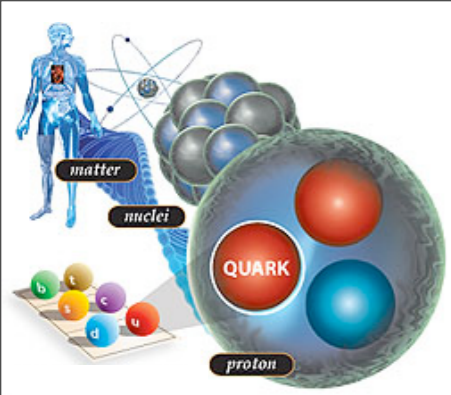




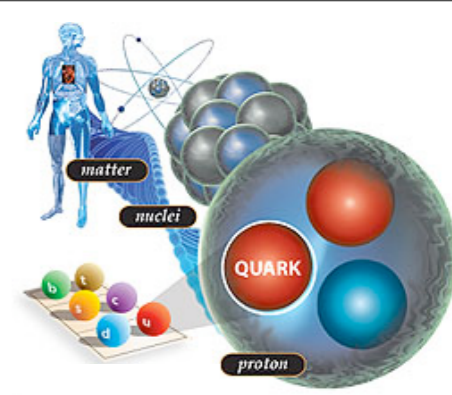
**S. Durr *et al.***  
**Science**

# Nuclear Physics from Lattice QCD

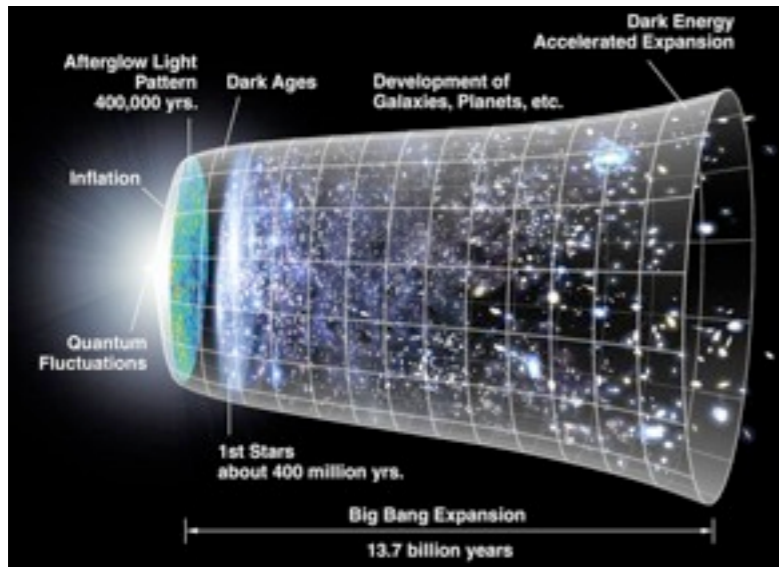
Martin J. Savage  
September 2011  
Erice, Sicily



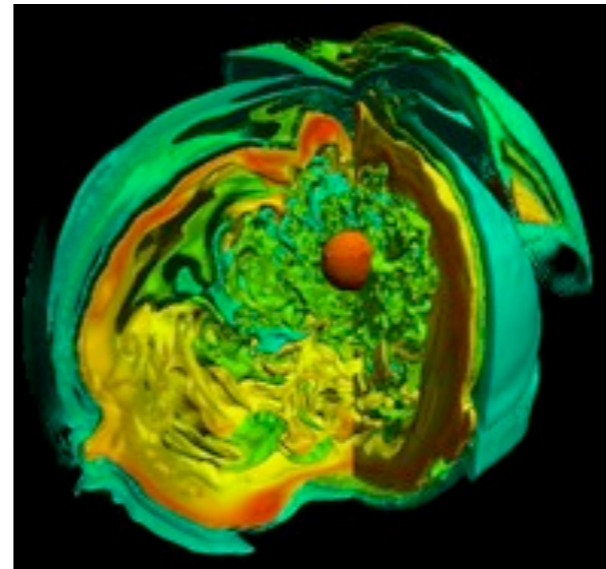
# Nuclear Physics Length Scales



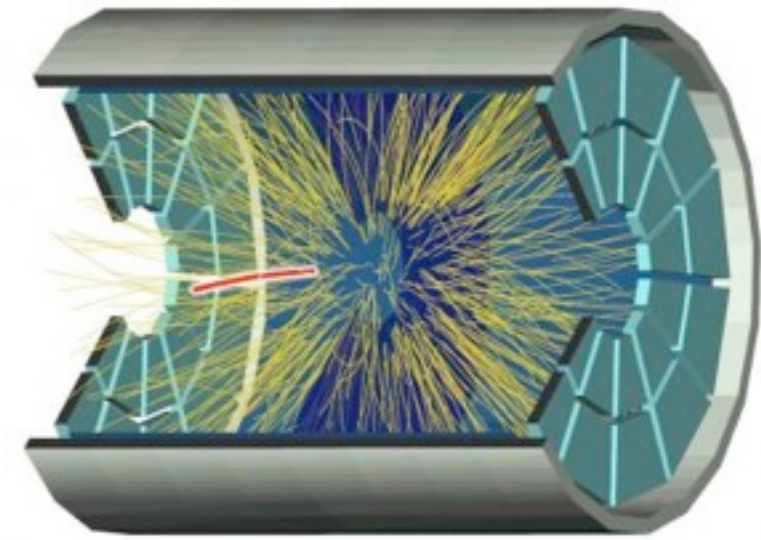
Phase transition(s) at early times,  
light sources at later times



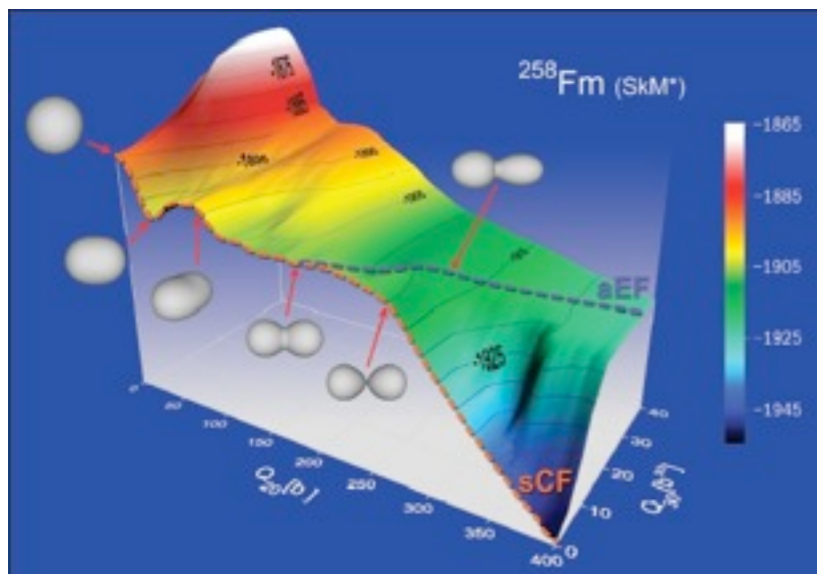
Production of most  
elements in the cosmos



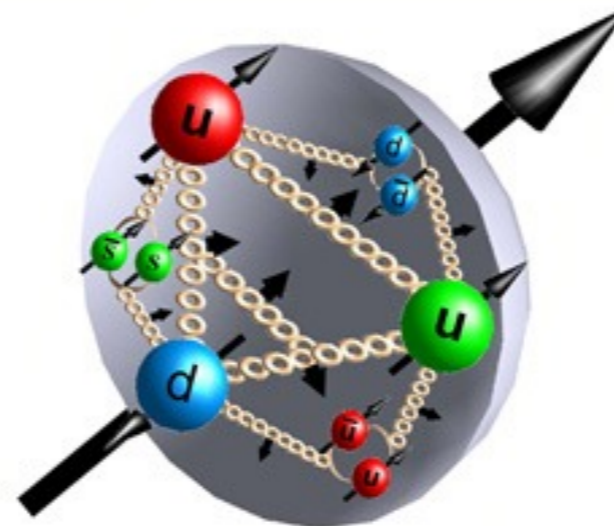
Matter under  
extreme conditions



Nuclei and their reactions



The structure and  
reactions of nucleons



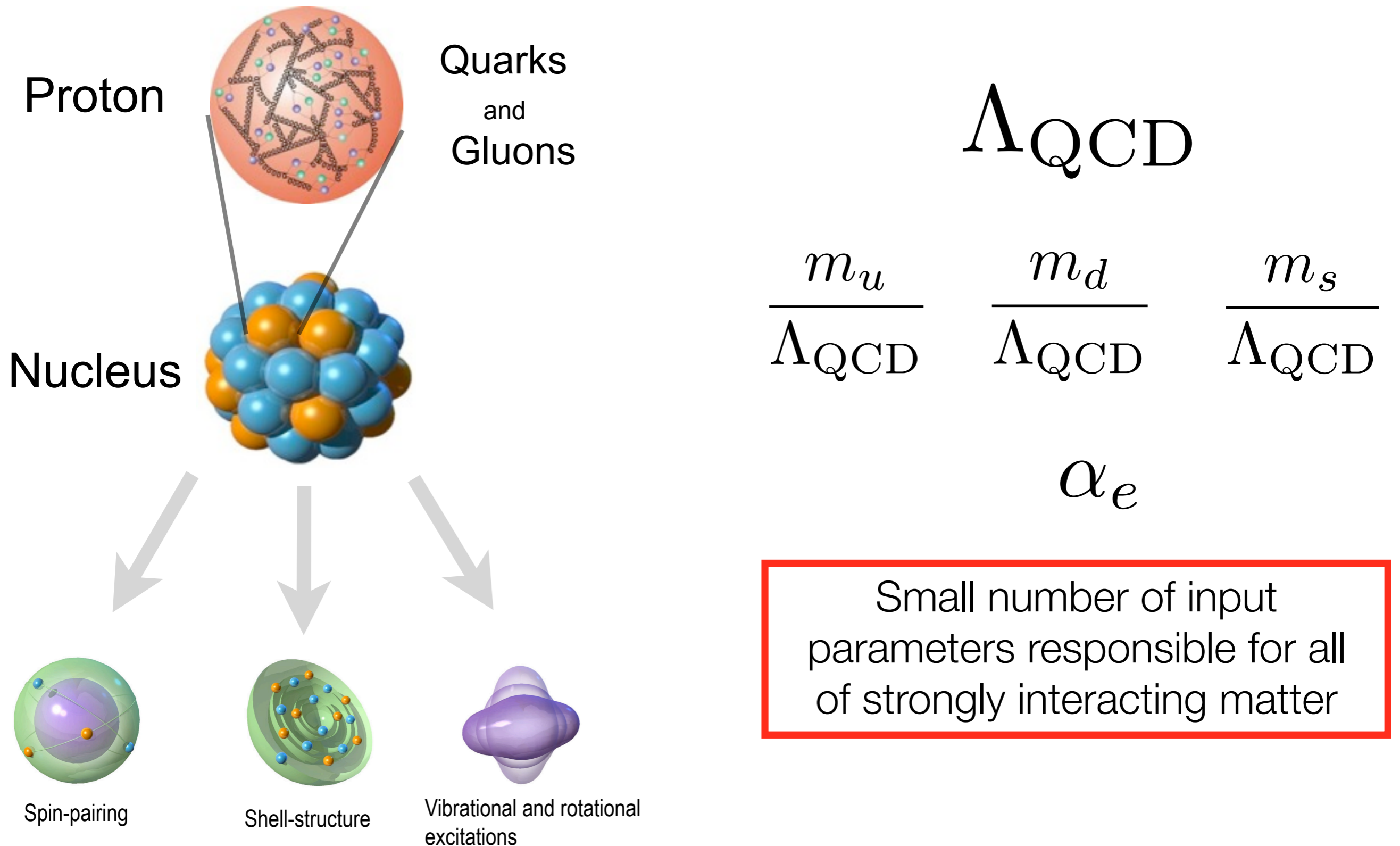
Nature's building  
blocks

	I	II	III	
Leptons	$\nu_e$	$\nu_\mu$	$\nu_\tau$	$Z$
	$e$	$\mu$	$\tau$	$W$
Quarks	$u$	$c$	$t$	$\gamma$
	$d$	$s$	$b$	$g$

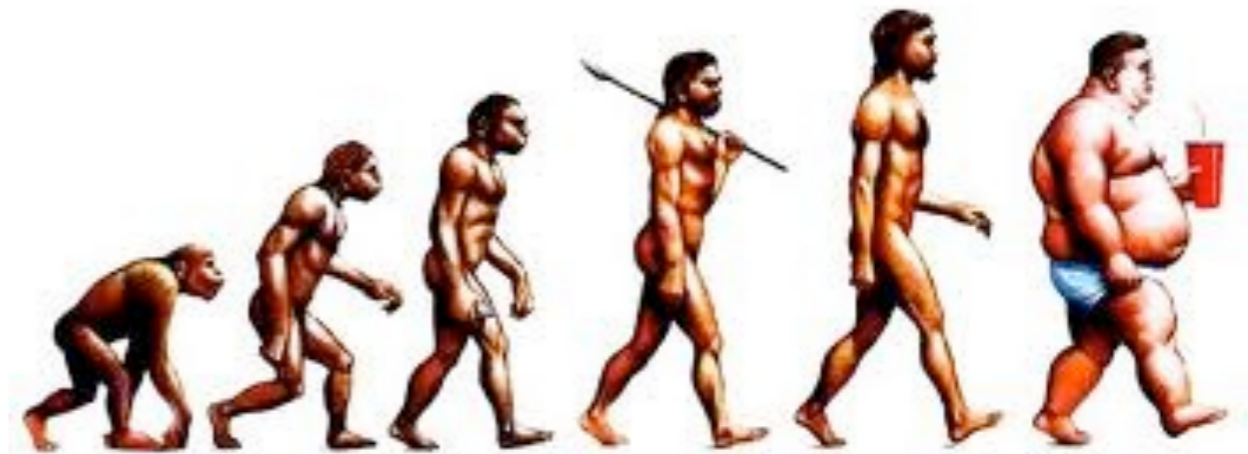
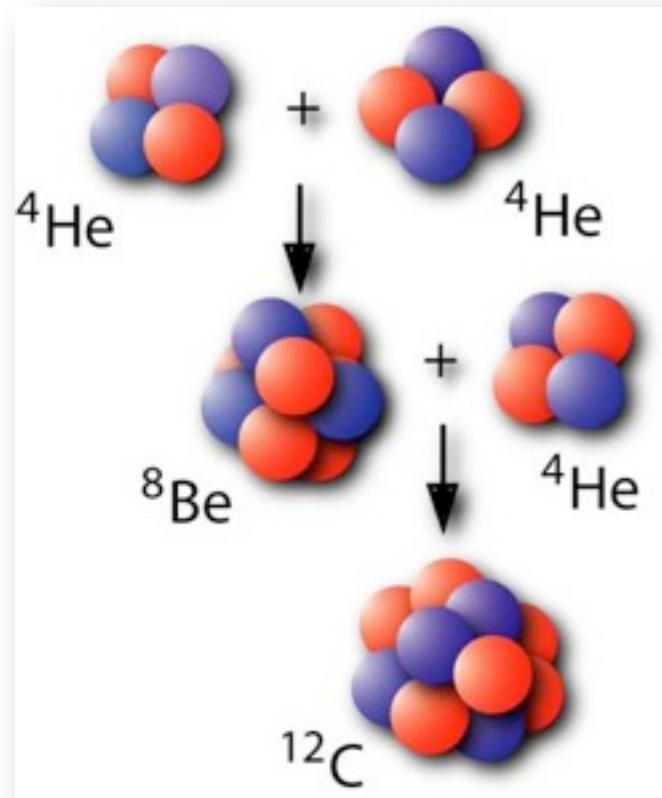
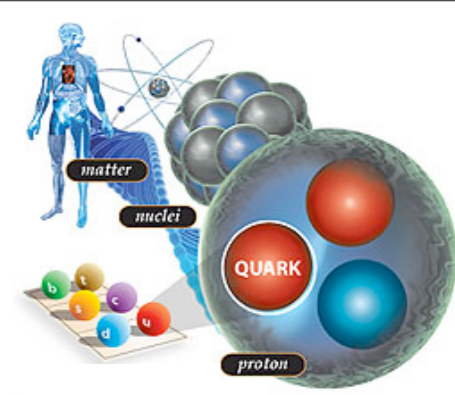
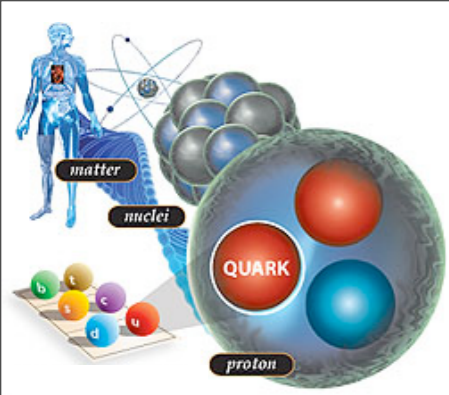
Force Carriers

Three Generations of Matter

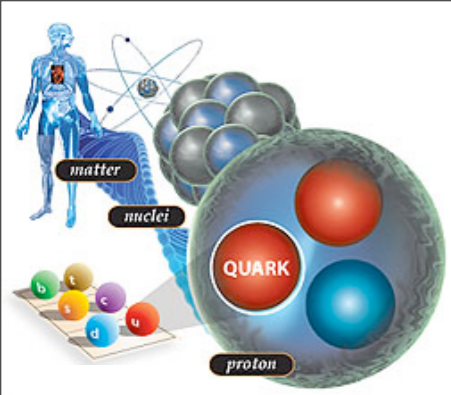
# The Structure and Interactions of Matter from Quantum Chromodynamics



