$

$$\frac{d\sigma_\gamma}{d\cos\theta} = \frac{\pi\alpha^2}{2s\beta}\left[(1+\cos^2\theta)|G_M|^2 + \frac{\sin^2\theta}{\tau}|G_E|^2\right]$$





difficult to perform separation $|G_M|$ and $|G_E|$ (low statistic)

used assumption

$|G_E|=|G_M| = 1$

fit: $|G_{eff}|(s) \simeq \frac{C}{s^2\ln^2[s/\Lambda^2]}$ with C=66.8GeV^2 Λ=300MeV

QCD factorization at large $q^2>0$

- general philosophy is the same: factorize hard modes in the region of moderate q^2
- unknown quantities: SCET FFs f_1 and f_2 defined by the same SCET operators

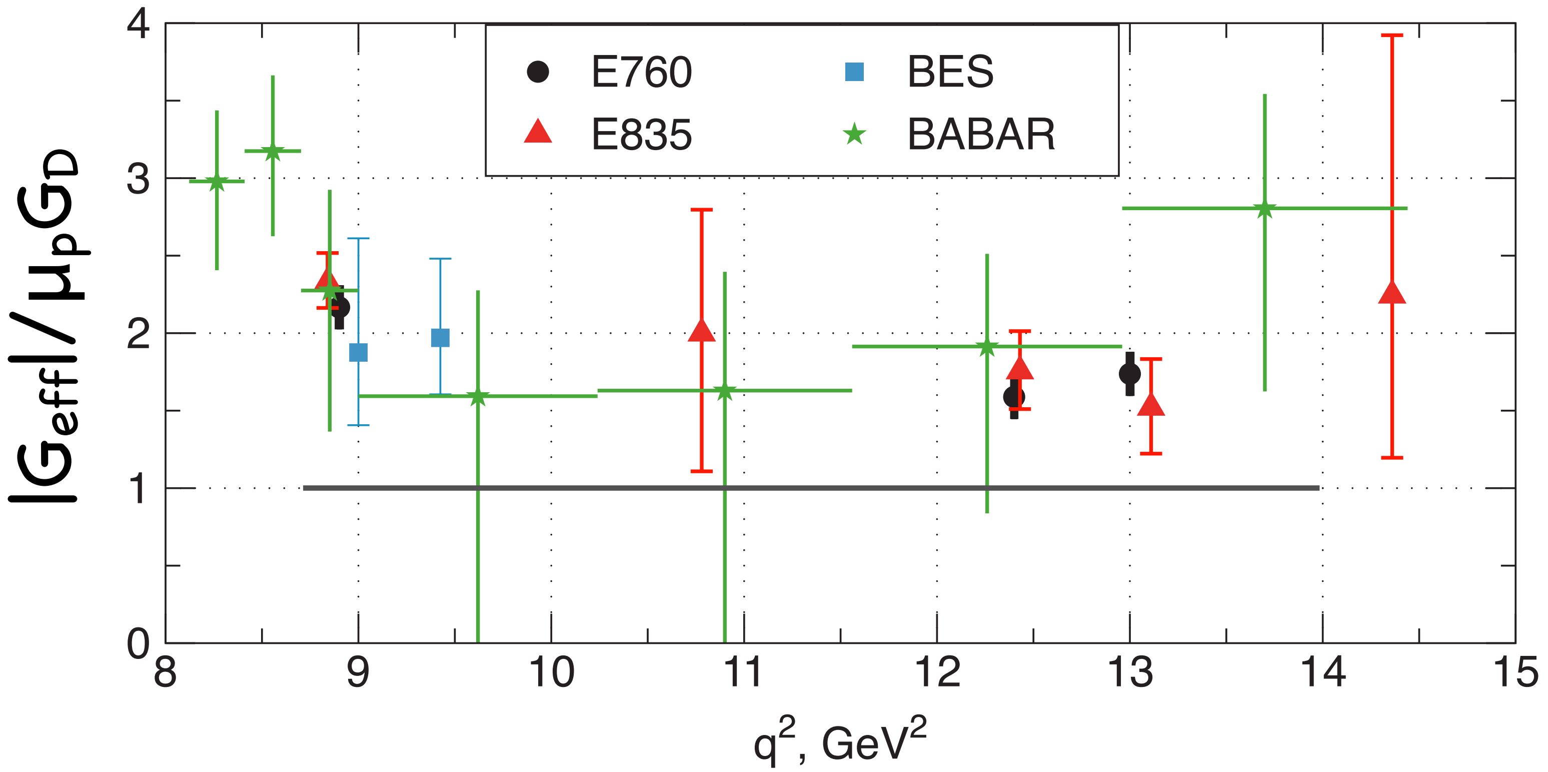
$$\langle p'p|\bar{\chi}_n(0)\gamma_{\perp\mu}\chi_{\bar{n}}(0)|0\rangle_{SCET} = \bar{N}(p')\,\gamma_{\perp\mu}\,U(p)\,f_1(q\Lambda)$$

