Hadron Interactions from Lattice QCD

-- from Quarks to Neutron Stars --

1. Introduction
2. Lattice Nuclear Force
3. Lattice Hyperon Force
4. Summary

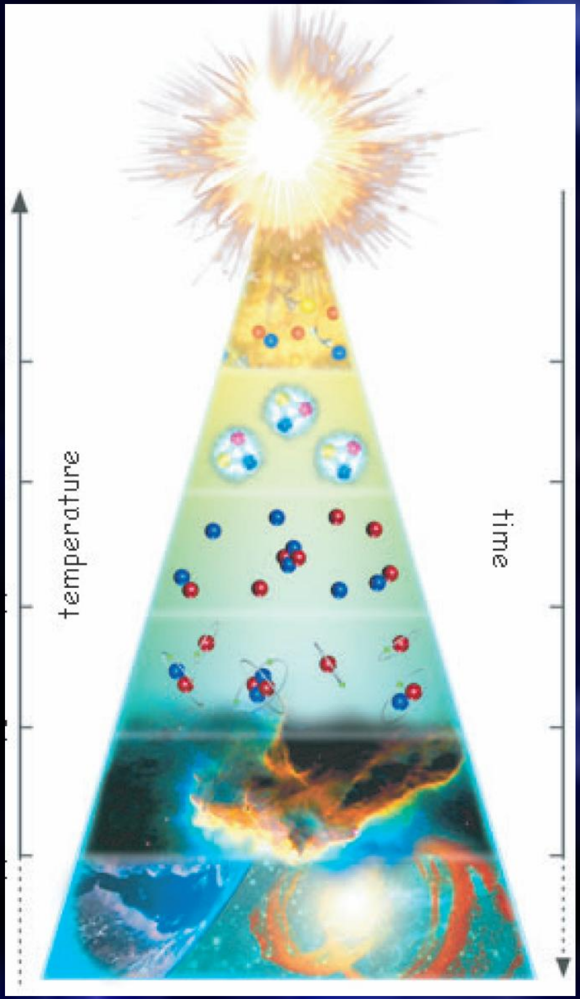
T. Hatsuda
(Univ. Tokyo / RIKEN)

THE UNIVERSITY OF TOKYO

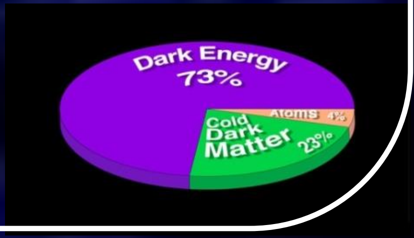
RIKEN

Lattice image by Delorfirith

Major challenges in QCD

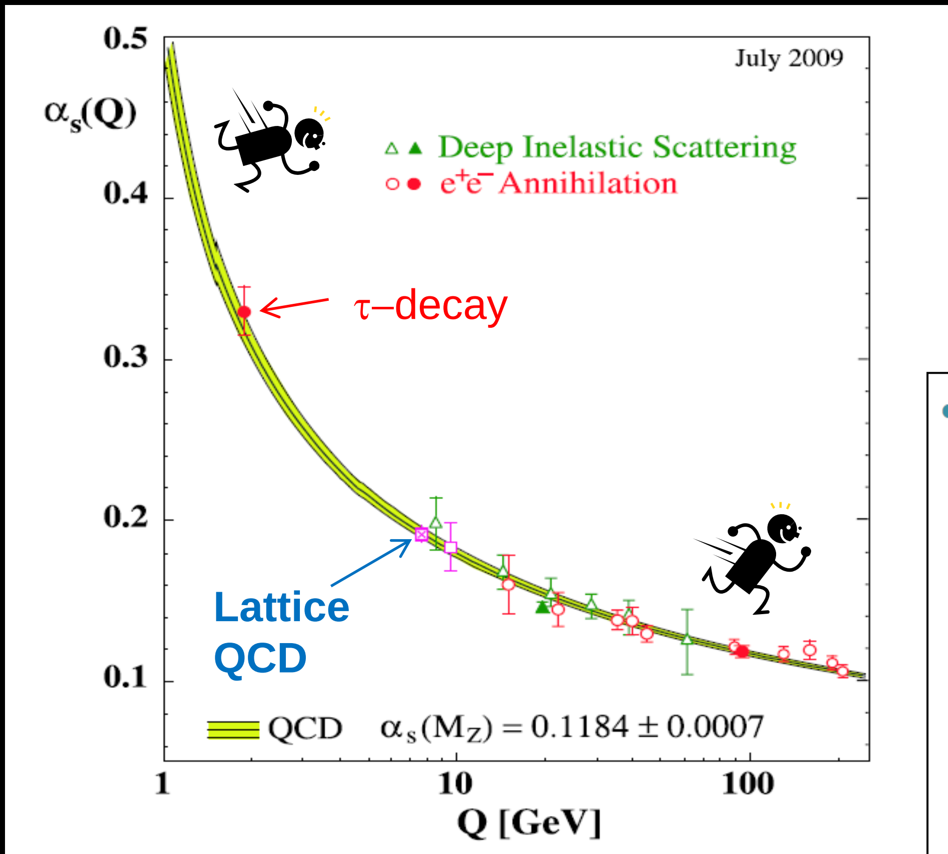


- Primordial form of matter
  quark-gluon plasma
- Origin of heavy elements
  in explosive astrophysical phenomena
- Super dense matter
  neutron star, exotic matter, …
- Inputs for "new physics" search
  dark matter, …



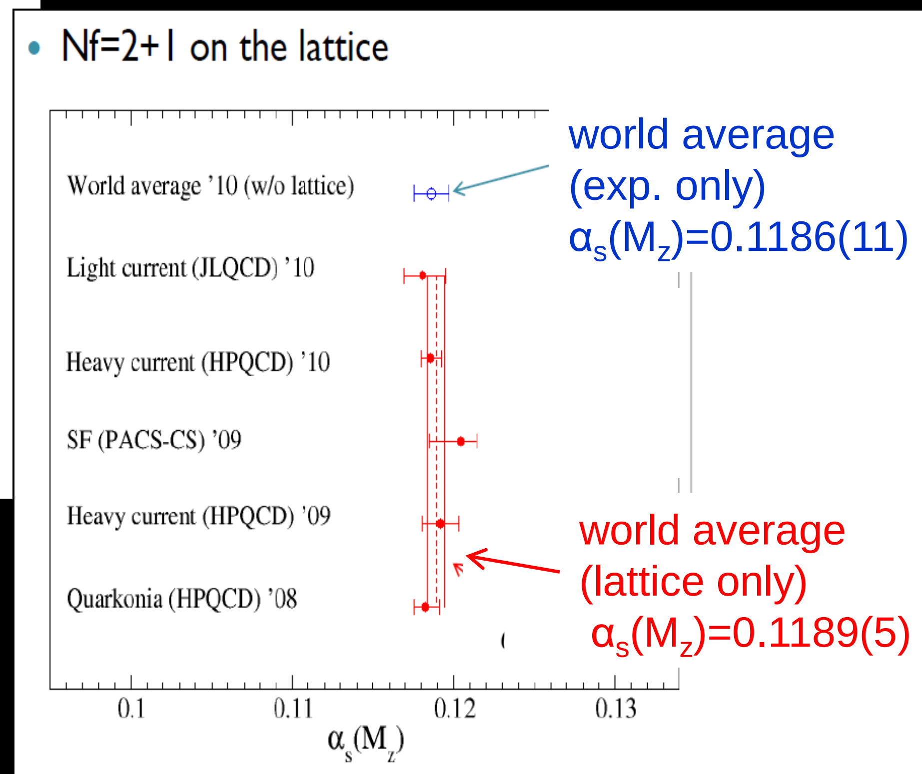
Lattice QCD provides
(1) precision calculations (2) qualitative pictures

QCD running coupling @ 2009



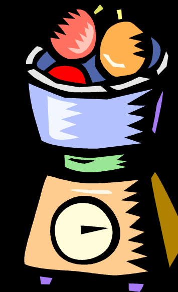
Bethke, Eur. Phys. J, C(2009)64:689

QCD running coupling @ 2011



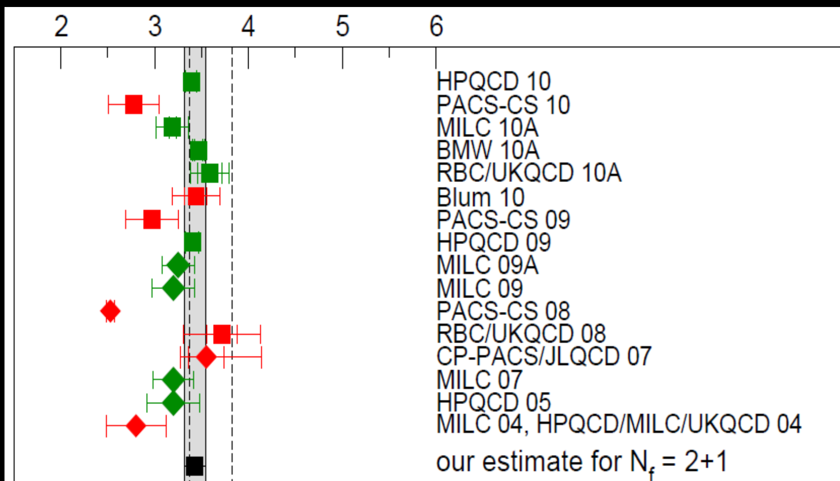
summarized by Shintani (Lattice2011)

Light quark masses (MSbar, @2GeV)

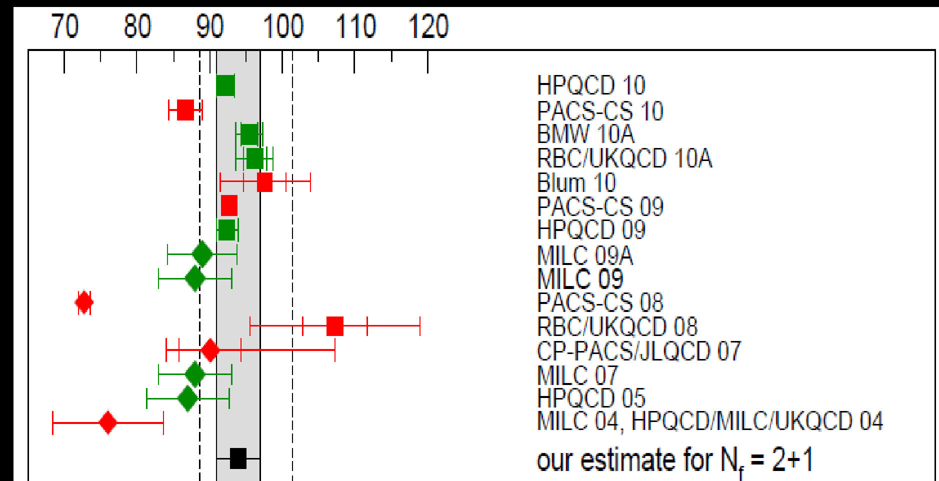


Summary by FLAG working group, arXiv:1011.4408[hep-lat]

m_{ud} [MeV]



m_s [MeV]



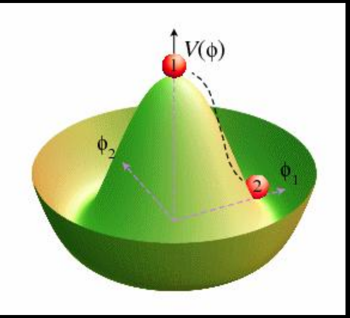
QCD(simulation)+QED(estimate)

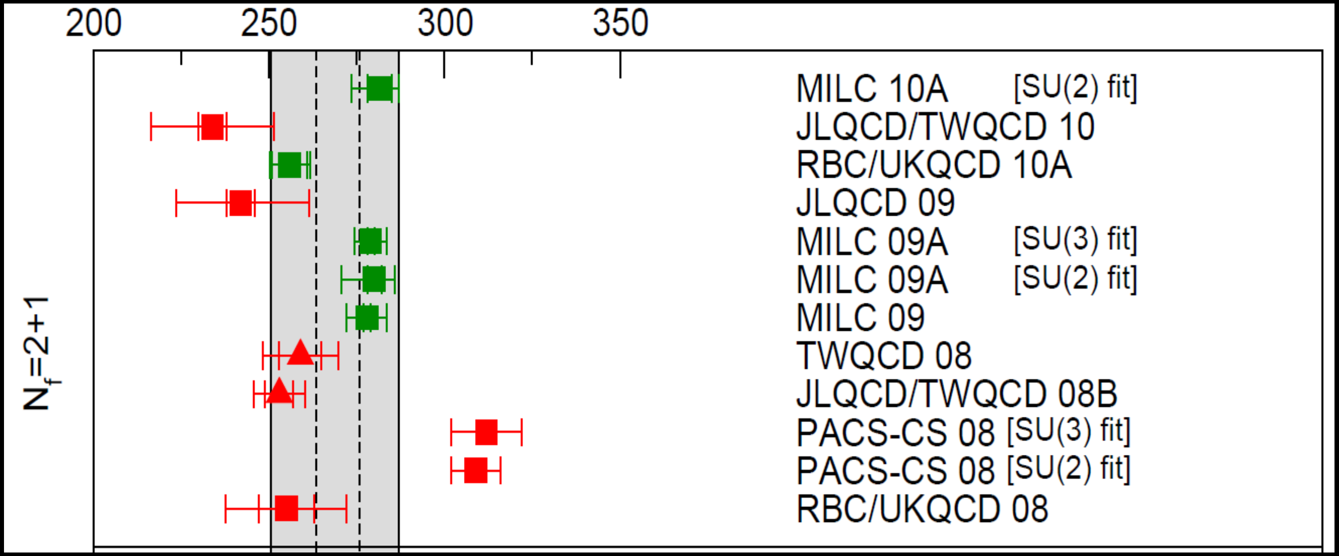
N_f	m_u [MeV]	m_d [MeV]	m_{ud} [MeV]	m_s [MeV]	m_s/m_{ud}
2+1	2.19(15)	4.67(20)	3.42(11)	94(3)	27.4(4)

QCD+QED simulation has also been started

Blum et al., Phys. Rev. D82 (2010) 094508

Low energy constants



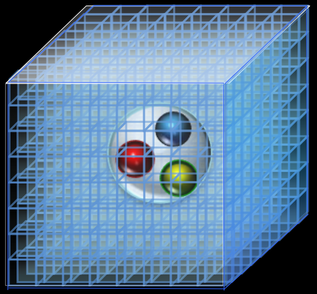


$$[-\langle \bar{q}q \rangle_{2\mathrm{GeV}}]^{1/3} = (250 - 275)[\mathrm{MeV}]$$

$$L_{4-8}, f_+(0), f_K/f_\pi, B_K, \mathrm{etc}$$

More in arXiv:1011.4408 [hep-lat]
(FLAG working group)

Hadron masses @ 2009

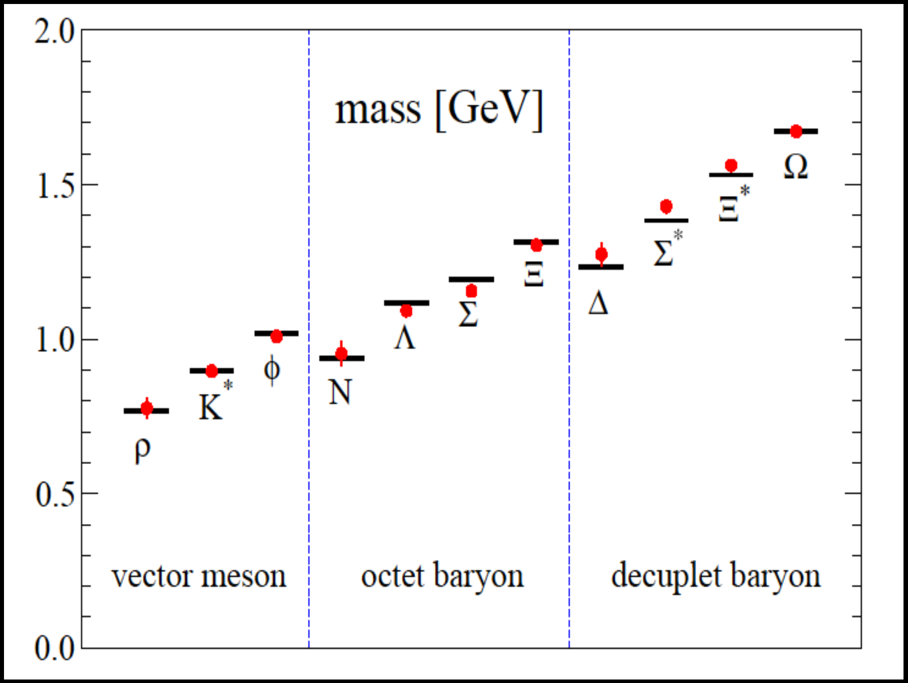


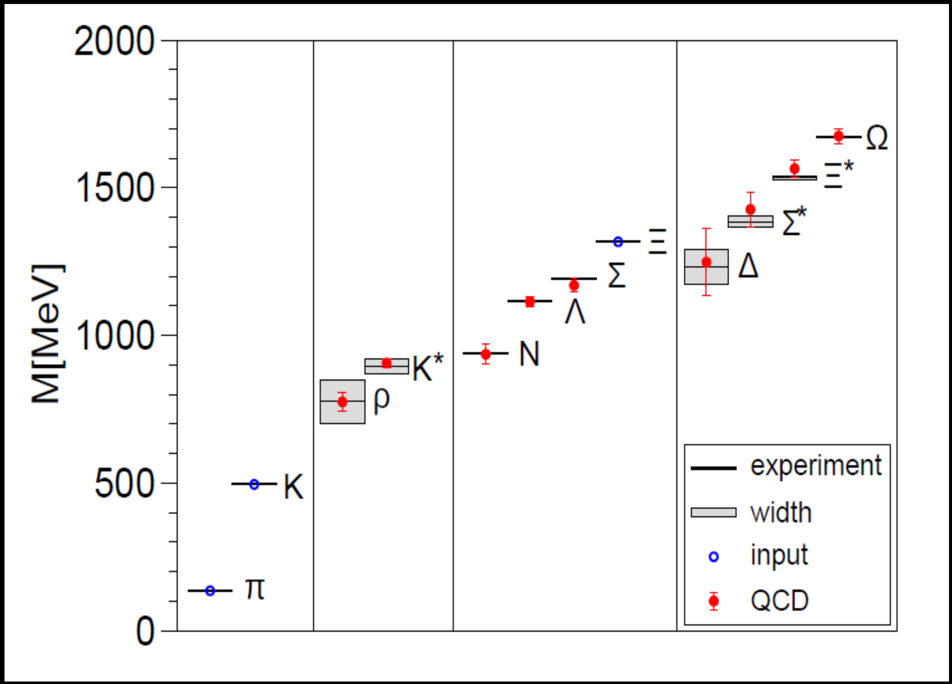
PACS-CS Collaboration,
Phys.Rev.D79(2009)034503

(2+1)-flavor, Wilson
L =2.9 fm, a =0.09 fm
m_π(min) =156 MeV

BMW Collaboration,
Science 322 (2008) 1224.

