INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS
33rd Course

RECENT DEVELOPMENTS IN NJL-JET MODEL: TMD

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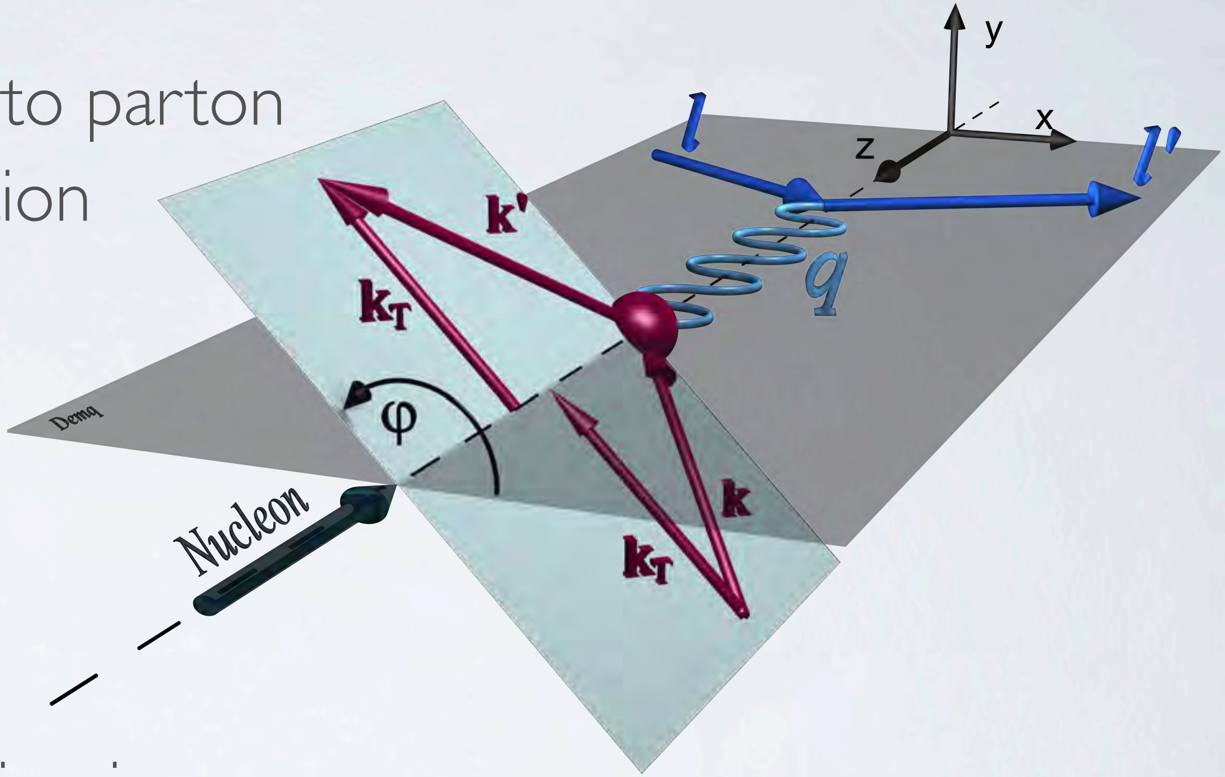
OUTLOOK

- Motivation
- Short Overview of the NJL-jet model and Monte-Carlo approach:
  - Strange quark and Kaons, Vector mesons, Nucleon-Antinucleon channels, secondary hadrons from the decays of resonances.
- Transverse Momentum Dependent FF, Hadron TM in SIDIS.
- Future Plans.

EXPLORING HADRON STRUCTURE

A. Kotzinian, Nucl. Phys. B441, 234 (1995).

- Semi-inclusive deep inelastic scattering (SIDIS): e *N* → e *h* *X*
- Cross-section factorizes into parton distribution and fragmentation functions.



Access to hadron structure:

- Ex., unpolarized cross section is ~

$$\sum_q e_q^2 \int d^2\mathbf{k}_T f_1^q(x, k_T^2)\pi y^2 \frac{\hat{s}^2 + \hat{u}^2}{Q^4} D_q^h(z, p_\perp^2)$$

- NJL provides a sound framework for calculating both!

MOTIVATION

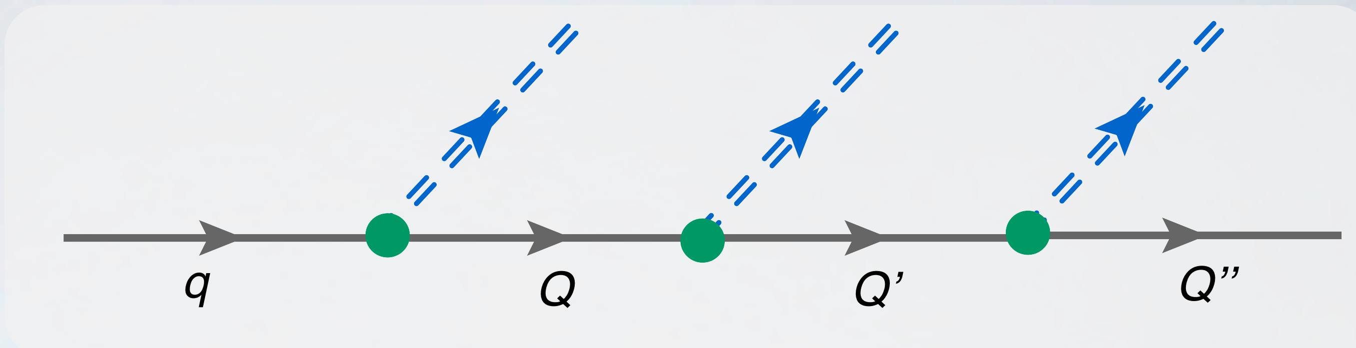
- Providing guidance based on a sophisticated model for applications to problems where phenomenology is difficult/ inadequate.
- Unfavored fragmentation functions from the model that goes beyond a single hadron emission approximation.
- Automatically satisfies the sum rules (at the model scale).
- Transverse-momentum dependent (TMD) fragmentations in the same model where structure functions (both unpolarized and polarized) were calculated.

