NEW VALUE OF PROTON CHARGE *rms* RADIUS

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Outline

Recent **progress in muon beams and laser technology** have enabled

- R.Pohl et al, Nature Vol. 466 (2010) 213

to carry out at the $\pi E5$ beam-line of the proton accelerator in **Paul Scherrer Institute** (PSI) (Switzerland) and **measurement of the muon hydrogen atom Lamb shift** and to determine the **proton charge** *rms* **radius**

$$r_p = 0.84184(67)\text{fm}, \tag{1}$$

disproving in this manner the value of Review of Particle Physics (2010)

$$r_p = 0.87680(690)\text{fm} \tag{2}$$

to be obtained mainly by a **precision spectroscopy of electron hydrogen atom** and calculations of bound states in QED.

This result **evoked stormy discussion**

- A.De Rujula, Phys. Lett. B693 (2010) 555
- A.De Rujula, Phys. Lett. B697 (2011) 26
- M.O.Distler, J.C.Bernauer, T.Walcher, Phys. Lett. B696 (2011) 343

if the **experimental value from muon hydrogen atom** spectroscopy is correct at all.

In our opinion, the **discussion is no more topical**, as the global analysis of the JLab proton polarization data on
$\mu G_{Ep}(Q^2)/G_{Mp}(Q^2)$
with all other existing nucleon EM FF data in space-like and time-like regions **provides a true behavior of** $G_{Ep}(Q^2)$, from which **compatible proton charge** *rms* **radius** with the **muon hydrogen atom value** is obtained.

More detail will be presented in next paragraphs.

However, before **we remind some concepts to be used in our discussion**.

The positively charged **proton** is compound of **(u,u,d)-quarks**

- $\Rightarrow$ **non-point-like** with some nonzero **charge distribution**
- and in EM interactions it **manifests EM structure** (similarly) the **neutron** to be compound of **(d,d,u)-quarks**
- the **size** of **proton charge distribution** is described by the **charge root-mean-square (*rms*) radius** in a specific (Breit) reference frame

$$r_p = \sqrt{< r_{Ep}^2 >}. \tag{3}$$

The **charge distribution** inside of the proton is given by the **Fourier transformation** of the **proton electric form factor** (FF) $G_{Ep}(t),\ t = q^2 = -Q^2$

$$\rho_{ch}(\mathbf{r}) = \frac{1}{(2\pi)^3} \int G_{Ep}(Q^2) e^{\mathbf{iQr}} d^3 r. \tag{4}$$

The **inverse Fourier transformation** then gives

$$G_{Ep}(Q^2) = \int \rho_{ch}(\mathbf{r}) e^{-\mathbf{iQr}} d^3 r. \tag{5}$$

If $\rho_{ch}(\mathbf{r})$ is **spherically symmetric distribution** $\Rightarrow$ the previous relation **can be rewritten into the spherical coordinates** and by the **integration over** θ **and** ϕ angles one gets

$$G_{Ep}(Q^2) = 4\pi \int_0^\infty \frac{\sin Qr}{Qr} \rho_{ch}(r) r^2 dr. \tag{6}$$

For the case of $Qr << 1$

$$\sin Qr \simeq Qr - \frac{(Qr)^3}{6} + \dots \tag{7}$$

and

$$G_{Ep}(Q^2) = 4\pi \int_0^\infty r^2 \rho_{ch}(r) dr - \frac{4\pi Q^2}{6} \int_0^\infty r^4 \rho_{ch}(r) dr + \dots \tag{8}$$

Now, taking into account the **charge distribution density normalization**

$$\int_0^\infty \rho_{ch}(r) 4\pi r^2 dr = 1 \tag{9}$$

and the **definition of charge-mean square radius**

$$< r_{Ep}^2 >= \int_0^\infty r^2 \rho_{ch}(r) 4\pi r^2 dr \tag{10}$$

one finally gets

$$G_{Ep}(Q^2) = 1 - \frac{Q^2}{6} < r_{Ep}^2 > + ... \qquad (11)$$

from where the **relation** between **charge mean-square radius** and **proton electric FF**

$$< r_{Ep}^2 >= 6 \frac{dG_{Ep}(t)}{dt} |_{t \to 0} \qquad (12)$$

is obtained.