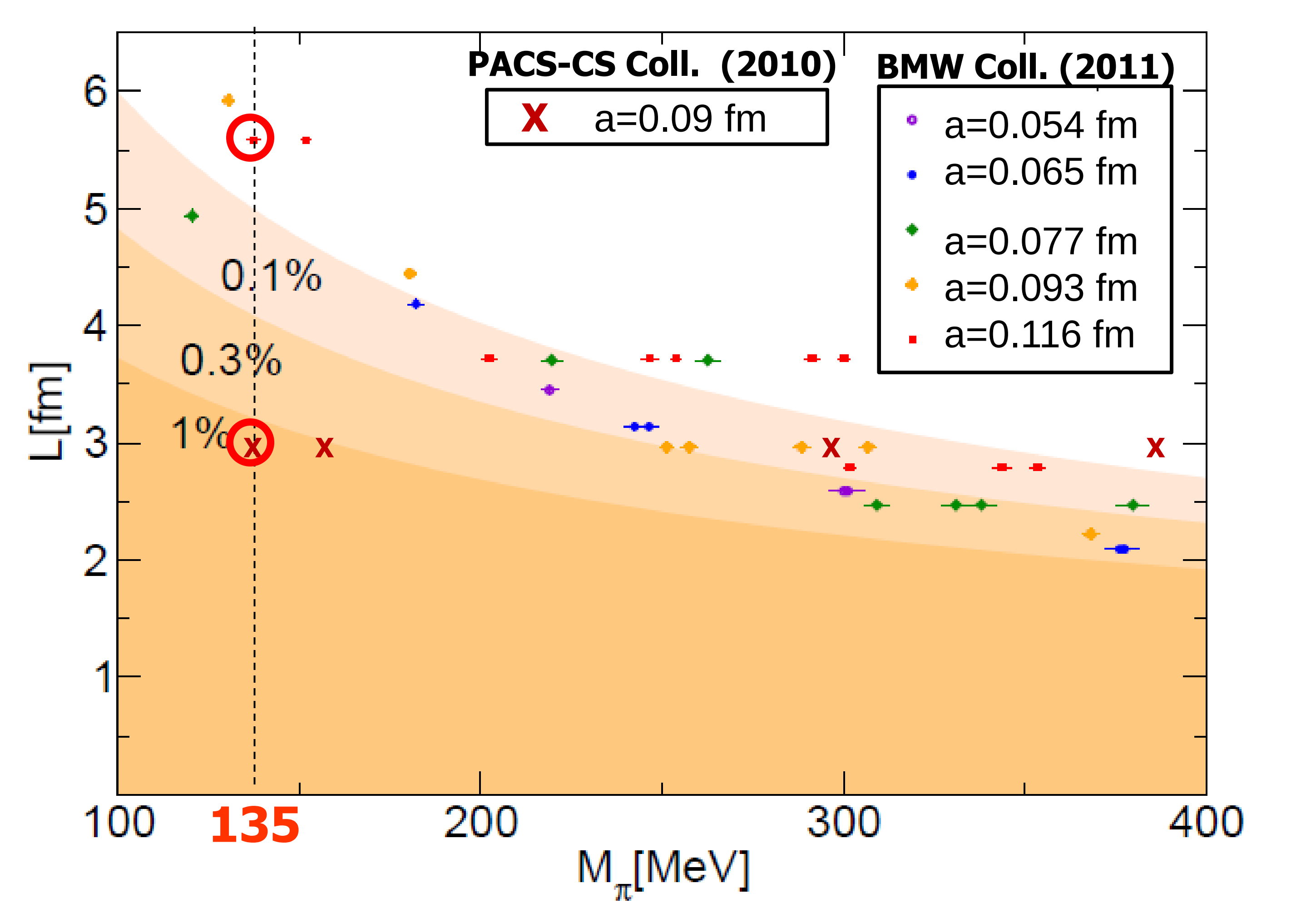
(2+1)-flavor, Wilson
L =(2.0- 4.1) fm, a =0.065, 0.085, 0.125 fm
m_π(min) =190 MeV





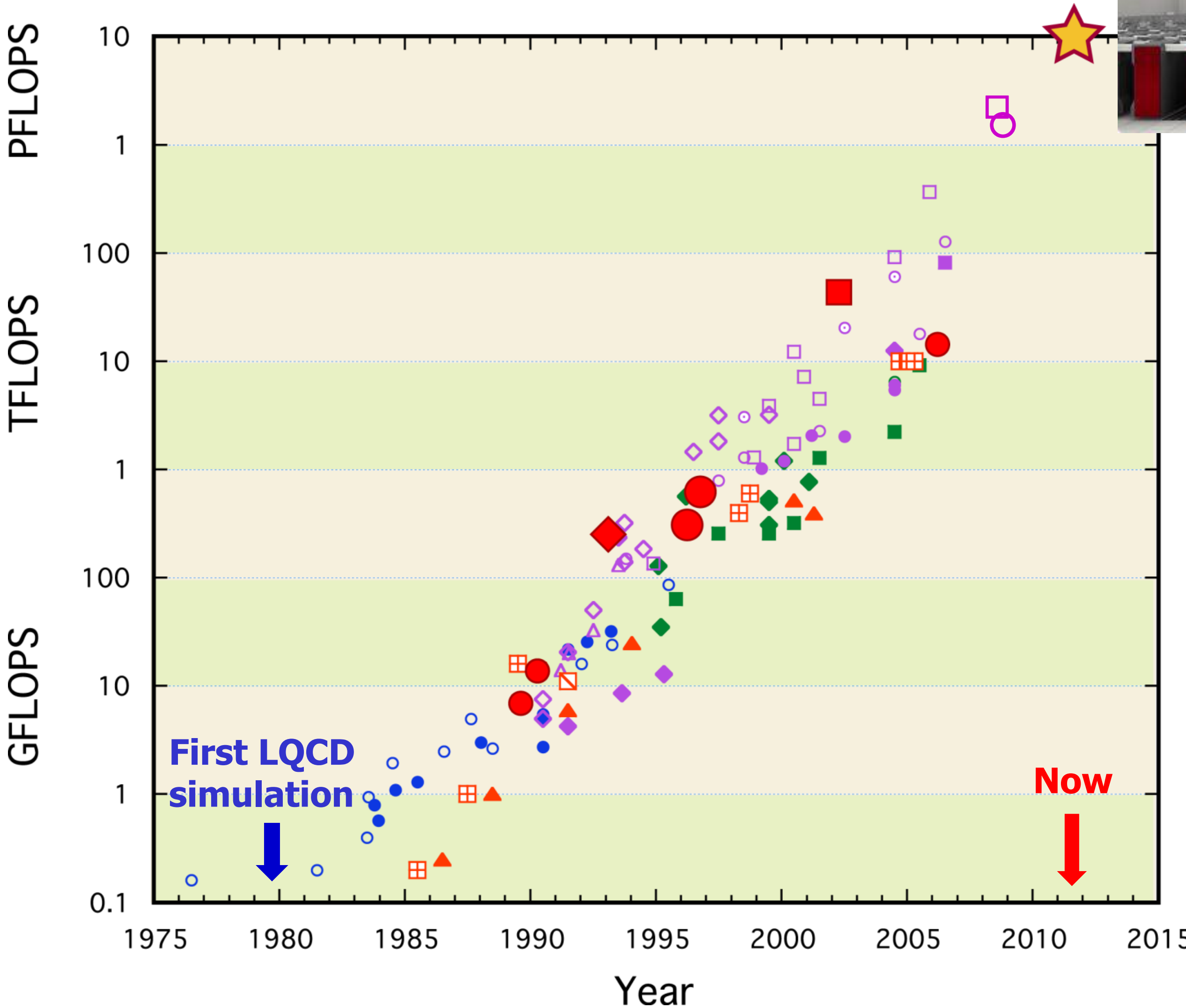
Hadron masses @ 2011
Physical point simulations in (2+1)-flavor QCD

PACS-CS Coll.: Phys. Rev.D81 (2010) 074503
BMW Coll.: Phys. Lett. B701 (2011) 265



⇒ Massive production of physical point config. at "KEI computer" from 2012

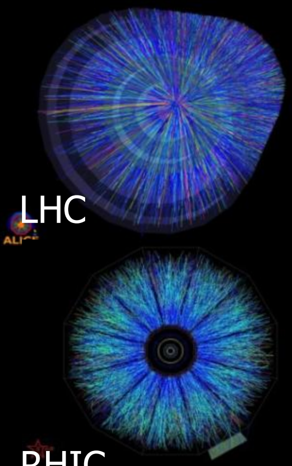
Supercomputer peak performance

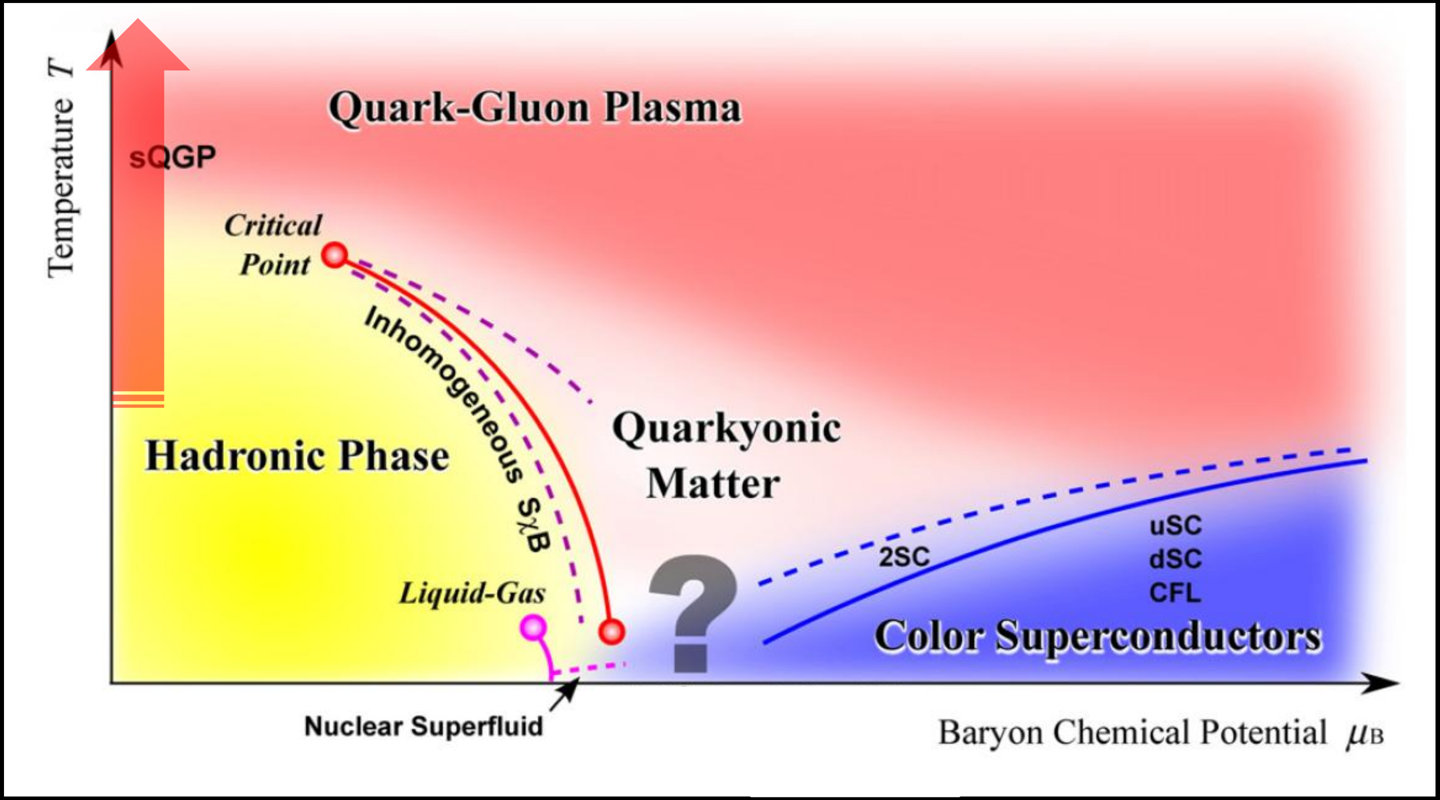


10PFlops “K computer” (RIKEN)

original plot by A. Ukawa

Hot QCD

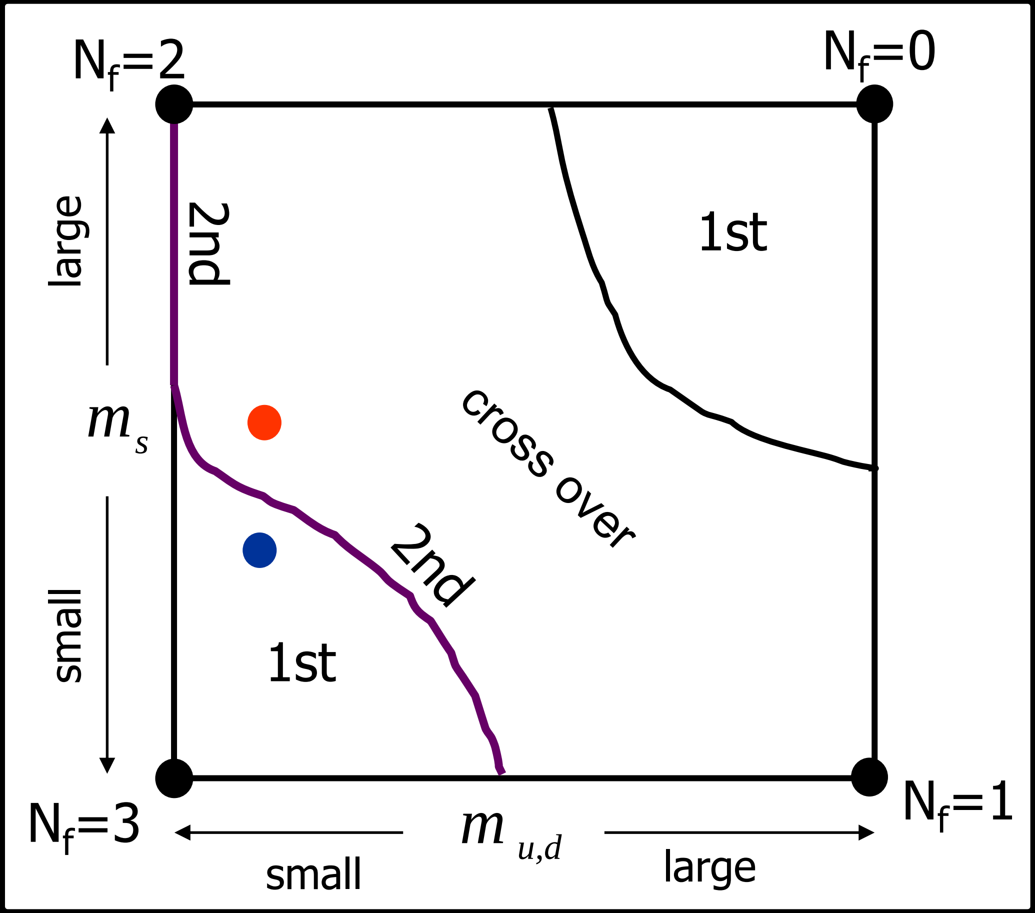




Fukushima and Hatsuda, Rep. Prog. Phys. 74 (2011)014001

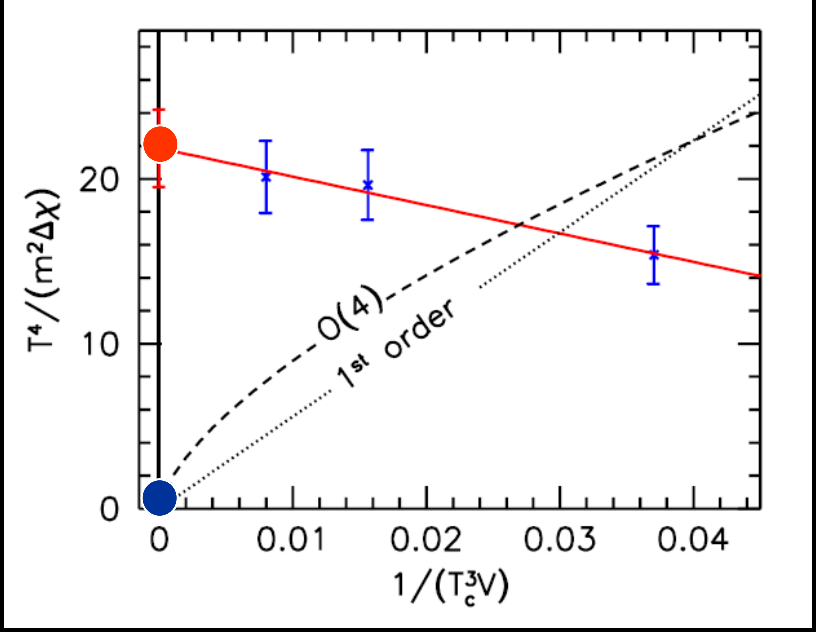
Thermal QCD transition at μ=0

Columbia plot



Finite size scaling

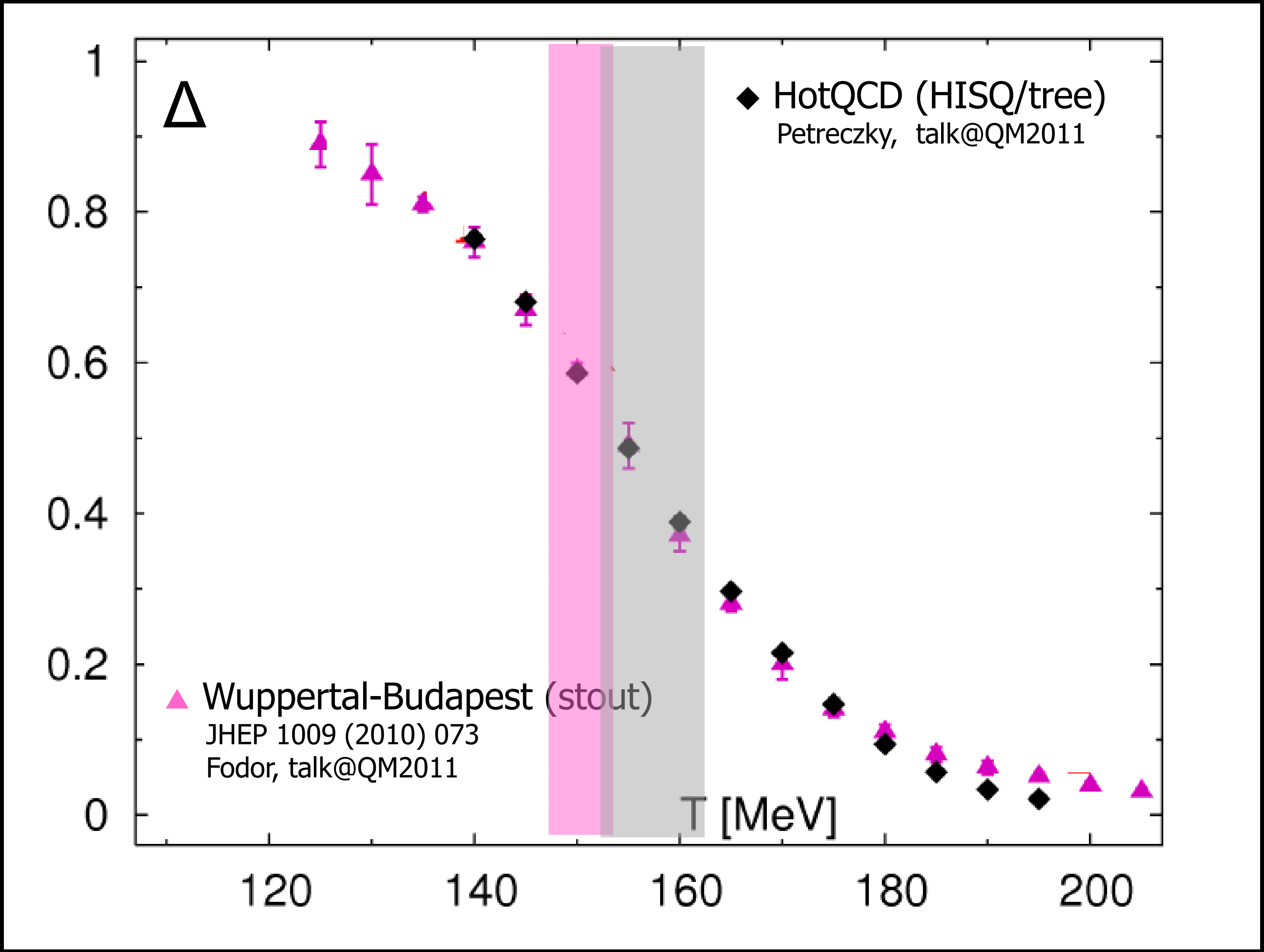
$$\chi_m = \frac{\partial^2 P}{\partial m_{\rm ud}^2} \sim \begin{cases} V & \text{1st order} \\ V^{2/3} & \text{2nd order} \\ V^0 & \text{crossover} \end{cases}$$