- timelike factorization is similar to the spacelike

$$F_1(q) \simeq e^{-S(q)}U_1(q)\,f_1(q\Lambda) \qquad F_2(q) \simeq e^{-S(q)}U_2\,(q)\,\frac{m_N^2}{-q^2}f_2(q\Lambda)$$

- large timelike logarithms generate imaginary contribution

FFs ratio: timelike vs. spacelike region

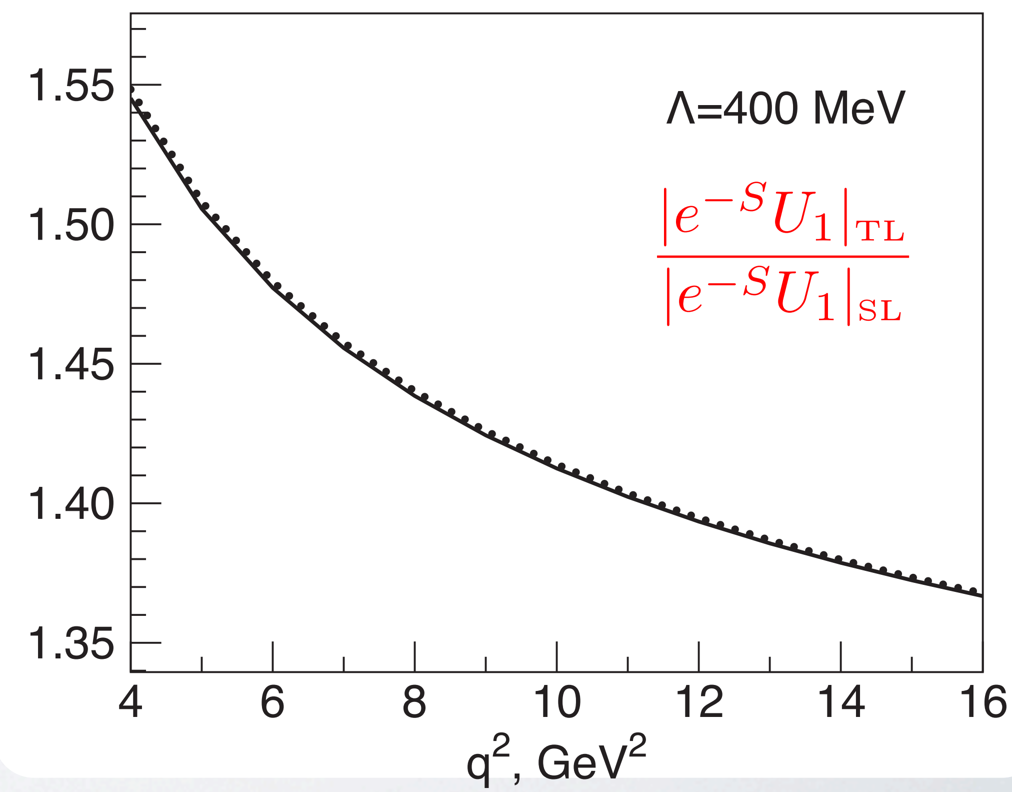


enhancement
in TL region

$$\frac{|G_{eff}|}{\mu_p G_D} \simeq 2$$

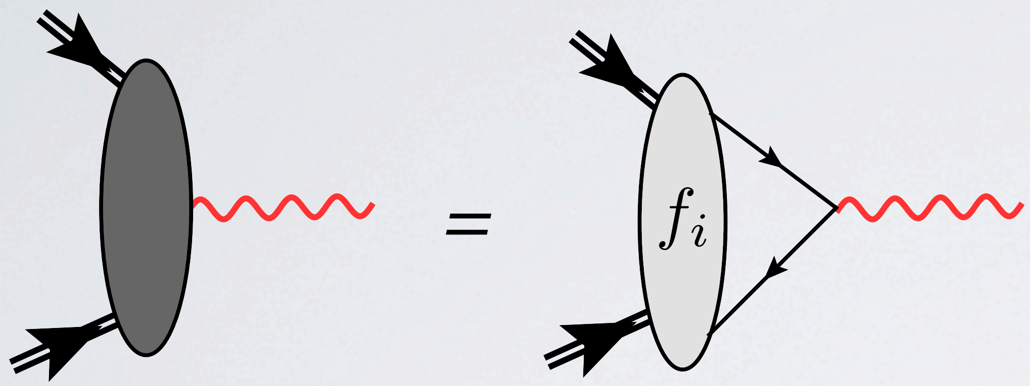