THE QUARK JET MODEL

Field, Feynman.Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



The probability of finding mesons m with mom. fraction z in a jet of quark q

$$D_q^m(z)dz = \hat{d}_q^m(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^m\left(\frac{z}{y}\right)\frac{dz}{y}$$

Probability of emitting the meson at link l

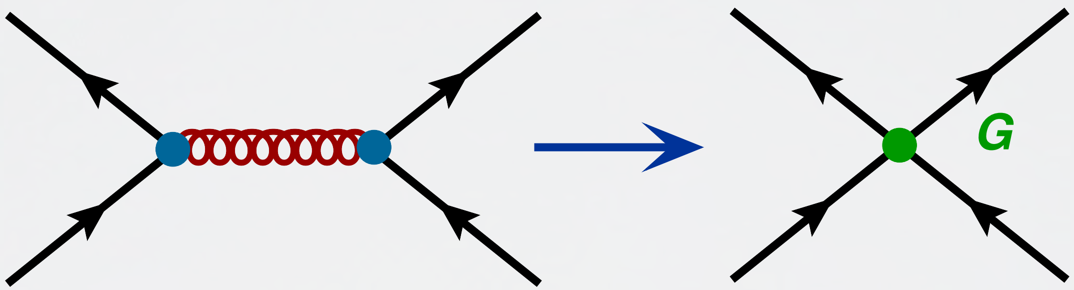
Probability of Momentum fraction y is transferred to jet at step l

The probability scales with mom. fraction

NAMBU--JONA-LASINIO MODEL

Effective Quark model of QCD

- Effective Quark Lagrangian $\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$



- Only 4-point interactions.
- Covariant, has the same flavor symmetries as QCD.
- Dynamically Generated Quark Mass from GAP Eqn.
- Lepage-Brodsky (LB)Invariant Mass Cutoff Regularization

$$M_{12} \leq \Lambda_{12} \equiv \sqrt{\Lambda_3^2 + M_1^2} + \sqrt{\Lambda_3^2 + M_2^2}$$

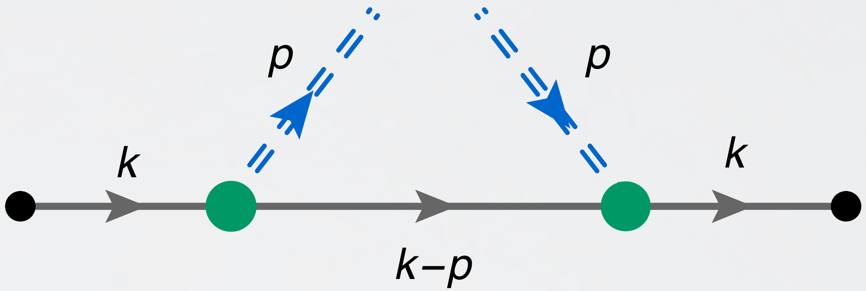
- No ad-hoc parameters: Taking Λ_3 and M_u as input, all masses and couplings fixed reproducing hadronic properties.

NJL-JET: ELEMENTARY SPLITTINGS

- One-quark truncation of the wavefunction:

$$d_q^m(z) : q \to Qm$$

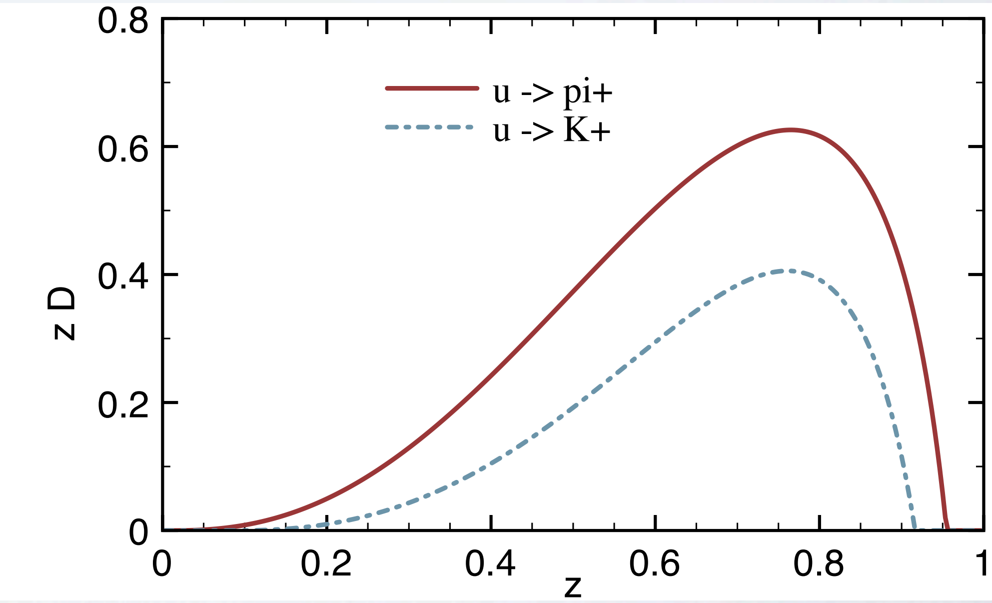
$$m = q\bar{Q} \quad z = p_-/k_-$$



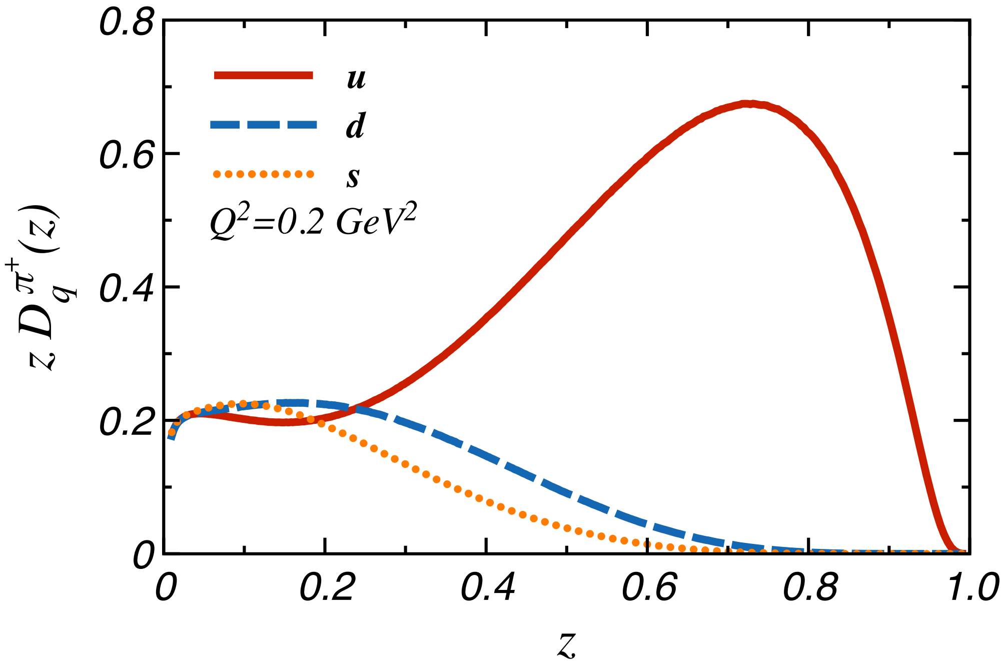
$$d_q^h(z) = \frac{1}{6} dp_- \int d^2p_\perp \sum_\alpha \frac{\left\langle k(\alpha)|a_h^\dagger(p)a_h(p)|k(\alpha)\right\rangle}{\langle k(\alpha)|k(\alpha)\rangle}$$