Budapest group, Nature 443 (2006) 675
Staggered, (2+1)-flavor, physical mass

Thermal chiral condensate in (2+1)-flavor QCD at m_π=135 MeV (staggered fermion)

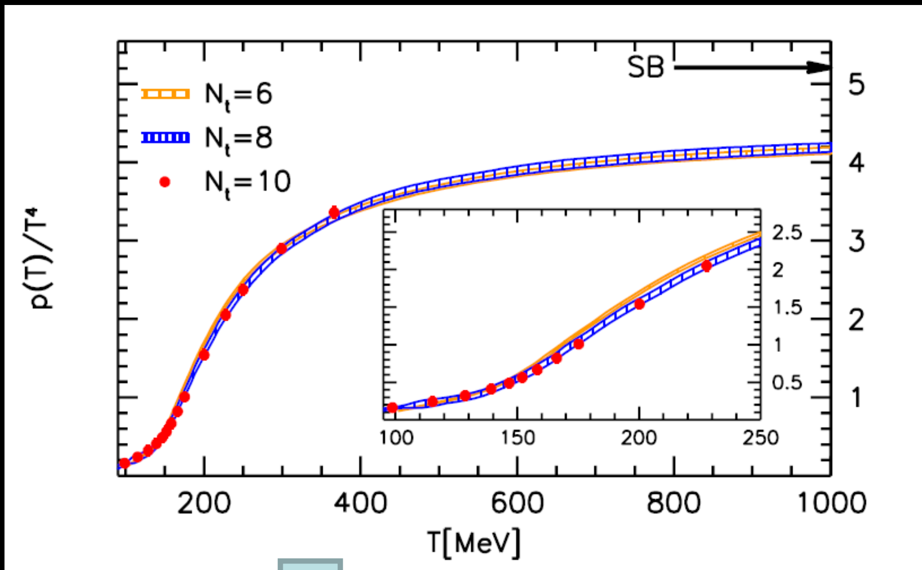
$$\Delta = \frac{\langle \bar{u}u \rangle_T - \frac{m_u}{m_s}\langle \bar{s}s \rangle_T}{\langle \bar{u}u \rangle_0 - \frac{m_u}{m_s}\langle \bar{s}s \rangle_0}$$

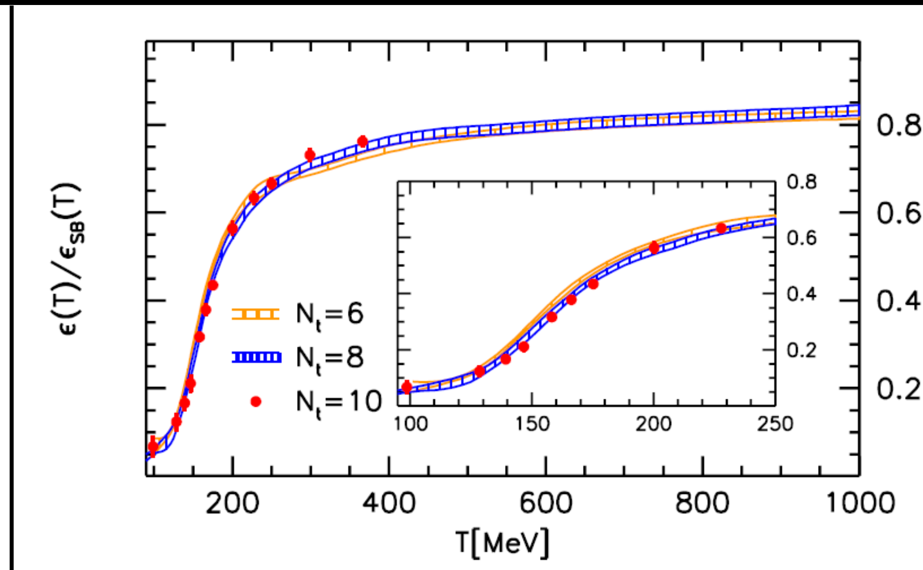


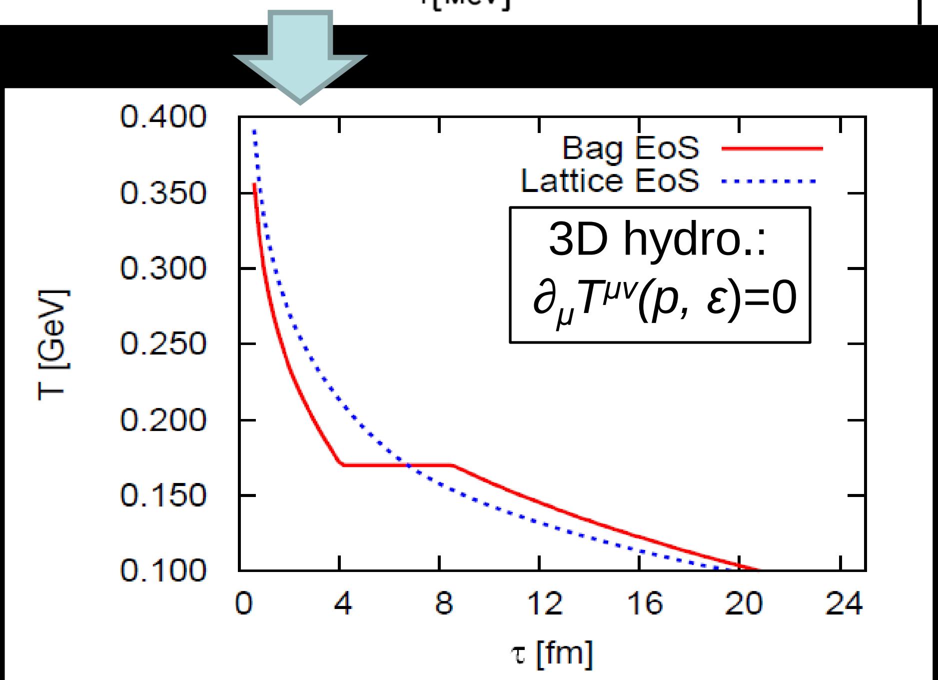
Chiral susceptibility peak ⇒ T_{pc}=150-160MeV

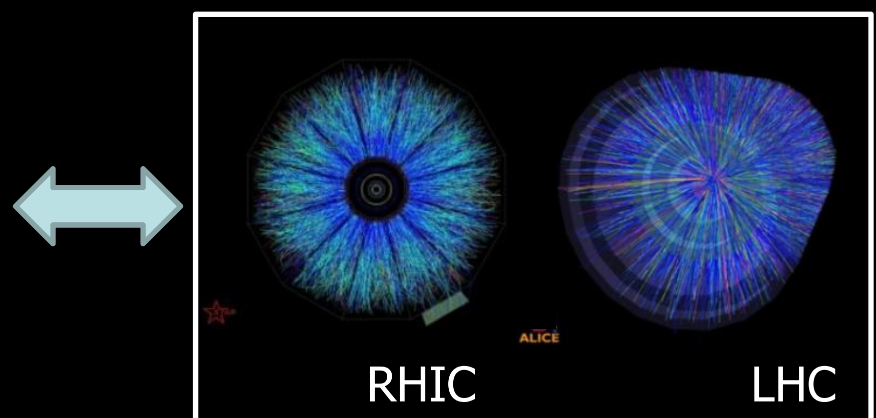
Lattice EOS: p(T), ε(T)

Borsanyi et al., JHEP 1011 (2010) 77



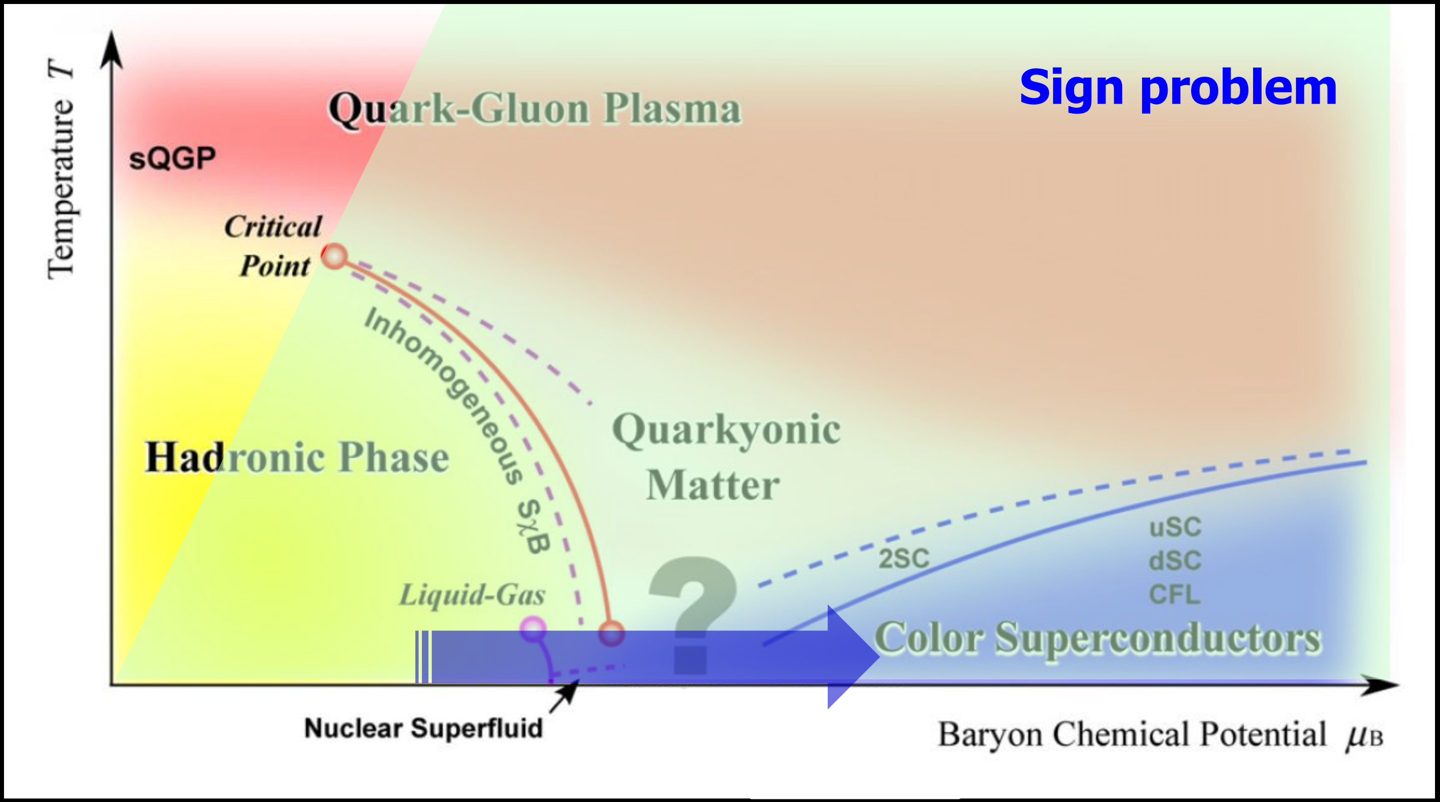




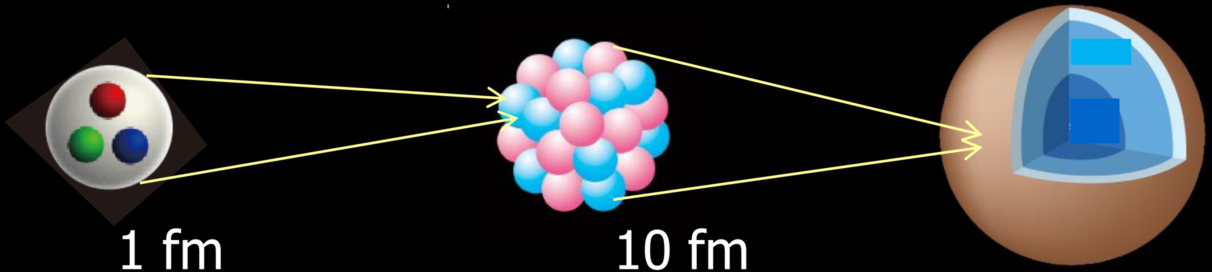


Akamatsu, Hamagaki, Hirano, Hatsuda, arXiv:1107.36[nucl-th]

Dense QCD



Fukushima and Hatsuda, Rep. Prog. Phys. 74 (2011)014001

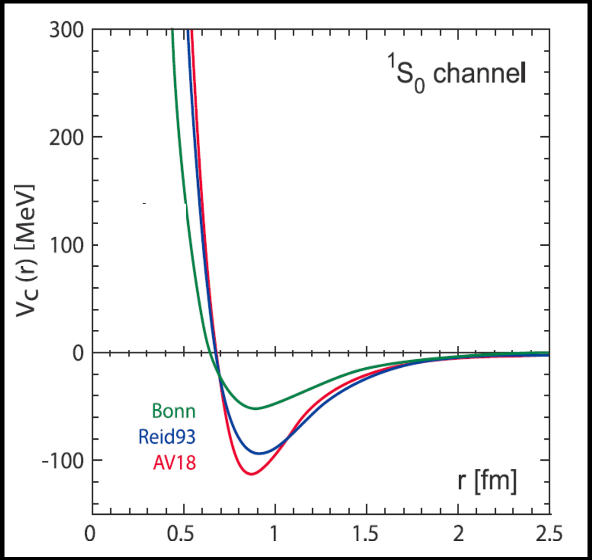


phenomenological nuclear forces

- NN int.: about 4500 np and pp scatt. data

"high precision" NN interactions		# of parameters
CD Bonn	(p space)	38
AV18	(r space)	40
EFT in N^3LO	($n\pi$+contact)	24

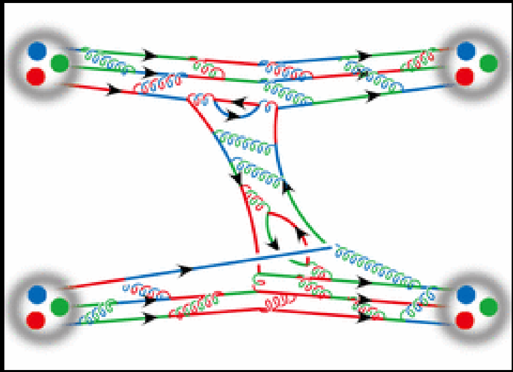
R. Machleidt, arXiv:0704.0807 [nucl-th]



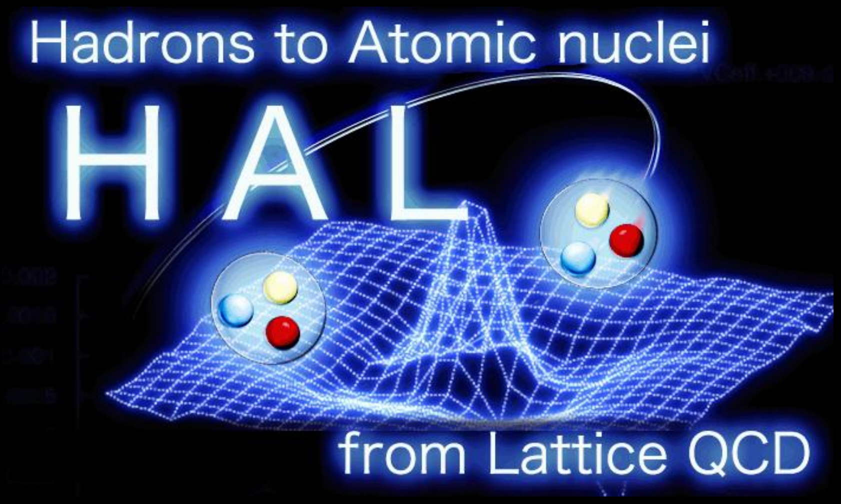
- NNN, YN, YY : data limited
- YYN, YNN, YYY : none

QCD has only <u>four</u> parameters :

m_u, m_d, m_s, Λ_{QCD}



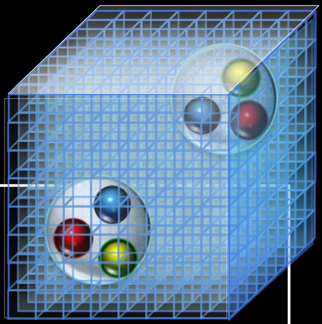
Lattice Nuclear Force



Tohoku Univ.	H. Nemura
Univ. Tsukuba	S. Aoki, N. Ishii, K. Sasaki
RIKEN	K. Murano, T. Hatsuda
Nihon Univ.	T. Inoue
Tokyo Inst. Tech.	Y. Ikeda
CNS, Univ. Tokyo	T. Doi

NN force from LQCD	Ishii, Aoki, Hatsuda, PRL 99 (2007), PTP123(2010)
YN and YY forces from LQCD	HAL QCD Coll., PTP 124 (2010)
H-dibaryon from LQCD	HAL QCD Coll., PRL 106 (2011)
NNN force from LQCD	HAL QC Coll., arXiv 1106.2276[hep-lat]

Basic strategy



1. NN wave function from lattice QCD

$$\phi_n(\vec{r},t) = \langle 0|N(\vec{x}+\vec{r},t)N(\vec{x},t)|E_n\rangle$$

$$\phi(\vec{r},t) = \sum_n c_n \phi_n(\vec{r},t)$$

1. NN potential from the NN wave function

$$\left(-\frac{\partial}{\partial t} - H_0\right)\phi(\vec{r},t) = \int U(\vec{r},\vec{r}')\phi(\vec{r}',t)d^3r'$$

3. Derivative expansion

$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta^3(\vec{r}-\vec{r}')$$

$$V(\vec{r},\nabla) = V_{\mathrm{C}}(r) + S_{12}V_{\mathrm{T}}(r) + \vec{\mathrm{L}}\cdot\vec{\mathrm{S}}\; V_{\mathrm{LS}}(r) + \{V_{\mathrm{D}}(r),\nabla^2\} + \cdots$$

LO LO NLO NNLO

Ishii, Aoki, Hatsuda, Phys.Rev.Lett. 99 (2007) 022001
Ishii et al. (HAL QCD Coll.) in preparation

- Potential is a nice <u>tool</u> to calculate observables
- Potential is <u>volume insensitive</u> (=Lattice Friendly)

Properties of lattice NN potential *U(r,r')*

$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta^3(\vec{r}-\vec{r}')$$

[1] $U(r,r')$ is $N(x)$-dependent

QM : (ψ, V) → observables
QFT : (asymptotic field, vertices) → observables
($N(x)$, $U(r,r')$) → observables

[2] $U(r,r')$ is E-independent

non-locality can be determined order by order

[3] $U(r,r')$ has minor volume dependence

Wave function is sensitive to the volume
Potential is insensitive to the volume
remember the deuteron !