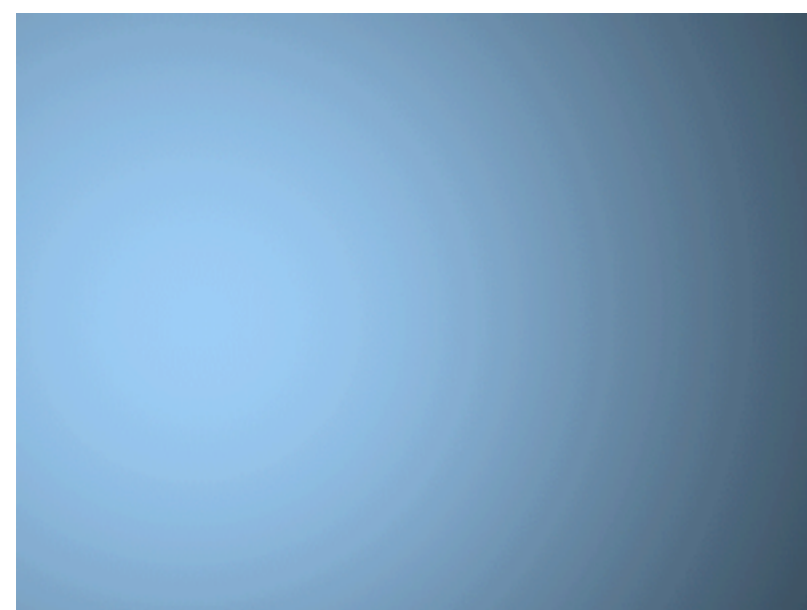
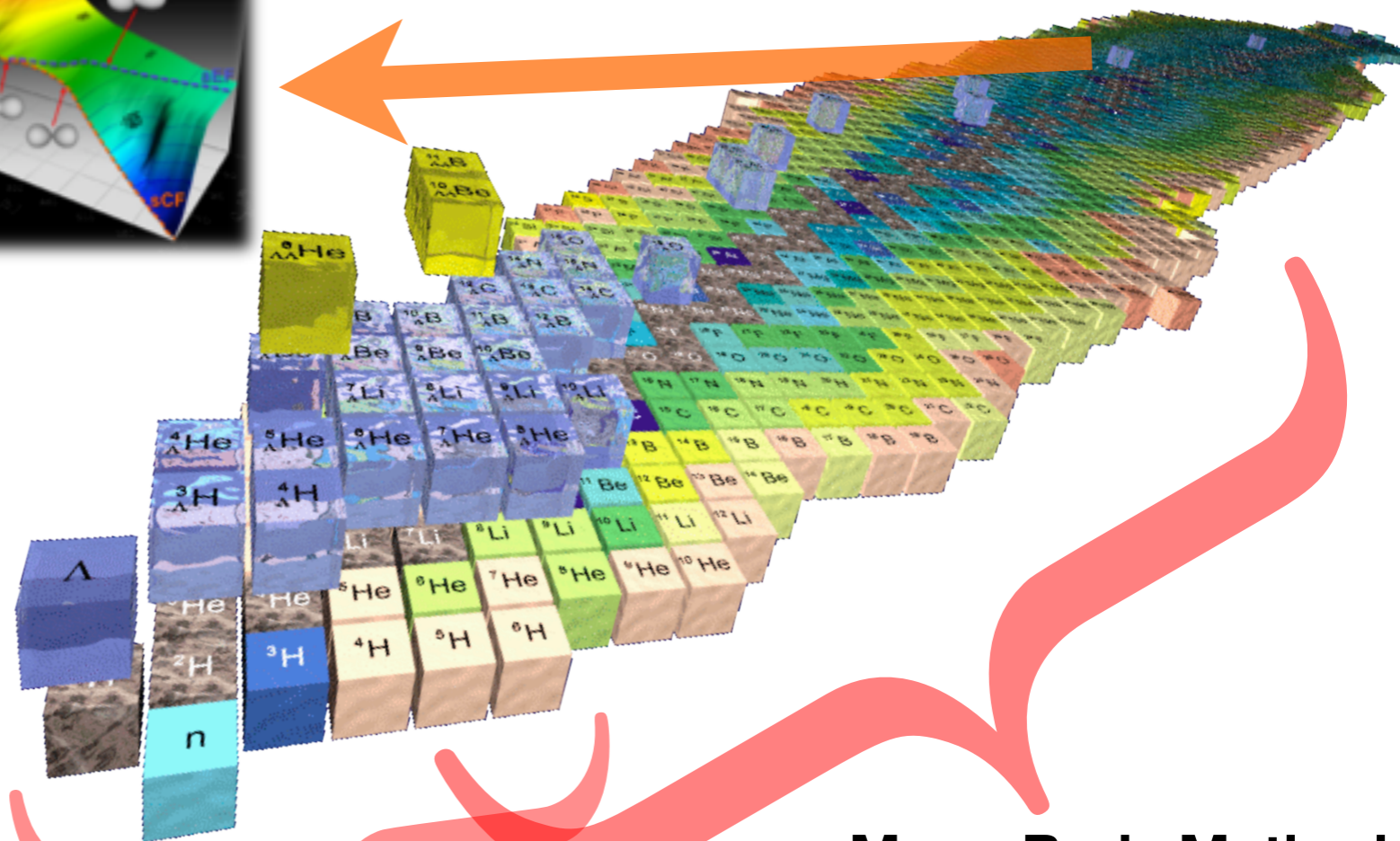
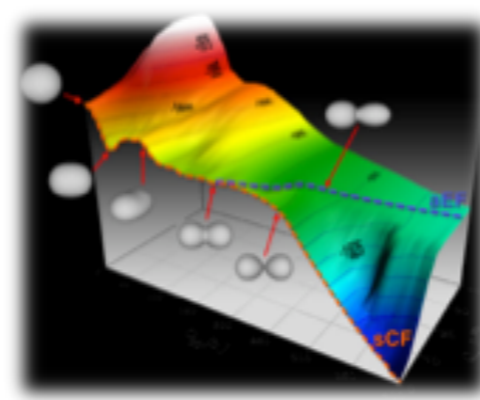
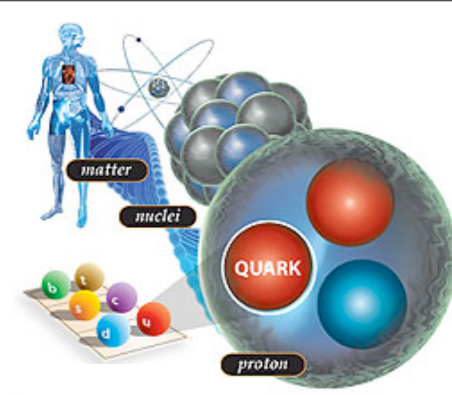
# Fundamental Question(s) about our Universe



- Nuclear physics exhibits fine-tunings
  - *Why ??*
  - *Range of quark masses that can produce sufficient carbon ?*



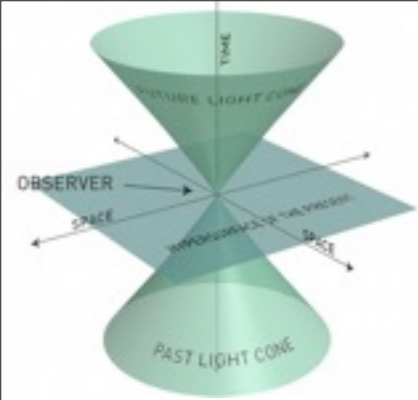
# ( Partial ) Unification of Nuclear Physics - Quantifiable Uncertainties



**Lattice QCD**

**Many-Body Methods  
EFT, LatticeEFT,  
GFMC, NCSM**

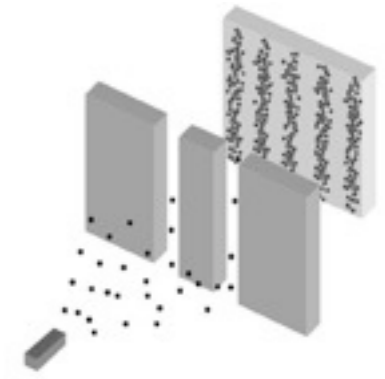
$g_s$   $m_u$   $m_d$   $m_s$   $e$   
Other Universes ?



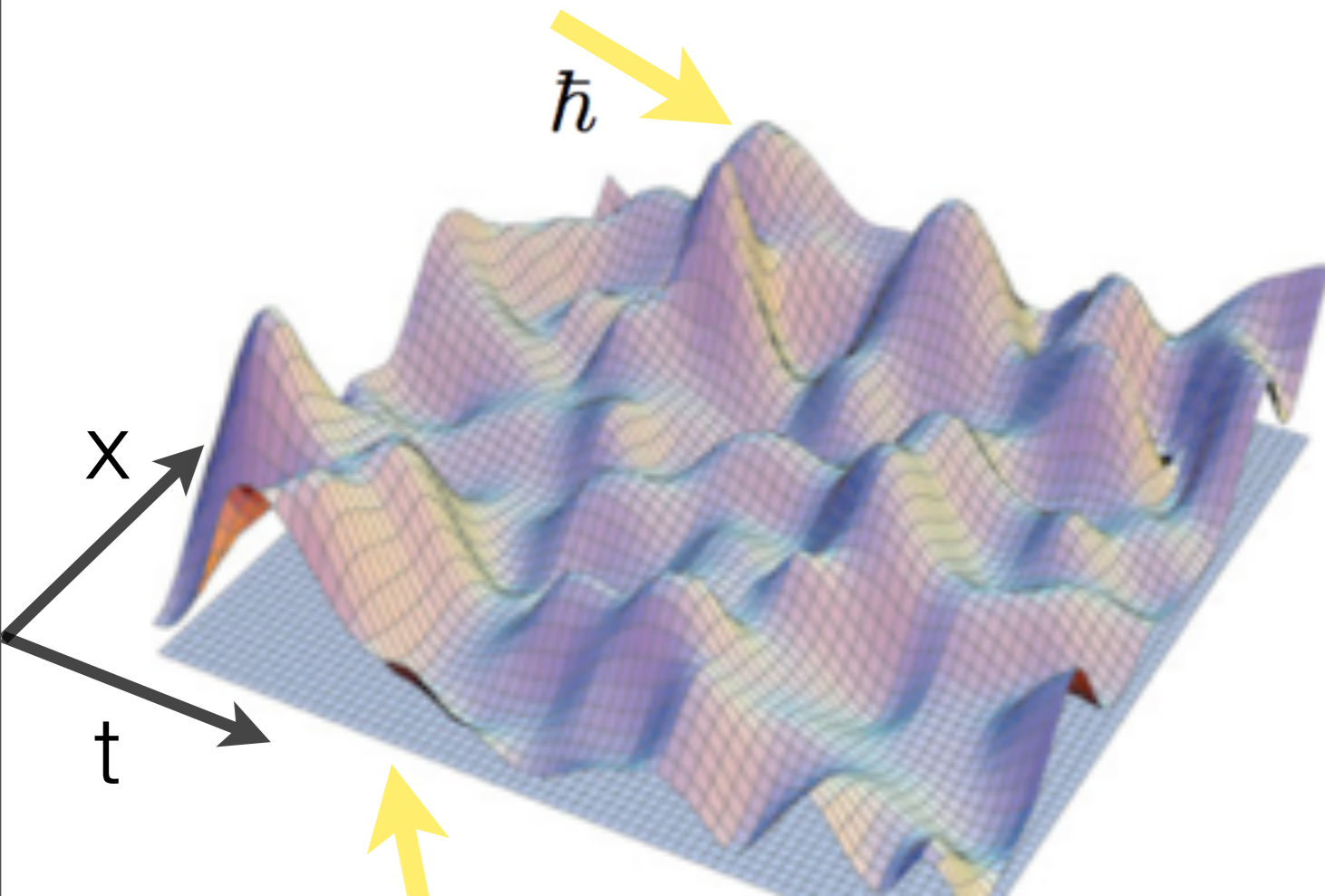
# Quantum Field Theory

## Scalar Field

a number at each point in space-time

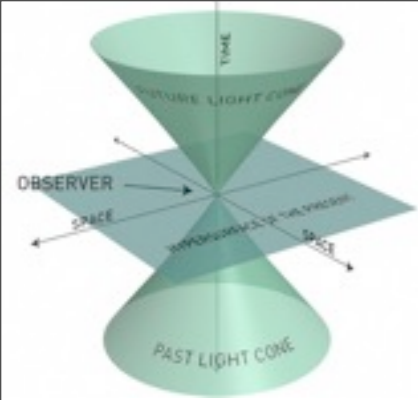


### Quantum Vacuum



$$e^{\frac{i}{\hbar} S[\phi]}$$

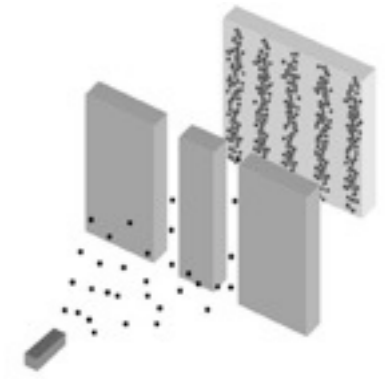
Classical Vacuum



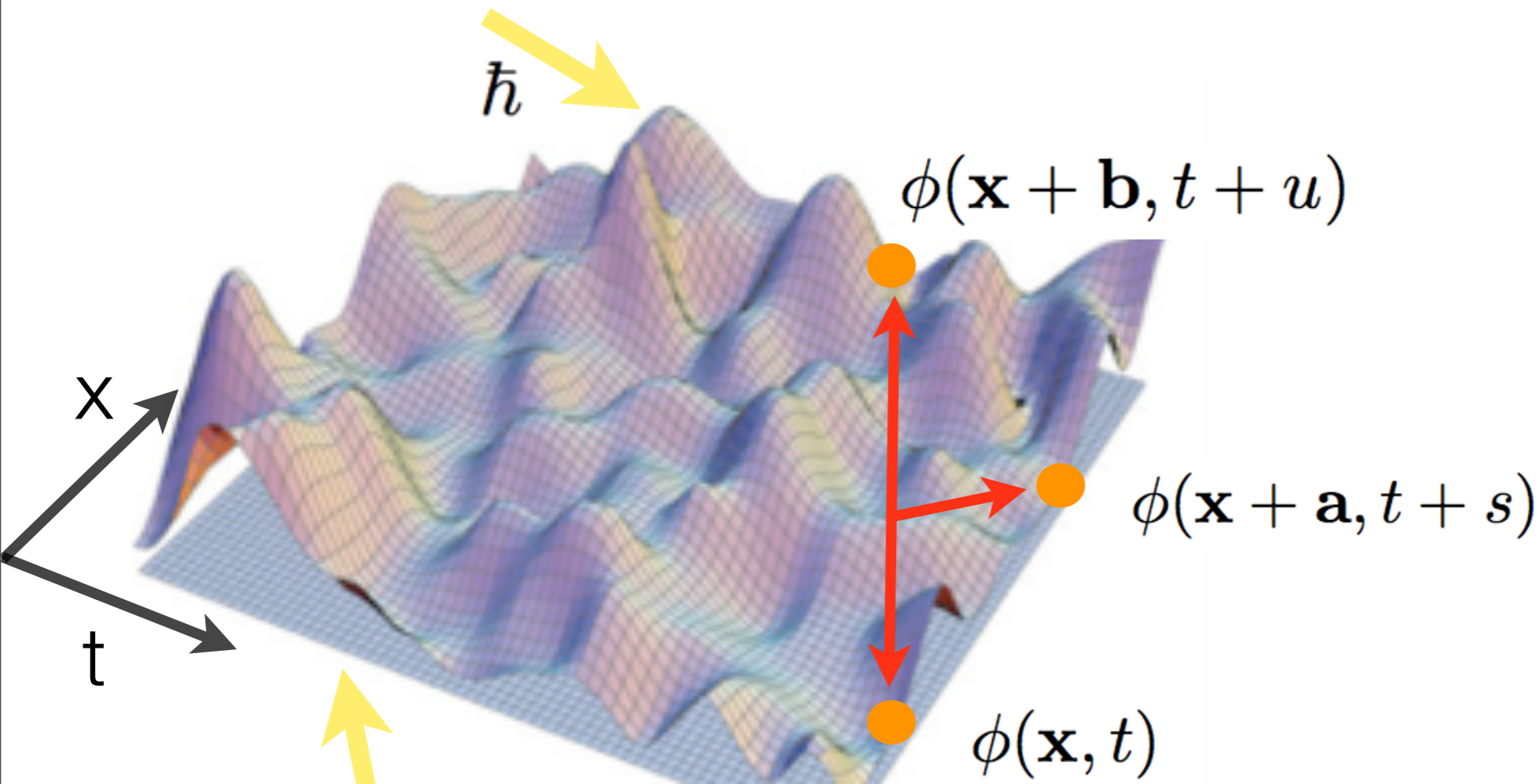
# Quantum Field Theory

## Scalar Field

a number at each point in space-time



### Quantum Vacuum



$$e^{\frac{i}{\hbar} S[\phi]}$$

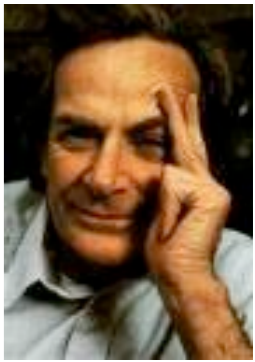
Classical Vacuum

e.g.  $\langle \phi(\mathbf{x}, t) \phi(\mathbf{x} + \mathbf{a}, t + s) \phi(\mathbf{x} + \mathbf{b}, t + u) \rangle$

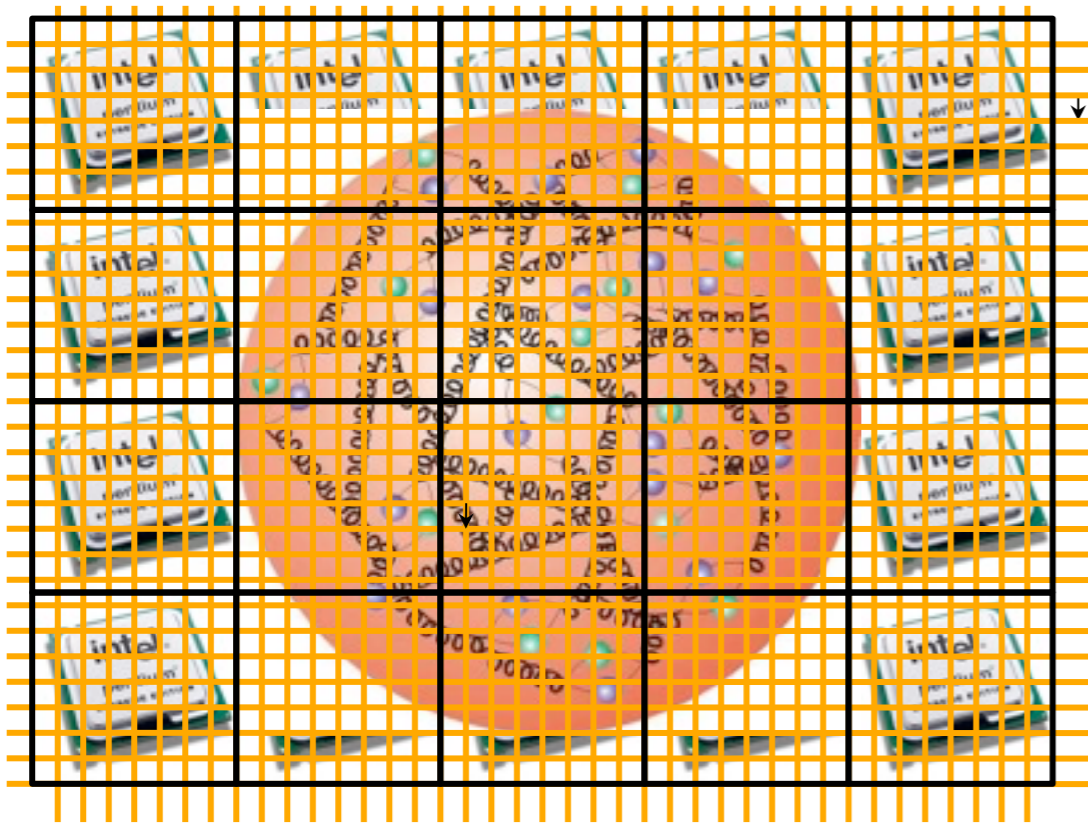
Quantum Fluctuations in the Vacuum Dictate Observables



# Lattice QCD



## Monte-Carlo Evaluation of QCD Path Integral



Lattice Spacing :  
 $a \ll 1/\Lambda\chi$   
 (Nearly Continuum)

Lattice Volume :  
 $m_\pi L \gg 2\pi$   
 (Nearly Infinite Volume)

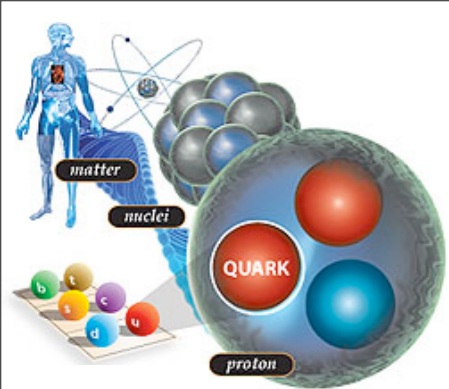
Effective Field Theory gives form of extrapolation  $a = 0$  and  $L = \infty$

Propagators

Gauge Configurations

$$\langle \hat{\theta} \rangle \sim \int \mathcal{D}\mathcal{U}_\mu \hat{\theta}[\mathcal{U}_\mu] \det[\kappa[\mathcal{U}_\mu]] e^{-S_{YM}} \rightarrow \frac{1}{N} \sum_{\text{gluon cfgs}}^N \hat{\theta}[\mathcal{U}_\mu]$$

