Electromagnetic Transition Form Factors of Mesons

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Motivation

Standard vector meson dominance (VMD) fails to describe the data.

Problem in QCD

Running coupling constant in QCD
- high energies: can use perturbation theory
- low energies: cannot use perturbation theory

Possible solution: effective theories
↔ hadrons as relevant degrees of freedom
Effective theories for light mesons

Calculation of hadronic reactions and decays:

- **low-energy region**: Chiral perturbation theory (ChPT)
  - Goldstone bosons ($\pi, K, \eta$) only active degrees of freedom
  - vector mesons are heavy
  
  $\Rightarrow$ not applicable for energy range of hadronic resonances ($\rho, \omega, K^*, \phi$)

- energy range of hadronic resonances:
  
  so far only phenomenologically successful models

**Aim**: effective field theories for higher energy range
New counting scheme for Goldstone bosons and light vector mesons

- masses of both vector mesons and pseudoscalar mesons are treated as soft, i.e. \( \sim q \)
- decays: all involved momenta are smaller than the mass of the decaying meson, i.e. \( \sim q \)

Possible justification:

other low-lying mesons are dynamically generated from interactions of Goldstone bosons and light vector mesons (hadrogenesis conjecture)
Leading-order Lagrangian

The leading-order chiral Lagrangian for the decay $V \to P\gamma^{(*)}$ is:

\[
\mathcal{L}_{\text{indir.}} = -\frac{1}{16f} h_A \varepsilon^{\mu\nu\alpha\beta} \text{tr}\{[V_{\mu\nu}, (\partial^\tau V_{\tau\alpha})] + \partial_\beta \Phi\} - \frac{1}{16f} b_A \varepsilon^{\mu\nu\alpha\beta} \text{tr}\{[V_{\mu\nu}, V_{\alpha\beta}] + [\Phi, \chi_0]_+\} - \frac{e_V m_V}{4} \text{tr}\{V^{\mu\nu} Q \partial_\mu A_\nu\}
\]

$\Rightarrow$ only decays via virtual vector mesons allowed

Decay photon into dilepton: usual QED
Uncertainties of our method

Rough estimate: one particular next-to-leading-order term

\[ \mathcal{L}_{\text{dir.}} = - \frac{1}{4f m_V} e_A \varepsilon^{\mu \nu \alpha \beta} \text{tr}\{[Q, (\partial^{\tau} V_{\tau \alpha})] + \partial_{\beta} \Phi \partial_{\mu} A_{\nu}\} \]

\Rightarrow \text{direct decay } V \rightarrow P\gamma^{(*)}

Parameters fixed by two-body decays \( V \rightarrow P\gamma \)

\[ \Leftarrow \text{parameter sets (P1) with } e_A = 0 \text{ and (P2) with } e_A \neq 0 \]

For decays into dileptons: no additional parameters needed

\Rightarrow \text{predictive power}
Decay $\omega \rightarrow \pi^0 l^+ l^-$

**Isospin conservation:** decay only possible via virtual $\rho^0$-meson

$\Rightarrow$ standard VMD (with invariant mass $|q|$ of the dilepton):

$$F_{\omega \pi^0}^\text{VMD}(q) = \frac{m_\rho^2}{m_\rho^2 - q^2}$$

$\hookrightarrow$ our calculations yield an additional constant term:

$$F_{\omega \pi^0}(q) \sim -h_A + \frac{(m_\omega^2 + m_\rho^2) h_A - 8 b_A m_\pi^2}{m_\rho^2 - q^2}$$
data taken by NA60 for decay
\[ \omega \rightarrow \pi^0 \mu^+ \mu^- \]

standard VMD fails to explain data

our calculations miss only the last three data points

\[
\Gamma_{\omega \rightarrow \pi^0 \mu^+ \mu^-} = (9.85 \pm 0.58) \times 10^{-7} \text{ GeV}
\]

\[
\Gamma_{\omega \rightarrow \pi^0 e^+ e^-} = (6.93 \pm 0.09) \times 10^{-6} \text{ GeV}
\]

Decay $\phi \rightarrow \eta e^+ e^-$

Relatively large error bars
$\rightarrow$ no assessment possible

$\Gamma_{\phi \rightarrow \eta e^+ e^-} = (4.64 \pm 0.26) \cdot 10^{-7}$ GeV

$\Gamma_{\phi \rightarrow \eta e^+ e^-}^{\text{exp.}} = (4.90 \pm 0.47) \cdot 10^{-7}$ GeV

Prediction:
$\Gamma_{\phi \rightarrow \eta \mu^+ \mu^-} = (2.75 \pm 0.29) \cdot 10^{-8}$ GeV

Summary and Outlook

Transition $\omega \to \pi^0$ and $\phi \to \eta$:

- form factor data well described
- $\omega \to \pi^0$ form factor much better described than with VMD
- widths agree well with experimental ones

Outlook:

- systematic inclusion of $\eta'$ meson
- next-to-leading-order calculations
Thanks for your attention.
Backup
\[ V_{\mu\nu} = \begin{pmatrix} \rho_{\mu\nu}^0 + \omega_{\mu\nu} & \sqrt{2}\rho_{\mu\nu}^+ & \sqrt{2}K_{\mu\nu}^+ \\ \sqrt{2}\rho_{\mu\nu}^- & -\rho_{\mu\nu}^0 + \omega_{\mu\nu} & \sqrt{2}K_{\mu\nu}^0 \\ \sqrt{2}K_{\mu\nu}^- & \sqrt{2}\bar{K}_{\mu\nu}^0 & \sqrt{2}\Phi_{\mu\nu} \end{pmatrix} \]

\[ \Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{pmatrix} + \sqrt{\frac{2}{3}}\eta_1I_{3\times3} \]
\[ \chi_0 = \begin{pmatrix} \bar{m}_\pi^2 & 0 & 0 \\ 0 & \bar{m}_\pi^2 & 0 \\ 0 & 0 & 2\bar{m}_K^2 - \bar{m}_\pi^2 \end{pmatrix} \]

\[ Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} \]
Form factor for the $\phi \rightarrow \eta$ transition:

$$f_{\phi\eta}(q) = \frac{2m_\phi}{3\sqrt{6}fm_ve} \left[ -e_A - 2b_Ae_Vm_V^2 \frac{2\bar{m}_K^2 - \bar{m}_\pi^2}{m_\phi^2} S_\phi(q^2) ight]$$

$$+ \frac{1}{4}e_Vh_Am_V^2 \left( 1 + \frac{q^2}{m_\phi^2} \right) S_\phi(q^2) \right]$$
Results

Decay of vector mesons (predictions):

\[ \omega \rightarrow \eta \mu^+ \mu^- : \]
\[ \Gamma^{\text{calc}} = (8.51 \pm 0.01) \cdot 10^{-12} \text{ GeV} \]

\[ \omega \rightarrow \eta e^+ e^- : \]
\[ \Gamma^{\text{calc}} = (2.72 \pm 0.09) \cdot 10^{-8} \text{ GeV} \]

\[ \phi \rightarrow \eta \mu^+ \mu^- : \]
\[ \Gamma^{\text{calc}} = (2.75 \pm 0.29) \cdot 10^{-8} \text{ GeV} \]