Key channels in NN scattering ($^{2s+1}L_J$)

$$V(\vec{r},\nabla) = V_C(r) + S_{12}V_T(r) + \vec{L}\cdot\vec{S}\,V_{LS}(r) + \{V_D(r),\nabla^2\} + \cdots$$

LO LO NLO NNLO

$^1S_0$ Central force ⟺ nuclear BCS pairing

Bohr, Mottelson & Pines, Phys. Rev. 110 (1958)

$^3S_1$-$^3D_1$ Tensor force ⟺ deuteron binding

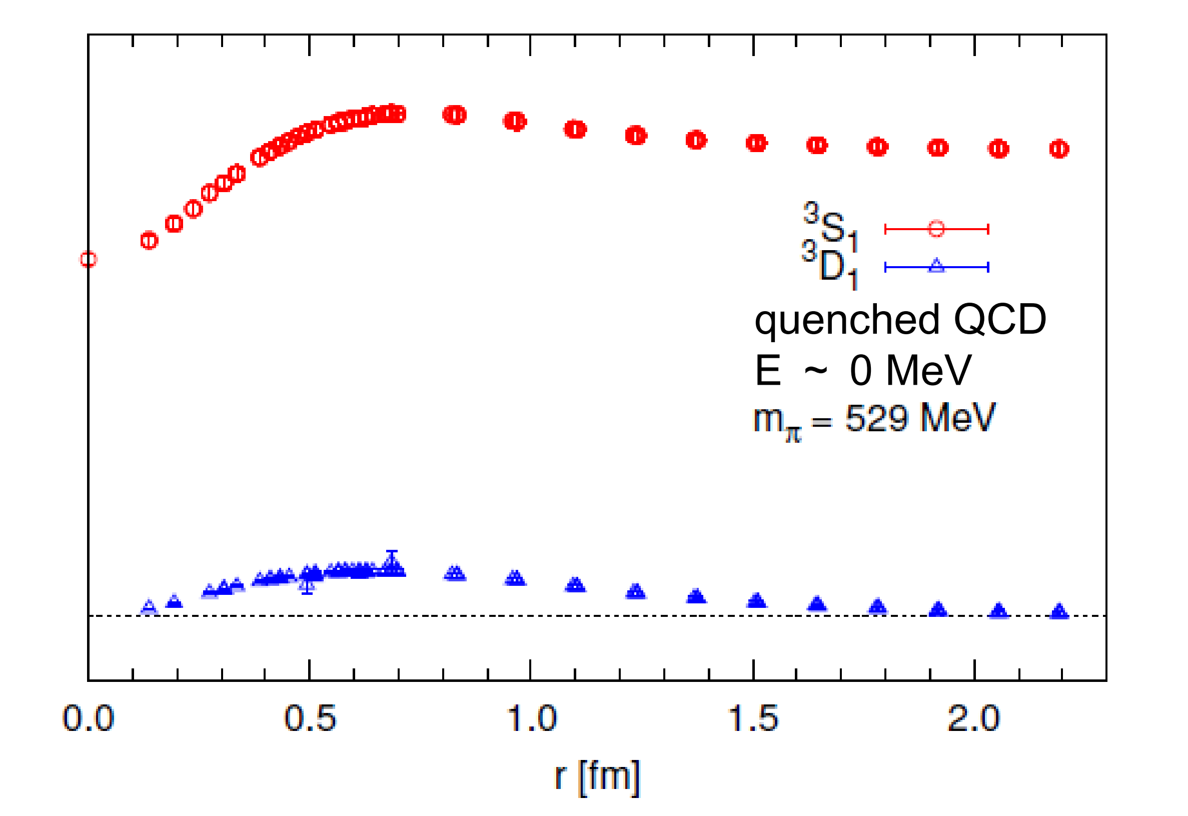
Schwinger, Phys. Rev. 55 (1939), Bethe, ibid.57 (1940)
Rarita & Schwinger, ibid. 59 (1941)

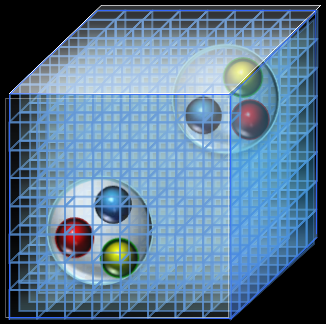
$^3P_2$-$^3F_2$ LS force ⟺ neutron superfluidity in neutron stars

Tamagaki, Prog. Theor. Phys. 44 (1970)
Hoffberg et al., Phys. Rev. Lett. 24 (1970)

Density profile of the deuteron with $S_z=\pm1$
Pandharipande et al., (1998)

[Exercise 1] LO potentials : $V_C(r)$ & $V_T(r)$





Aoki, Ishii & Hatsuda,
Prog. Theor. Phys. 123 (2010) 89

- Rapid quark-mass dependence of $V_T(r)$
- Evidence of the one-pion-exchange
- Consistent with operator product expansion at r～0

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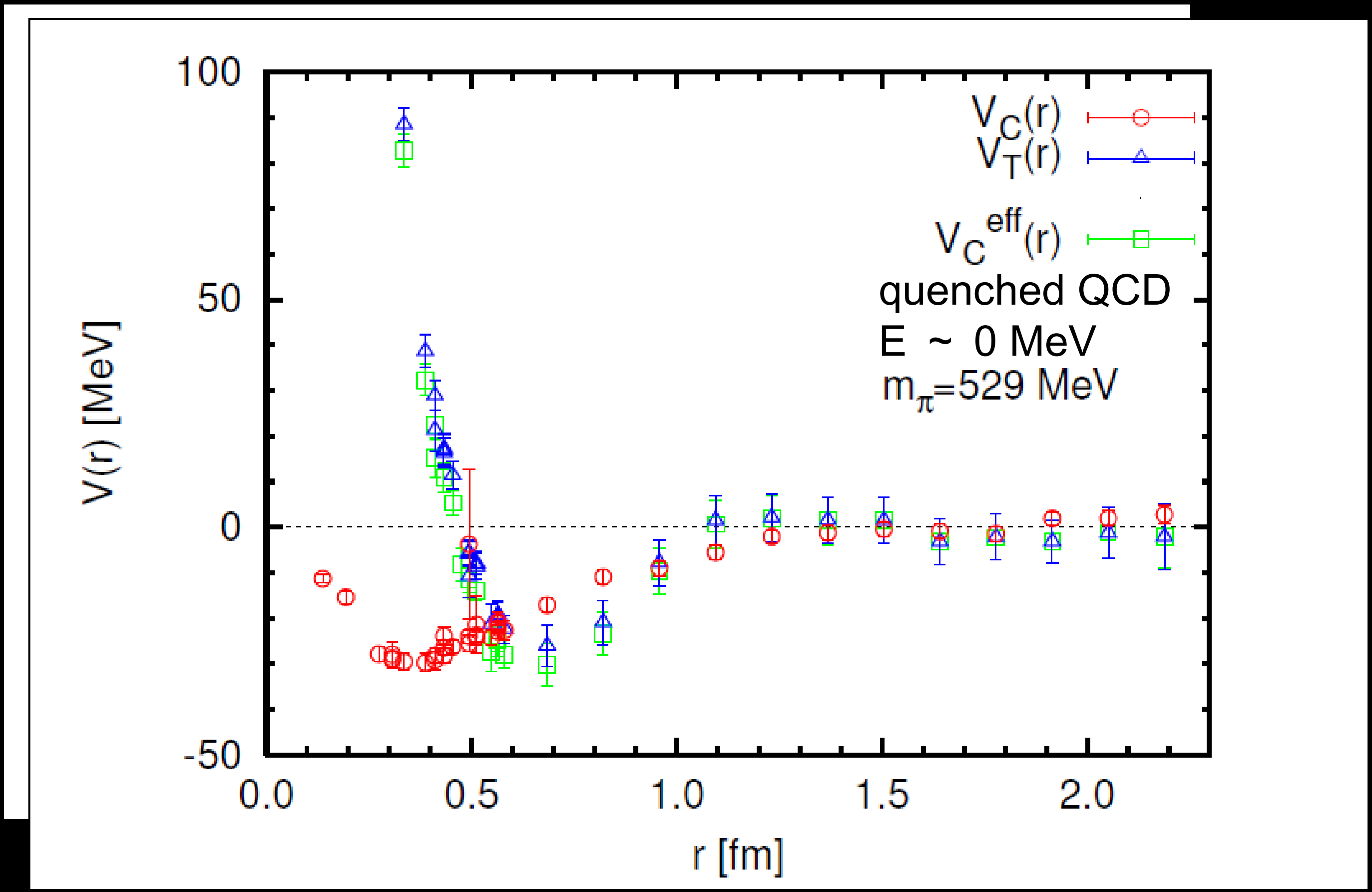

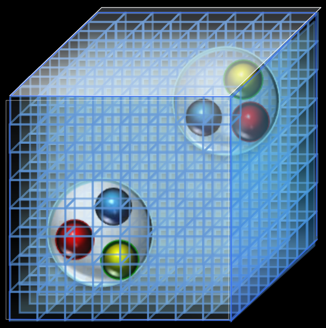

Aoki, Ishii & Hatsuda,
Prog. Theor. Phys. 123 (2010) 89

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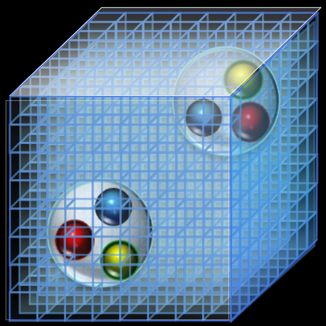

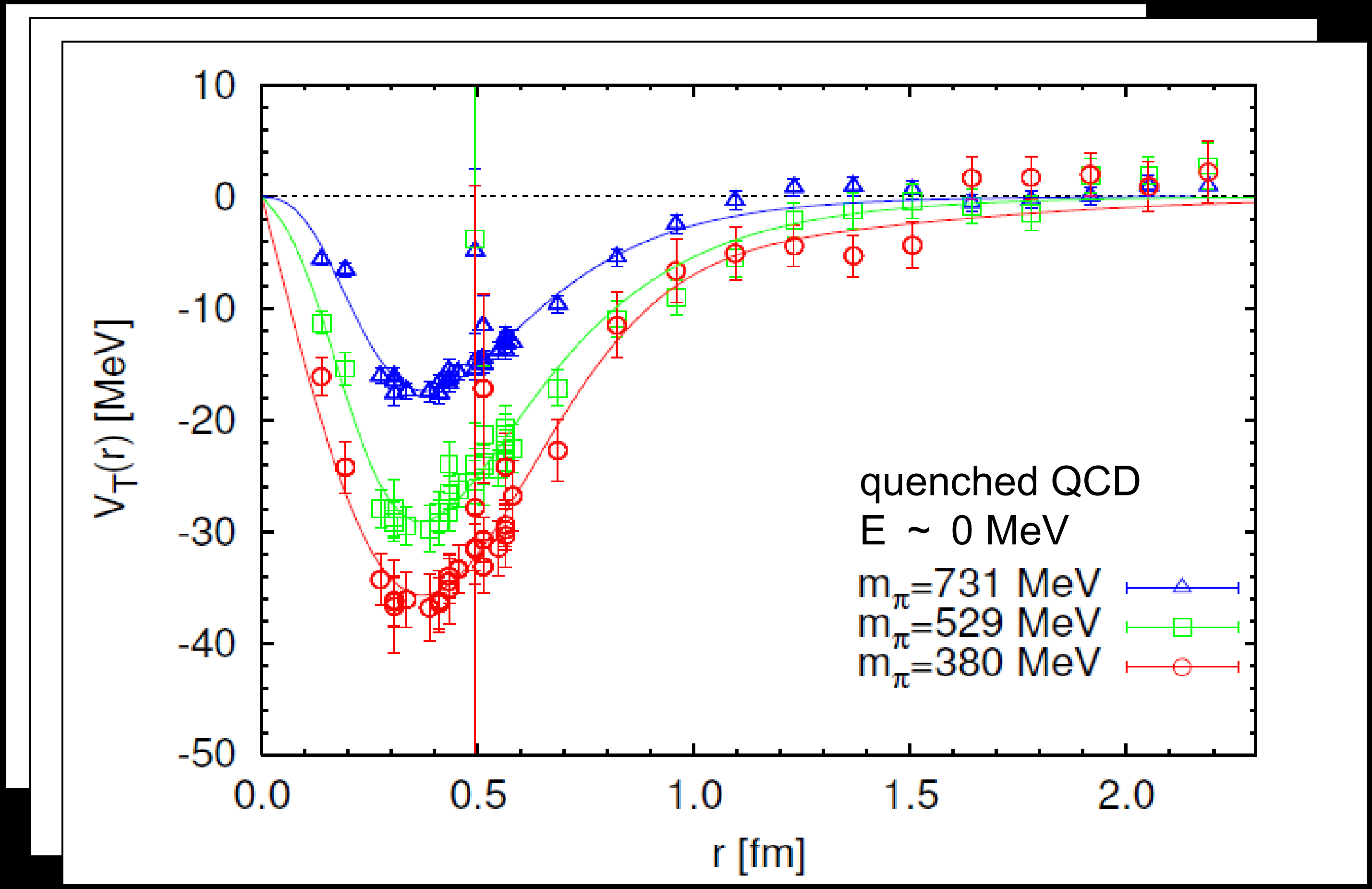

Aoki, Ishii & Hatsuda,
Prog. Theor. Phys. 123 (2010) 89

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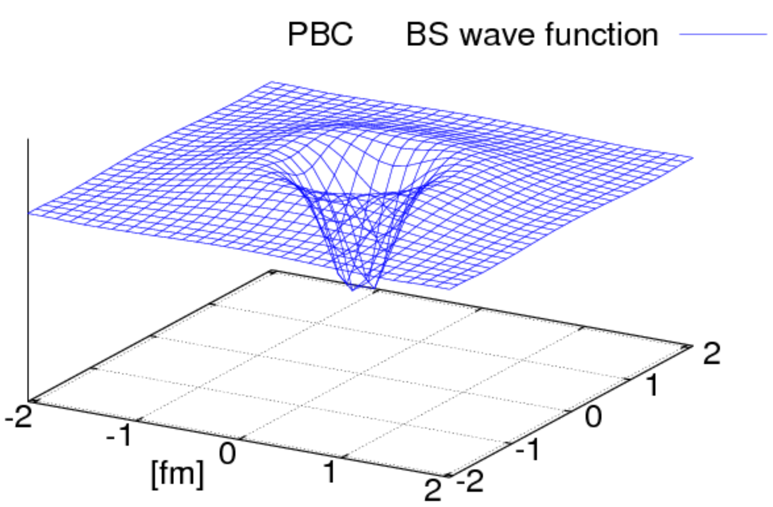
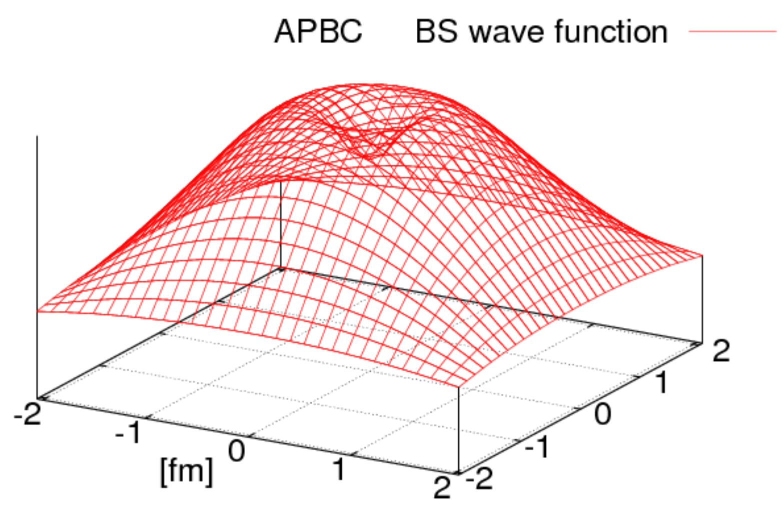
[Exercise 2] Magnitude of the non-locality

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\ V_{\rm LS}(r) + \{V_{\rm D}(r),\nabla^2\} + \cdots$$