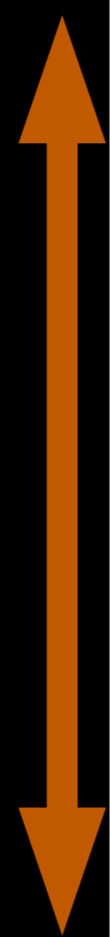
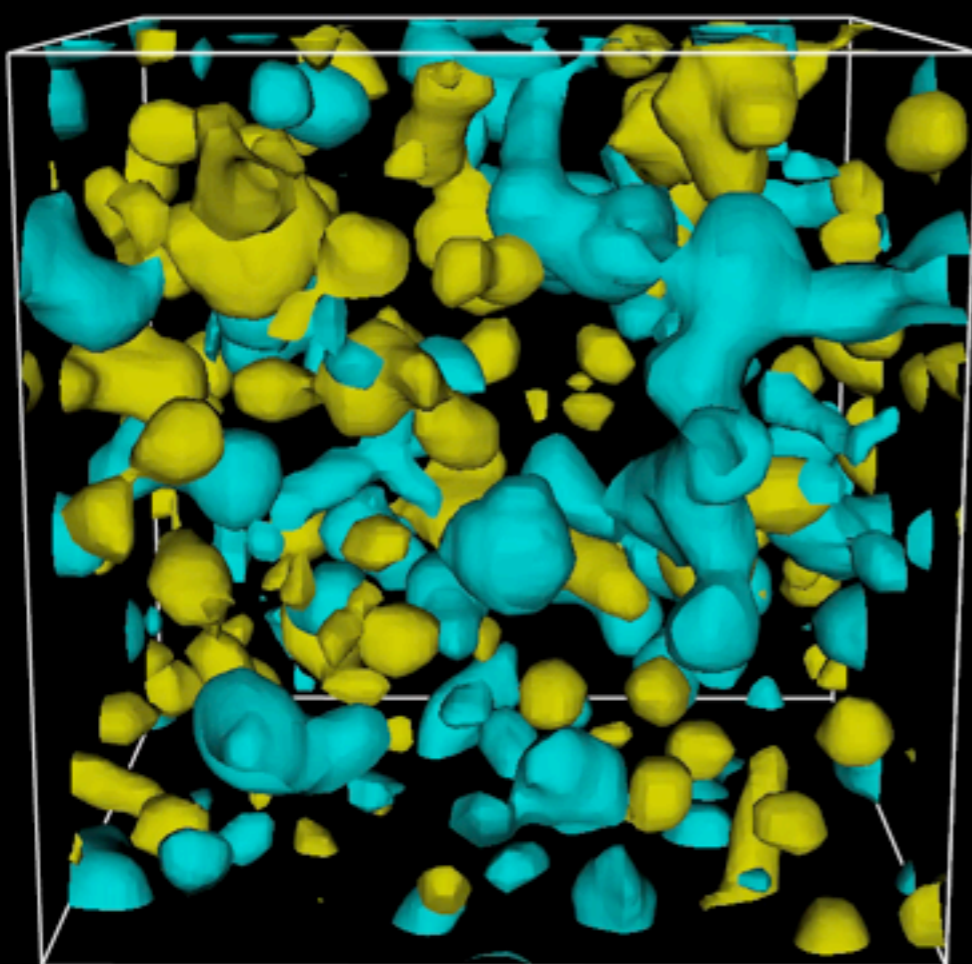




# At the Heart of Visible Matter

## The Vacuum is Complex

### The Quantum Vacuum

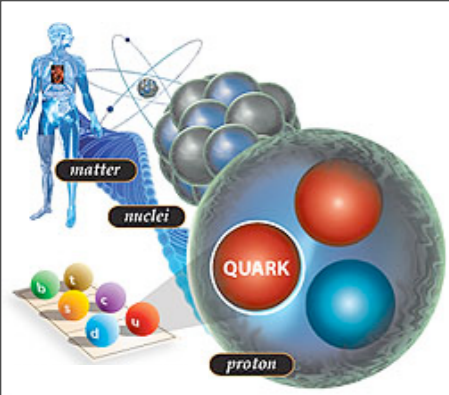


$$L \sim 4 \text{ fm}$$

$$\Delta t \sim 6 \times 10^{-24} \text{ s}$$

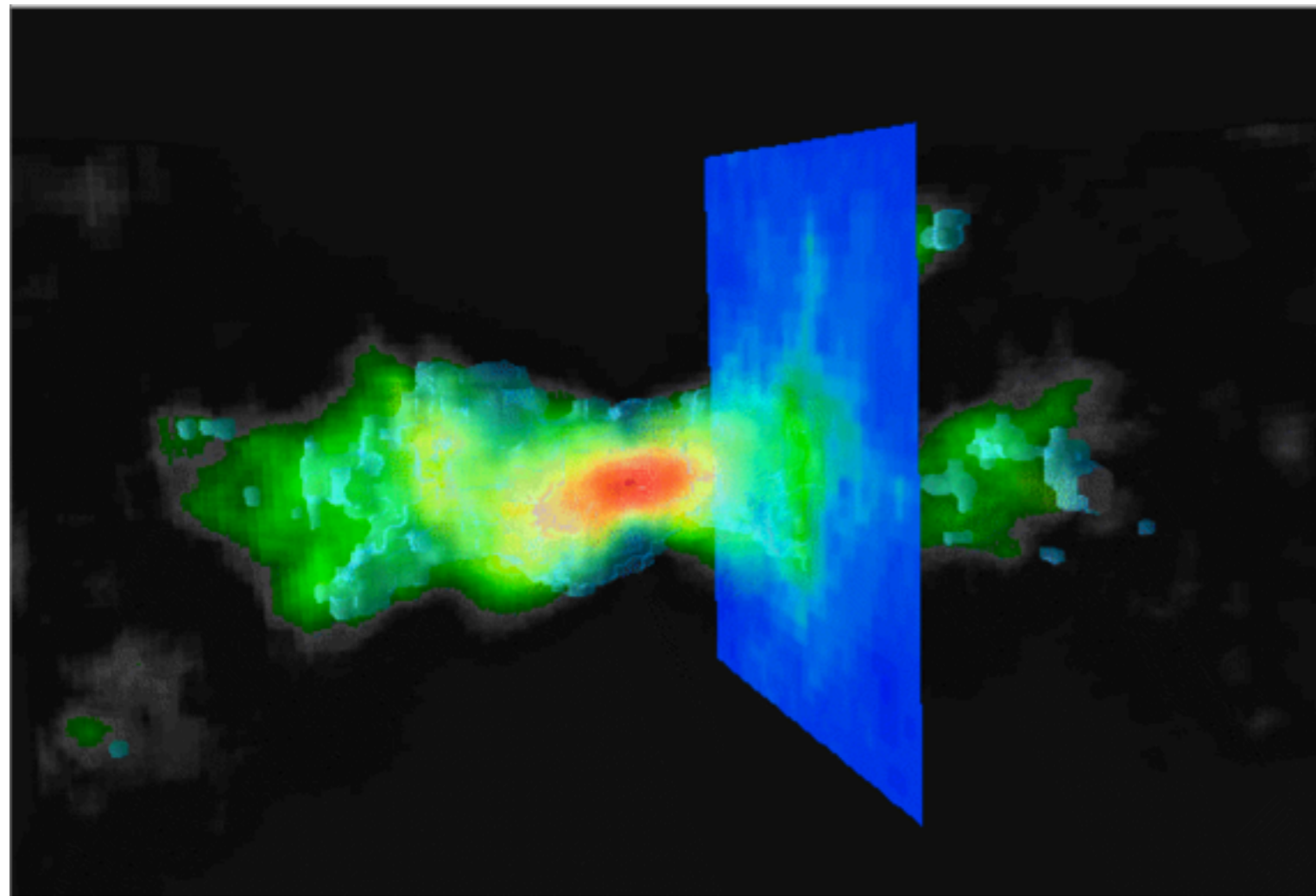
$$\text{“Pixelation”} \sim (0.12 \times 10^{-15} \text{ m})^3$$

**Topological Charge Density**  
**Massimo DiPierro**



# At the Heart of Visible Matter

## Quarks and Gluons are Confined ( $T=0$ )



Rajan Gupta *et al*

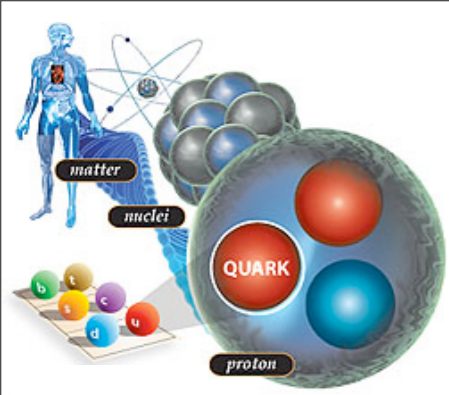
Quark Propagator on One Gauge  
Configuration

No isolated (free) quarks

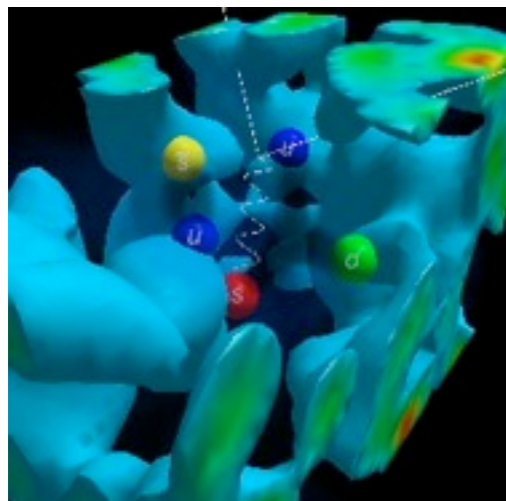
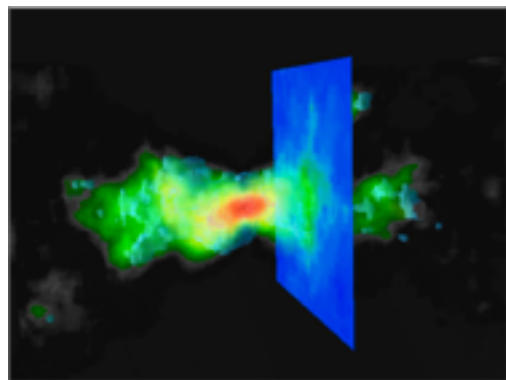
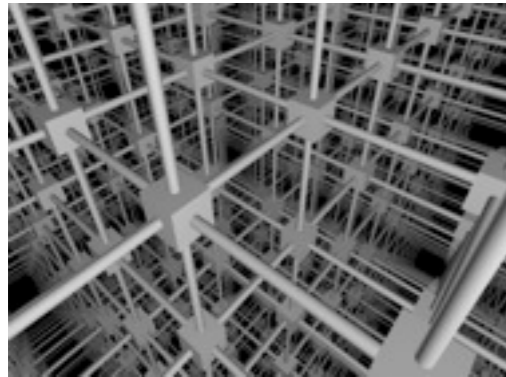


Cancellation of  
Probability Amplitudes

Pion, Nucleon from  
same propagators

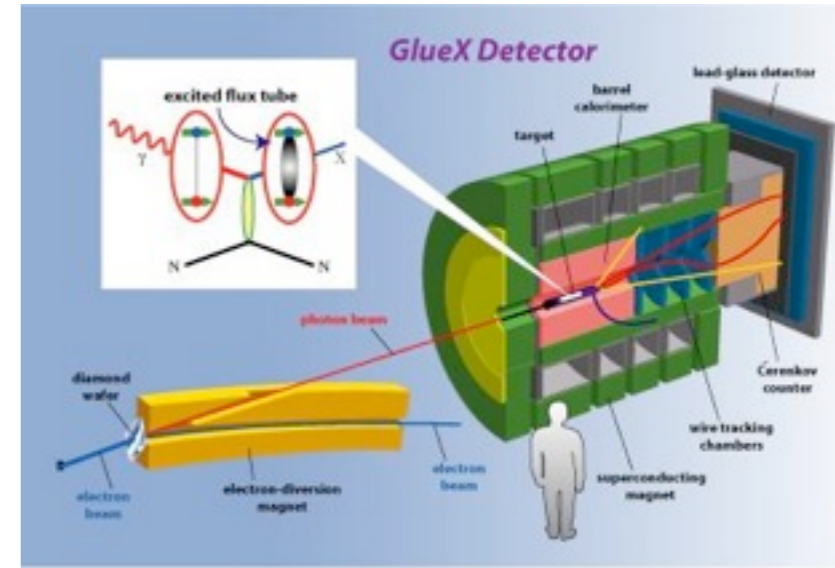
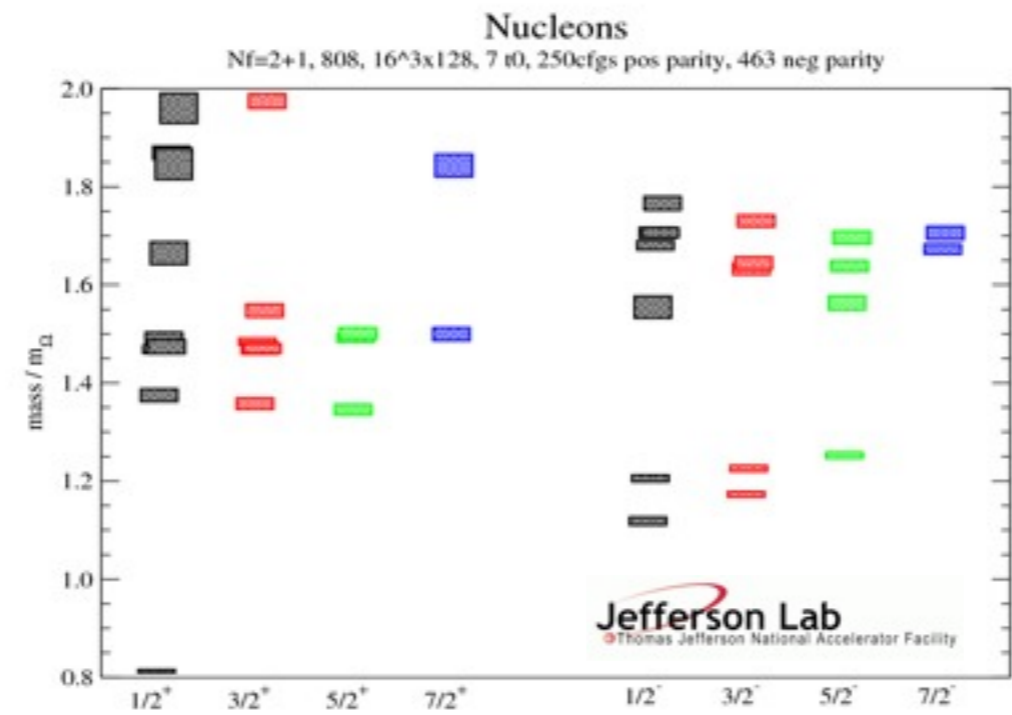
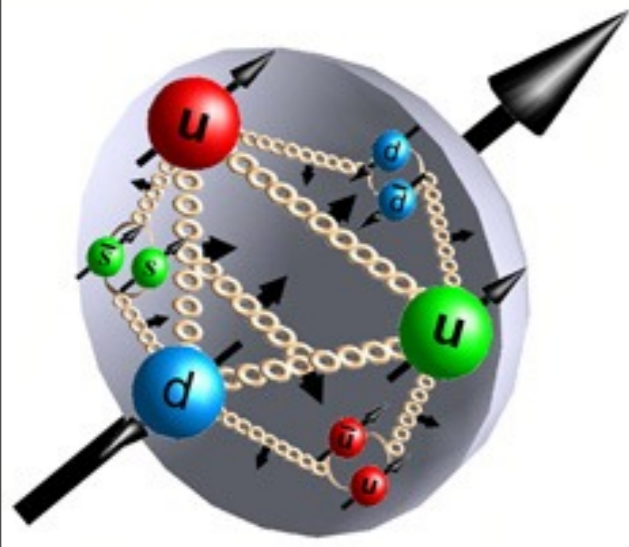
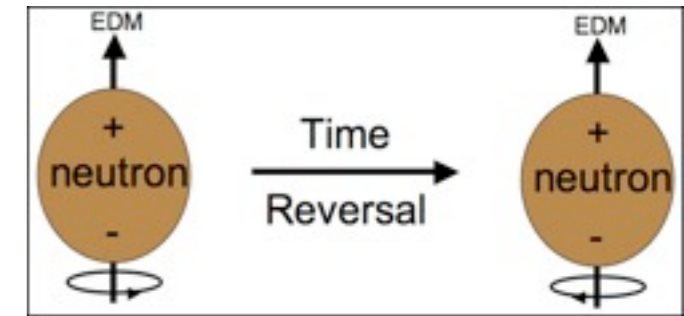
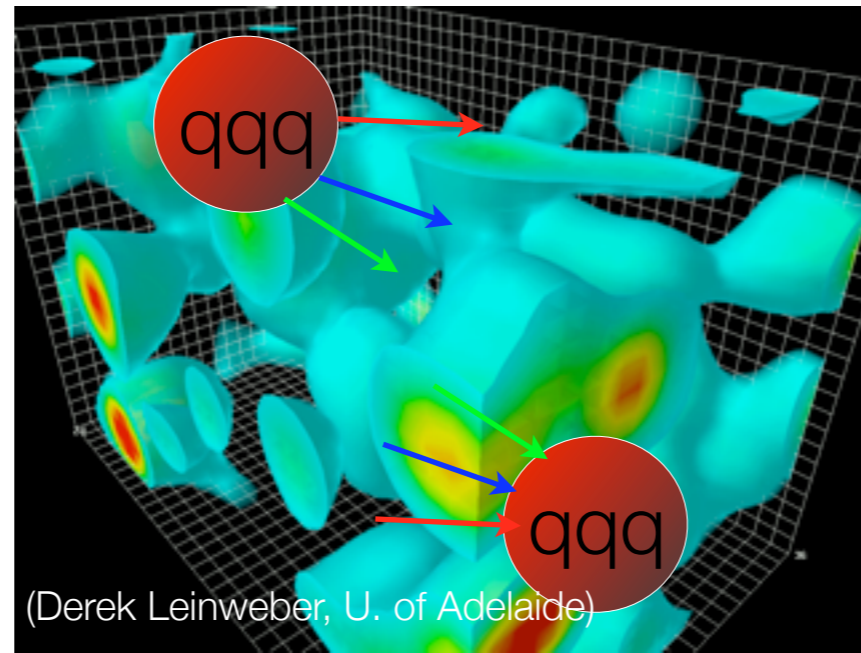


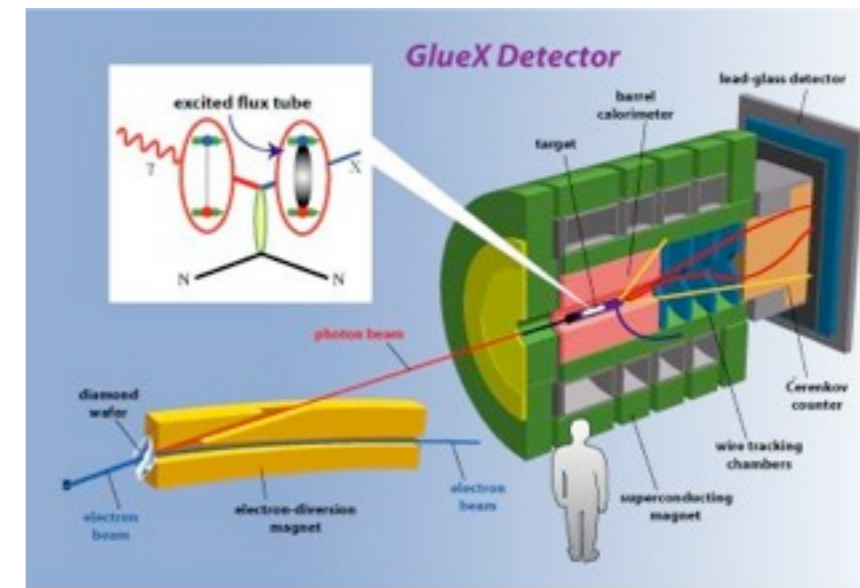
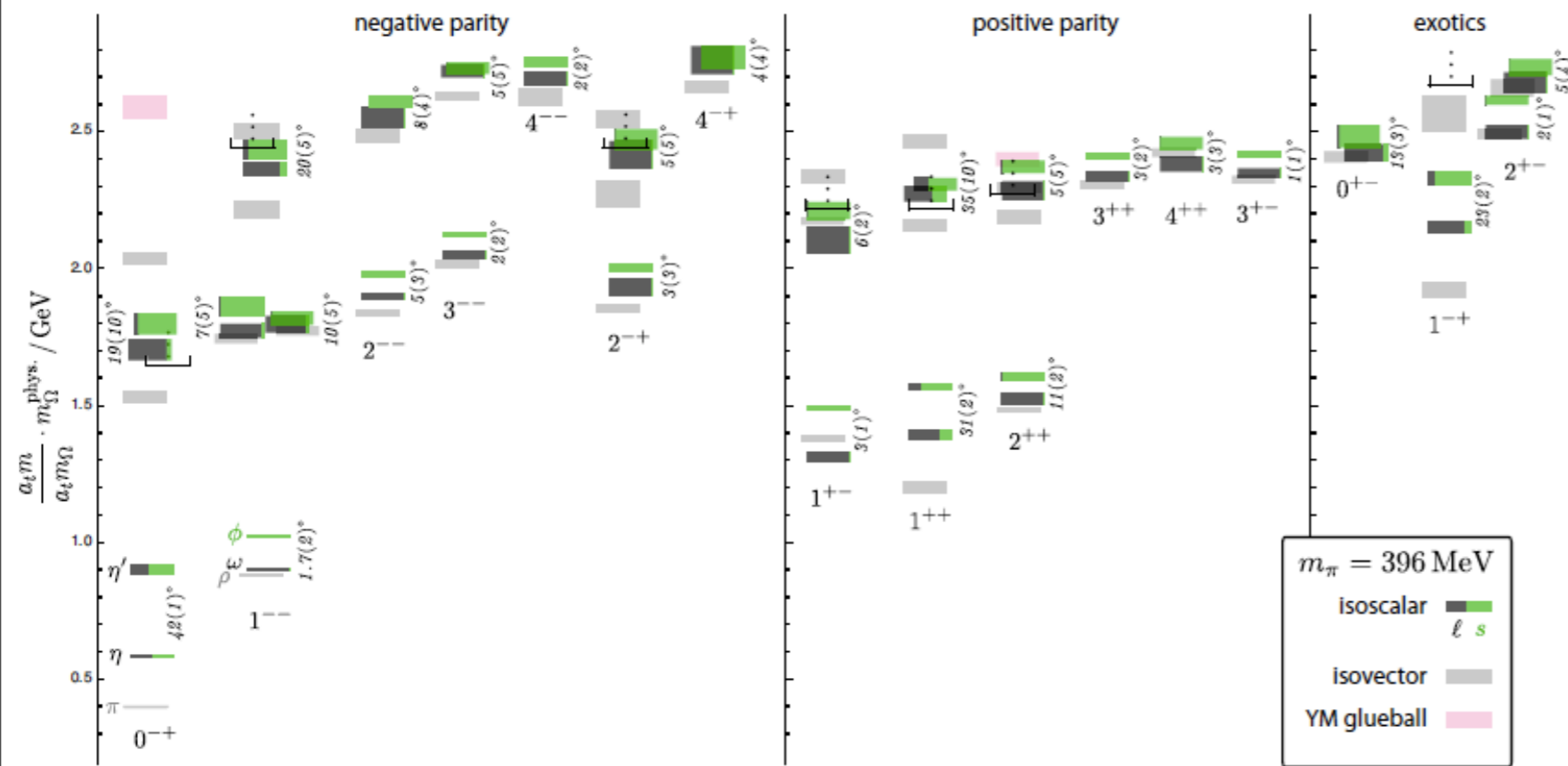
# LQCD Calculations Today



- Configurations
  - HMC, 2+1 (+1) flavors , isotropic or anisotropic
  - Domain-Wall, Clover, Staggered : det of matrix  $\gg 10^8 \times 10^8$
  - $\geq$  physical quark masses
  - L from  $\sim 2.5$  fm to  $> 12$  fm , a from  $\sim 0.1$  fm to  $< 0.05$  fm
  - No EM and degenerate light quarks
  - $< \sim 128$ K cpu cores ,  $\sim 10$ K trajectories,  $\sim 1$ K cfgs ,  $< 10$  GB/cfg
  - generated once, saved for use by many (USQCD)
  
- Propagators
  - $< \sim 64$ K cpu and  $\sim 256$  gpu (parallel code)
  - generated, used for correlation functions, deleted
  - $< 100$  GB
  - 1 (HEP) to 500 (NP) propagators per cfg
  - invert Dirac operator : deflation, multi-grid, ...
  