$$\frac{|F_1|_{\rm TL}}{|F_1|_{\rm SL}} \simeq \frac{|e^{-S}U_1|_{\rm TL}}{|e^{-S}U_1|_{\rm SL}} \frac{|f_1(q)|_{\rm TL}}{|f_1(Q)|_{\rm SL}}$$

Sudakov logs provide
enhancement at large time-like q^2



Exploring universality of SCET FFs

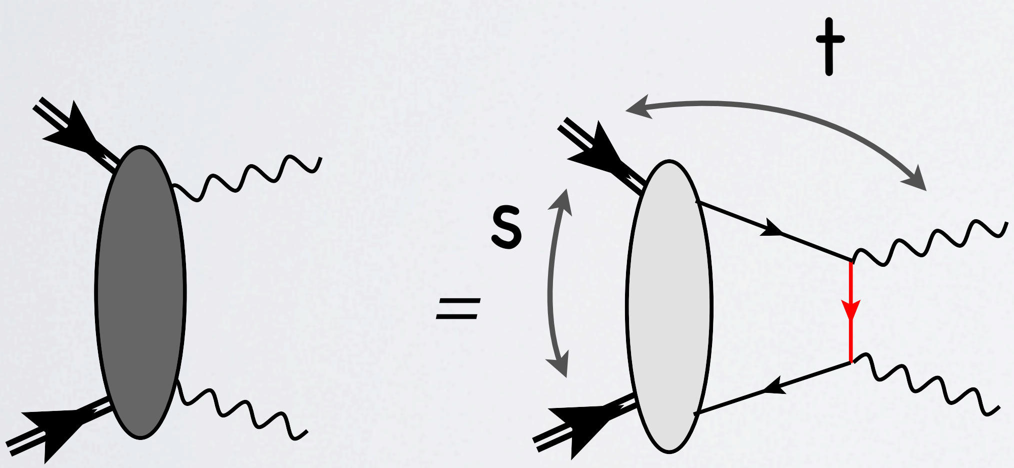
- SCET FFs appear in different reactions in the different flavor combinations



$F_1 \sim f_1 = e_u f_1^u + e_d f_1^d$

$F_2 \sim f_2 = e_u f_2^u + e_d f_2^d$ subleading in 1/Q

wide angle annihilation $p\bar{p} \to \gamma\gamma$ or production $\gamma\gamma \to p\bar{p}$