There are **two completely different sources** of experimental information on proton charge *rms* radius r_p

- the precision spectroscopy of **atomic hydrogen**
- elastic **electron-proton scattering**

This method is based on **precision measurement of Lamb shift**

- W.E.Lamb and R.C.Retherford, Phys. Rev. 72 (1947) 241

i.e. the **energy difference measurement between** $2S_{1/2}$ **and** $2P_{3/2}$ **states in muon hydrogen atom** and its **comparison with a theoretical formula for energy difference**, dependent explicitly on the squared charge *rms* radius r_p of the proton and the **third Zemach moment** $< r_p^3 >_{(2)}$ as follows

$$\Delta E = [209.9779(49) - 5.2262\frac{r_p^2}{\text{fm}^2} + 0.00913\frac{\langle r_p^3 \rangle_{(2)}}{\text{fm}^3}]\text{meV}. \quad (13)$$

The **third Zemach moment** $< r_p^3 >_{(2)}$ is defined by the relation

$$< r_p^3 >_{(2)} = \int d^3 r r^3 \rho_{(2)}(r) \tag{14}$$

where $\rho_{(2)}(r)$ is the **convolution** of the proton charge distribution $\rho_{ch}(r)$

$$\rho_{(2)}(r) = \int d^3 r_2 \rho_{ch}(| \vec{r} - \vec{r_2} |) \rho_{ch}(r_2). \tag{15}$$

NOTE:

Consequently, the **third Zemach moment is unknown without the knowledge of the** $\rho_{ch}(r)$ to be directly related with the Fourier transformation of $G_{Ep}(Q^2)$.

From the previous relations one comes to the conclusion:

in order to determine the proton charge *rms* radius in the spectroscopy of the muon hydrogen atom, one has to know $G_{Ep}(Q^2)$**, from which one can determine directly** $r_p = \sqrt{< r_{Ep}^2 >}$ **by the relation** $< r_{Ep}^2 >= -6\frac{dG_{Ep}(Q^2)}{dQ^2}|_{Q^2 \to 0}$.

Just at this moment discussion was created - mentioned at the **INTRODUCTION**.

The proton charge *rms* radius determined at the **muon hydrogen atom spectroscopy** could depend on the choice of the shape of $G_{Ep}(Q^2)$.

In the **muon hydrogen atom experiment**

- R.Pohl et al, Nature Vol. 466 (2010) 213

the **standard dipole form** has been exploited, leading to the **difference energy theoretical formula**

$$\Delta E = [209.9779(49) - 5.2262\frac{r_p^2}{\mathrm{fm}^2} + 0.0346\frac{r_p^3}{\mathrm{fm}^3}]\mathrm{meV} \qquad (16)$$

with $r_p = \sqrt{< r_{Ep}^2 >}$ given in fm.

The measured transition $2S^{F=1}_{1/2} - 2P^{F=2}_{3/2}$ has been found to be 49881.88(76)GHz, which corresponds to the energy difference $\Delta E = 206.2949(32)\text{meV}$.

By a **comparison of this result with the previous theoretically estimated energy difference (16)** one finds **10 times more precise** proton charge *rms* radius

$$r_p = 0.84184(67)\text{fm}$$

than the value of Review of Particle Physics (2010)

$$r_p = 0.87680(690)\text{fm},$$

- K.Nakamura et al (Particle Data Group), J. of Phys. G37 (2010) 075021

to be obtained by an **electron hydrogen atom spectroscopy**, where the **energy difference is 6.5 million smaller** in comparison with the same quantity in **muon hydrogen atom spectroscopy**.