- Contractions
  - Hundreds of different correlation functions per propagator
  - permutations - recursion - needs algorithmic development, arprec
  - one xml file saved for subsequent analysis

# The Structure of Hadrons





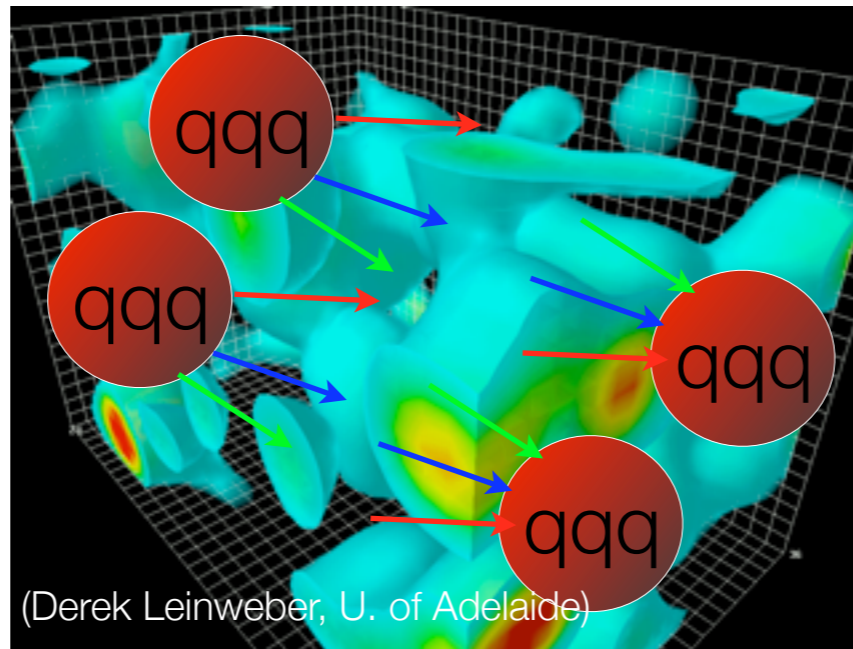
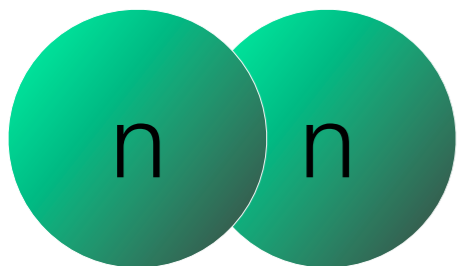
Dudek *et al* , arXiv:1102.4299

Lattice QCD will predict the exotic spectrum before  
or during the GlueX experiment  
(with sufficient compute resources)

**NSAC Milestone 2018 HP15:** The first results on the search for exotic mesons using photon beams will be completed.

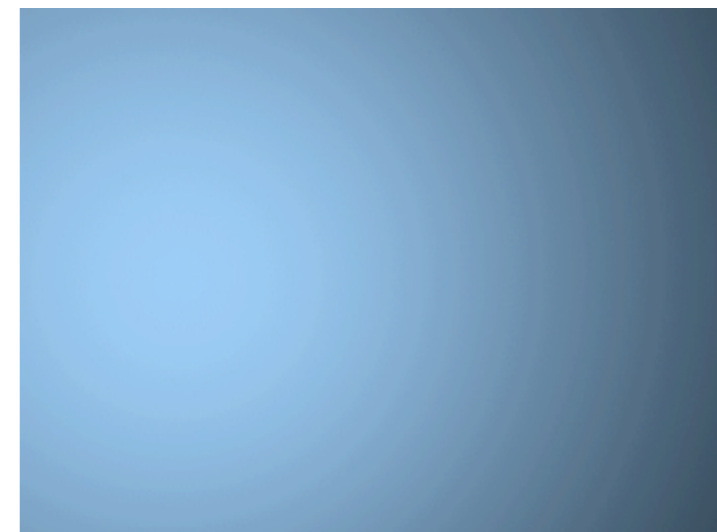
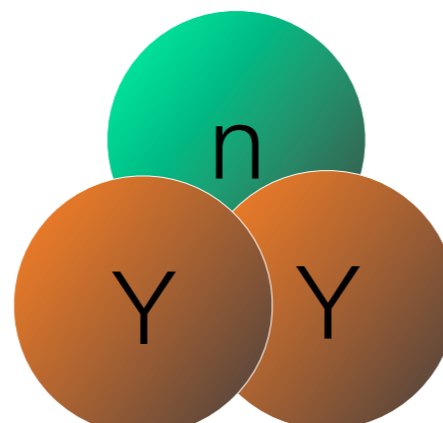
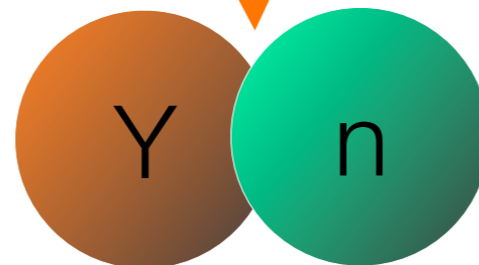
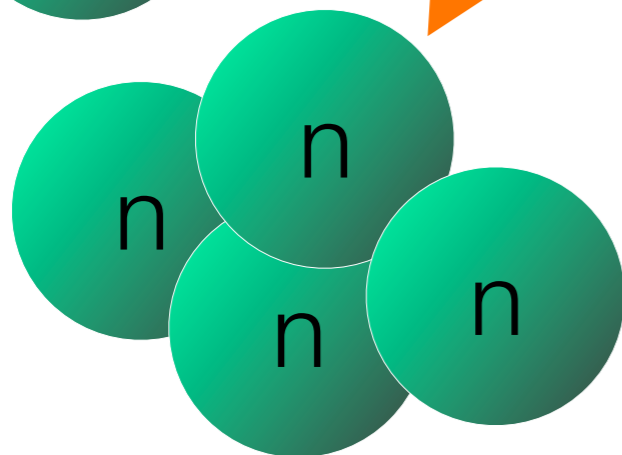
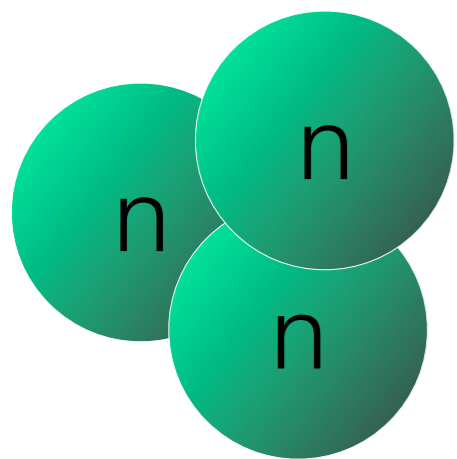
# Multi-Hadron Systems

NN-interaction  
verification



Hadron-Hadron

- 1) T-Matrix ( $E$ ) ala Luscher  
NPLQCD
- 2)  $U(E, \mathbf{r}, \mathbf{r}', \text{sink})$   
HALQCD



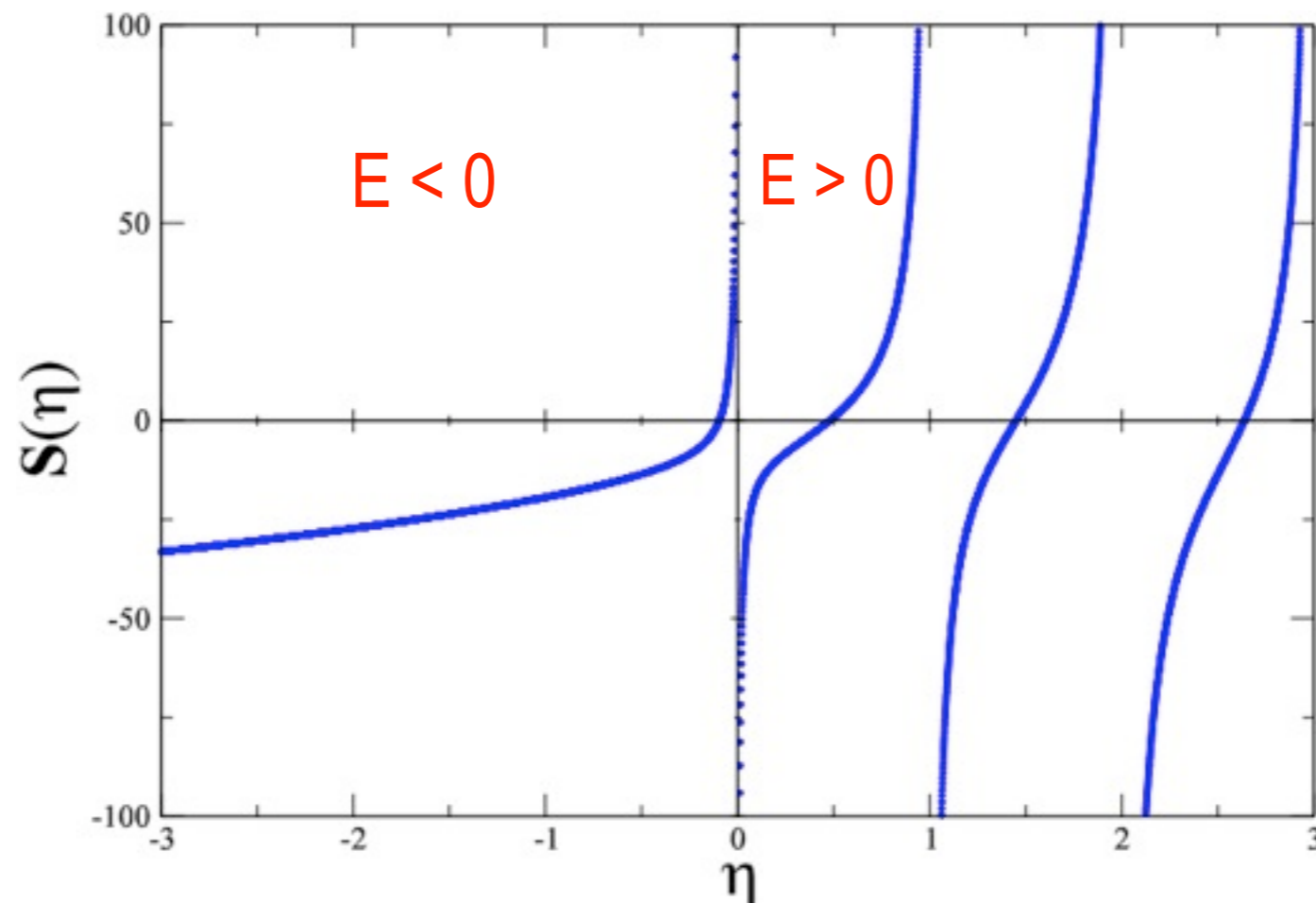
# Energy Eigenvalues and the Luscher Relation

Below Inelastic Thresholds :  
Measure on lattice

$$\delta E = 2\sqrt{p^2 + m^2} - 2m$$

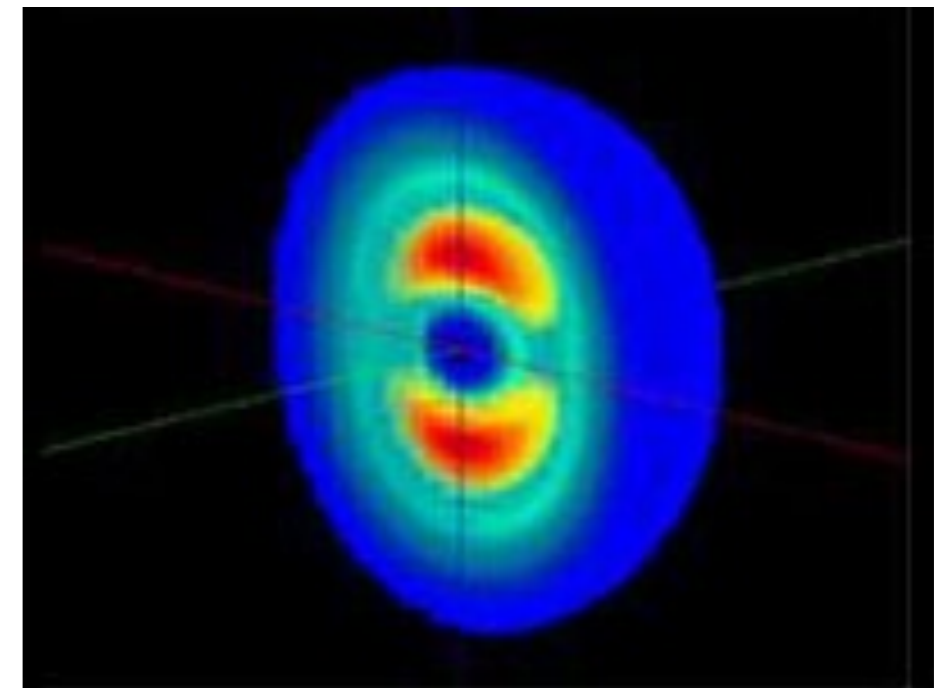
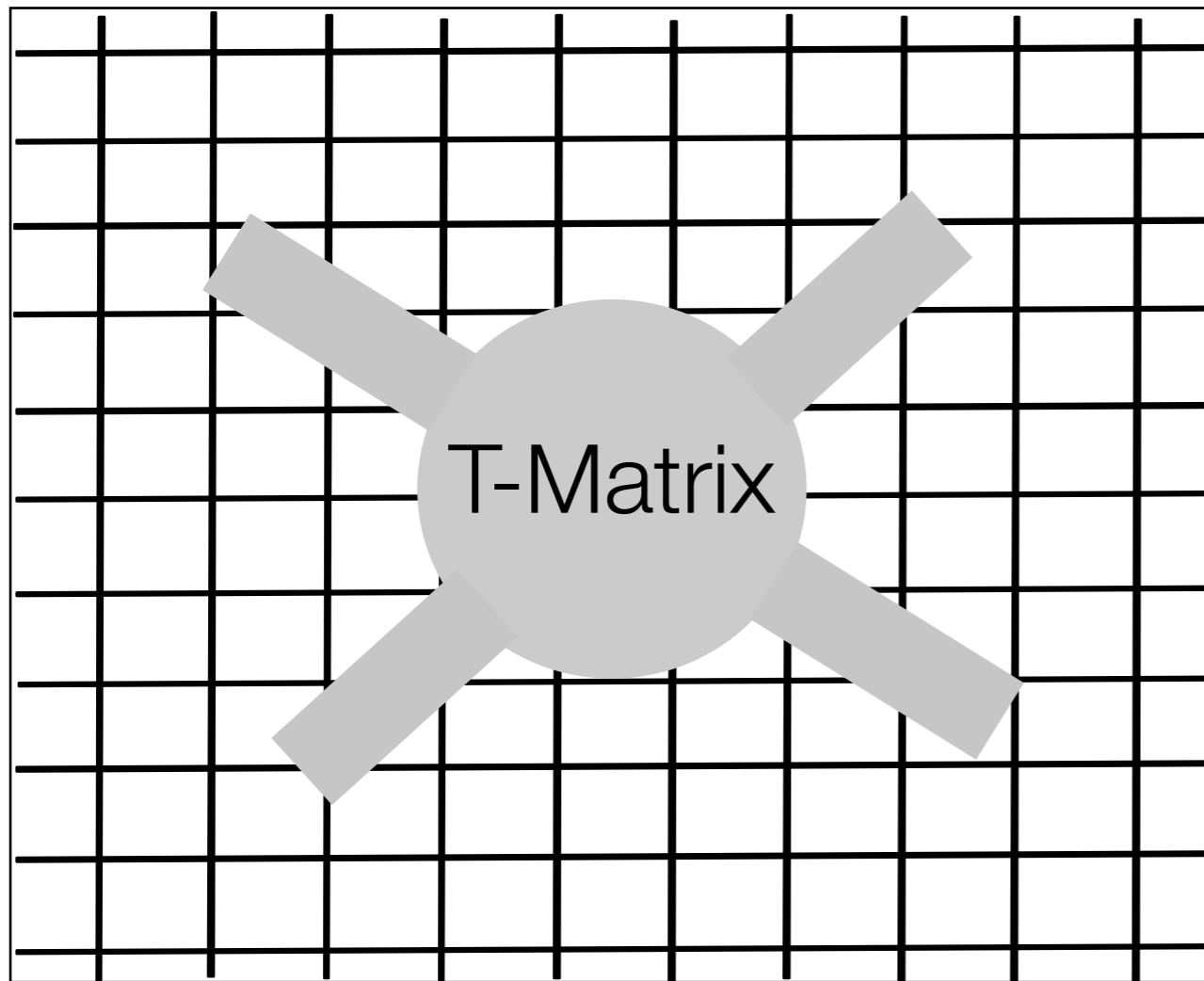
$$p \cot \delta(p) = \frac{1}{\pi L} \mathbf{S} \left( \left( \frac{Lp}{2\pi} \right)^2 \right)$$

$A_1^+$





Large Scattering Lengths are OK !