$-t, \, -u, \, s \gg \Lambda$

6 amplitudes: $T_{1\text{-}6}$

hel. cons. $T_{2,4,6} \sim \mathcal{F}_1 = e_u^2 f_1^u + e_d^2 f_1^d$

hel.flip. $T_{1,3,5} \sim \mathcal{F}_2 = e_u^2 f_2^u + e_d^2 f_2^d$

subleading in 1/Q

NK, Vanderhaeghen (to appear)

Exploring universality of SCET FFs

NK, Vanderhaeghen (to appear)

$\gamma\gamma \to p\bar{p}$

$$\frac{d\sigma^{\gamma\gamma\to p\bar{p}}}{d\cos\theta} \simeq \frac{2\pi\alpha^2}{s}\frac{1+\cos^2\theta}{\sin^2\theta}|\mathcal{F}_1|^2$$

$\mathcal{F}_1 = e_u^2 f_1^u + e_d^2 f_1^d$

θ-independent!

in order to compare with data:

- kinematical power corrections have been added
  - assume $|G_E|=|G_M|$ and use $|G_{eff}|$ from the FF data
    - unknown: $\Delta\phi$ relative phase between F_1 and F_2 $(\cos\Delta\phi < 0)$

      ratio of the abs. values of the quark ffs $r = |f_1^d|/|f_1^u|$

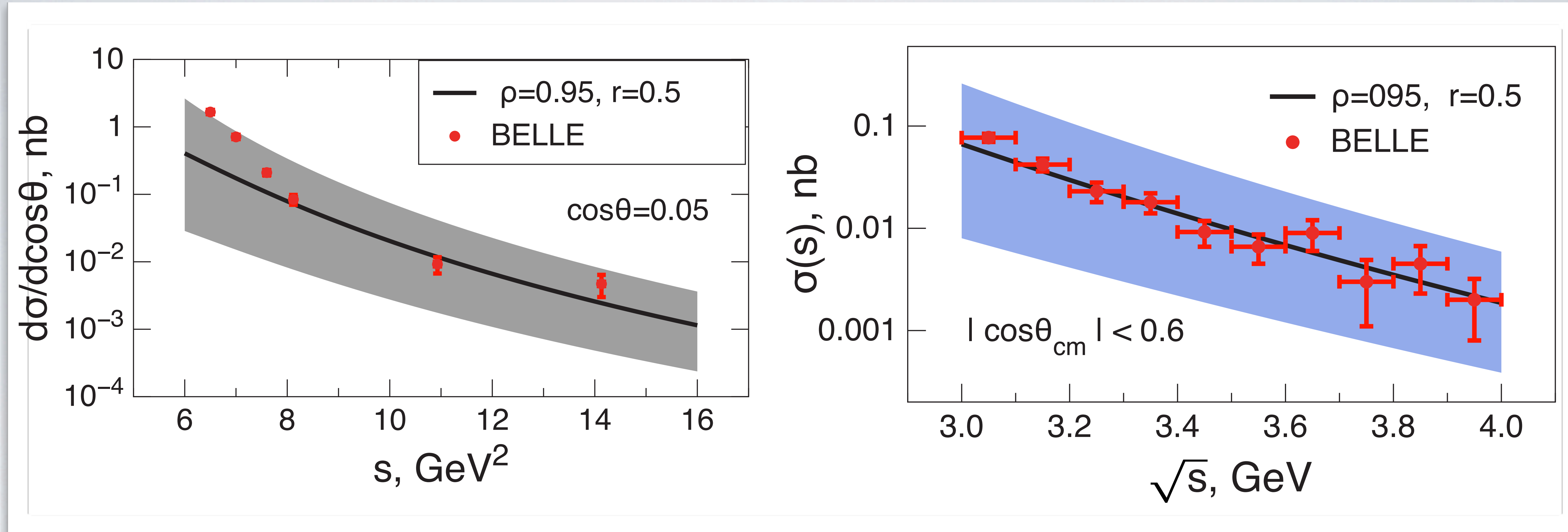
      δ relative phase between f_1^d and f_1^u