The reasons are:

- the muon is about 200 times heavier than the electron
- the atomic Bohr radius is correspondingly about 200 times smaller in μp in comparison with *ep* hydrogen atom
- the reduced mass with muon is 186.3 times larger than the reduced mass with electron
- therefore in this manner **effects of the finite size of the proton** on the muon hydrogen atom spectroscopy **are thus enhanced**

In the paper

- M.O.Distler, J.C.Bernauer, T.Walcher, Phys. Lett. B696 (2011) 343

the authors have defended the **muon hydrogen atom spectroscopy result for proton charge *rms* radius** as it does not depend too much on the chosen parametrization of $G_{Ep}(Q^2)$, with the exception of the extreme example proposed by De Rujula.

Nevertheless, further we demonstrate that **in the global analysis of nucleon EM FF data** one finds the true behavior of $G_{Ep}(Q^2)$, which gives a consistent value of r_p with muon hydrogen spectroscopy experiment result.

There are **two types of elastic processes** used for obtaining of experimental information on $G_{Ep}(t)$:

- **unpolarized** elastic scattering $e^- p \to e^- p$
- the **longitudinally polarized electron beam** in the polarization transfer process $\overrightarrow{e} p \to e \overrightarrow{p}$

from which proton charge *rms* radius can be determined.

Unpolarized $e^- p \to e^- p$ scattering

In the case of unpolarized $e^- p \to e^- p$ scattering The corresponding **differential cross-section in Lab. system** in one-photon-exchange approximation, with E_0 - electron beam energy and θ - electron scattering angle,

is

$$\frac{d\sigma(e^- p \to e^- p)}{d\Omega} = \frac{d\sigma}{d\Omega_{Mott}}[A(t) + B(t)\tan^2\theta/2] \qquad (17)$$

where $A(t)$, $B(t)$ - elastic proton structure functions and

$$\frac{d\sigma}{d\Omega_{Mott}} = \frac{\alpha^2}{4E_0^2} \frac{\cos^2\theta/2}{\sin^4\theta/2} \frac{1}{1+(2E_0/m_p)\sin^2\theta/2} \tag{18}$$

is the **relativistic Rutherford cross-section** of massless electron scattering from point-like proton.

The **structure of the proton** (similarly of the neutron) is **completely described** by electric $G_{Ep}(t)$ and magnetic $G_{Mp}(t)$ FFs (spin 1/2 particle) and

$$A(t) = \frac{G^2_{Ep}(t) - t/4m^2_p G^2_{Mp}(t)}{1 - t/4m^2_p}; \quad B(t) = -2.t/4m^2_p G^2_{Mp}(t). \tag{19}$$

Note, that the ratio

$$\frac{d\sigma(e^- p \to e^- p)}{d\Omega} / \frac{d\sigma}{d\Omega_{Mott}} \tag{20}$$

when **evaluated for a fixed squared momentum transfer** $-t$ and **plotted against** $\tan^2 \theta/2$ yields a **straight line**.

The **slope** and **intercept** of this line **yields information** on $G_{Ep}(t)$ and $G_{Mp}(t)$ - **Rosenbluth technique**.

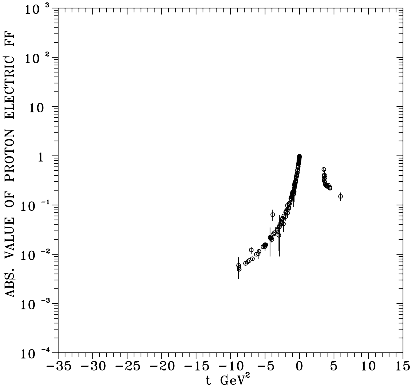

Figure: 1 Experimental data on proton electric form factor.

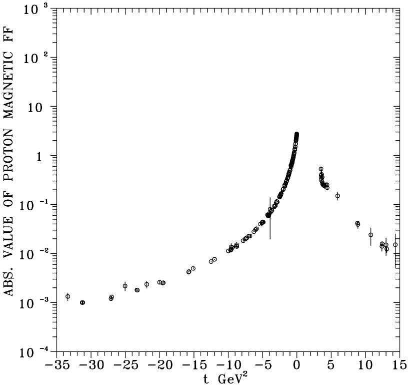

Figure: 2 Experimental data on proton magnetic form factor.