$$u \longrightarrow d\pi^+$$

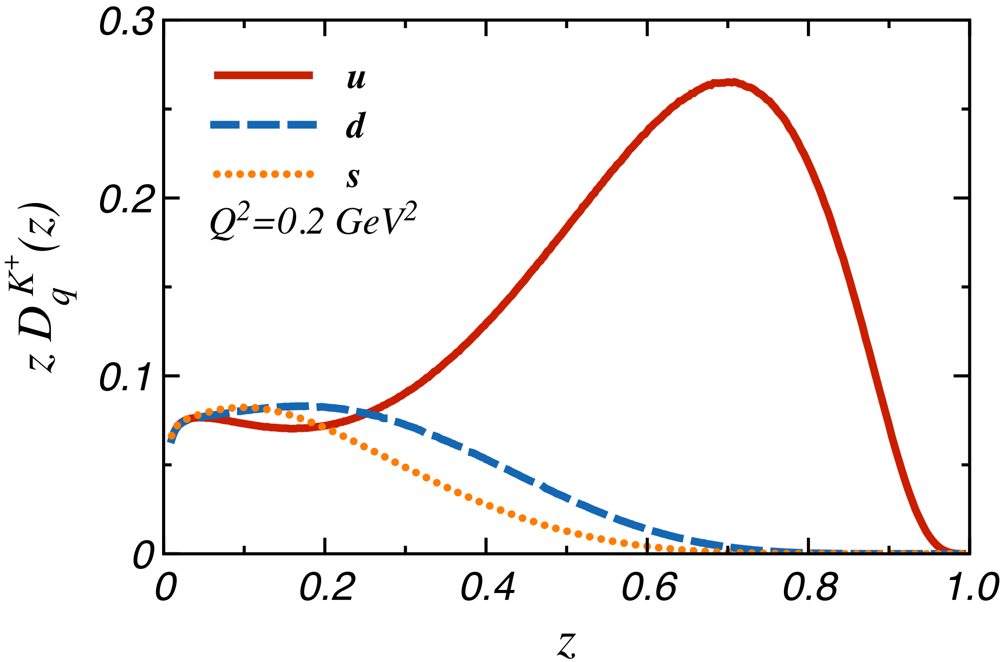
$$u \longrightarrow sk^+$$



SOLUTIONS OF THE INTEGRAL EQUATIONS



π^+



K^+

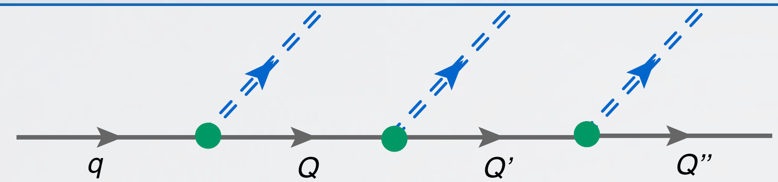
MONTE-CARLO (MC) APPROACH

- Simulate decay chains to extract number densities.
- Allows for inclusion of TMD and experimental cut-offs.
- Numerically trivially parallelizeable (MPI, GPGPU).

FRAGMENTATIONS FROM MC

STARTING WITH PIONS

- Assume Cascade process:



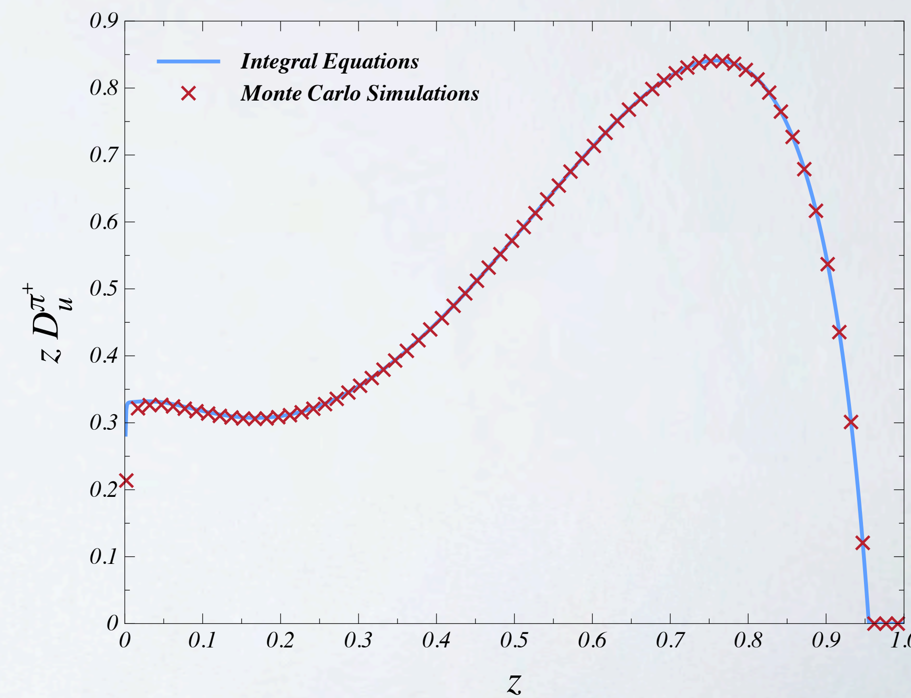
$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

- Sample the emitted hadron according to splitting weight.

- Randomly sample z from input splittings.

- Evolve to sufficiently large number of decay links.

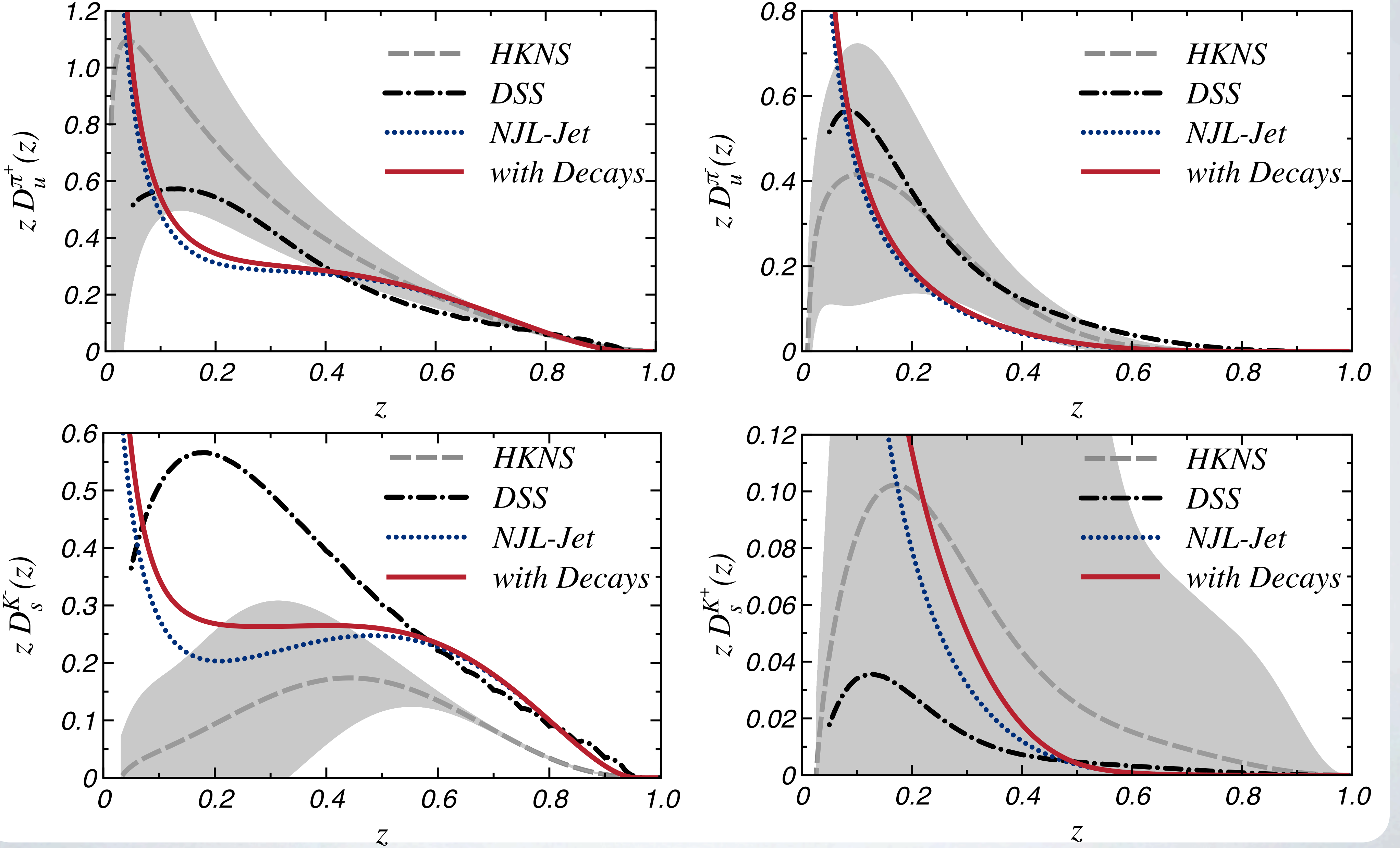
- Repeat for decay chains with the same initial quark.