- PBC (T_{Lab}~0 MeV)
- APBC (T_{Lab}~100 MeV)





Murano et al. (HAL QCD Coll.), Prog. Theor. Phys. 125 (2011) 1225

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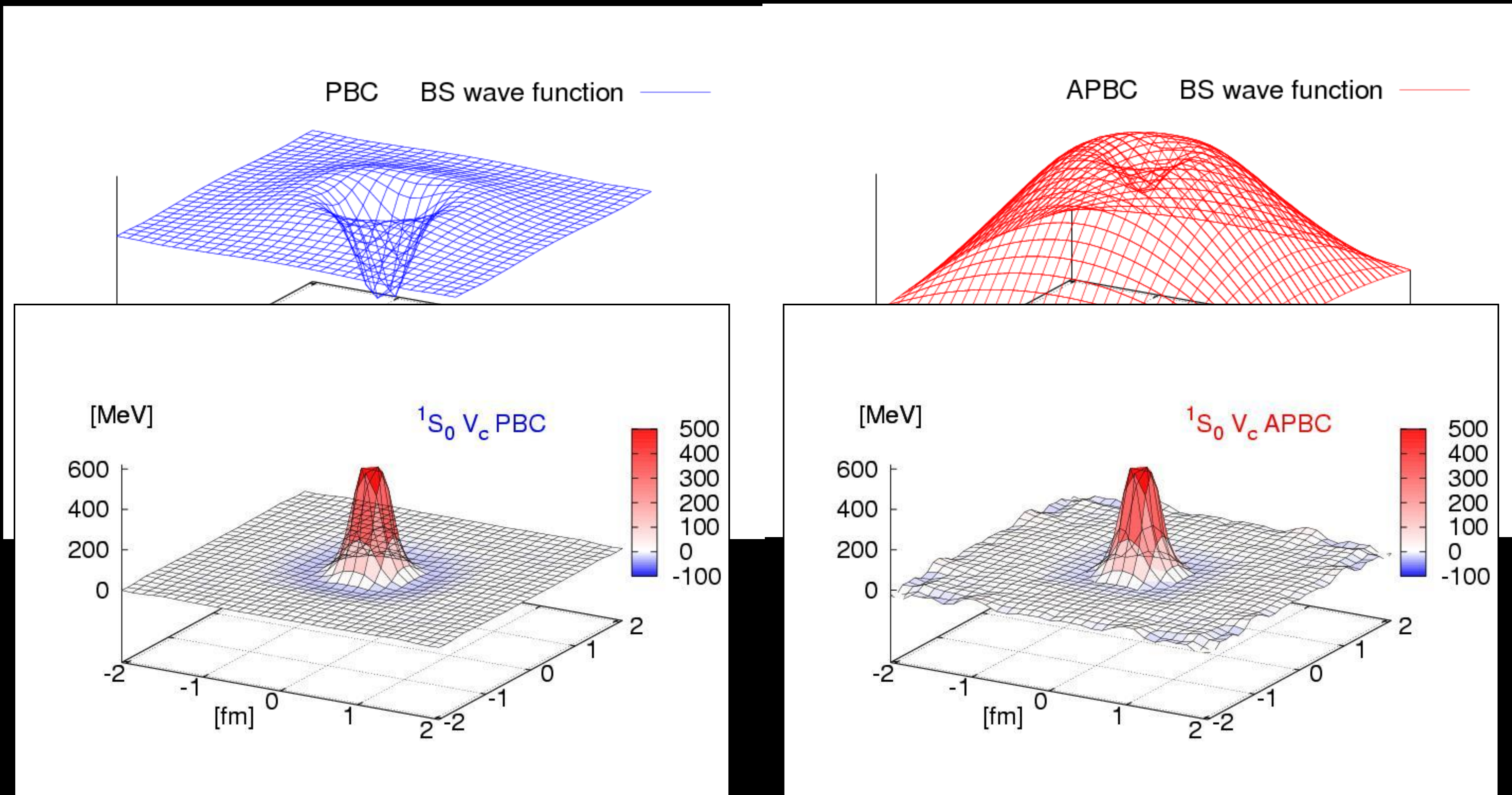
- PBC (T_{Lab} ~ 0 MeV)
- APBC (T_{Lab} ~ 100 MeV)



Murano et al. (HAL QCD Coll.), Prog. Theor. Phys. 125 (2011) 1225

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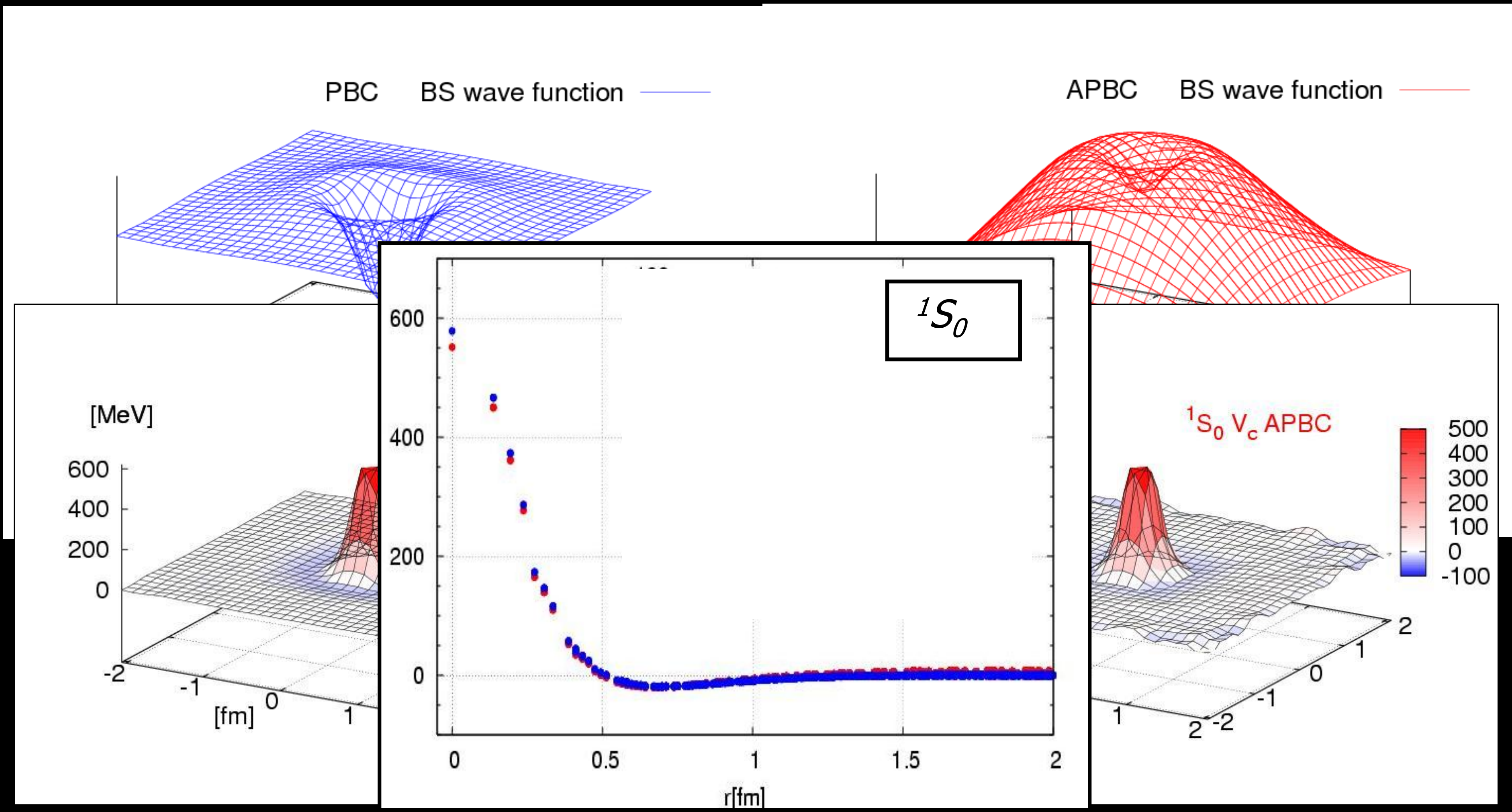
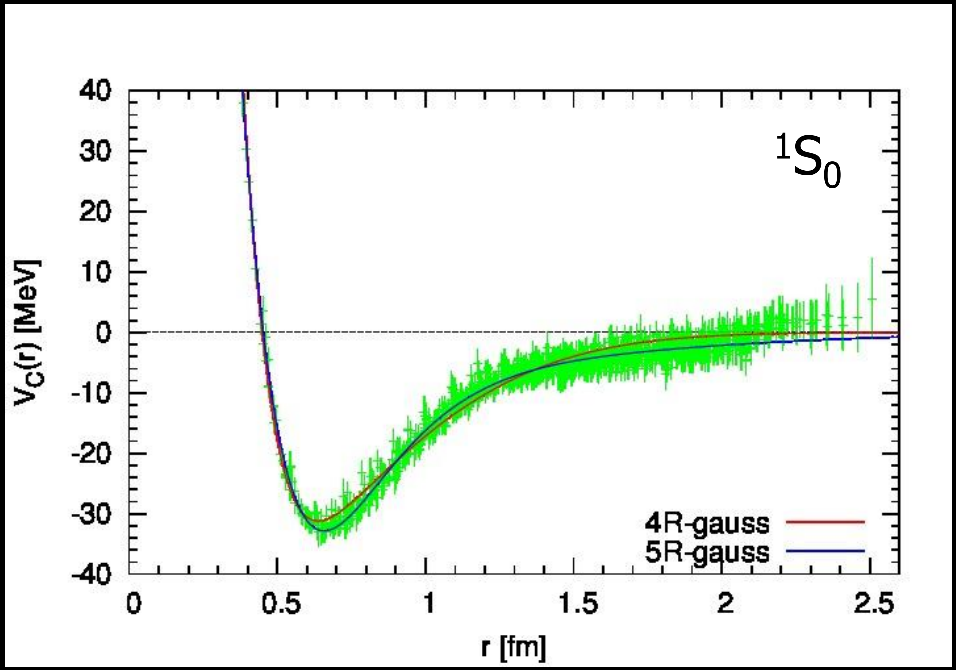
- PBC ($T_{Lab} \sim 0$ MeV)
- APBC ($T_{Lab} \sim 100$ MeV)



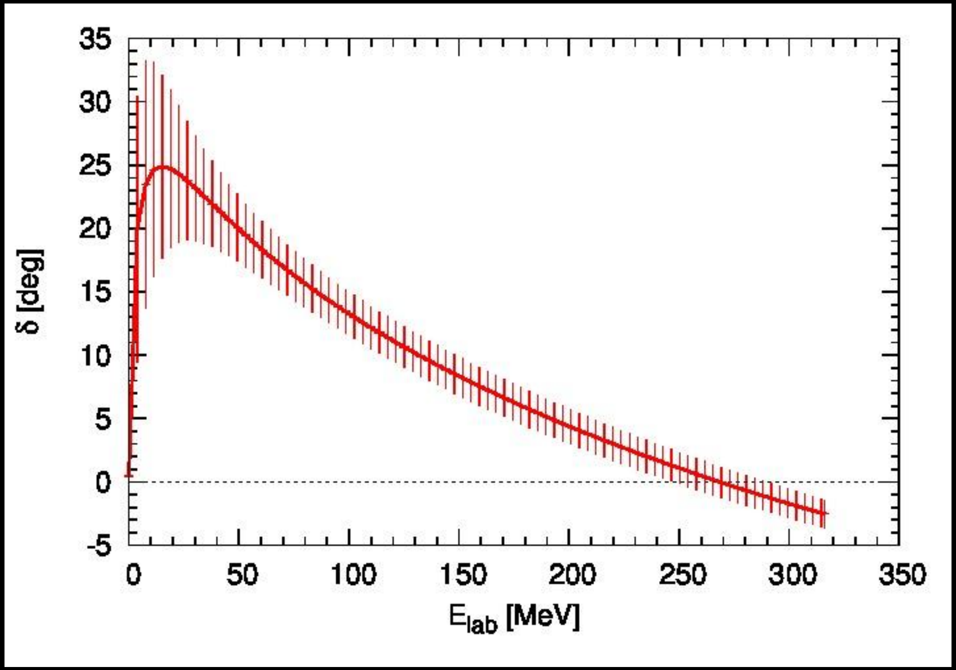
Murano et al. (HAL QCD Coll.), Prog. Theor. Phys. 125 (2011) 1225

Central potential in (2+1)-flavor QCD

HAL QCD Coll., in preparation

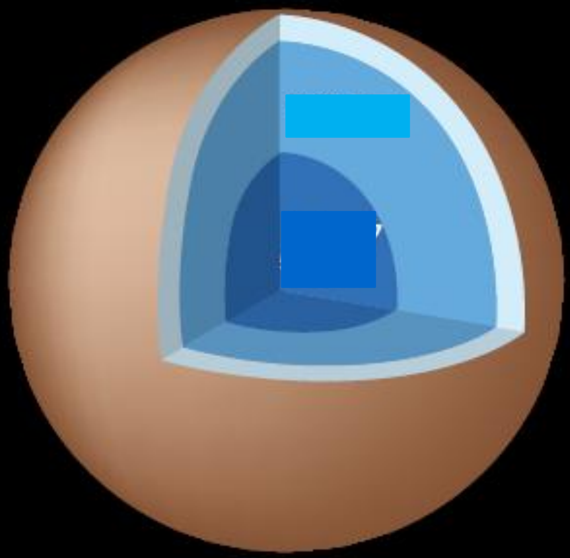


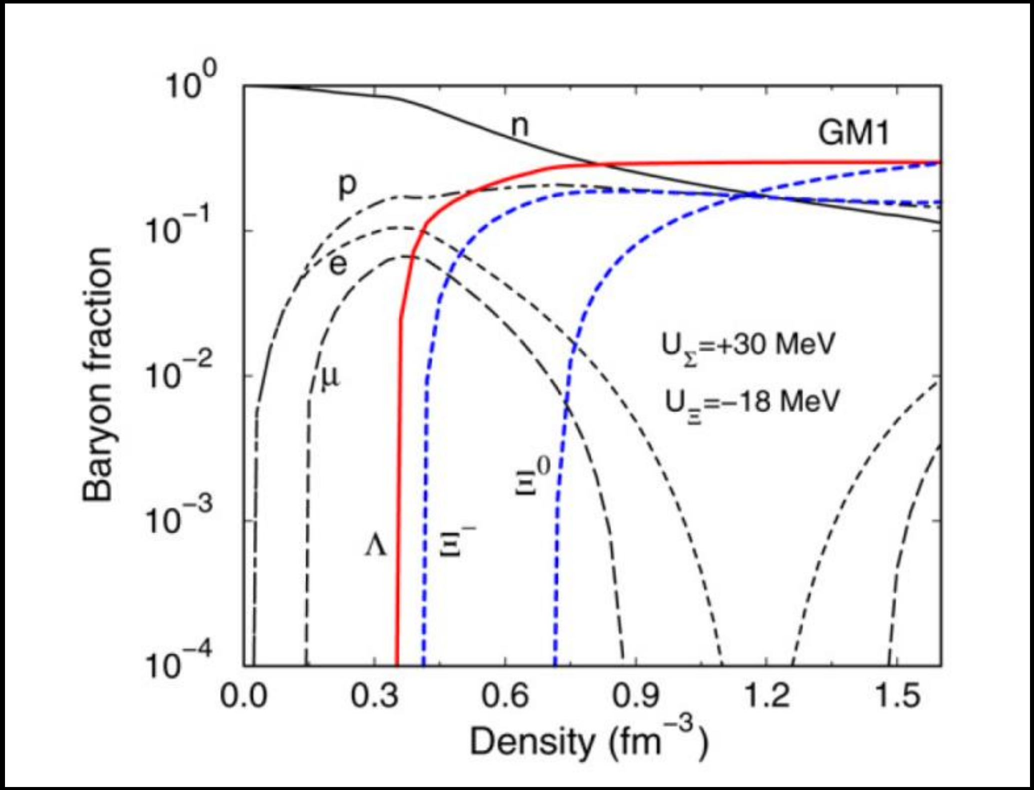
PACS-CS gauge config.
(Clover + Iwasaki)
a= 0.09 fm, L=2.9 fm
m_π= 700 MeV



Physical point simulations (m_π=135MeV with L=6fm & 9fm) will be carried out at KEI computer

Inner core of neutron stars
-- role of 2-body and 3-body forces --

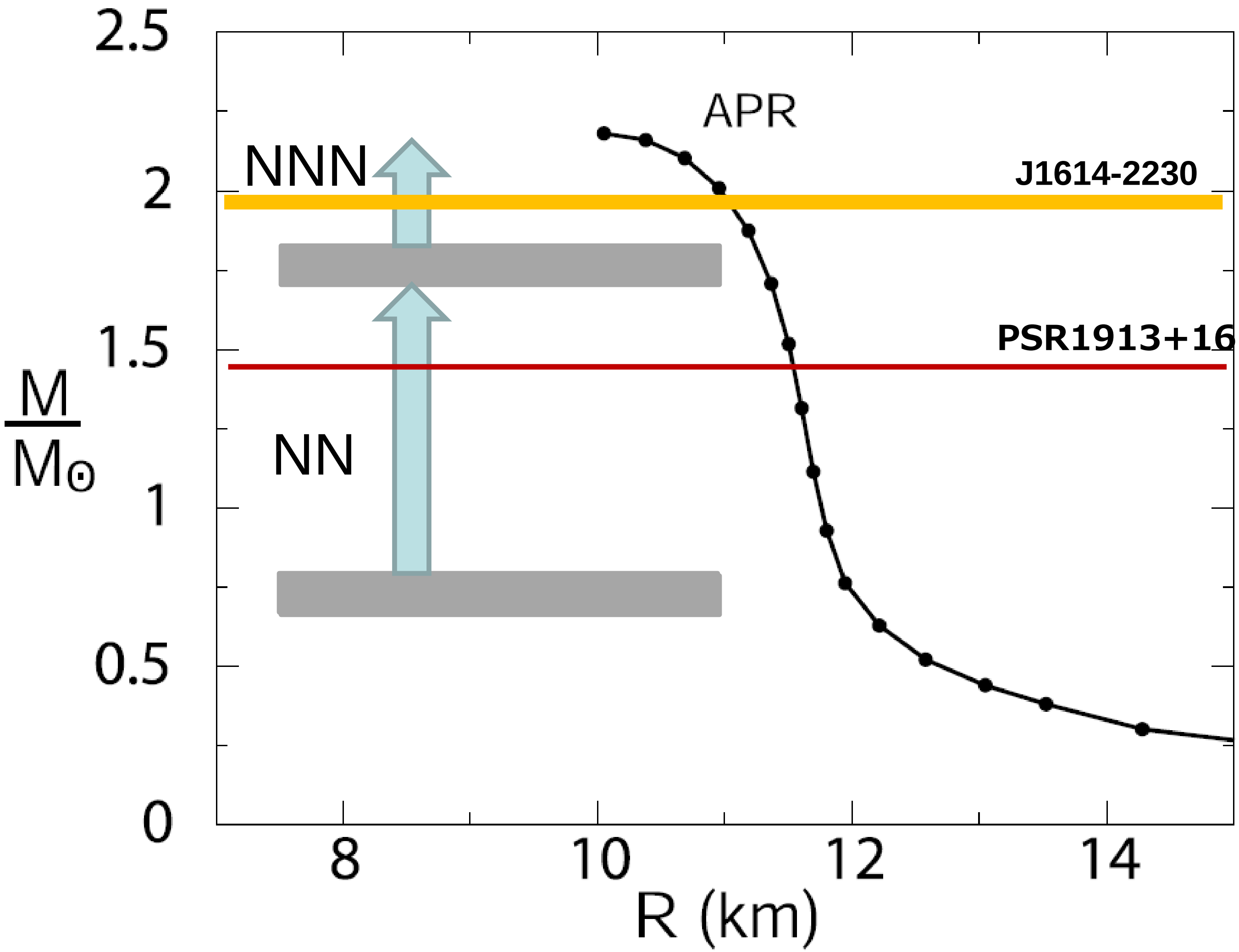


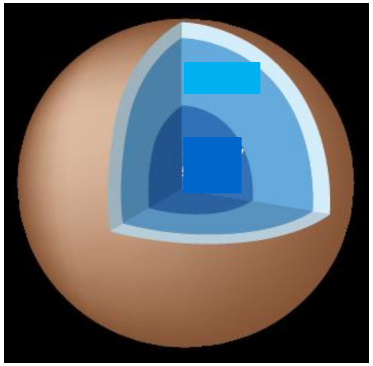


Schaffner-Bielich, Nucl. Phys.A 835, 279 (2010)