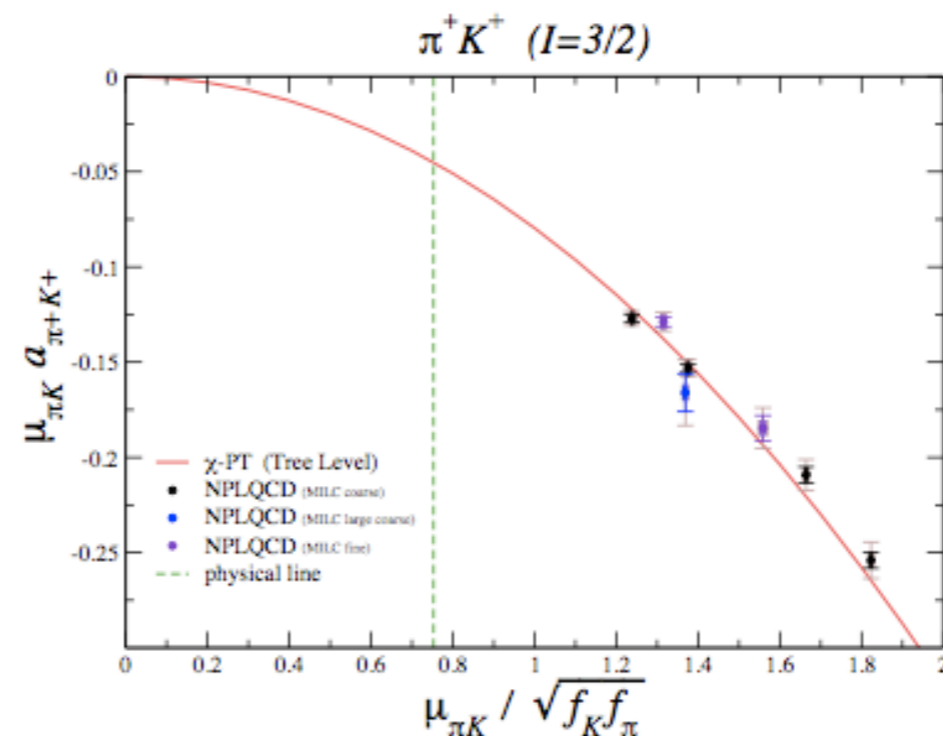
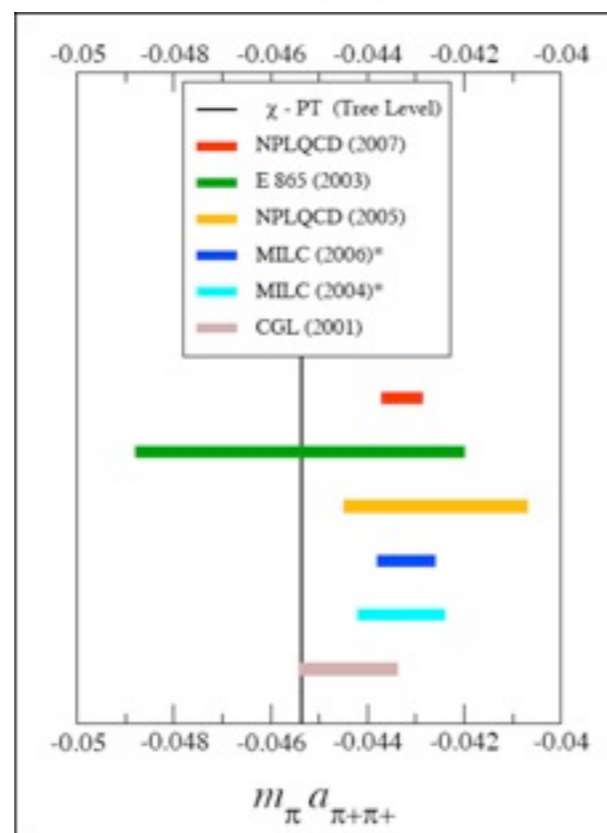
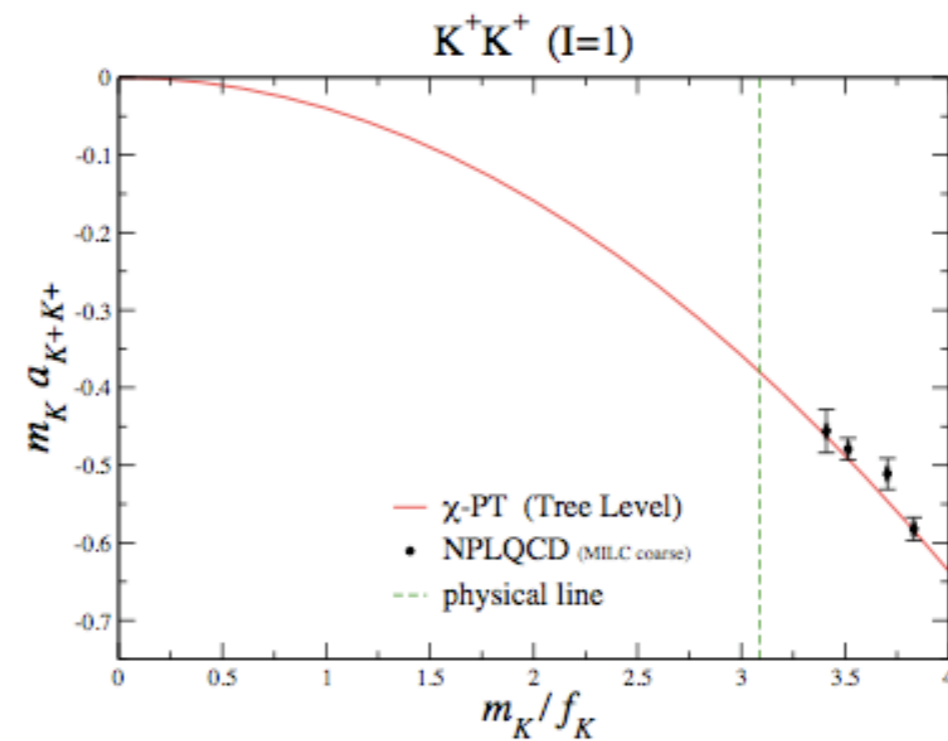
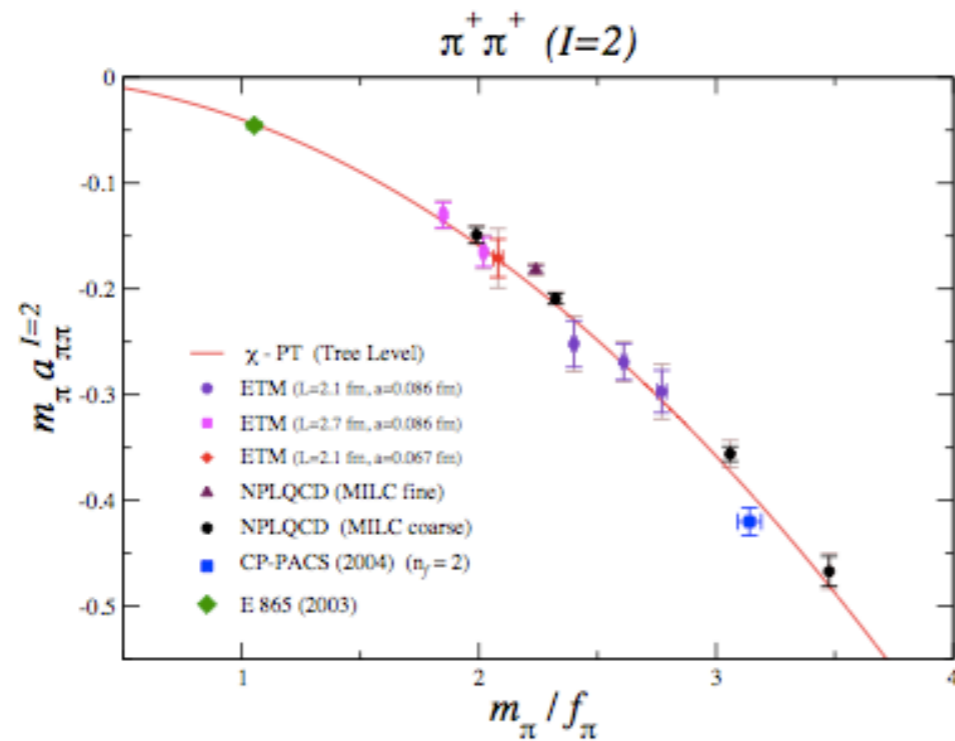


Require :  $L \gg r_0$  but **ANY**  $a$



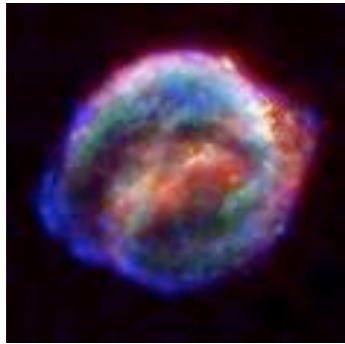


# Lattice QCD and the Simplest Hadronic Interactions



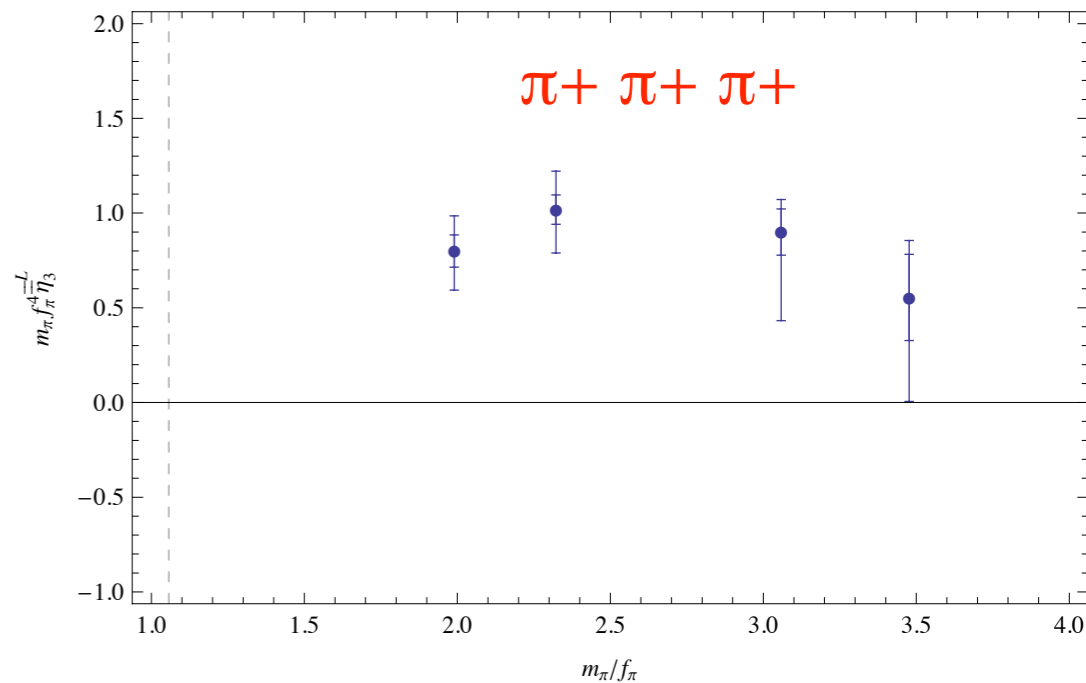
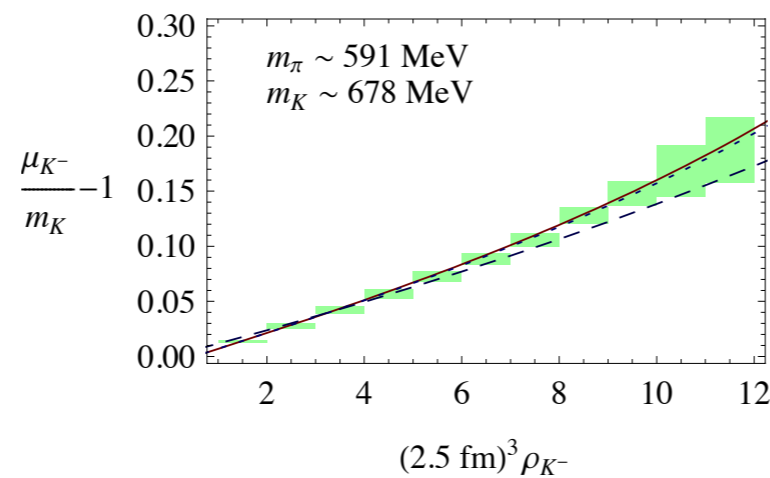
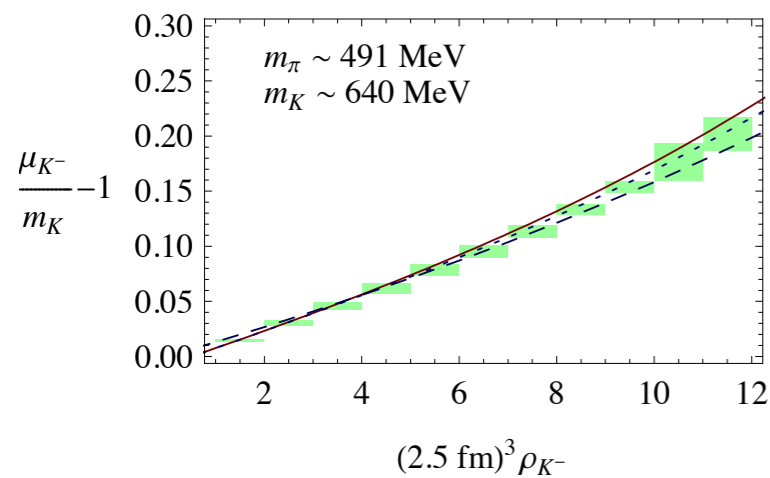
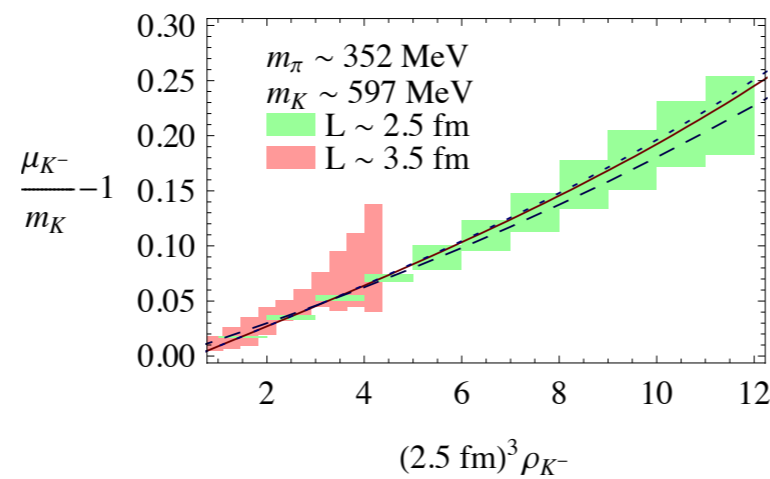
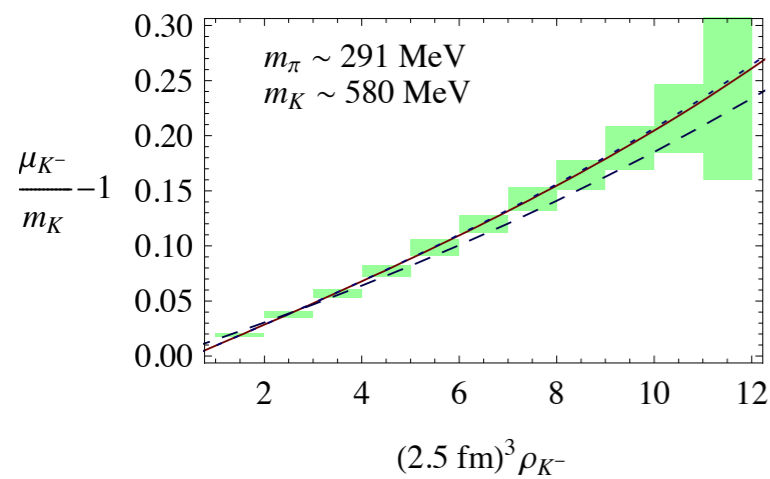
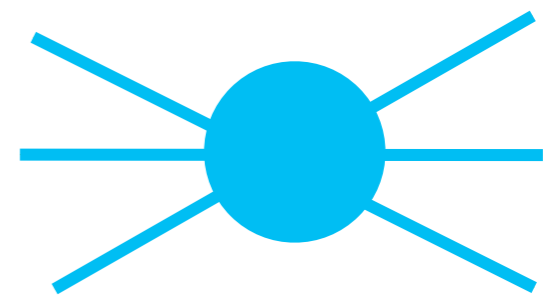


# Bose-Einstein Condensates of pions and kaons : Many-Body Physics



Kaon Condensates

Pion 3-Body Interaction



- Systems with kaons and pions : Detmold + Smigielski



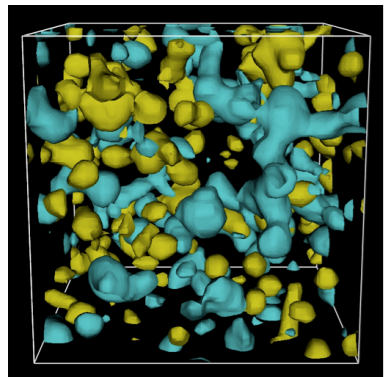
# Multi-Volume Study by NPLQCD : 2009 - 2011