      considered as a free parameters

Exploring universality of SCET FFs

$\gamma\gamma \to p\bar{p}$ data Belle collab., 2005

NK, Vanderhaeghen (to appear)



$$|G_{eff}|(s) \simeq \frac{C}{s^2 \ln^2[s/\Lambda^2]}$$ with C=66.8GeV² Λ=300MeV

shaded area $0 < r = |f_1^d|/|f_1^u| < 1$

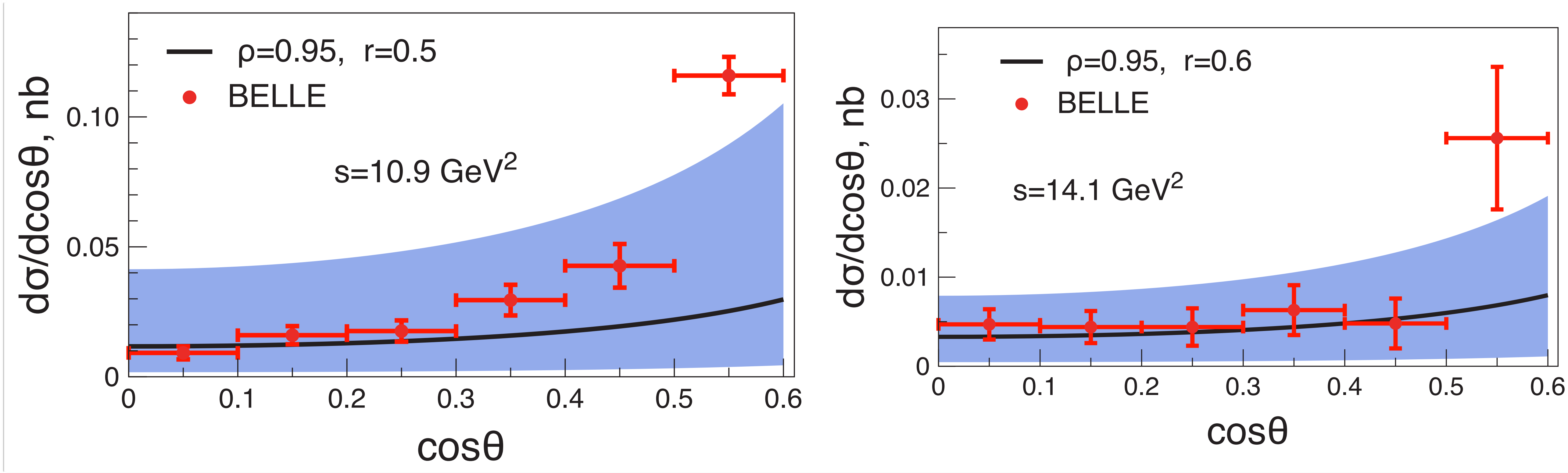
$0 < \rho = -\cos\Delta\phi < 1$ relative phase between F_1 and F_2

$\cos\delta = 1$ relative phase between f_1^u and f_1^d fixed for simplicity

Exploring universality of SCET FFs

$\gamma\gamma \to p\bar{p}$ data Belle collab., 2005

NK, Vanderhaeghen (to appear)



$$|G_{eff}|(s) \simeq \frac{C}{s^2 \ln^2[s/\Lambda^2]}$$ with C=66.8GeV² Λ=300MeV