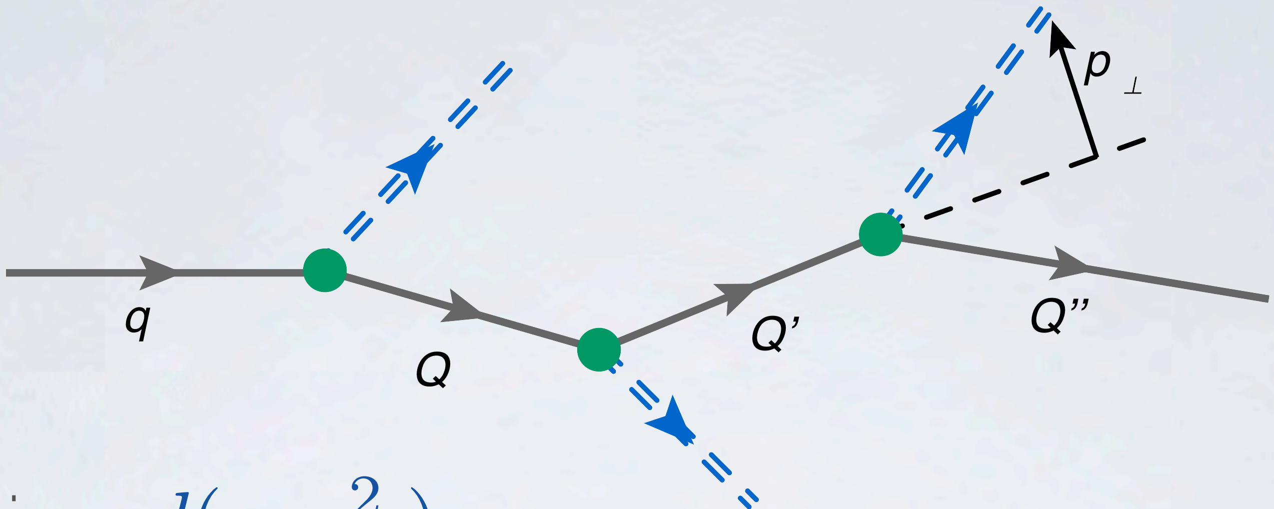
Results with vector mesons, N-Nbar: $Q^2 = 4\ \mathrm{GeV}^2$

Favored

Unfavored



INCLUDING THE TRANSVERSE MOMENTUM

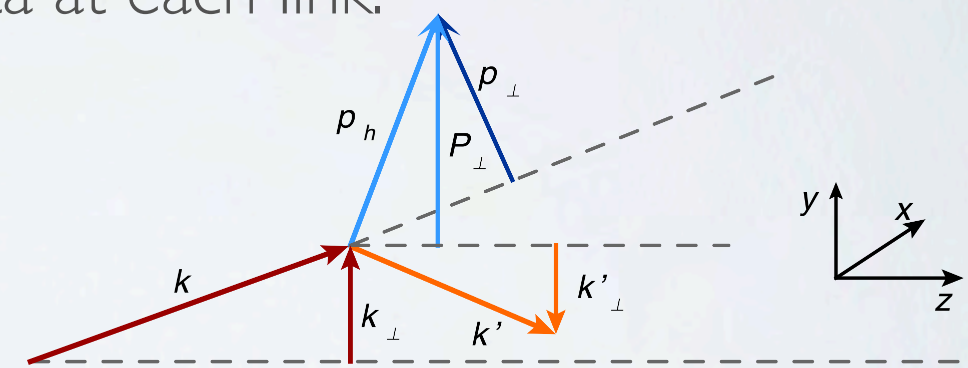


- TMD splittings: $d(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\boxed{\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp}$$