Experimental data for neutron FF

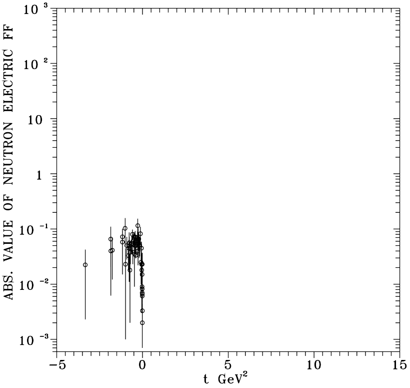

Figure: 3 Experimental data on neutron electric form factor.

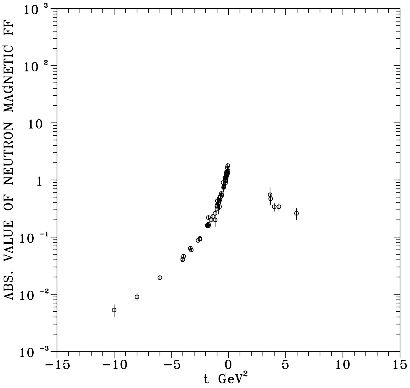

Figure: 4 Experimental data on neutron magnetic form factor.

However - **in the differential cross-section** (17) with (19) $G^2_{Mp}(t)$ is multiplied by $(-t/4m^2_p)$ - factor, $\Rightarrow$ with $-t$ increased the **measured cross-section** (17) becomes dominant by $G^2_{Mp}(t)$-part contribution - **making the extraction of $G^2_{Ep}(t)$ more and more difficult**.

Therefore the **first method** of a determination of $G^p_E(t)$ is considered to be **unreliable!**

In despite of this fact, the **data on unpolarized elastic electron-proton scattering for very low values of** $-t$ have been used

- I.Sick, Phys. Lett B576 (2003) 62
- P.G.Blunden, I.Sick, Phys. Rev. C72 (2005) 057601

for determination of the proton charge *rms* radius. A sophisticated data analysis, based on a **continued-fraction expansion** of $G_{Ep}(t)$, was carried out and very large value

$$r_p = 0.895(18)\text{fm} \tag{21}$$

of the proton charge *rms* radius has been found.

This result was criticized by

- A.De Rujula, Phys. Lett. B693 (2010) 555

because it is **based on an extrapolation of data with a large spread** and a poor χ^2 per degree of freedom.

Polarization transfer process $\overrightarrow{e}\,p \to e\,\overrightarrow{p}$

Utilization of the **longitudinally polarized electron beams** in the polarization transfer process $\overrightarrow{e}\,p \to e\,\overrightarrow{p}$ - makes possible to measure $G_{Ep}(t)$ **with very high precision**.

The **polarization** $\overrightarrow{\mathbf{P}}$ of the recoil proton **has only two nonzero components**

- A.I.Akhiezer, M.P.Rekalo, Sov. J. Part. Nucl. 4 (1974) 277.

- **perpendicular** P_t to the proton momentum in scattering plane

$$P_t = \frac{h}{I_0}(-2)\sqrt{\tau(1+\tau)}G_{Ep}G_{Mp}\tan\theta/2 \qquad (22)$$

- **parallel** P_l to the proton momentum in the scattering plane

$$P_l = \frac{h(E_e + E_{e'})}{I_0 m_p}\sqrt{\tau(1+\tau)}G_{Mp}^2\tan^2\theta/2, \qquad (23)$$

where h is the **electron beam helicity**,
$I_0 = G_{Ep}^2 + \tau[1 + 2(\tau + 1)\tan^2\theta/2]G_{Mp}^2$ and $\tau = Q^2/4m_p^2$.
$\Rightarrow$

$$\frac{G_{Ep}}{G_{Mp}} = -\frac{P_t}{P_l}\frac{(E_e + E_{e'})}{2m_p}\tan\theta/2 \qquad (24)$$

Recently **in Jlab**

- M.K.Jones et al, Phys. Rev. Lett. 84 (2000) 1398
- O.Gayon et al, Phys. Rev. Lett. 88 (2002) 092301
- V.Panjabi et al, Phys. Rev. C71 (2005) 055202

have measured simultaneously P_t and P_l of the recoil proton in the polarization process $\overrightarrow{e}\,p \rightarrow e\,\overrightarrow{p}$.