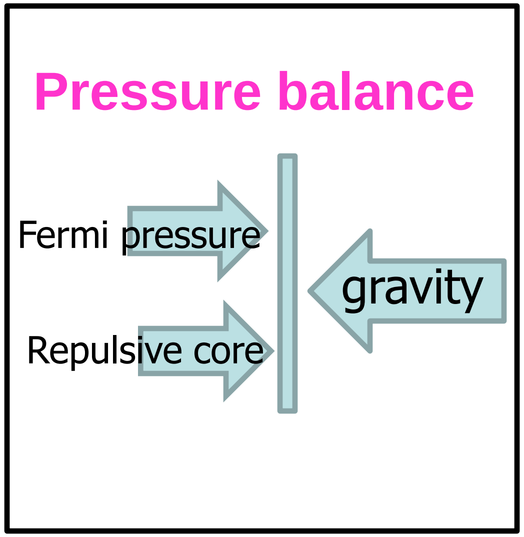
YN interaction	⇔	onset of hyperon mixture
NNN (BBB) interaction	⇔	max mass (e.g. 1.97(4) $M_\odot$)

Nuclear Force and Neutron Star



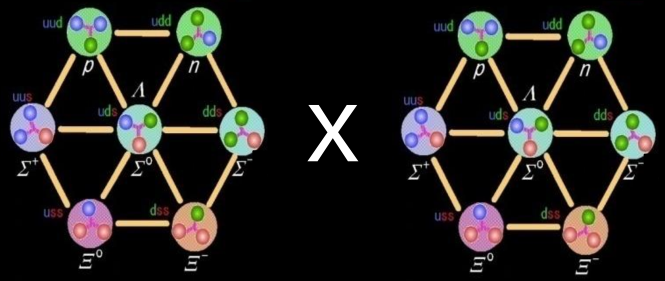


(ρ_{max} ~ $6\rho_0$)



BB interactions in 3-flavor LQCD

1. Numerical experiments of YN & YY interactions
   (not easily accessible in laboratory experiments)
2. Physical origin of the "short range NN repulsion"
3. Fate of H-dibaryon



$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{Symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{Anti-symmetric}}$$

<u>Six</u> independent potentials in the flavor-basis

irreducible BB source operator

$$\overline{BB^{(27)}} = +\sqrt{\frac{27}{40}}\,\overline{\Lambda\Lambda} - \sqrt{\frac{1}{40}}\,\overline{\Sigma\Sigma} + \sqrt{\frac{12}{40}}\,\overline{N\Xi} \quad \text{or} \quad +\sqrt{\frac{1}{2}}\,\overline{p\,n} + \sqrt{\frac{1}{2}}\,\overline{n\,p}$$

$$\overline{BB^{(8s)}} = -\sqrt{\frac{1}{5}}\,\overline{\Lambda\Lambda} - \sqrt{\frac{3}{5}}\,\overline{\Sigma\Sigma} + \sqrt{\frac{1}{5}}\,\overline{N\Xi}$$

$$\overline{BB^{(1)}} = -\sqrt{\frac{1}{8}}\,\overline{\Lambda\Lambda} + \sqrt{\frac{3}{8}}\,\overline{\Sigma\Sigma} + \sqrt{\frac{4}{8}}\,\overline{N\Xi}$$

with

$$\overline{\Sigma\Sigma} = +\sqrt{\frac{1}{3}}\,\overline{\Sigma^+\Sigma^-} - \sqrt{\frac{1}{3}}\,\overline{\Sigma^0\Sigma^0} + \sqrt{\frac{1}{3}}\,\overline{\Sigma^-\Sigma^+}$$

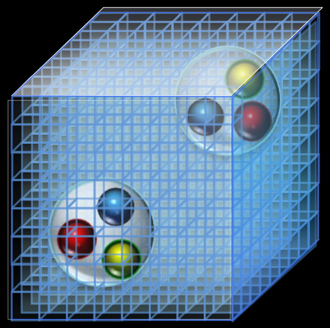
$$\overline{N\Xi} = +\sqrt{\frac{1}{4}}\,\overline{p\,\Xi^-} + \sqrt{\frac{1}{4}}\,\overline{\Xi^-\,p} - \sqrt{\frac{1}{4}}\,\overline{n\,\Xi^0} - \sqrt{\frac{1}{4}}\,\overline{\Xi^0\,n}$$

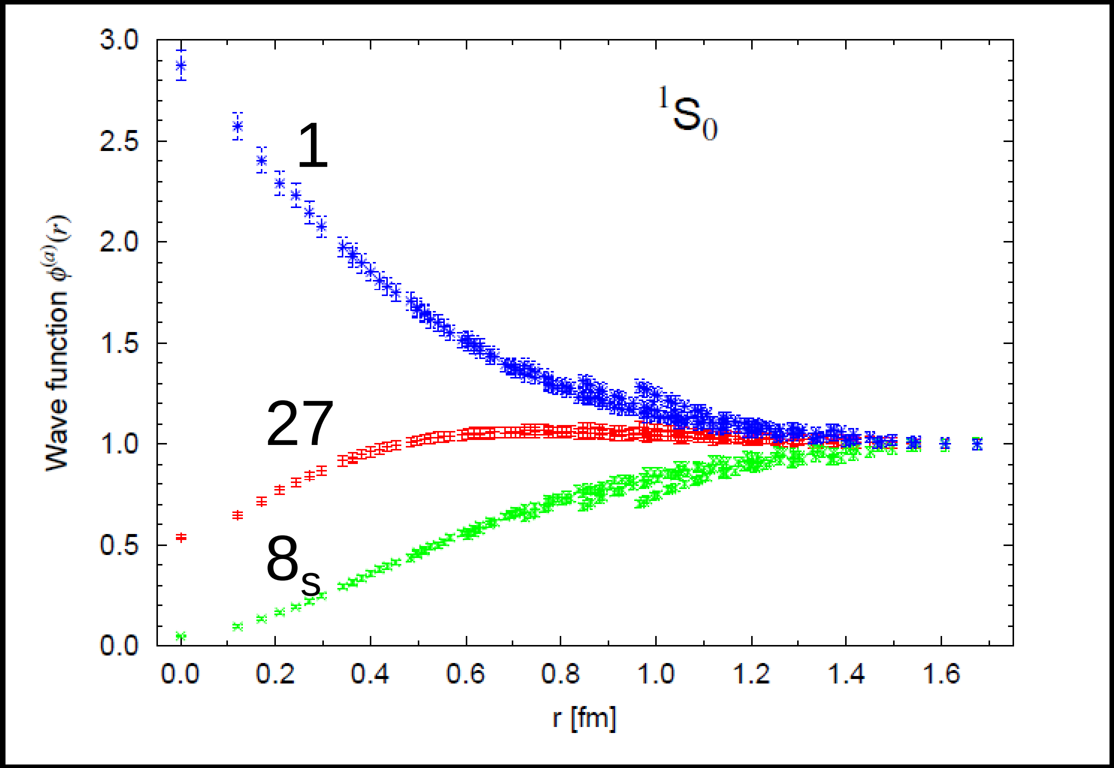
$$\overline{BB^{(10^*)}} = +\sqrt{\frac{1}{2}}\,\overline{p\,n} - \sqrt{\frac{1}{2}}\,\overline{n\,p}$$

$$\overline{BB^{(10)}} = +\sqrt{\frac{1}{2}}\,\overline{p\,\Sigma^+} - \sqrt{\frac{1}{2}}\,\overline{\Sigma^+\,p}$$

$$\overline{BB^{(8a)}} = +\sqrt{\frac{1}{4}}\,\overline{p\,\Xi^-} - \sqrt{\frac{1}{4}}\,\overline{\Xi^-\,p} - \sqrt{\frac{1}{4}}\,\overline{n\,\Xi^0} + \sqrt{\frac{1}{4}}\,\overline{\Xi^0\,n}$$

BB wave functions in flavor-basis





Iwasaki + clover
(CP-PACS/JLQCD config.)
L=1.9 fm, a=0.12 fm, 16^3x32
m_π=835 MeV, m_B=1752 MeV

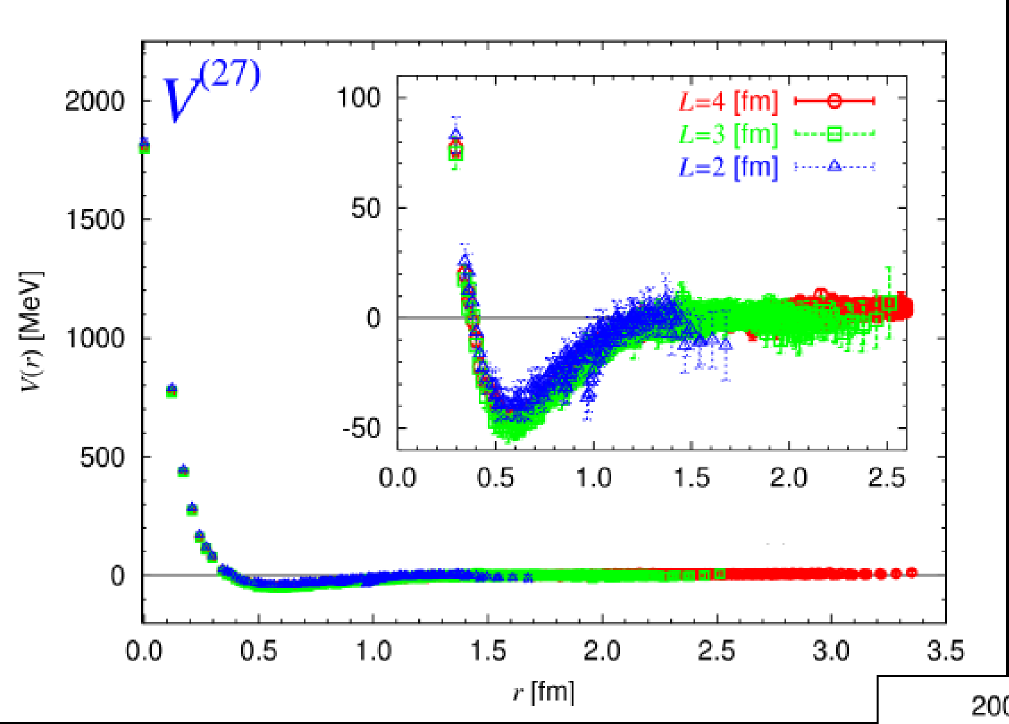
Inoue et al. (HAL QCD Coll.)
Prog. Theor. Phys. 124 (2010) 591

Short range BB int. ⇔ Quark Pauli principle

1 : allowed, 27 : partially blocked, 8_s : blocked

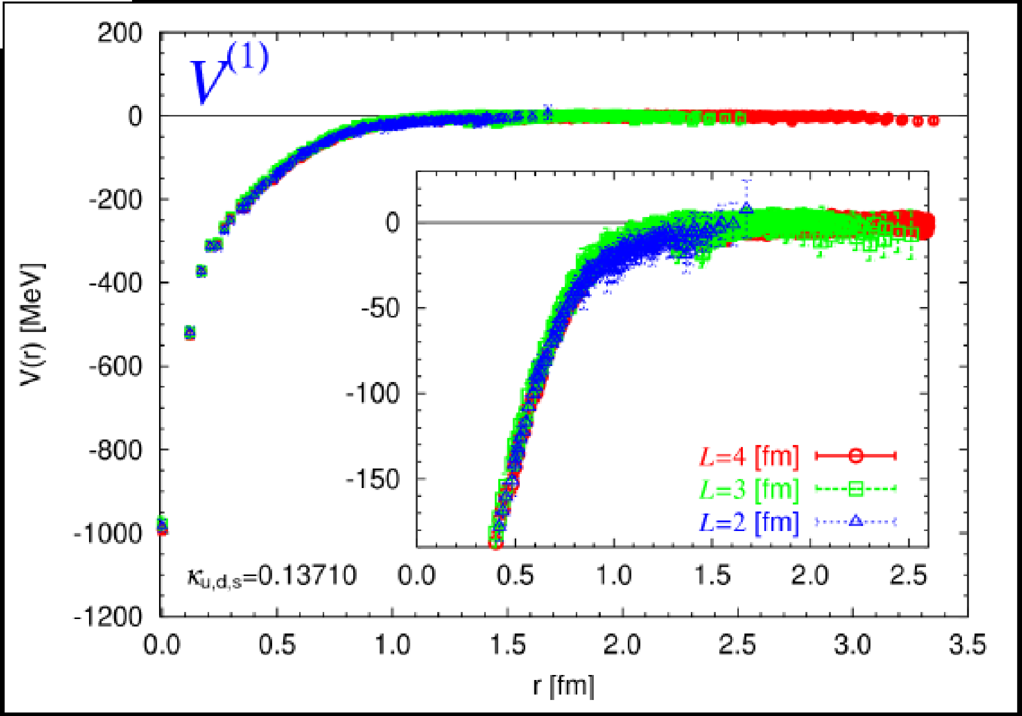
c.f. quark cluster model (Oka-Yazaki, Faessler-Shimizu Toki, ...)