shaded area $0 < r = |f_1^d|/|f_1^u| < 1$

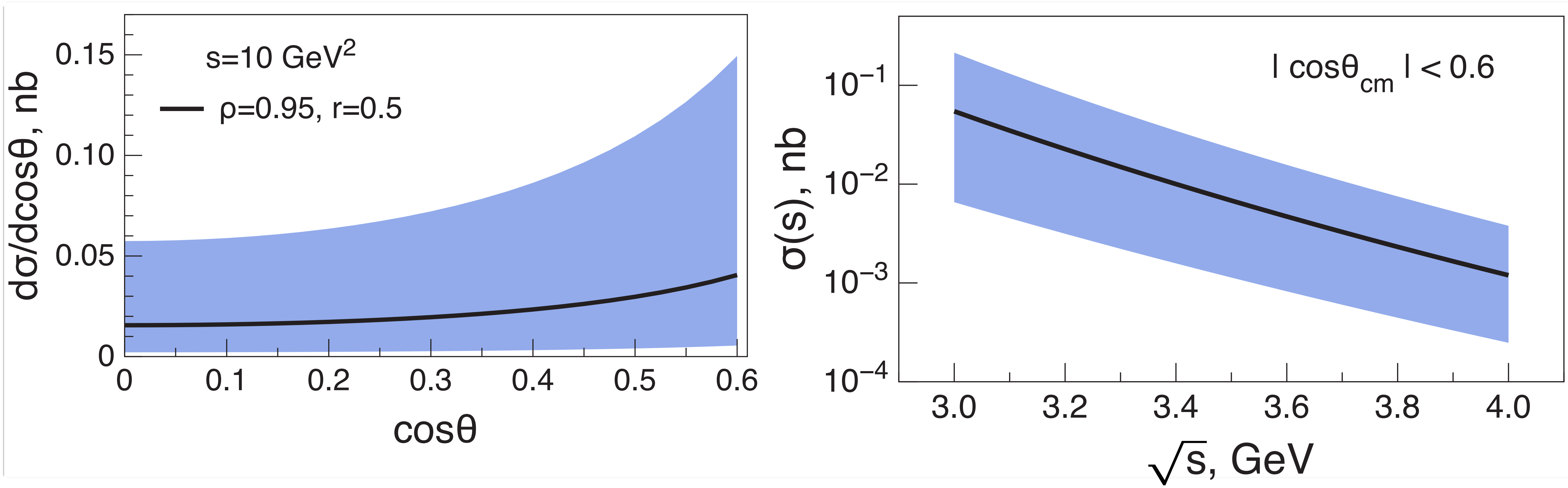
$0 < \rho = -\cos\Delta\phi < 1$ relative phase between F_1 and F_2

$\cos\delta = 1$ relative phase between f_1^u and f_1^d fixed for simplicity

Exploring universality of SCET FFs

PANDA $\quad p\bar{p} \to \gamma\gamma$

predictions NK, Vanderhaeghen (to appear)



$$|G_{eff}|(s) \simeq \frac{C}{s^2 \ln^2[s/\Lambda^2]}$$ with C=66.8GeV² Λ=300MeV

