Resonance/Parton Duality in Electroproduction of Pions

Murat Kaskulov, Ulrich Mosel
Pion e.m. Formfactor

- Pion e.m. FF at larger $Q^2$ mainly from JLAB, extracted from model for long. X-section

\[
\sigma_L \propto \left[ \frac{F_\pi(Q^2)}{t - m_\pi^2 + i0^+} \right]^2
\]

VGL model: $t$-channel exchange + Born-graph and $F_\rho = F_\pi$

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Electro-Pion production

\[ e' + p \rightarrow e' + e + \pi^- + p \]

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Long and Transv Pion Production

- VGL model
  very good for $L$
  dramatically bad for $T$

Nagging thought: can $F_{\pi}$ be reliably extracted from a model that fails for $T$?

Data from $\pi$-CT Exp. at JLAB,
T. Horn et al., PR C78, 058201 (2008)
General Motivation

- Extraction of pion formfactor relies on model for pion-electroproduction: how good is it?
- QCD (handbag) predicts: $\frac{\sigma_L}{\sigma_T} \propto Q^2$ but $\sigma_T$ is large, and grows, in JLAB, HERMES data
- Where does pQCD start to work: at JLAB@6 or JLAB@12 or ...?
- Description in terms of partons (GPDs) or hadrons?

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Extended VGL Model

- Current for $\pi^+$ production

\[
-i J^\mu_s(\gamma^* p \rightarrow \pi^+ n) = \sqrt{2} g_{\pi NN} \bar{u}_s(p') \gamma^5 \left[ \frac{F_{\gamma \pi \pi}(Q^2, t)}{t - m^2_{\pi} + i0^+} (k + k')^\mu 
+ \frac{F_s(Q^2, s, t)}{s - M_p + i0^+} (p + q)_{\sigma} \gamma^\sigma \gamma^\mu + M_p \gamma^\mu 
+ \frac{(k - k')^\mu}{Q^2} \right] u_s(p),
\]

Reggeize pion propagator

Different e.m. formfactors for $\pi$ and $p$, still gauge invariant (Gross-Riska)

\[
D(t) = \frac{1}{t - m^2_{\pi} + i0^+} \Rightarrow R[\alpha_{\pi}(t)] 
= \left[ 1 + \frac{e^{-i\pi\alpha_{\pi}(t)}}{2} \right] (-\alpha'_{\pi}) \Gamma[-\alpha_{\pi}(t)] e^{g_{\pi}(t) \ln(\alpha'_{\pi})},
\]

Will this cure T problem??

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Extended VGL Model

- Extended VGL model very good for $L$
- still dramatically bad for $T$

Data: Jlab, Horn et al.
Electro-Pionproduction Facts

Observe

1. \( \sigma_T(e, e'\pi)(Q^2) \sim \sigma_{\text{DIS}}(Q^2) \)

2. \( \frac{\sigma_L}{\sigma_T} \rightarrow 0 \) for \( Q^2 \rightarrow \infty \)

Bebek et al, 1978 for DIS

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Transverse strength by DIS

- Take excl. limit of DIS:

\[ p(e, e'\pi^+)X \rightarrow p(e, e'\pi^+)n \quad \text{for} \quad z \rightarrow 1 \]

and add excl. DIS cross section:

\[ \sigma = \sigma(t - \text{channel} + \text{Born}) + \sigma(\text{DIS}) \]
exclusive DIS describes $T$

- Use PYTHIA to calculate DIS
- Excellent Description of $L$, determined by $t$-channel + Born (red)
- Excellent description of $T$, determined by DIS (green)
Improvement: Amplitudes

- Model works very well, but cannot give mixed $LT$ X-sections

- String breaking gives only X-sections, not amplitudes
Partons in Electro-Pionproduction

- Use PYTHIA to calculate excl. DIS
- Stringbreak $\rightarrow$ DIS
- DIS = $N^*$ $(W > 2$ GeV, resonances overlap)
Improvement: amplitudes

- Exps. (JLAB, HERMES) cover $W \sim 2 - 4$ GeV, region of overlapping nucleon resonances all with pion decay channels

- Add sum over (many) N*\(s\) to Born-term

- Use duality to connect N* properties with partons

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Electro-Pionproduction

\[ e + p + n \rightarrow e' + \pi^+ + \pi^- + p + n \]

\[ = \sum_{N^*} \]