Volume dependence of the BB potentials



Inoue et al. [HAL QCD Coll.]
Phys. Rev. Lett. 106 (2011) 162002

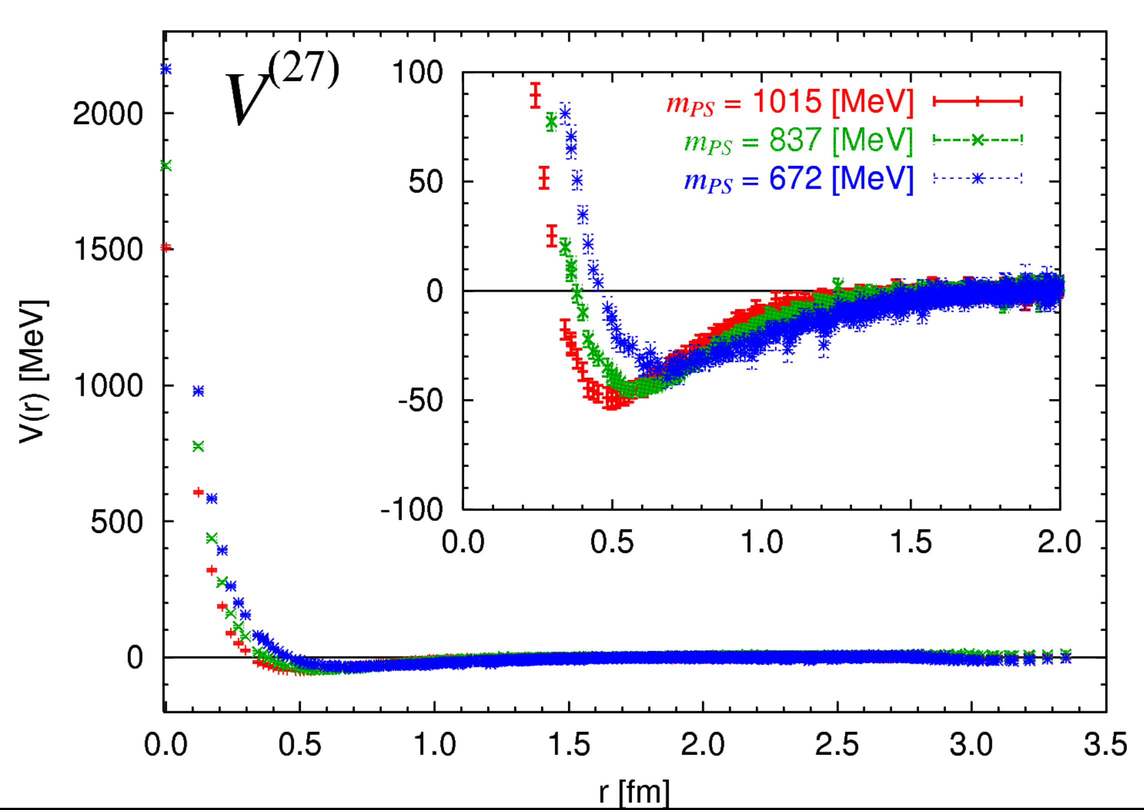
a = 0.12 fm, L= 2, 3, 4 fm
m_π= 837 MeV



Quark-mass dependence of the BB potentials

Inoue et al. [HAL QCD Coll.]
Phys. Rev. Lett. 106 (2011) 162002

a = 0.12 fm, L= 4 fm
m_π=469, 672, 837, 1015, 1170 MeV



Short range BB int. ⇔ Quark Pauli principle

1 : allowed, 27 : partially blocked, 8_s : blocked

Quark-mass dependence of the BB potentials

Inoue et al. [HAL QCD Coll.]
Phys. Rev. Lett. 106 (2011) 162002

a = 0.12 fm, L= 4 fm
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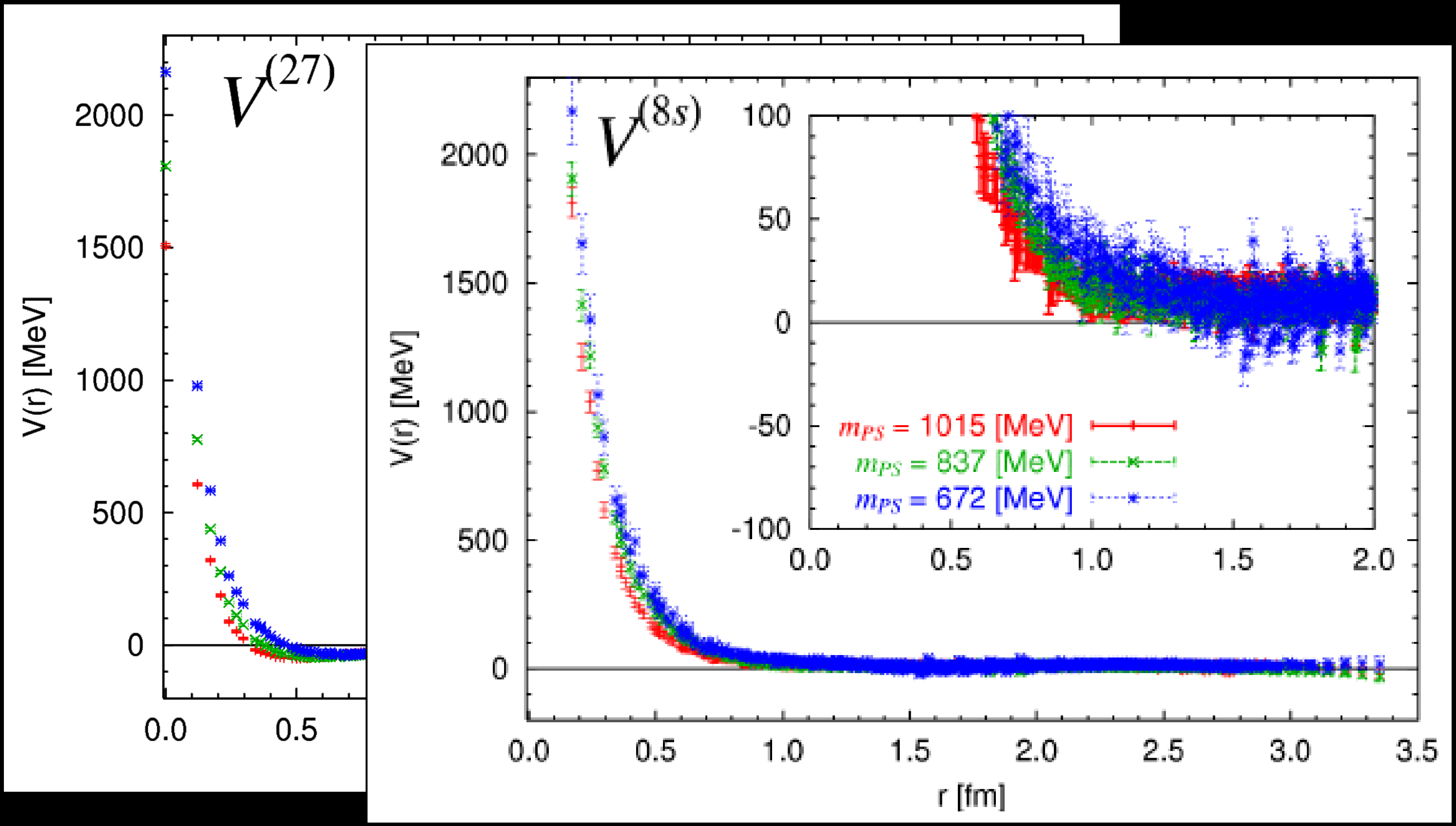

Short range BB int. ⇔ Quark Pauli principle

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Quark-mass dependence of the BB potentials

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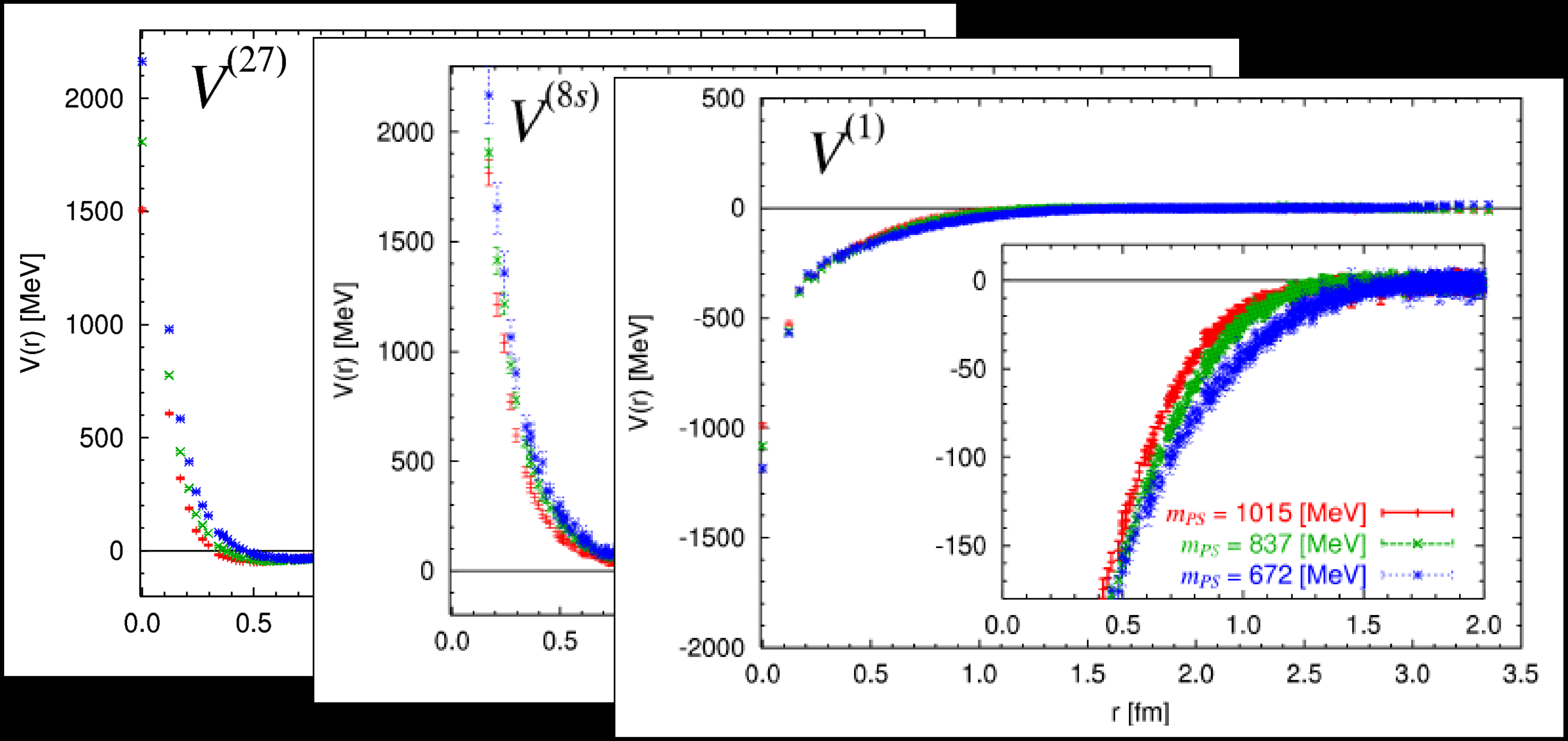
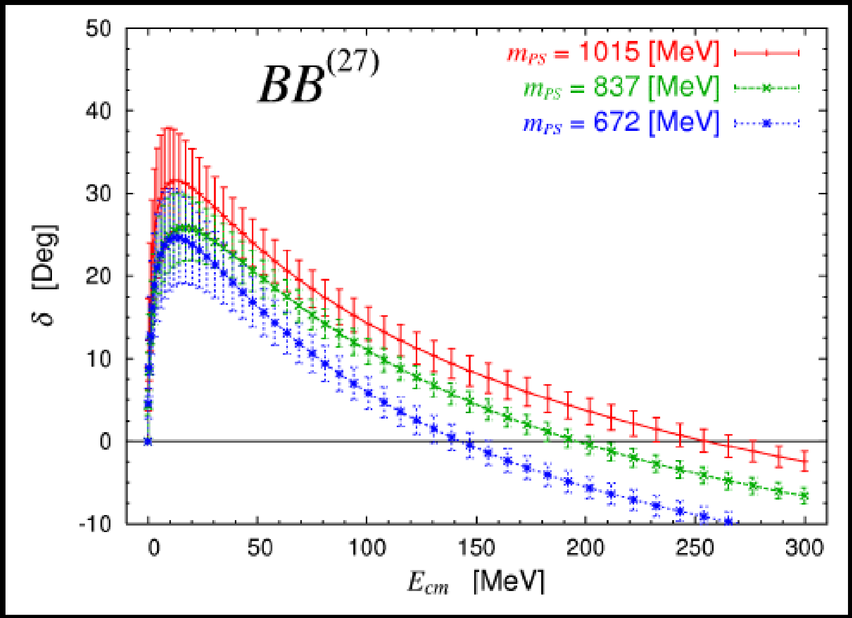
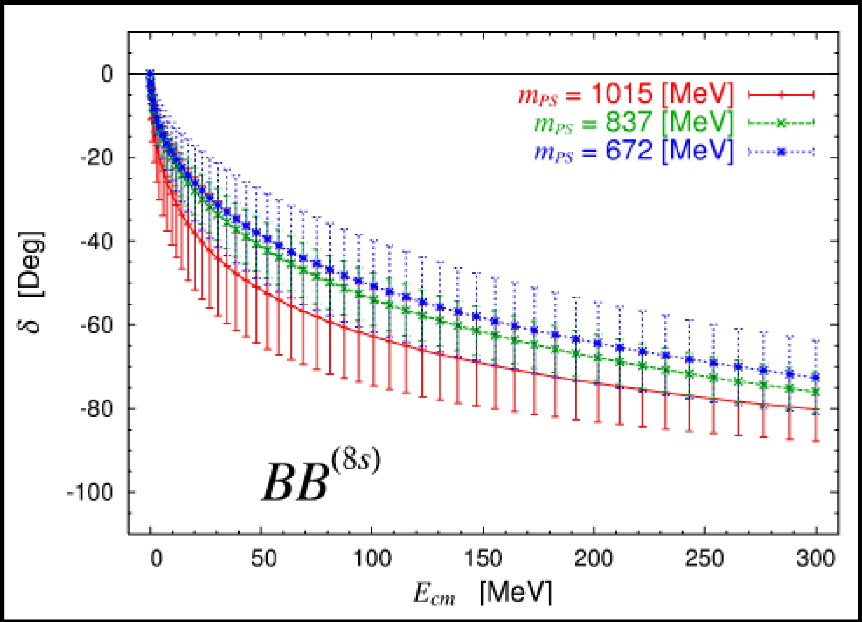


Short range BB int. ⇔ Quark Pauli principle

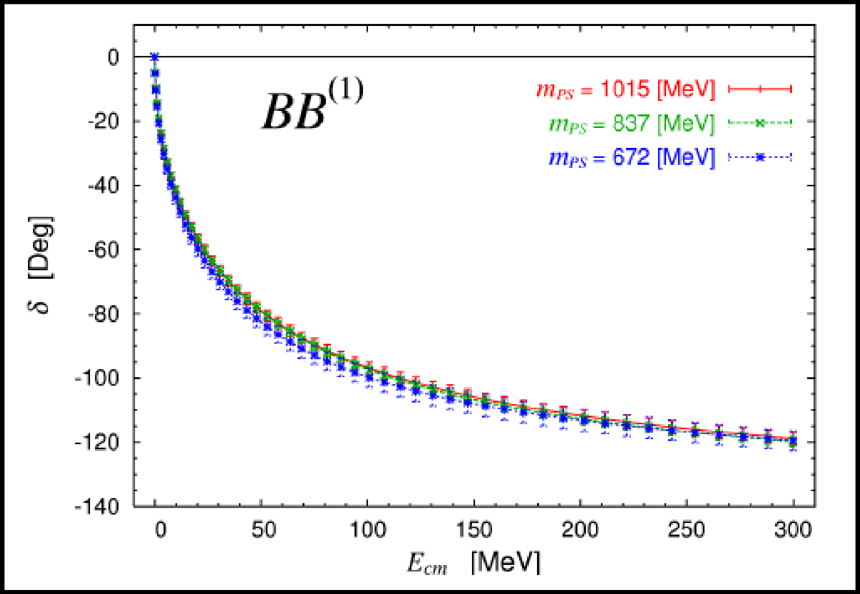
1 : allowed, 27 : partially blocked, 8_s : blocked

BB phase shifts in flavor-basis ($^1S_0$ channel)



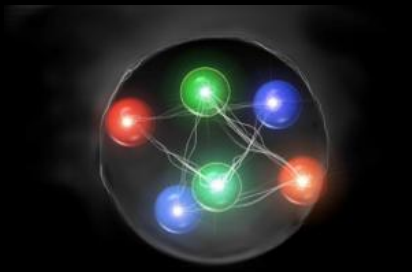


NN

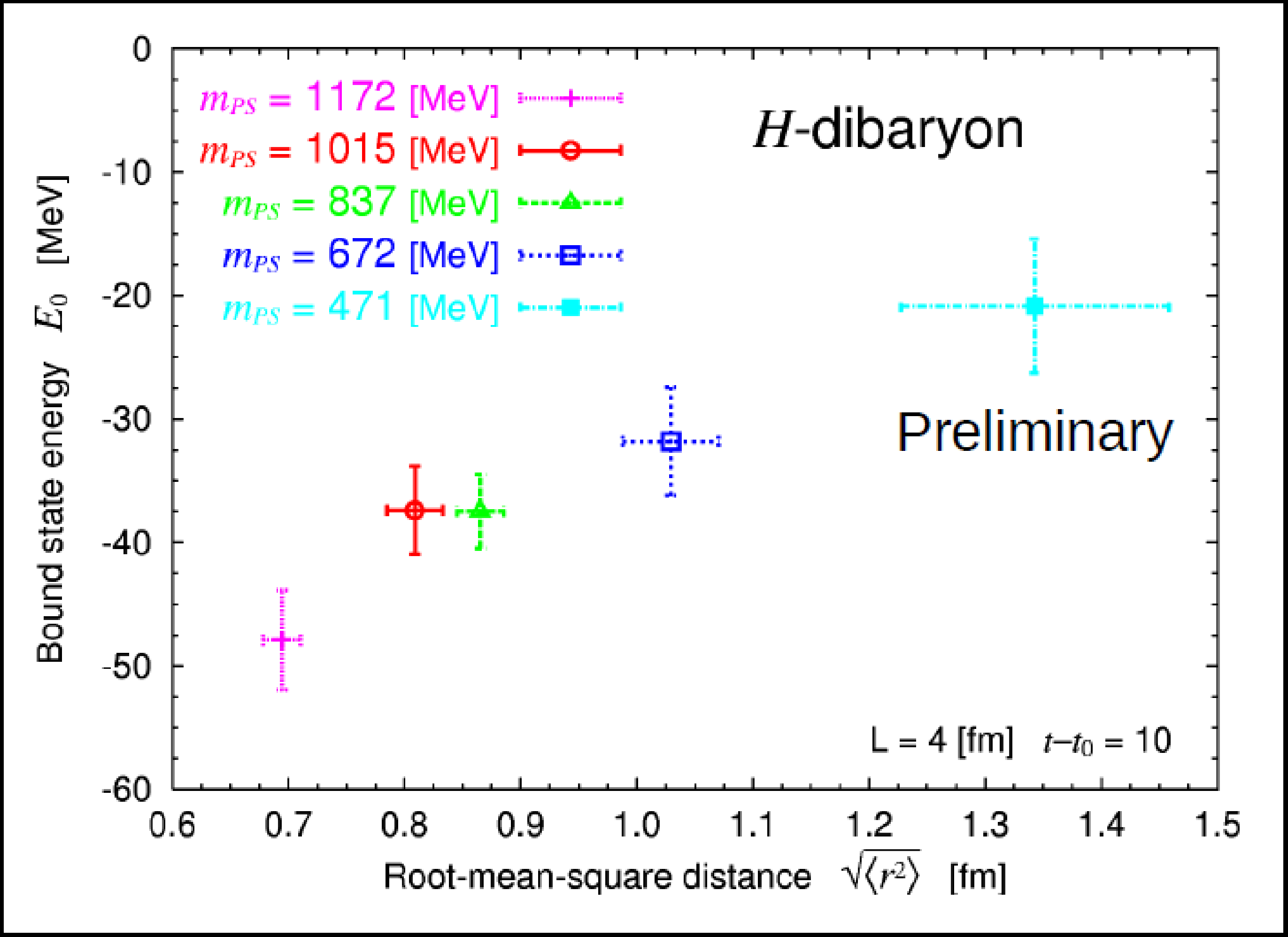


Inoue et al. [HAL QCD Coll.]
Phys. Rev. Lett. 106 (2011) 162002

H-dibaryon from LQCD
-- binding energy vs. size --

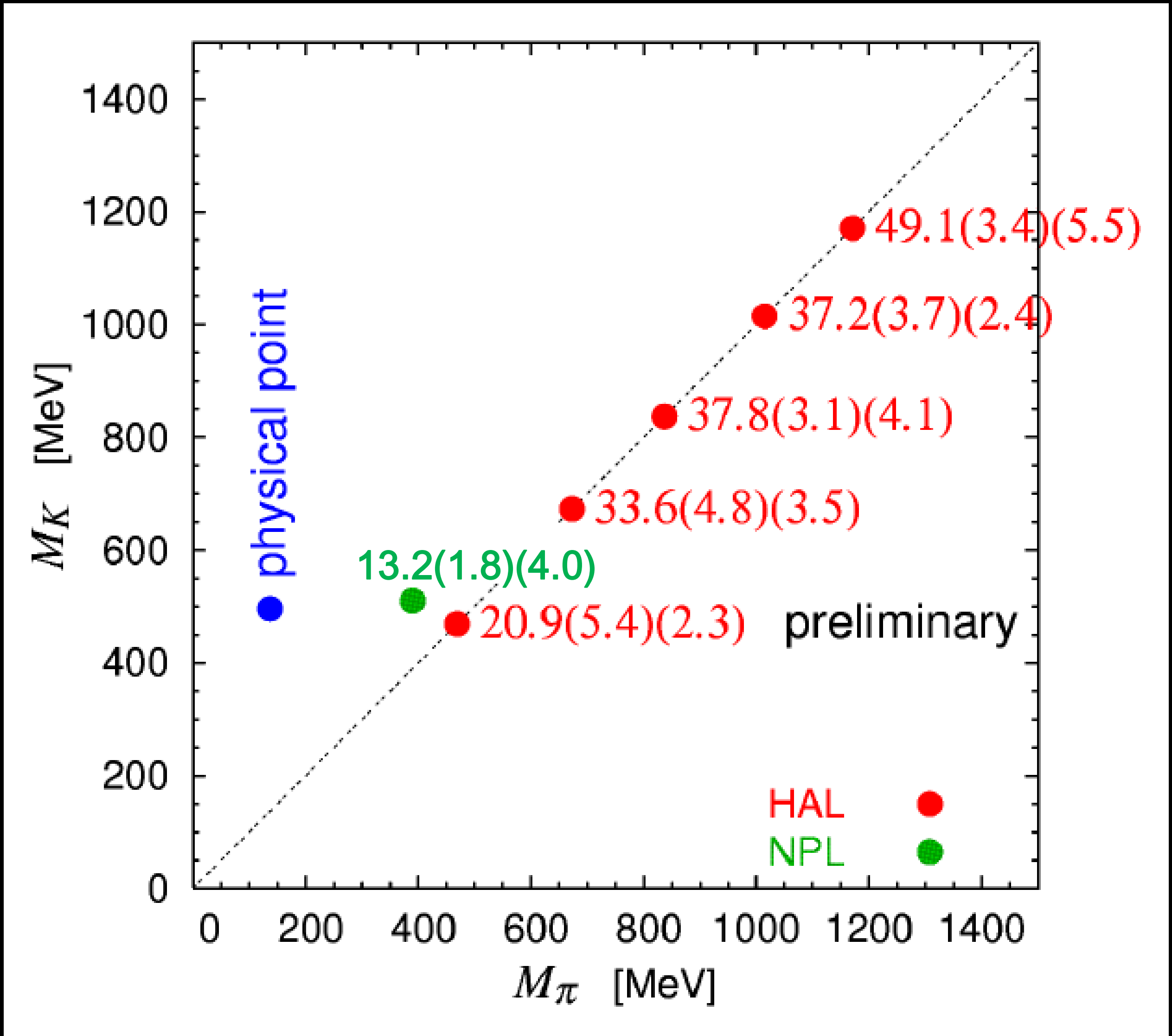


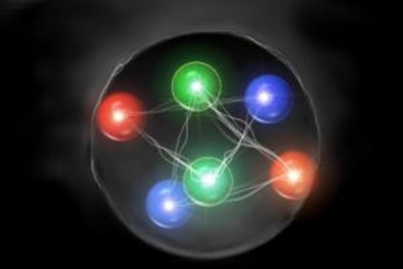
Jaffe, Phys. Rev. Lett. 38 (1977) 195



Inoue et al. [HAL QCD Coll.] Phys. Rev. Lett. 106 (2011) 162002

H binding energy [MeV]