shaded area $\quad 0 < r = |f_1^d|/|f_1^u| < 1$

$0 < \rho = -\cos\Delta\phi < 1$ relative phase between F_1 and F_2

$\cos\delta = 1$ relative phase between f_1^u and f_1^d

fixed for simplicity

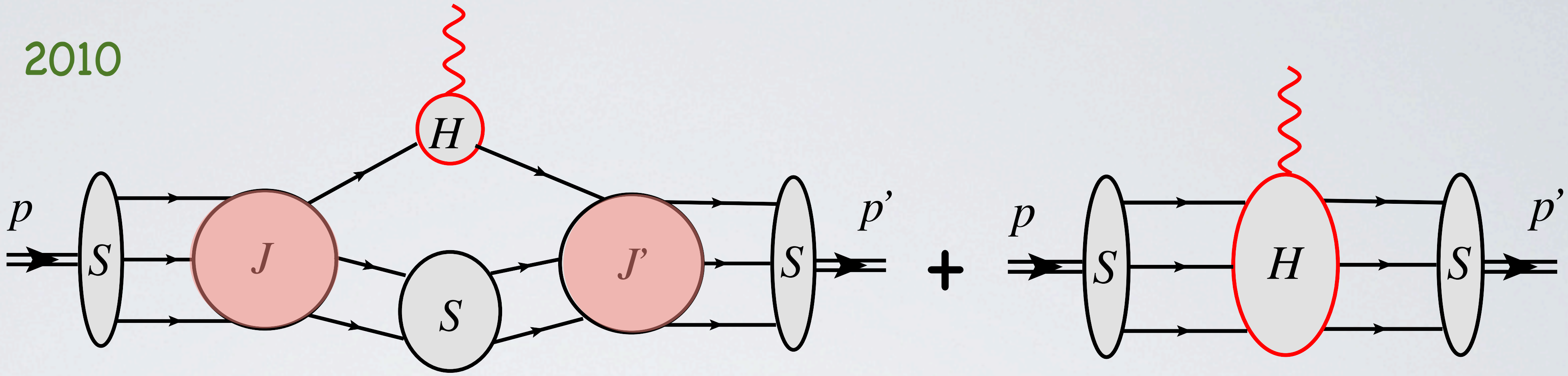
Conclusions

- QCD description of the soft spectator scattering for nucleon FFs involves 2 large scales: hard ~Q^2 and hard collinear ~$Q\Lambda$
- Intermediate $Q^2 \simeq 4 - 16 - ??\,\mathrm{GeV}^2$: the hard-collinear scale is not large enough $Q\Lambda \sim m_N^2$.
- non-perturbative dynamics is described in terms of SCET FFs $f_{1,2}(Q\Lambda)$ (they do not related to GPDs). In the large Q limit ($Q\Lambda \gg m_N^2$) can be farther factorized.
- Perturbative corrections associated with the hard scattering (hard ~Q^2) can be computed systematically. Well known hard 2-gluon contribution is the NNLO correction
- The same picture works also for the proton FFs in the timelike region. Sudakov logs provide enhancement.
- The same SCET FFs can describe the other processes but in the different flavor combination.

Soft rescattering at large Q^2 : factorization approach

NK, Vanderhaeghen, 2010

$$F_1(Q) =$$



$$F_1^{(s)}(Q^2) \simeq C_A(Q^2, \mu_I) \quad \mathbf{\Psi}'(y_i, \mu_{II}) * \int_0^\infty d\omega_1 d\omega_2 \, \mathbf{J}'(y_i, \omega_i, Q, \mu_I, \mu_{II})$$
$$\mathbf{\Psi}(x_i, \mu_{II}) * \int_0^\infty d\nu_1 d\nu_2 \, \mathbf{J}(x_i, \nu_i, Q, \mu_I, \mu_{II}) \quad \boldsymbol{S}(\omega_i, \nu_i; \mu_{II})$$

Soft correlation function: di-quark "propagator"

$$\boldsymbol{S}(\omega_i, \nu_i; \mu_{II}) = \int \frac{d\eta_1}{2\pi} \int \frac{d\eta_2}{2\pi} \, e^{-i\eta_1\nu_1 - i\eta_2\nu_2} \int \frac{d\lambda_1}{2\pi} \int \frac{d\lambda_2}{2\pi} e^{i\lambda_1\omega_1 + i\lambda_2\omega_2} \langle 0 | \, \mathbf{O}_S(\eta_i, \lambda_i) \, | 0 \rangle$$

$$\mathbf{O}_S(\eta_i, \lambda_i) \quad = \varepsilon^{i'j'k'} \left[S_n^\dagger(0)\right]^{i'l} \left[S_n^\dagger q(\lambda_1 n)\right]^{j'} C\Gamma \left[S_n^\dagger q(\lambda_2 n)\right]^{k'}$$
$$\times \varepsilon^{ijk} \left[S_{\bar n}(0)\right]^{li} \left[\bar q S_{\bar n}(\eta_1 \bar n)\right]^{j} \bar\Gamma C \left[\bar q S_{\bar n}(\eta_2 \bar n)\right]^{k}$$