$$\boxed{\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp}$$



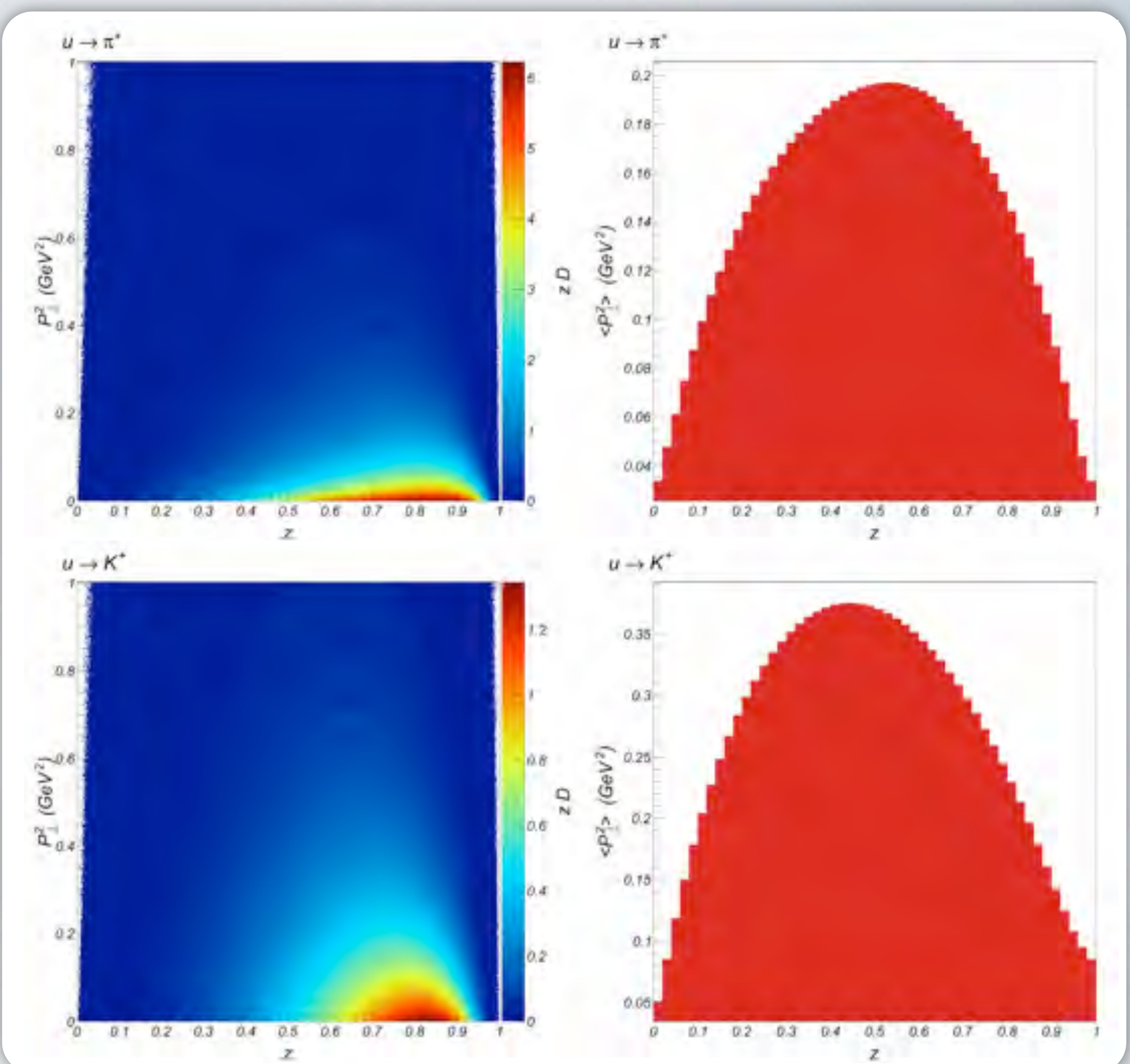
- Calculate the Number Density

$$D_q^h(z, P_\perp^2)\Delta z\ \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

TMD SPLITTING FUNCTIONS

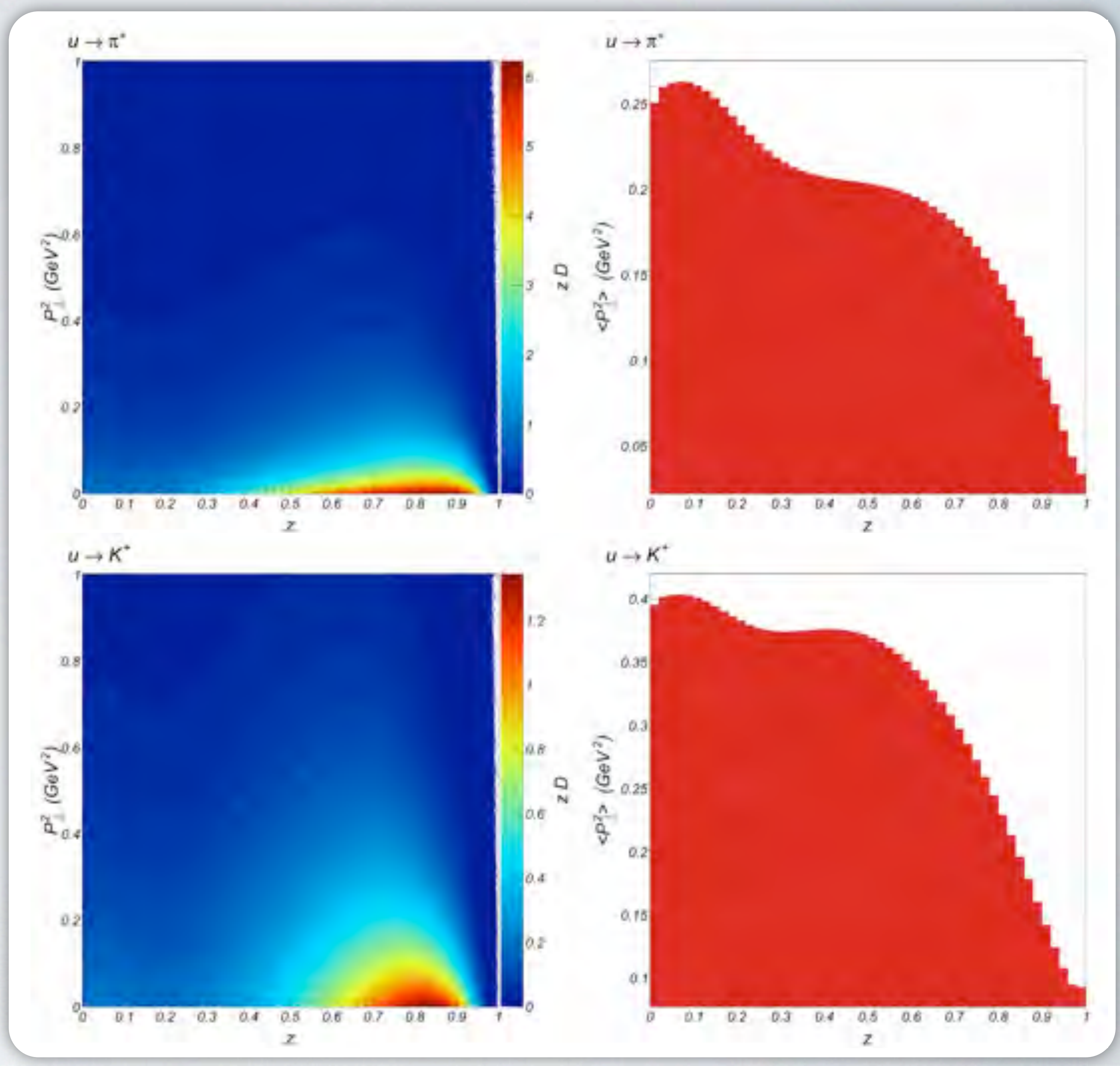
- TMD splittings from the NJL model
- Use dipole cutoff function with LB regularizations

$$\langle P_\perp^2 \rangle \equiv \frac{\int d^2\mathbf{P}_\perp \; P_\perp^2 D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp \; D(z, P_\perp^2)}$$