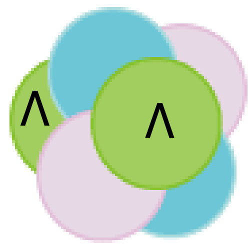


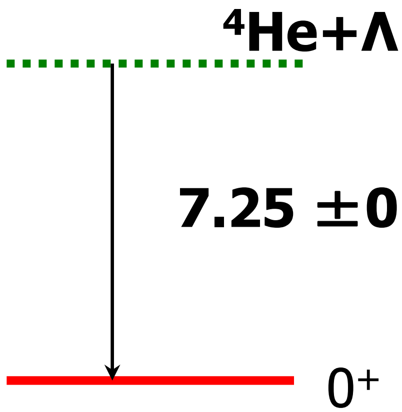
Jaffe, Phys. Rev. Lett. 38 (1977) 195

Inoue et al. [HAL QCD Coll.] Phys. Rev. Lett. 106 (2011) 162002; arXiv:1109.1620 [hep-lat]
Beane et al. [NPLQCD Coll.] Phys.Rev.Lett. 106 (2011) 162001; arXiv:1109.2889 [hep-lat]

double-Λ hypernuclei

$^{6}_{\Lambda\Lambda}He$

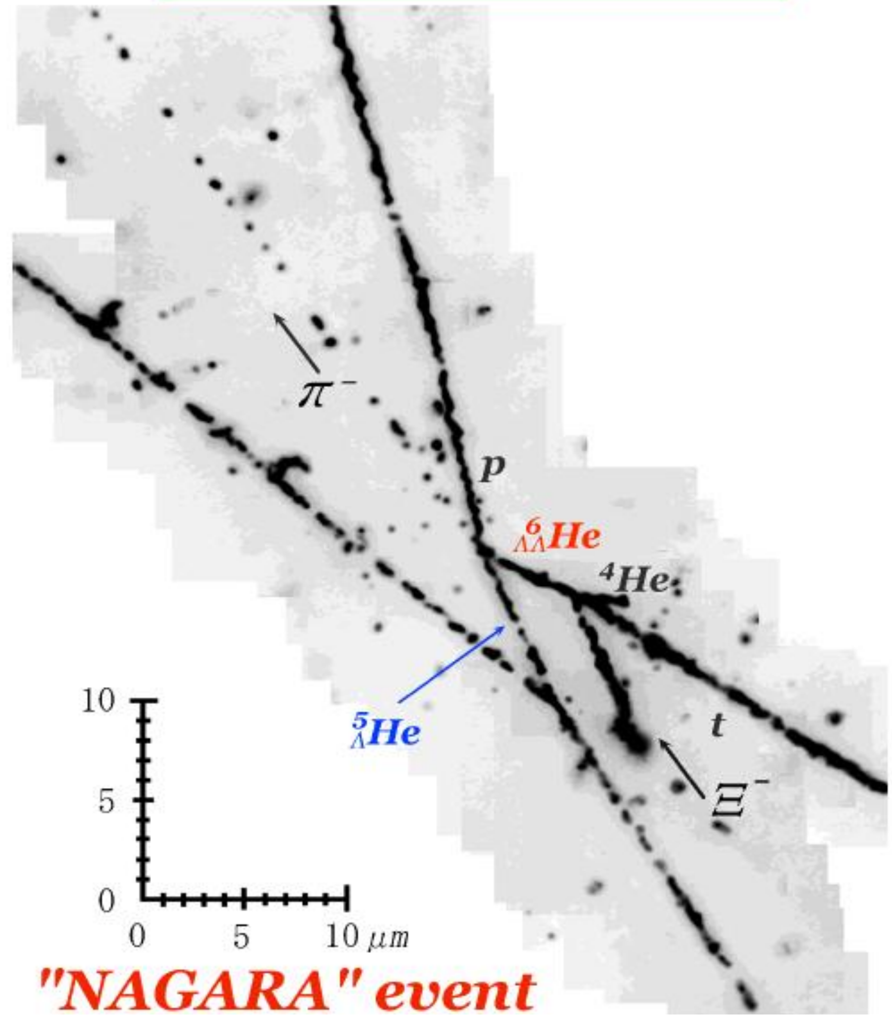




- ΛN attraction
- ΛΛ weak attraction
- B_H < 7.25 MeV

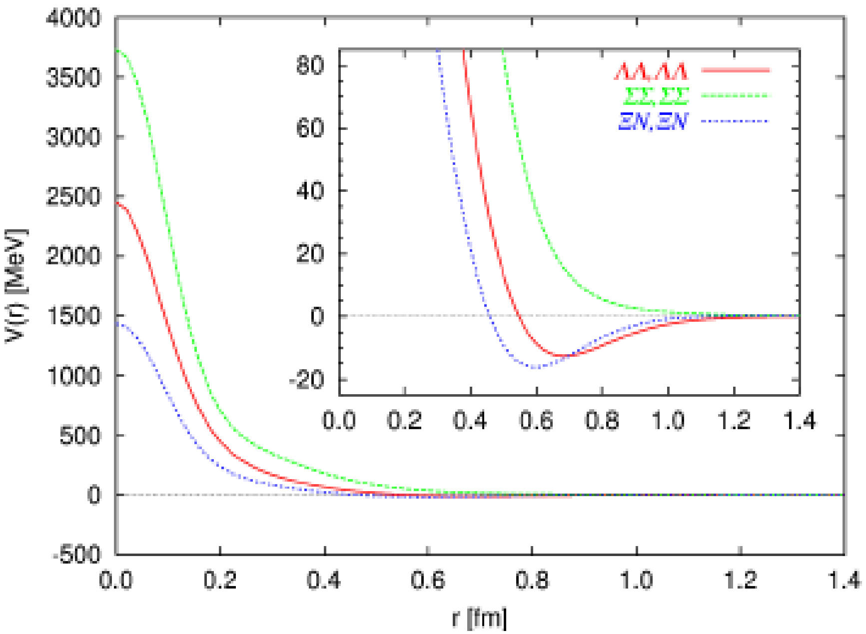
H. Takahashi *et al.*, PRL 87, 212502-1 (2001)

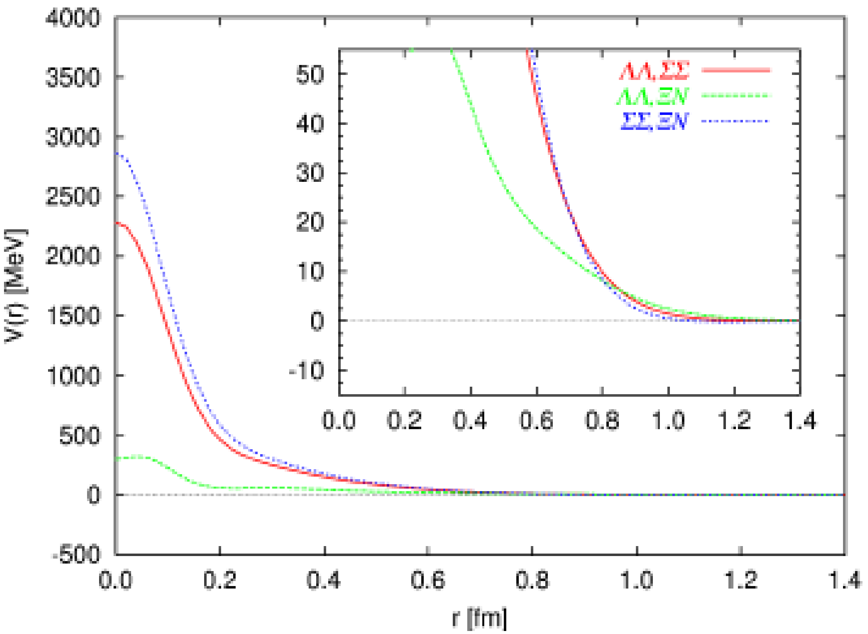
$$\Xi^- + {}^{12}C \rightarrow {}^{6}_{\Lambda\Lambda}He + {}^{4}He + t$$
$$ {}^{6}_{\Lambda\Lambda}He \rightarrow {}^{5}_{\Lambda}He + p + \pi^-$$



S=−2, I=0 BB $^1S_0$ potential

$$\begin{pmatrix} \Lambda\Lambda \\ \Sigma\Sigma \\ \Xi N \end{pmatrix} = U \begin{pmatrix} |27\rangle \\ |8\rangle \\ |1\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{(27)} & & \\ & V^{(8)} & \\ & & V^{(1)} \end{pmatrix} U^t \rightarrow \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{\Sigma\Sigma} & V^{\Lambda\Lambda}_{\Xi N} \\ & V^{\Sigma\Sigma} & V^{\Sigma\Sigma}_{\Xi N} \\ & & V^{\Xi N} \end{pmatrix}$$

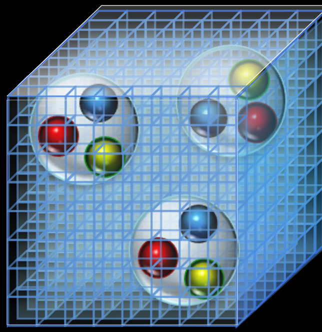


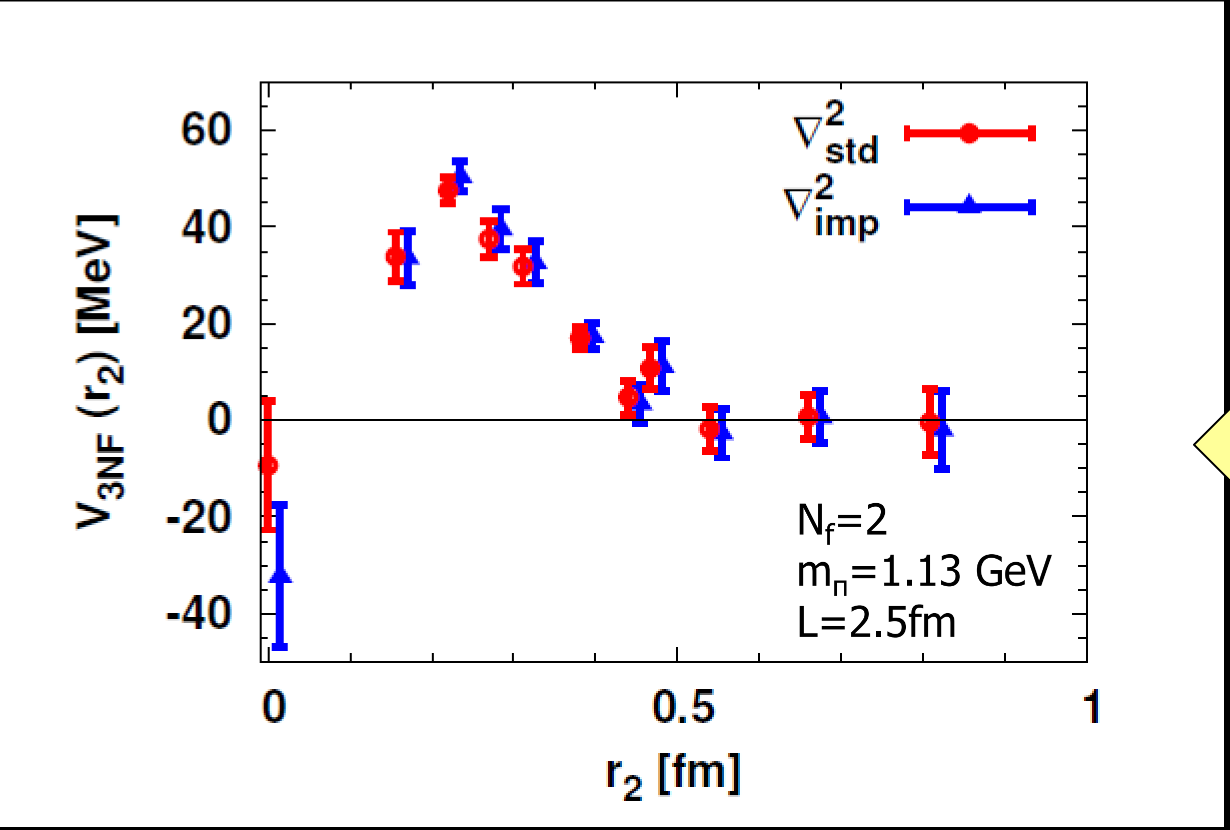


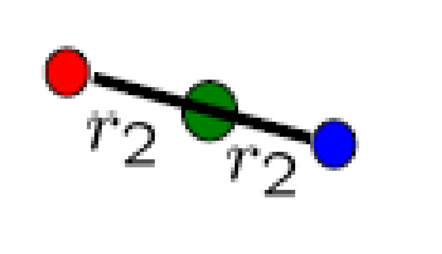
3N force (spin-isospin independent part) from LQCD

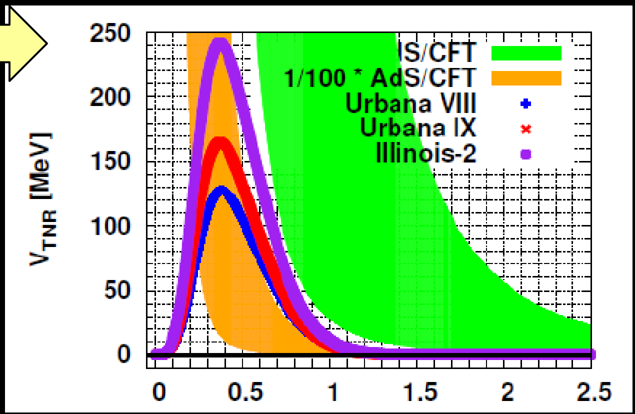
$$\psi_{3N}(\vec{r},\vec{\rho}) \quad \equiv \quad \langle 0|N(\vec{x}_1)N(\vec{x}_2)N(\vec{x}_3)|E_{3N}\rangle,$$

$$\left[-\frac{1}{2\mu_r}\nabla_r^2 - \frac{1}{2\mu_\rho}\nabla_\rho^2 + \sum_{i<j} V_{2N}(\vec{r}_{ij}) + V_{3NF}(\vec{r},\vec{\rho})\right]\psi_{3N}(\vec{r},\vec{\rho}) = E_{3N}\psi_{3N}(\vec{r},\vec{\rho}),$$





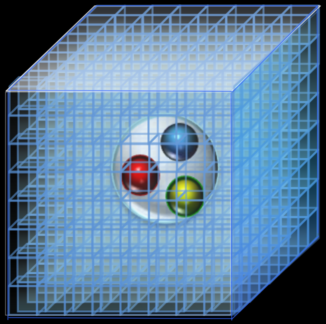




r_2 [fm]

T. Doi et al [HAL QCD Coll.], arXiv:1106.2276[hep-lat]

"Summary"

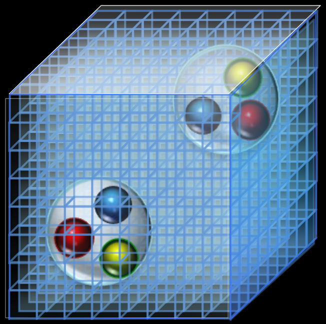


1. LQCD provides precision computations
   (2+1)-flavor, L=6fm, a=0.05fm, m_π=135MeV
   α_s , $m_{u,d,s,}$ low energy constants, …

2. LQCD provides inputs for N & P phenomenology
   - quark-gluon plasma (EOS, spectral function,…)
   - dark matter (e.g. ssbar in the nucleon)

3. LQCD provides qualitative pictures on
   - nucleon & hyperon forces
   - quark model (Kawanai 9/20)

Hyper nuclear physics
at J-PARC



"Future" (10 Pflops era from 2012)

In a few years, we would (like to) hear more on

1. Physical point simulations for many observables
2. Simulations with better fermions

   staggered, Wilson → domain wall, overlap

3. BB and BBB interactions

   → better understanding of nuclei and neutron stars from QCD

   Luecher, Nucl. Phys. B354 (1991) 531
   - Energy shift : NPL-QCD Coll. PACS-CS Coll.
   - Potential : HAL QCD Coll.

4. Sign problem in dense QCD