The **new data on the ratio** $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$ for $0.49\text{GeV}^2 \leq Q^2 \leq 5.54\text{GeV}^2$ **have been obtained** by (24) (see Fig. 5)

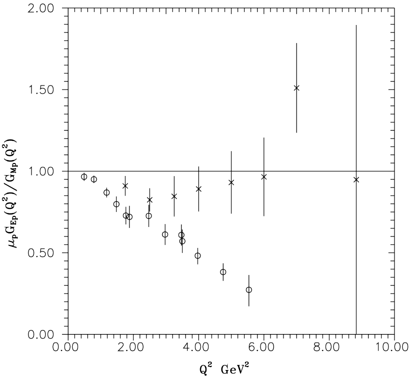

Figure: 5 New JLab polarization data on the ratio $\mu_p G_{Ep}(t)/G_{Mp}(t)$

They **demonstrate the Rosenbluth method** in a determination of $G_{Ep}(Q^2)$ space-like behavior from the $e^- p \to e^- p$ process to be **unreliable !**.

In order to find the corresponding behavior of $G_{Ep}(t)$ in extended space-like region, **we have analyzed all existing nucleon FF data** by our *U&A* model

- S.Dubnicka, A.Z.Dubnickova, P.Weisenpacher, J. Phys. G29 (2003) 405 of the nucleon EM structure

- excluding the **Rosenbluth space-like** $G_{Ep}(t)$ **data**
- and replacing them by **JLab proton polarization data** on the ratio $\mu_p G_{Ep}(t)/G_{Mp}(t)$.

The **results are presented in Figs. 6 7, 8 and 9 by full lines**.

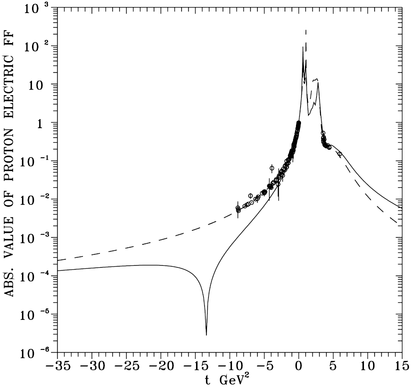

Figure: 6 Theoretical behavior of proton electric form factor.

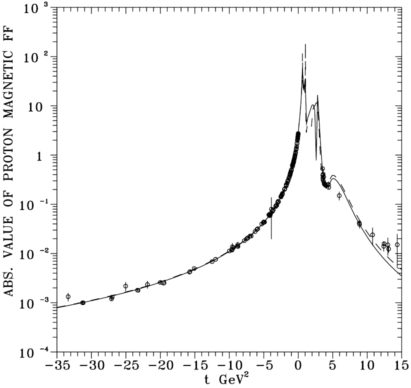

Figure: 7 Theoretical behavior of proton magnetic form factor.

Theoretical behavior of neutron and magnetic form factors

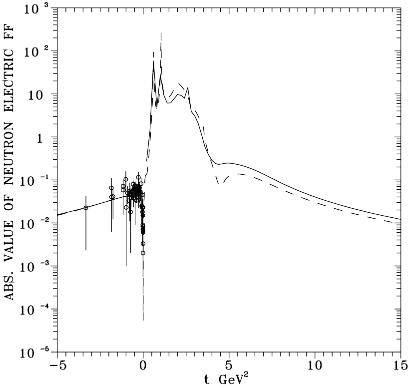

Figure: 8 Theoretical behavior of neutron electric form factor.

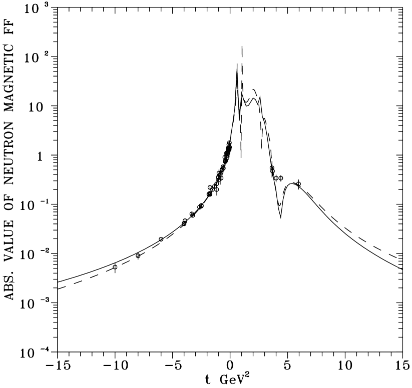

Figure: 9 Theoretical behavior of neutron magnetic form factor.