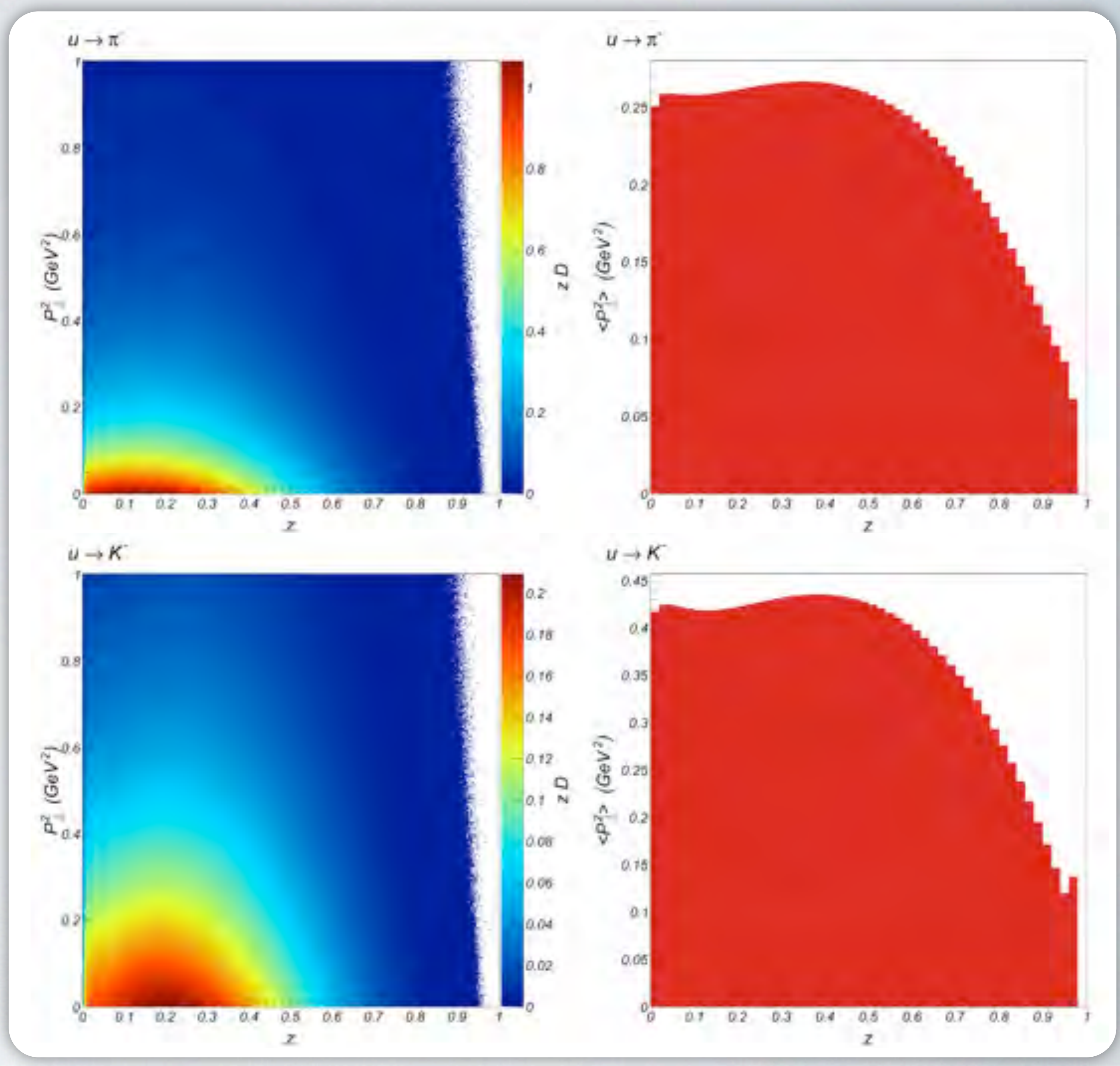
TMD FRAGMENTATION FUNCTIONS

- FAVORED

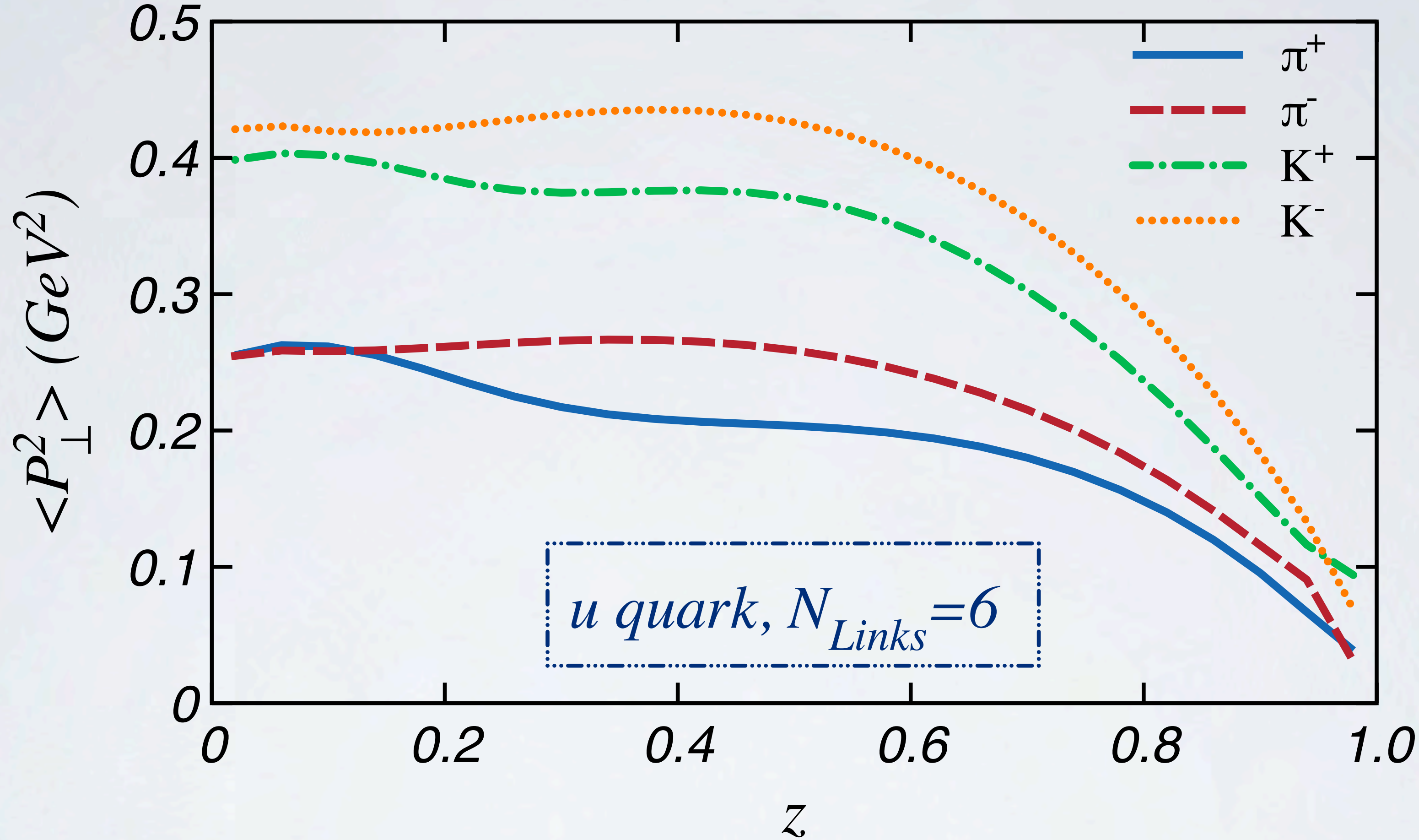


TMD FRAGMENTATION FUNCTIONS

- UNFAVORED

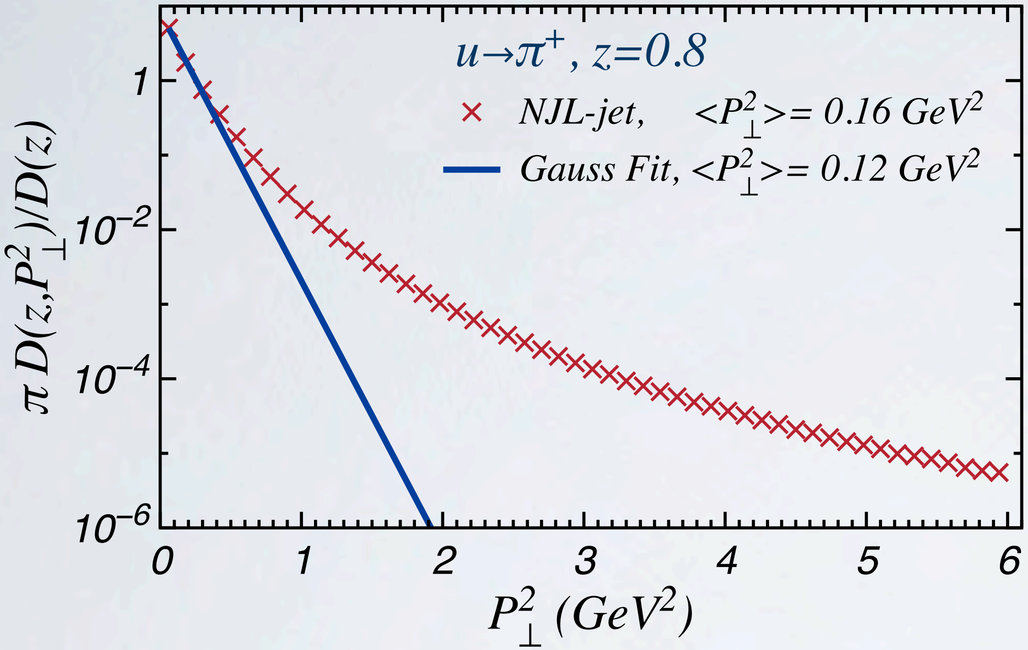


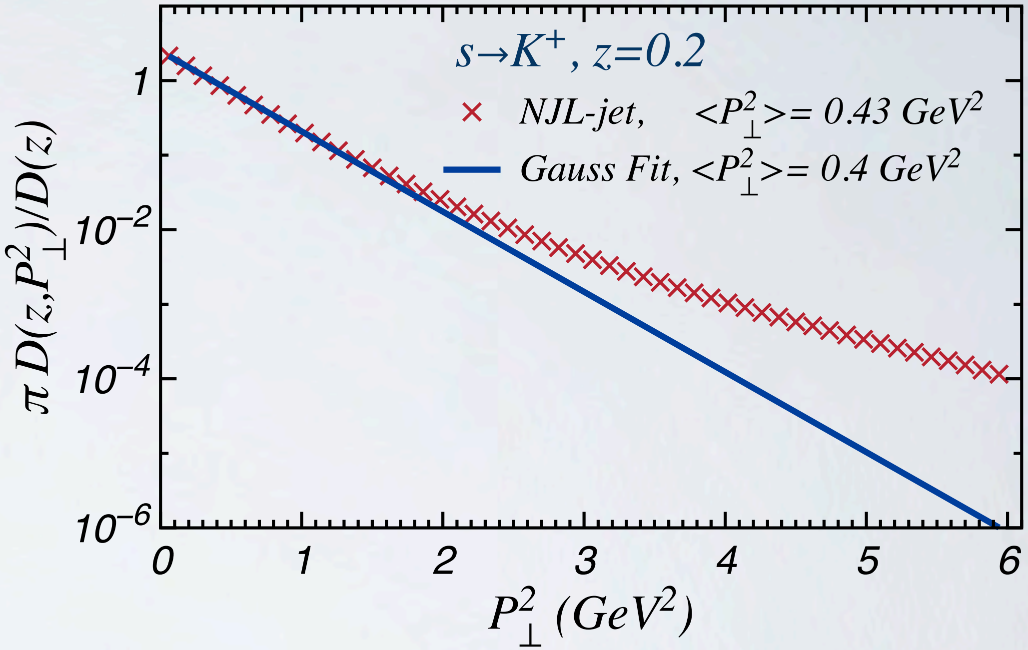
AVERAGE TRANSVERSE MOMENTA VS Z



- The average transverse momenta of kaons are larger than those of pions.