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N* contribution to $\pi^+$ production

- Current for $\pi^+$ production
- Replace

$\sum_i r(M_i) c(M_i) \frac{F(Q^2, M_i^2)}{s - M_i^2 + i0^+}$

$\sum_i \rightarrow \int_{M_p^2}^{\infty} dM_i^2 \rho(M_i^2)$

Density of N* resonances

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N*-parton connection

- Local Bloom-Gilman duality:

\[ F_2^p(x_B, Q^2) = \sum (M_i^2 - M_p^2 + Q^2) W(Q^2, M_i) \delta(s - M_i^2), \]

with

\[ W(Q^2, M_i) = r^2(M_i) [F(Q^2, M_i)]^2, \quad F(0, M_i) = 1. \]

- Integrate over \( M_i \)

\[ F_2^p(x_B, Q^2) = (s - M_p^2 + Q^2) r^2(s) [F(Q^2, s)]^2 \rho(s). \]

Density of resonances

Relation between structure function \( F_2 \) and formfactor \( F \)
Duality

\[ F_2^p(\omega') \propto (\omega' - 1)^3, \]
\[ \omega' = 1 + W^2/Q^2 \]

\[ F(Q^2, M_i^2) = \left( \frac{1}{1 + \xi \frac{Q^2}{M_i^2}} \right)^2, \]

\[ (\omega' - 1)^3 \propto Q^2(\omega' - 1)^4 \frac{r^2(s)\rho(s)}{\xi^4} \]

\[ r^2(s)\rho(s) \propto \frac{1}{Q^2(\omega' - 1)} = \frac{1}{s}. \]

e.m. coupling \( r(s) \) decreases with \( s \)
-> **Integral over all resonances converges**


**N* contribution to π⁺ production**

- 'effective Born-Term FF'

\[
F_s(Q^2, s) = \frac{\int_0^\infty ds_i \frac{s_i^{-\beta}}{s-s_i+i0^+} \left( \frac{1}{1+\xi \frac{Q^2}{s_i}} \right)^2 \int_0^\infty ds_i' \frac{s_i'^{-\beta}}{s-s_i'+i0^+}}{\int_0^\infty ds_i \frac{s_i^{-\beta}}{s-s_i+i0^+}}.
\]

Harder for higher resonances: consequence of BG duality

- Dashed: model
- Solid: free proton

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$Q^2$, $\varepsilon$-dependence

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N* contribution to $\pi^+$ production

Perfect agreement over wide range of $Q^2$

Green: $t$-channel
dashed: $t + \text{Born}$
solid: $t + \text{Born} + \text{res}$

Data from F-$\pi2$, $\pi$-CT expts.
Benchmark for JLAB@12

$W \sim 3 \text{ GeV}$

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\( \pi^+ \) production at HERMES

**Legend:**
- **Red:** resonance
- **Green:** L
- **Blue:** t-channel

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\( \pi_0 \) photo- + electroproduction

dash-dash-dotted: resonance, dash-dotted: \( t \)-channel, dashed: \( L \)-contrib

Data: HERMES


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$Q^2$ scaling of $L$ and $T$

**Hard scattering Prediction:**

$$\frac{\sigma_L}{\sigma_T} \sim Q^2$$

$L$ very different from hard scattering prediction (Horn et al)

Res/parton model predicts increase with $Q^2$ only for small $x$
Summary

- Any good reaction model of pion production has to describe both $T$ and $L$ components.
- Transverse strength from resonance contributions.
- Cornell, DESY and JLAB data are all described, with same model, same parameters.
- Transverse pion production does not follow QCD scaling law at JLAB@5 and JLAB@12.
- QCD scaling for $L/T$ holds only for small $x < 0.5$.
Neutral pion electroproduction in \( p(\text{e}, \text{e}'\pi^0)p \) above \( s > 2 \) GeV

Exclusive pion electroproduction off nucleons and nuclei.

Beam spin asymmetry in deeply virtual \( \pi \) production.

Deep exclusive charged \( \pi \) electroproduction above the resonance region.

Deep exclusive electroproduction of \( \pi^+ \) from data measured with the HERMES detector at DESY.

Deeply inelastic pions in the exclusive reaction \( p(\text{e}, \text{e}'\pi^+)n \) above the resonance region.