lattice spacing :  $b \sim 0.123$  fm

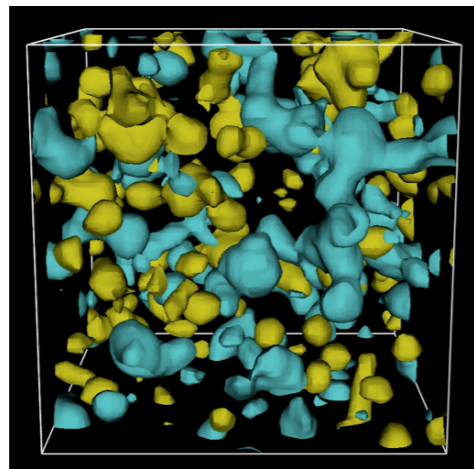
pion mass :  $m_\pi \sim 390$  MeV

fermion action : Clover

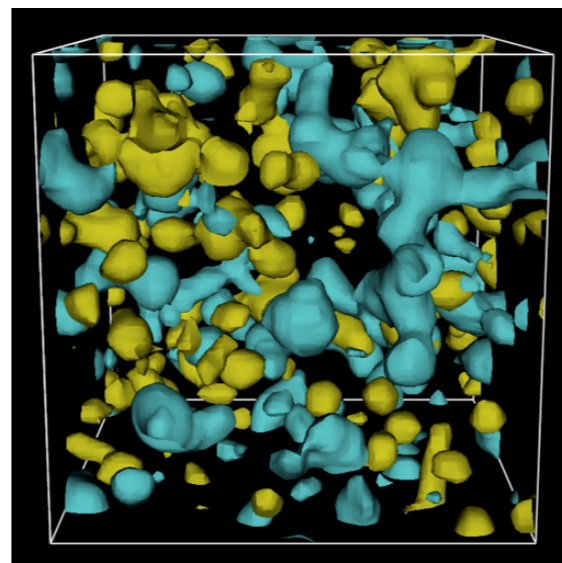
anisotropy :  $\xi_t \sim 3.5$



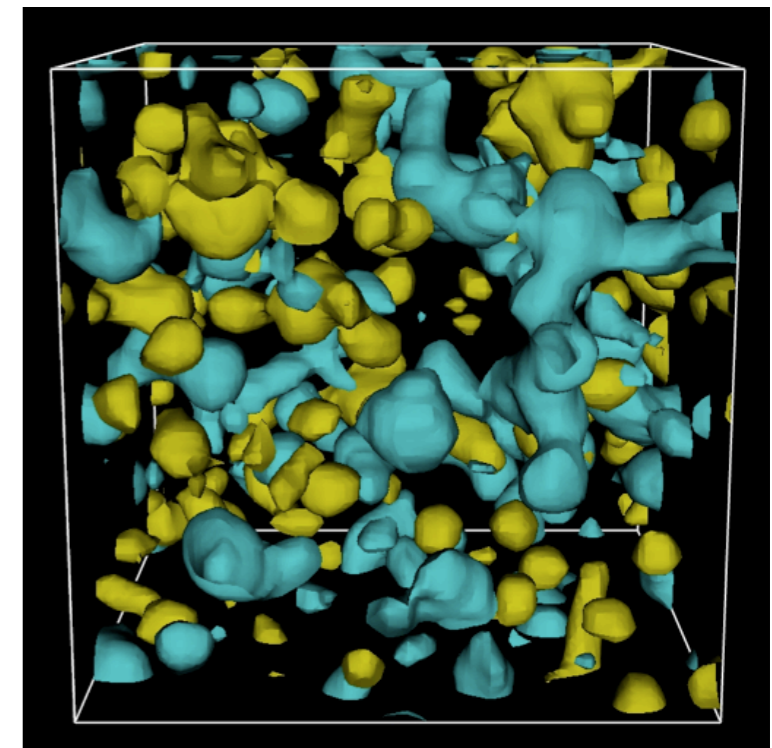
$L \sim 2$  fm



$L \sim 2.5$  fm



$L \sim 3$  fm



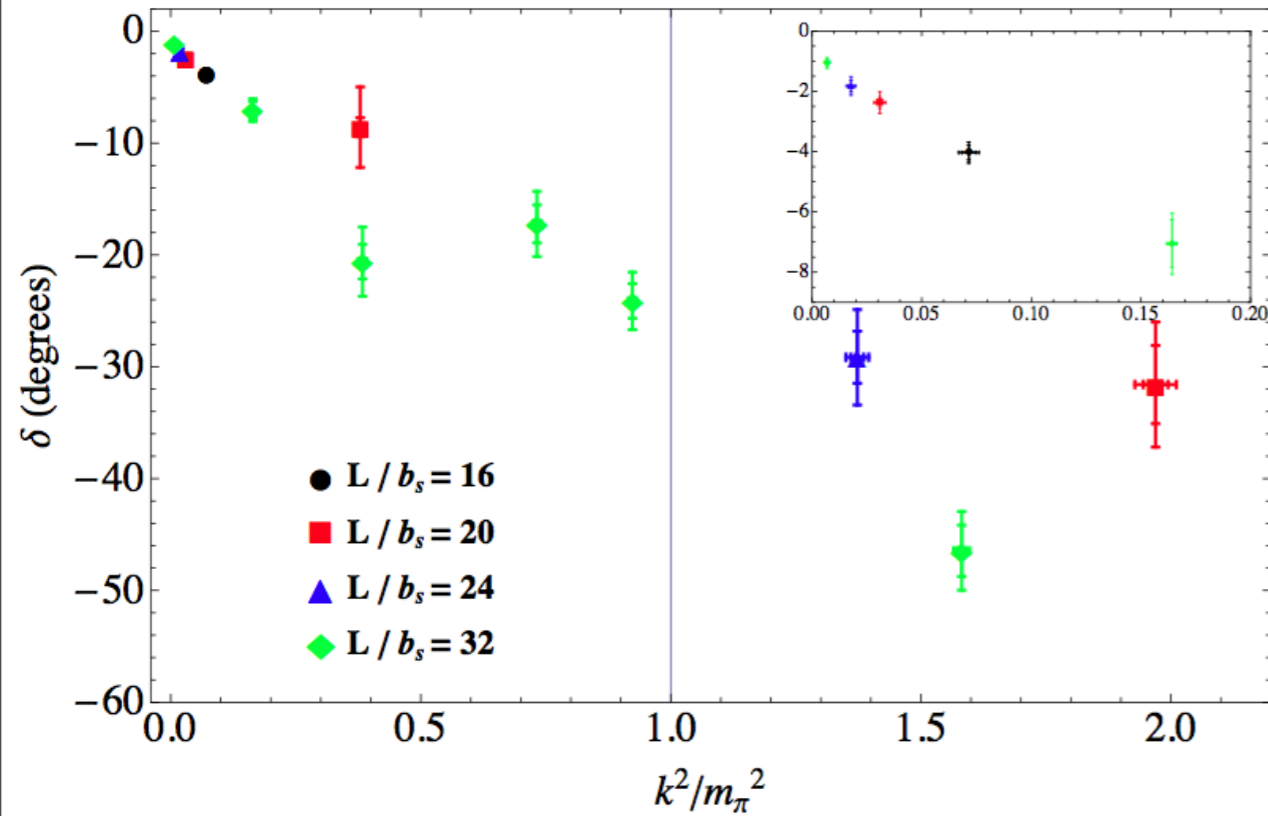
$L \sim 4$  fm

resources :  $\sim 80 \times 10^6$  core hrs

$m_\pi L \sim 4, 5, 6, 8$      $m_\pi T \sim 9, 9, 9, 18$



# I = 2 $\pi\pi$ Scattering Phase-Shift

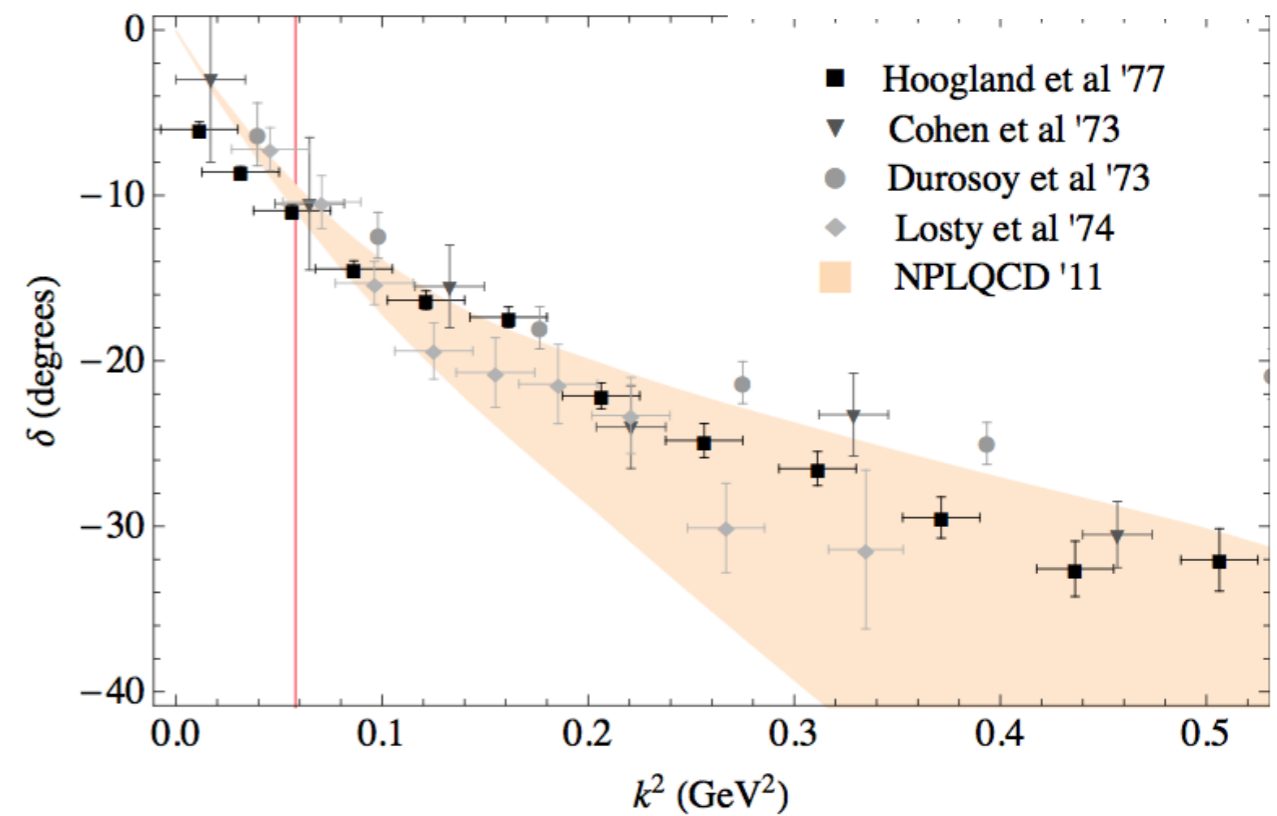
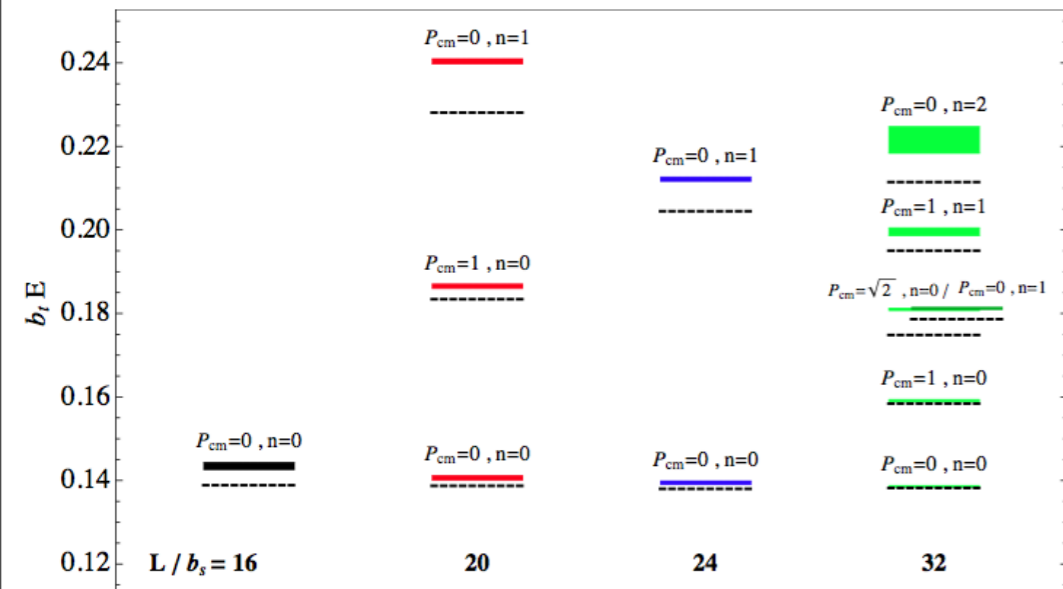


$m_\pi \sim 390$  MeV

$\chi$ PT

$m_\pi \sim 139$  MeV

lattice energy-levels



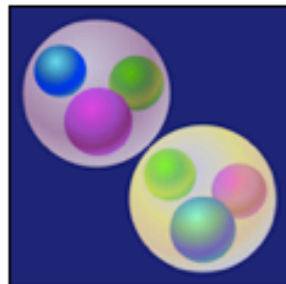


# H-Dibaryon An Exotic Nucleus



sics » Synopses » Binding baryons on the lattice

## Binding baryons on the lattice



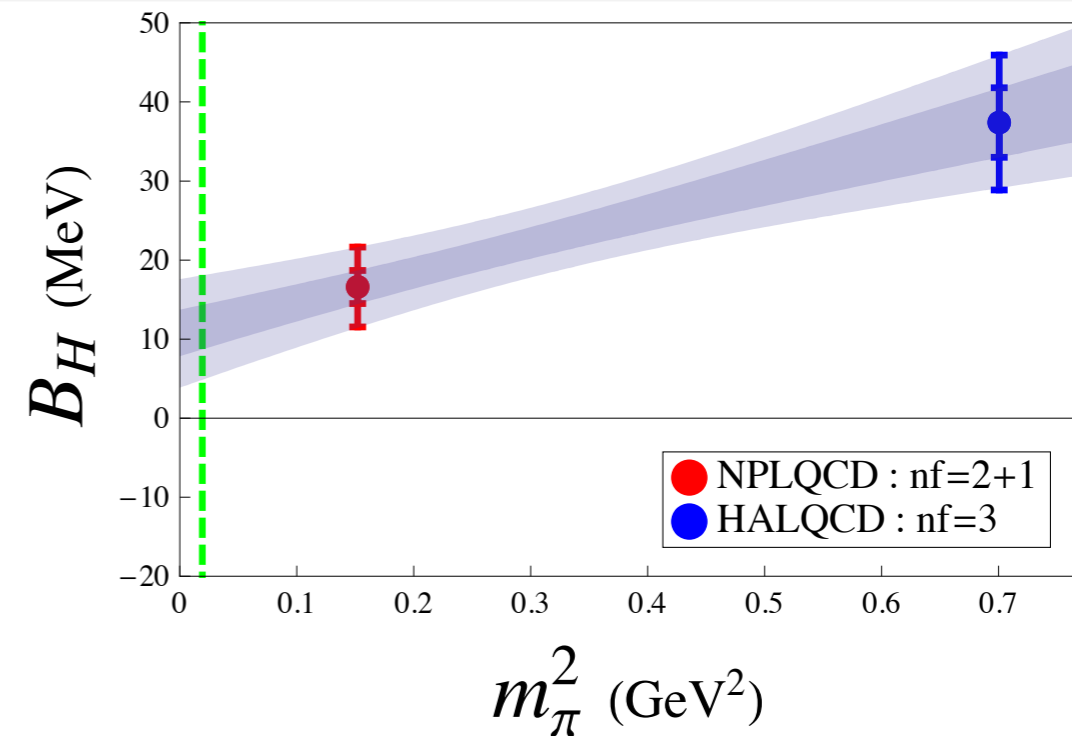
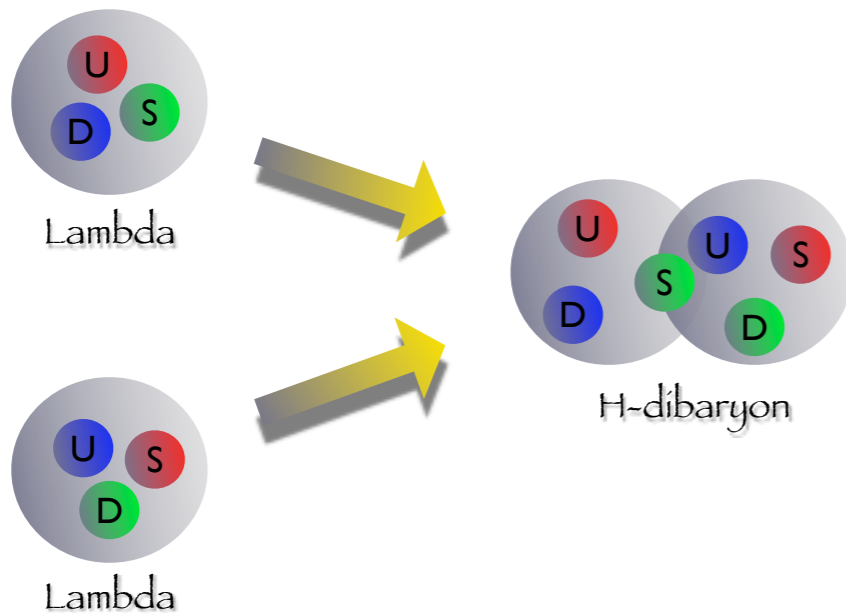
Credit: Alan Stonebraker

### Evidence for a Bound $H$ Dibaryon from Lattice QCD

S. R. Beane, E. Chang, W. Detmold, B. Joo, H. W. Lin, T. C. Luu, K. Orginos, A. Parreño, M. J. Savage, A. Torok, and A. Walker-Loud (NPLQCD Collaboration)  
Phys. Rev. Lett. **106**, 162001 (Published April 20, 2011)

### Bound $H$ Dibaryon in Flavor $SU(3)$ Limit of Lattice QCD

Takashi Inoue, Noriyoshi Ishii, Sinya Aoki, Takumi Doi, Tetsuo Hatsuda, Yoichi Ikeda, Keiko Murano, Hidekatsu Nemura, and Kenji Sasaki (HAL QCD Collaboration)  
Phys. Rev. Lett. **106**, 162002 (Published April 20, 2011)



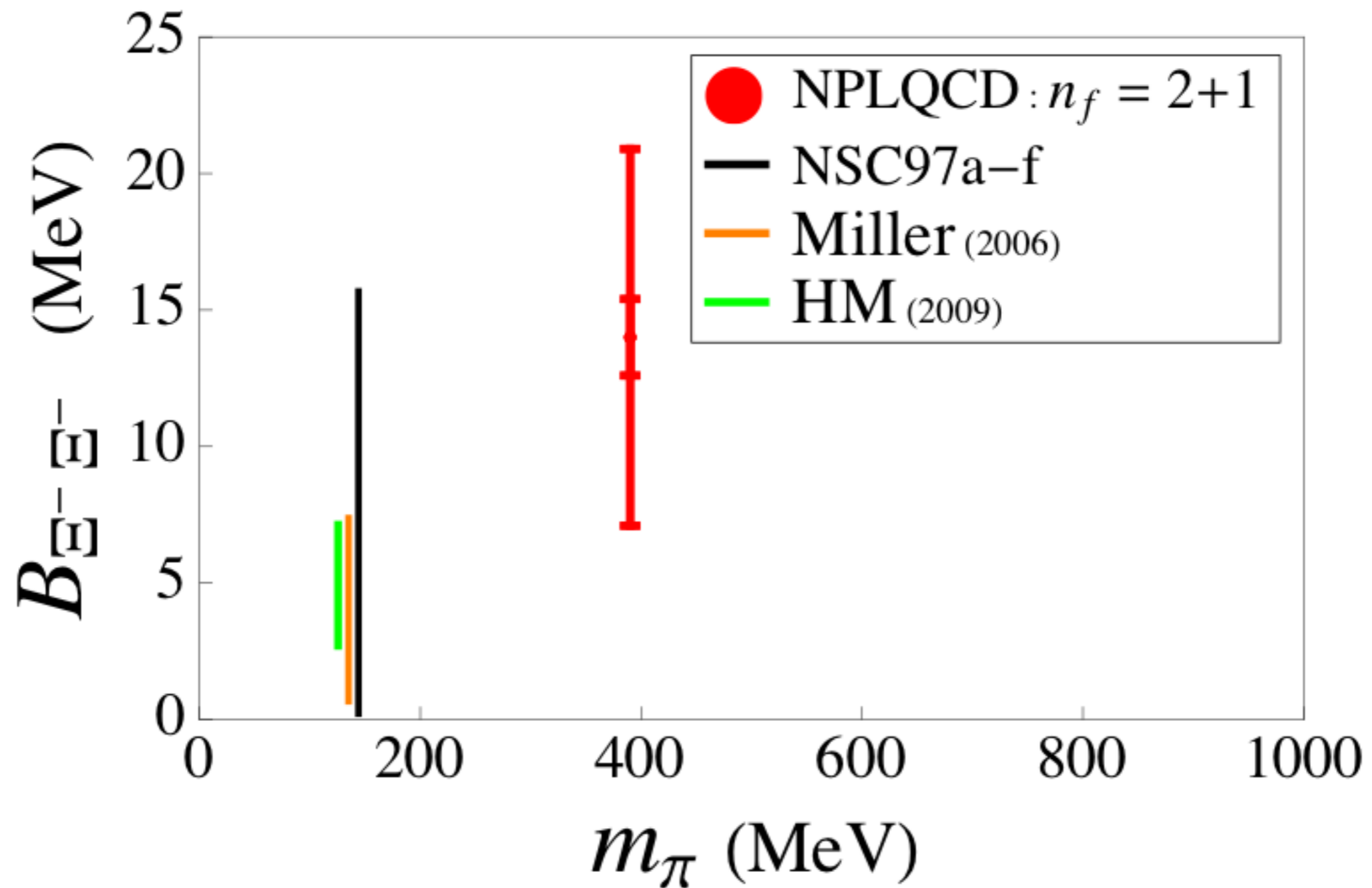
**NSAC Milestone 2014 HP10:** Carry out *ab initio* microscopic studies of the structure and dynamics of light nuclei based on two-nucleon and many-nucleon forces and lattice QCD calculations of hadron interaction mechanisms relevant to the origins of the nucleon-nucleon interaction



# $\Xi^- \Xi^-$ Bound State



NPLQCD arXiv:1109.2889 – this week!