- Relatively flat in mid-z region.

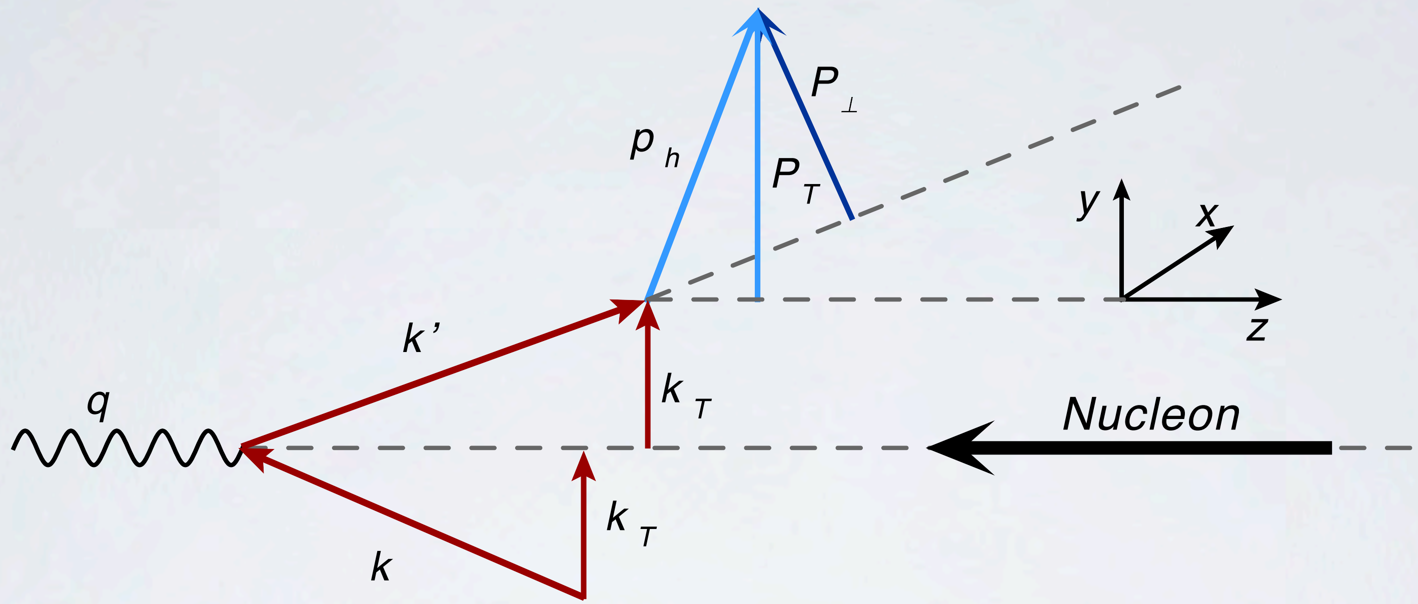
COMPARISON WITH GAUSSIAN ANSATZ





- Gaussian ansatz assumes: $D(z, P_\perp^2) = D(z)\dfrac{e^{-P_\perp^2/\langle P_\perp^2\rangle}}{\pi\langle P_\perp^2\rangle}$
- Unfavored fragmentation in low-z region agrees well with Gaussian.