Description of that ratio is presented in Fig. 10.
The corresponding **charge distribution inside of the proton**

- C.Adamuscin, S.Dubnicka, A.Z.Dubnickova, P.Weisenpacher, Prog. Part. Nucl. Phys. 55 (2005) 228

is presented on Fig.11 by full line, generating the
value of the proton charge mean-square radius
$\langle r_p^2 \rangle = 0.7207(117)\mathrm{fm}^2$.

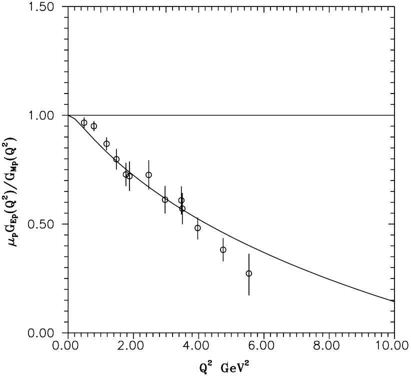

Figure: 10 JLab polarization data.

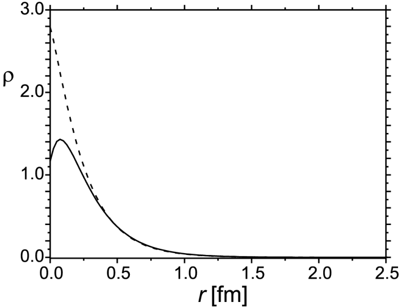

Figure: 11 Charge distribution behavior

Just the square-root of the latter gives
the **proton charge *rms* radius**

$$r_p = 0.84894(690)\text{fm}, \tag{25}$$

to be compatible with the muon hydrogen spectroscopy result

$$r_p = 0.84184(67)\text{fm}.$$

By integration of various shapes of $G^2_{Ep}(Q^2)$ one gets values

- M.O.Distler, J.C.Bernauer, T.Walcher, Phys. Lett. B696 (2011) 343

shown in the TABLE

The TABLE of $G^2_{Ep}(Q^2)$ various shapes results

G_{Ep}	$< r^2_{Ep} >$	$< r^3 >_{(2)}$	$f = \frac{<r^3>_{(2)}}{r^3_p}$
$dipole$	0.6581	2.023	3.789
$Friar - Sick$	0.801	2.71	3.78
$Arrington$	0.742	2.50	3.91
$Bernauer - Arrington$	0.774(8)	2.85(8)	4.18(13)
$DeRujula$	0.771	36.2	53.5
$non - dipole$	0.7207	2.38	3.89,

where **in the last line the results from our realistic non-dipole behavior of $G_{Ep}(t)$ are attached** for comparison.

For completeness, if instead of the **third Zemach moment in** ΔE the relation fr_p^3 from the last line of Table is substituted, one obtains slightly enlarged value $r_p = 0.84194(67)$fm in the muon hydrogen atom spectroscopy experiment.

In the **global analysis of all existing nucleon EM FF data**

- by a **sophisticated analytic model of nucleon EM structure**
- the **non-dipole behavior of** $G_{Ep}(Q^2)$ with the zero around $Q^2 = 13\mathrm{GeV}^2$ has been found
- such $G_{Ep}(Q^2)$ (through the relation $r_p = \sqrt{< r_{Ep}^2 >} = \sqrt{-6 \frac{dG_{Ep}(Q^2)}{dQ^2}|_{Q^2 \to 0}}$) **gives the value** $r_p = 0.84894(690)\mathrm{fm}$ **compatible with the value** $r_p = 0.84184(67)\mathrm{fm}$ obtained in the muon hydrogen atom spectroscopy experiment
- as a result **mutual consistency** between the **electron-proton scattering** and the **muon hydrogen atom spectroscopy** experiments is **finally found**.