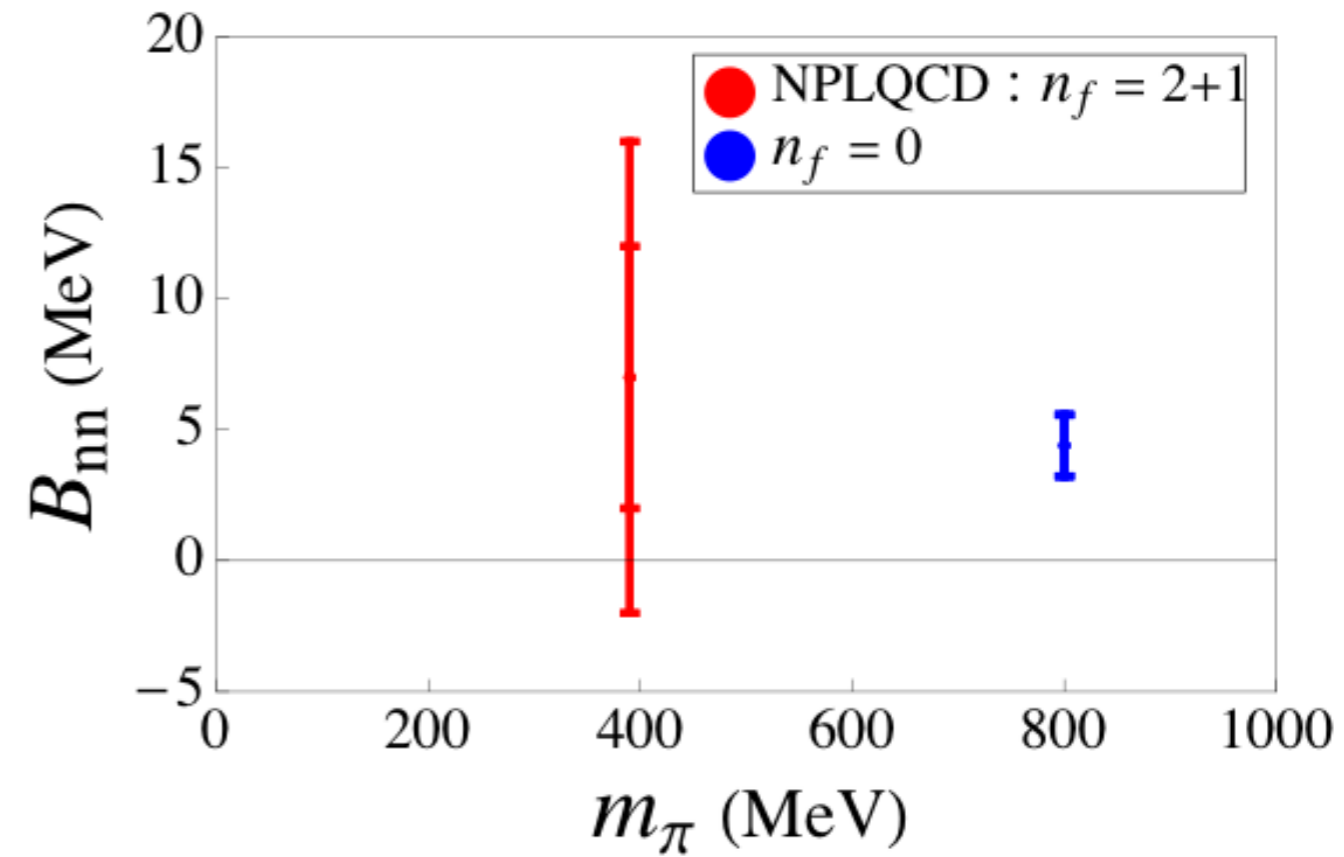
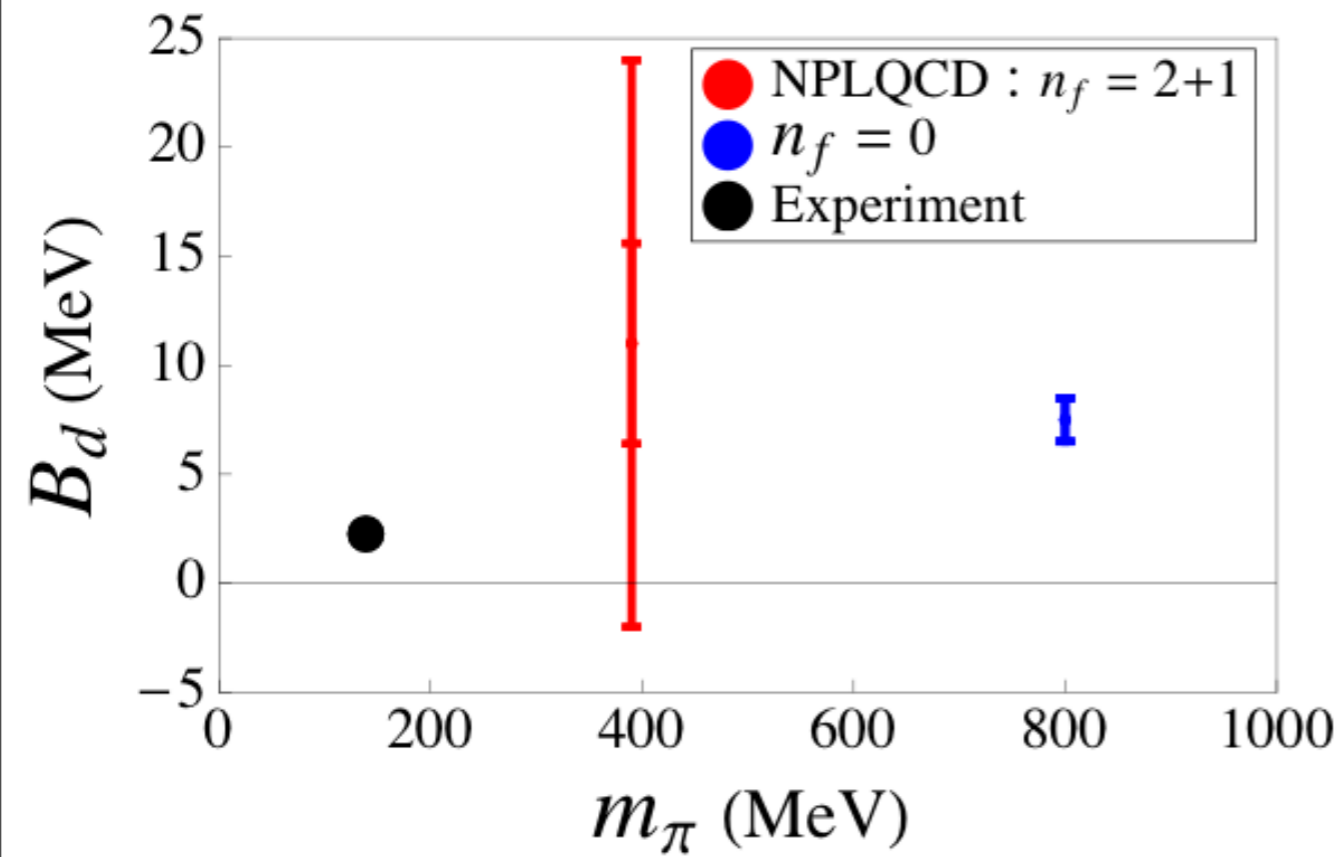




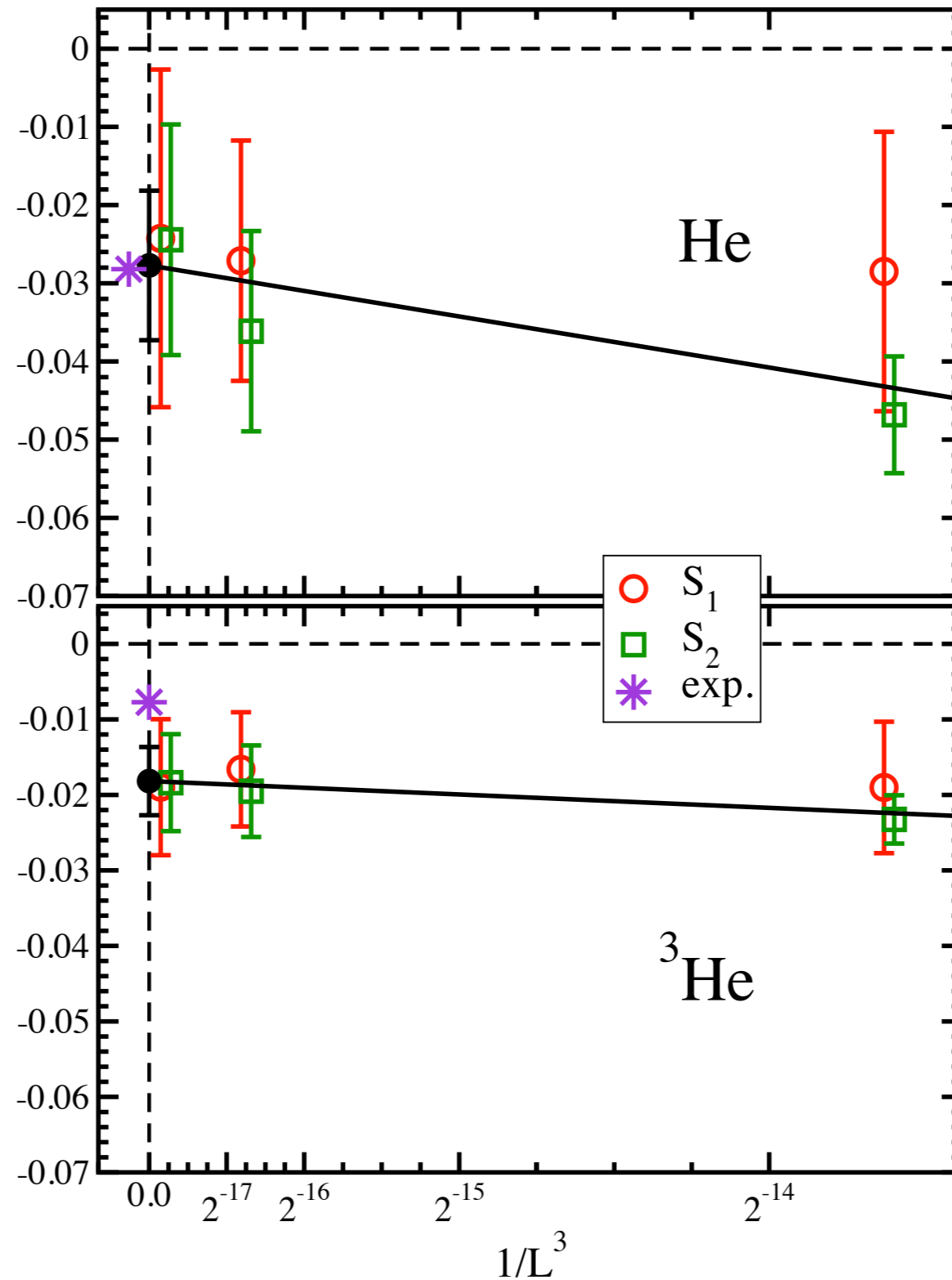
# NN Bound States



NPLQCD arXiv:1109.2889 – this week!

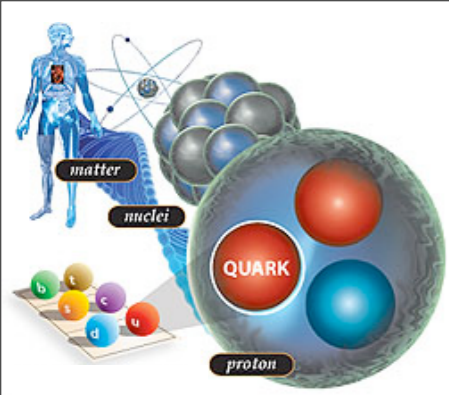


# Quenched $A = 3$ and $4$ , $m_\pi \sim 800$ MeV

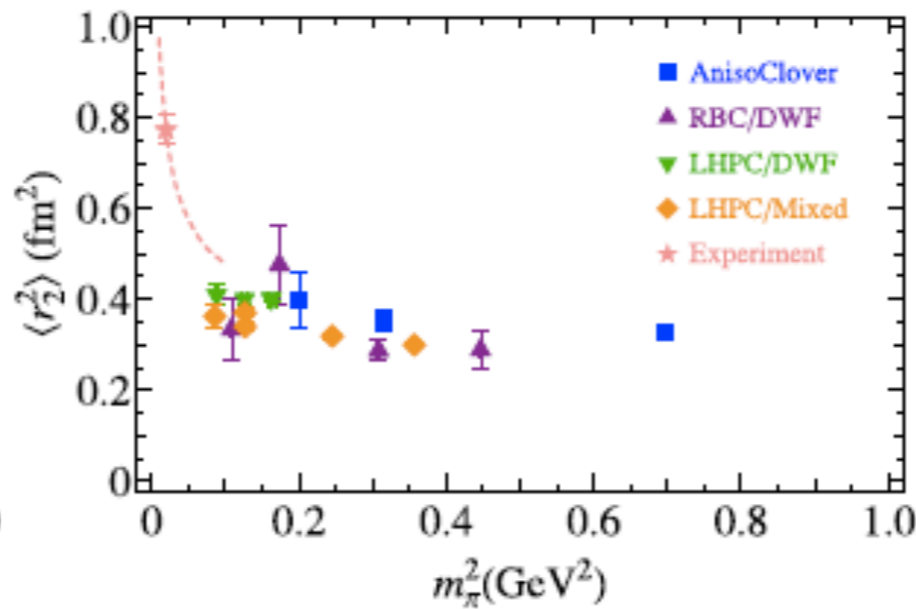
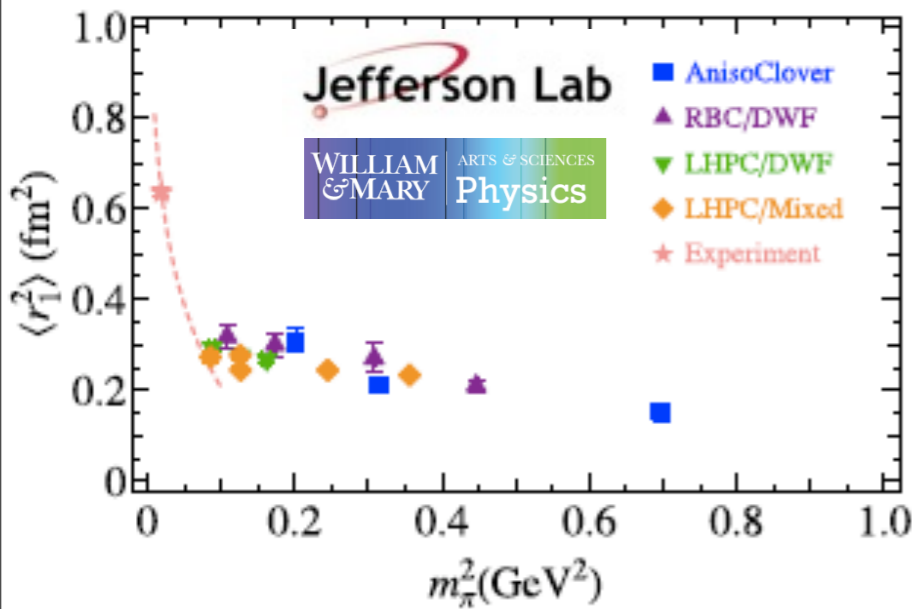
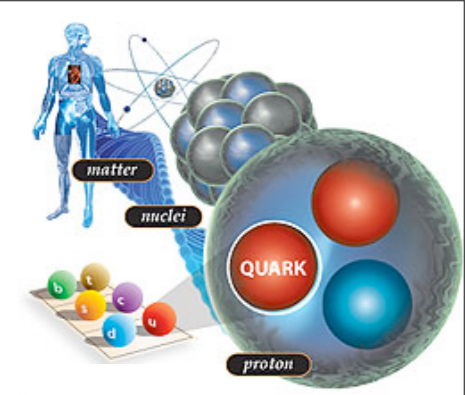


PACS-CS  
2009

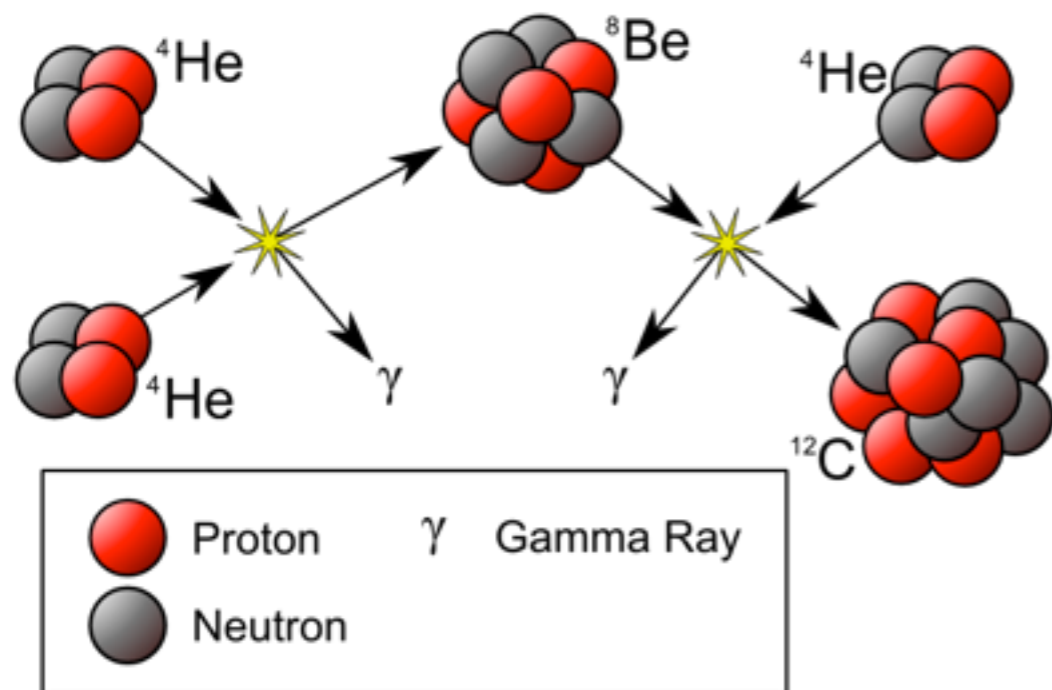




# Nuclear Physics at the Physical Pion Mass



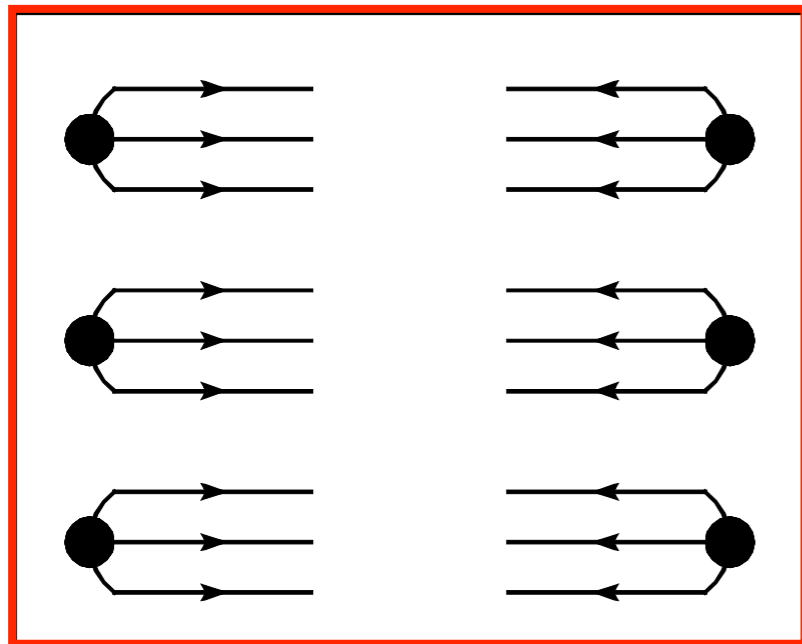
Chiral Behavior  
-QCD symmetries



Nuclear Physics is Fine-Tuned  
 -QCD input parameters  
 -Our universe is special ?

# Many Nucleons (Baryons)

Large number of Wick contractions



$$\text{Proton} : N^{\text{cont}} = 2$$

$${}^{235}\text{U} : N^{\text{cont}} = 10^{1494}$$

$$\begin{aligned} N_{\text{cont.}} &= u!d!s! \quad (\text{Naive}) \\ &= (A + Z)!(2A - Z)!s! \\ &\sim A^3 \quad (\text{Kaplan}) \end{aligned}$$

Symmetries provide significant reduction

$${}^3\text{He} : 2880 \rightarrow 93$$

Recursion Relations

# Simulated Calculations of the Deuteron

(NOT actual calculations)

Precision Level of Energy Shift	Bound State Energy (MeV)	1 <sup>st</sup> Continuum Level (MeV)
0%	-3.147	4.005
1%	$-3.111 \pm 0.031$	$4.015 \pm 0.040$
5%	$-2.95 \pm 0.16$	$4.24 \pm 0.20$
10%	$-2.66 \pm 0.31$	$3.65 \pm 0.40$

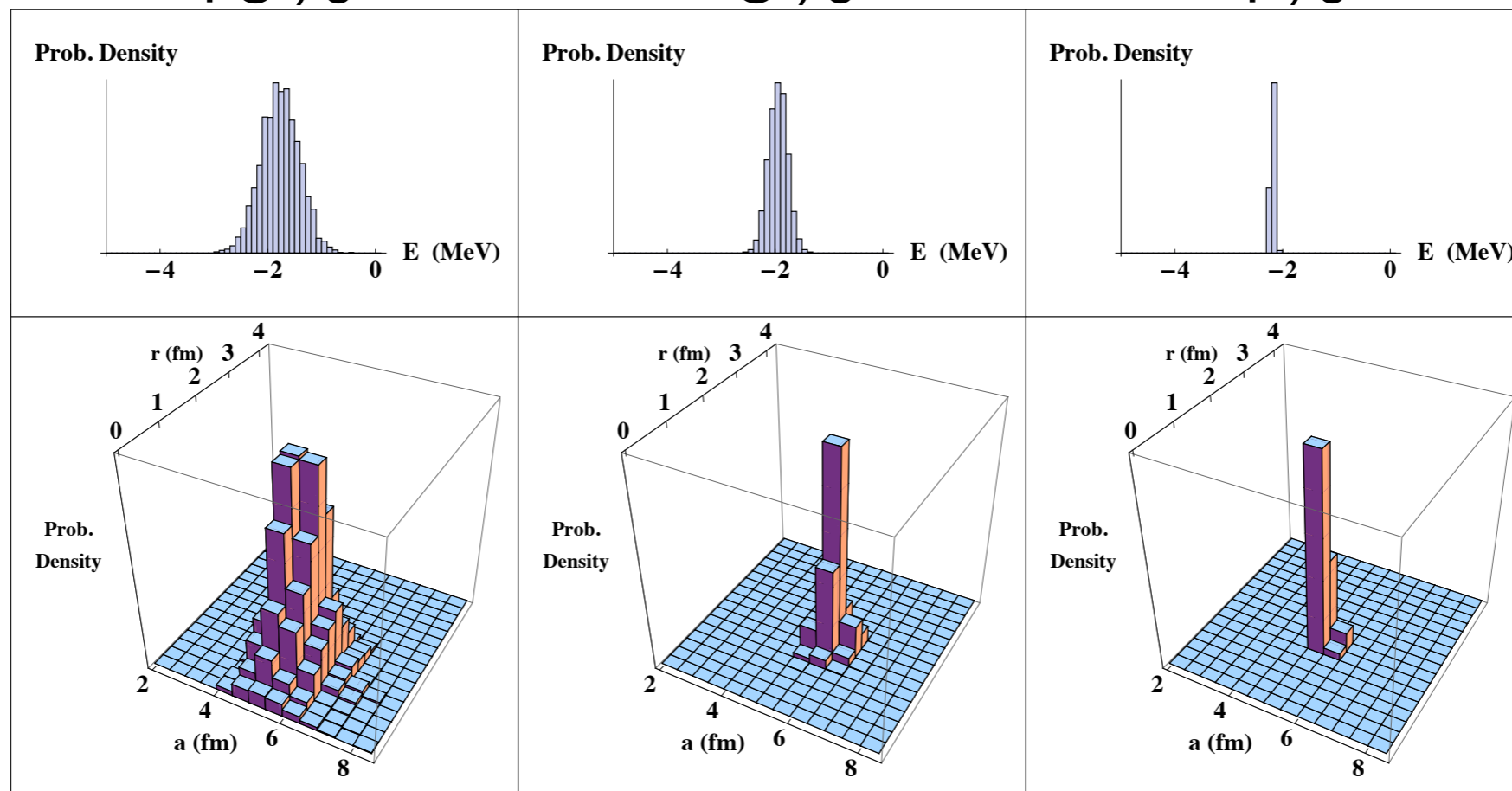
$$E_D \sim 2 \text{ GeV}$$

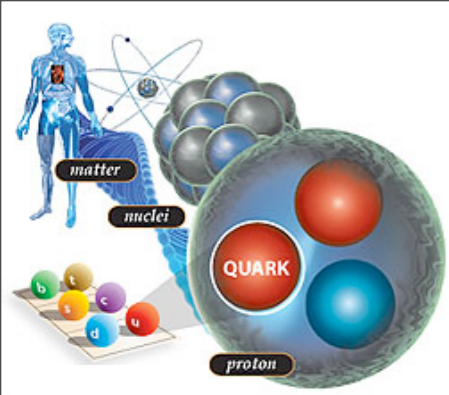
$$\Delta E_D \sim 2 \text{ MeV}$$

10%

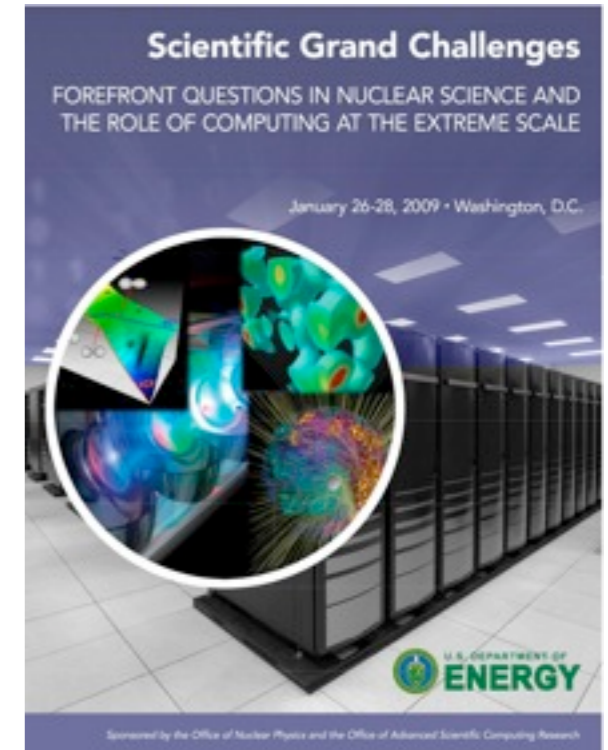
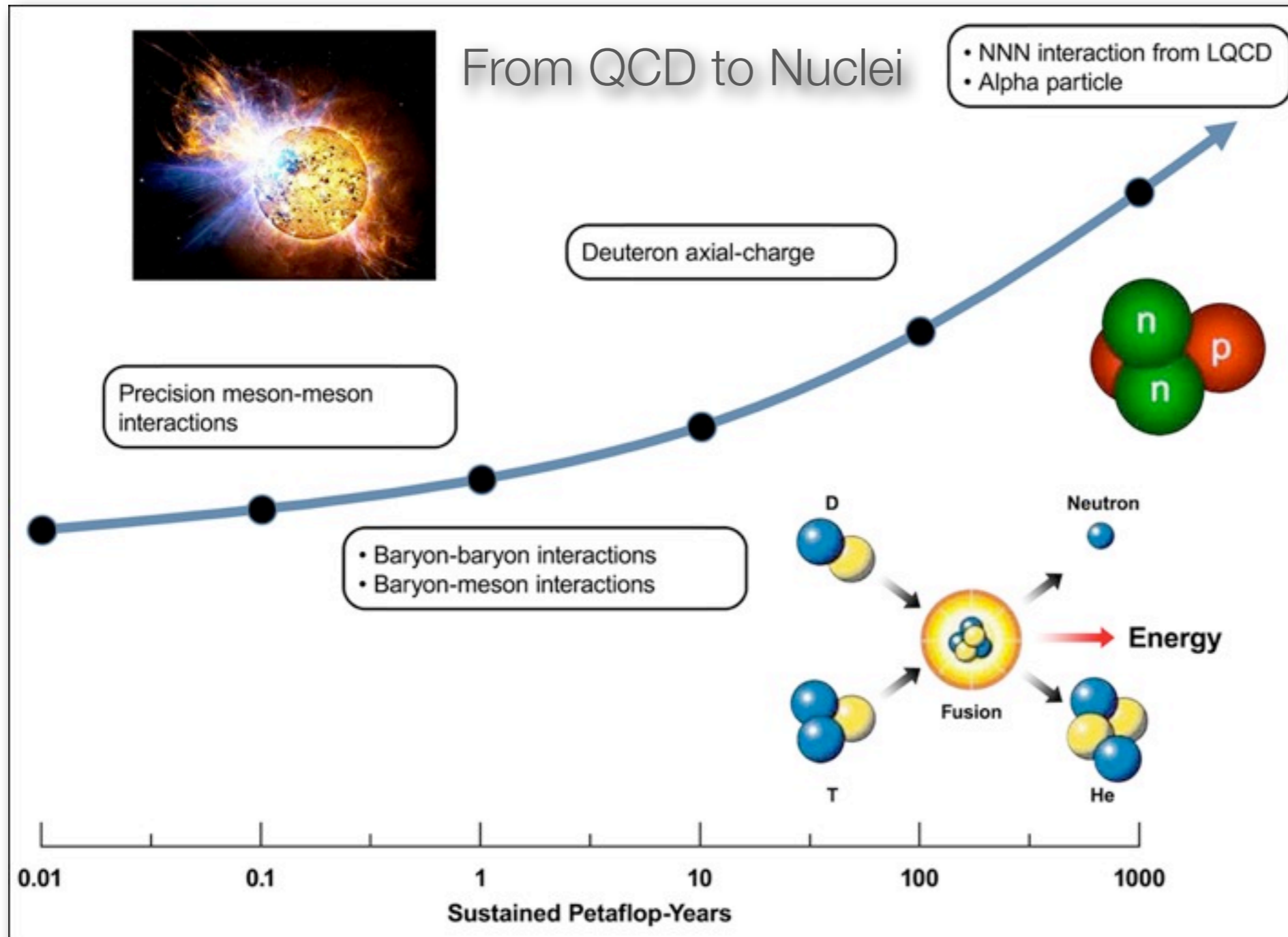
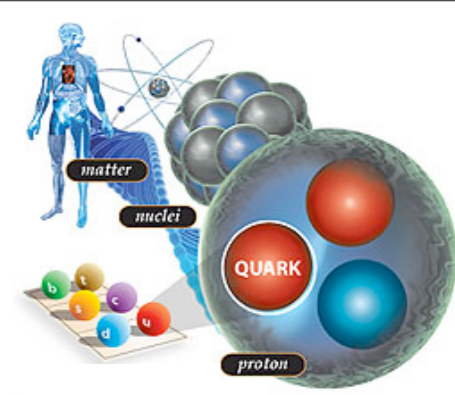
5%

1%





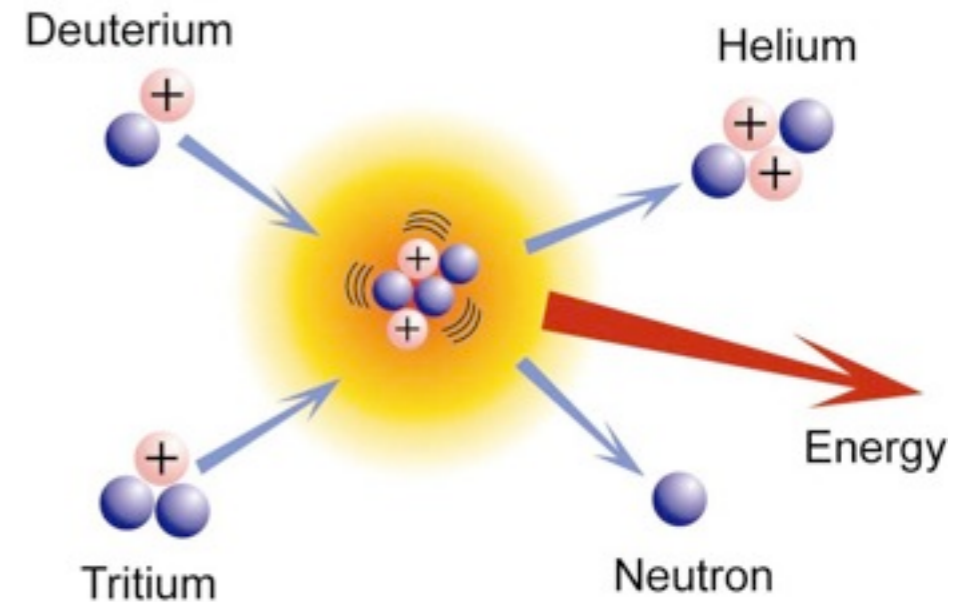
# Computational Requirements



Electromagnetism  
 Isospin Breaking  
 The Real Deal !

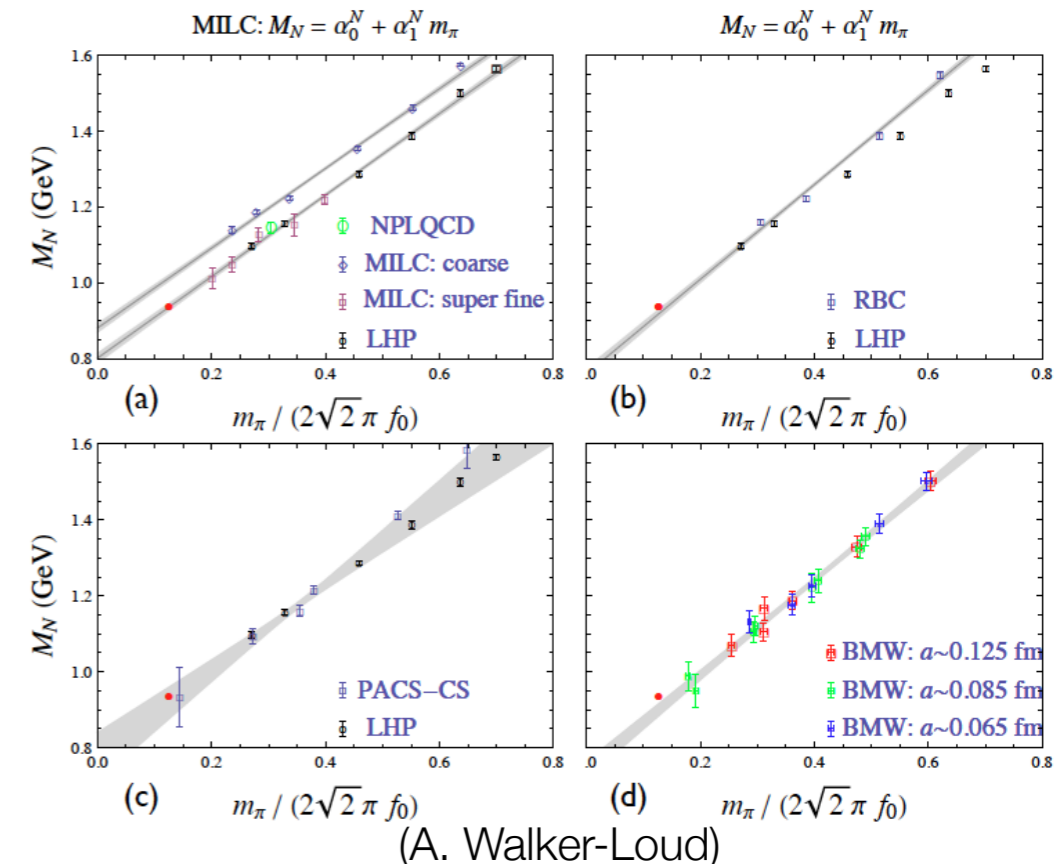
# Beyond Computational Requirements: Formal Issues , e.g.

What Lattice QCD calculations are required to predict multi-body nuclear reactions ?

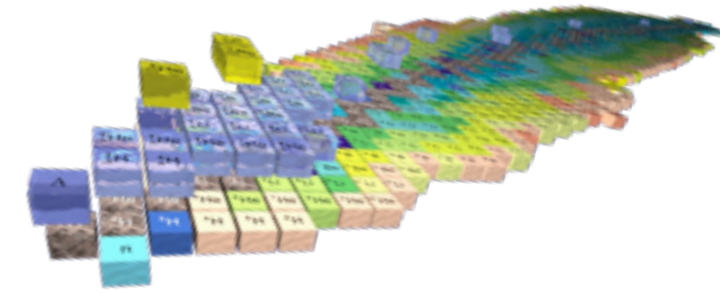


What length-scales determine the convergence of EFT expansions ?

(to predict more complex systems than LQCD can access)  
(quark masses, number of flavors?)

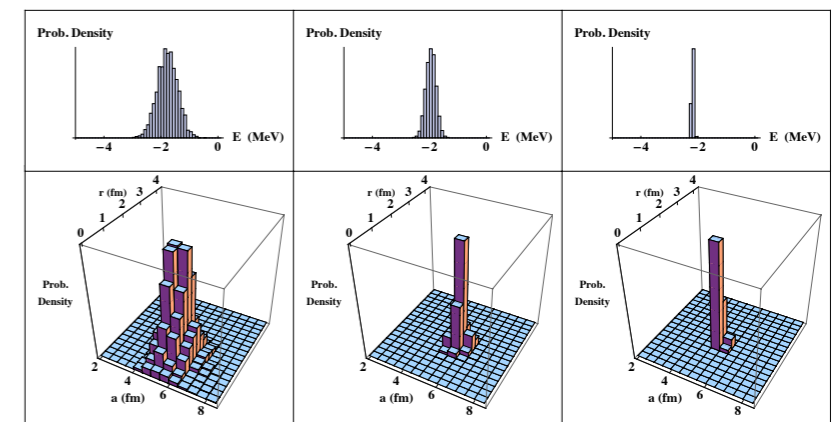


# Nuclear Theory Needs (physical $m_\pi$ )

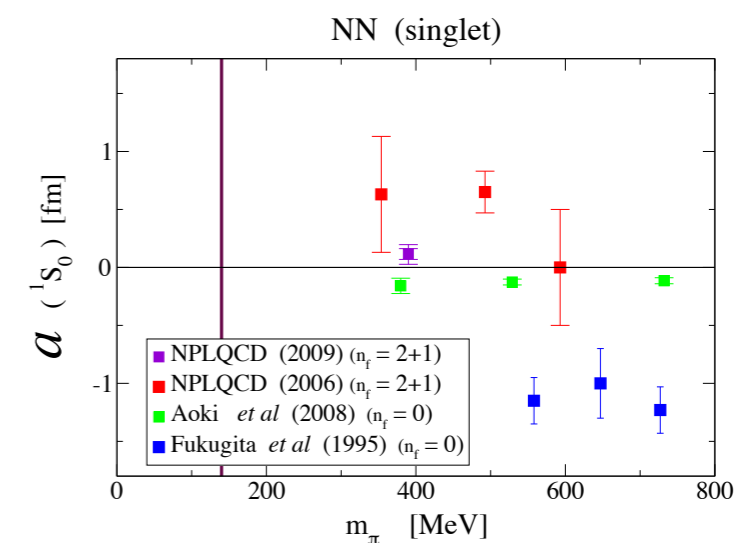


- What do we expect to observe in LQCD calculations in a finite volume
  - Need 3-body and 4-body spectrum (cubic volumes)
  - With and without background electroweak fields

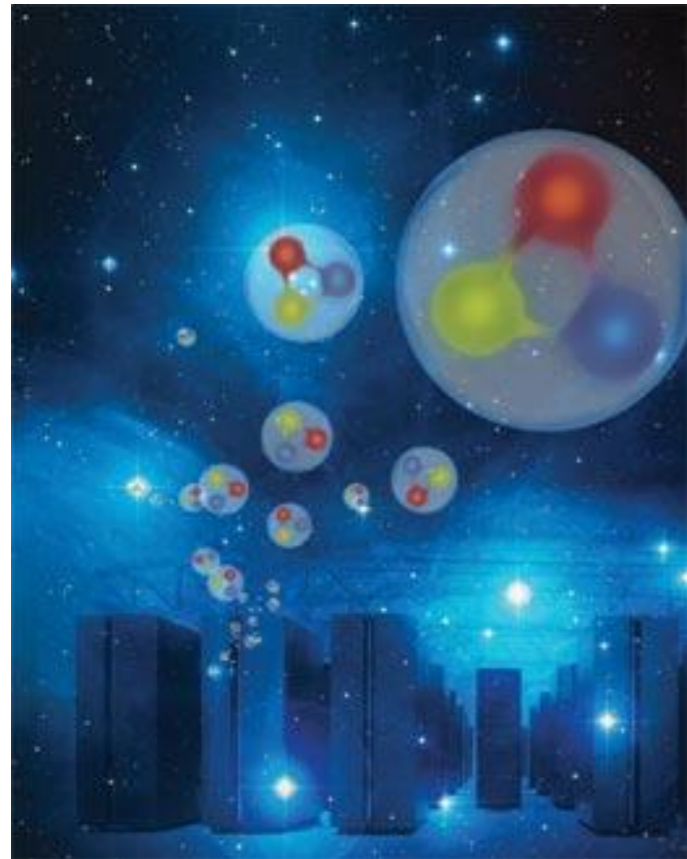
- How to optimally invert lattice data
  - what volumes should we calculate with ?



- Quark-mass dependence
  - Higher orders in EFT for few-body systems



# Closing Remarks



- Close to discovering how hadrons and nuclei emerge from quarks and gluons using Lattice QCD
- Moving toward light nuclear systems with quantifiable and removable uncertainties
  - input into larger scale nuclear calculations

Organizers : Thank You

END

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