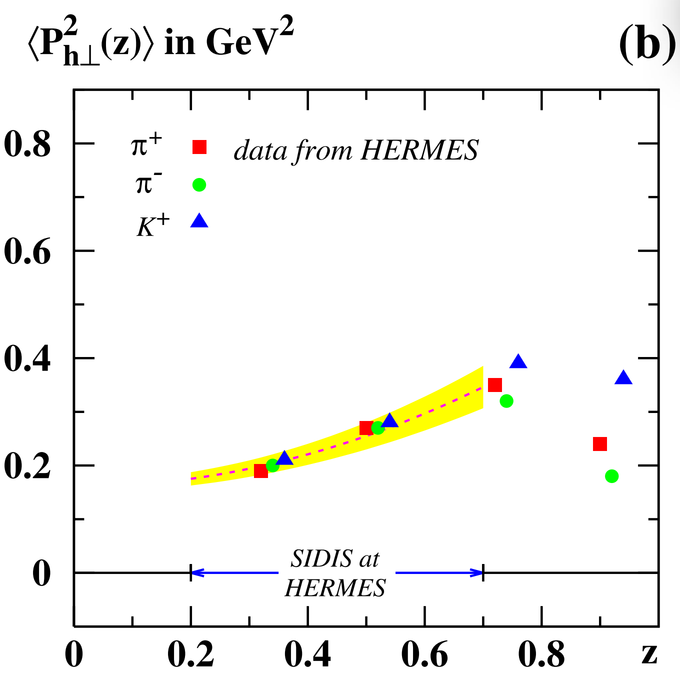
THE TRANSVERSE MOMENTA OF HADRONS IN SIDIS



- Use TMD quark distribution functions calculated in the NJL model .
- Transfer of the transverse momentum: $\boxed{\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}}$
- Evaluate $\langle P_T^2 \rangle$ using MC simulations to calculate the number densities

AVERAGE TRANSVERSE MOMENTA

$$\langle k_T^2 \rangle \equiv \frac{\int d^2\mathbf{k_T}\ k_T^2 f(x, k_T^2)}{\int d^2\mathbf{k_T}\ f(x, k_T^2)} \qquad \langle P_\perp^2 \rangle \equiv \frac{\int d^2\mathbf{P}_\perp\ P_\perp^2 D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp\ D(z, P_\perp^2)}$$



P. Schweitzer et al., Phys.Rev. D81, 094019 (2010).

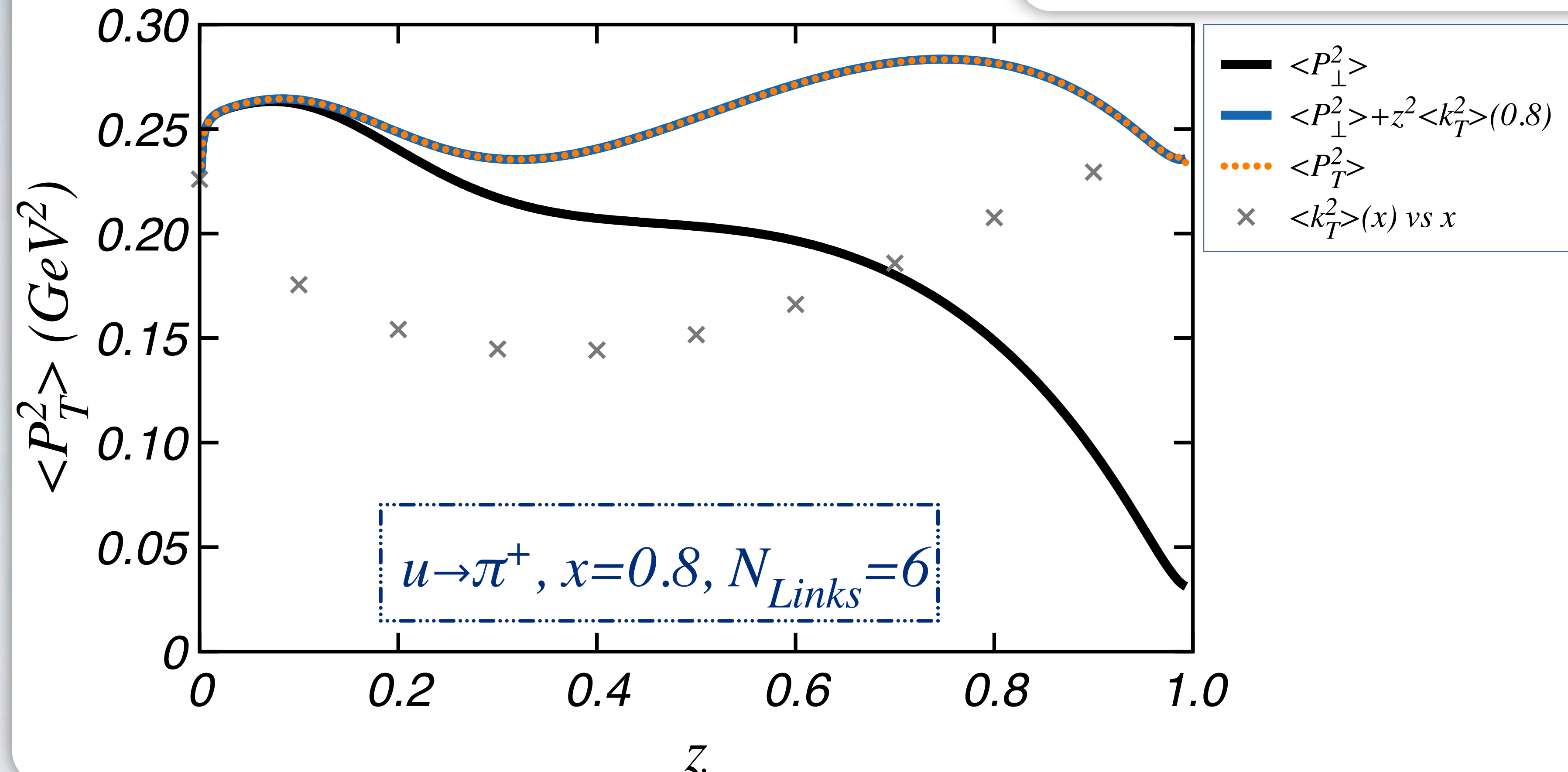
Using Gaussian Ansatz and:

$$\langle P_T^2 \rangle = \langle P_\perp^2 \rangle + z^2 \langle k_T \rangle$$

$$\boxed{\begin{aligned} \langle k_T^2 \rangle &= (0.38 \pm 0.06)\ \mathrm{GeV}^2 \\ \langle P_\perp^2 \rangle &= (0.16 \pm 0.01)\ \mathrm{GeV}^2 \end{aligned}}$$

AVERAGE TRANSVERSE MOMENTA

$$\langle P_T^2 \rangle \equiv \frac{\int d^2\mathbf{P_T}\; P_T^2\, \tilde{D}(z, P_T^2)}{\int d^2\mathbf{P_T}\; \tilde{D}(z, P_T^2)}$$

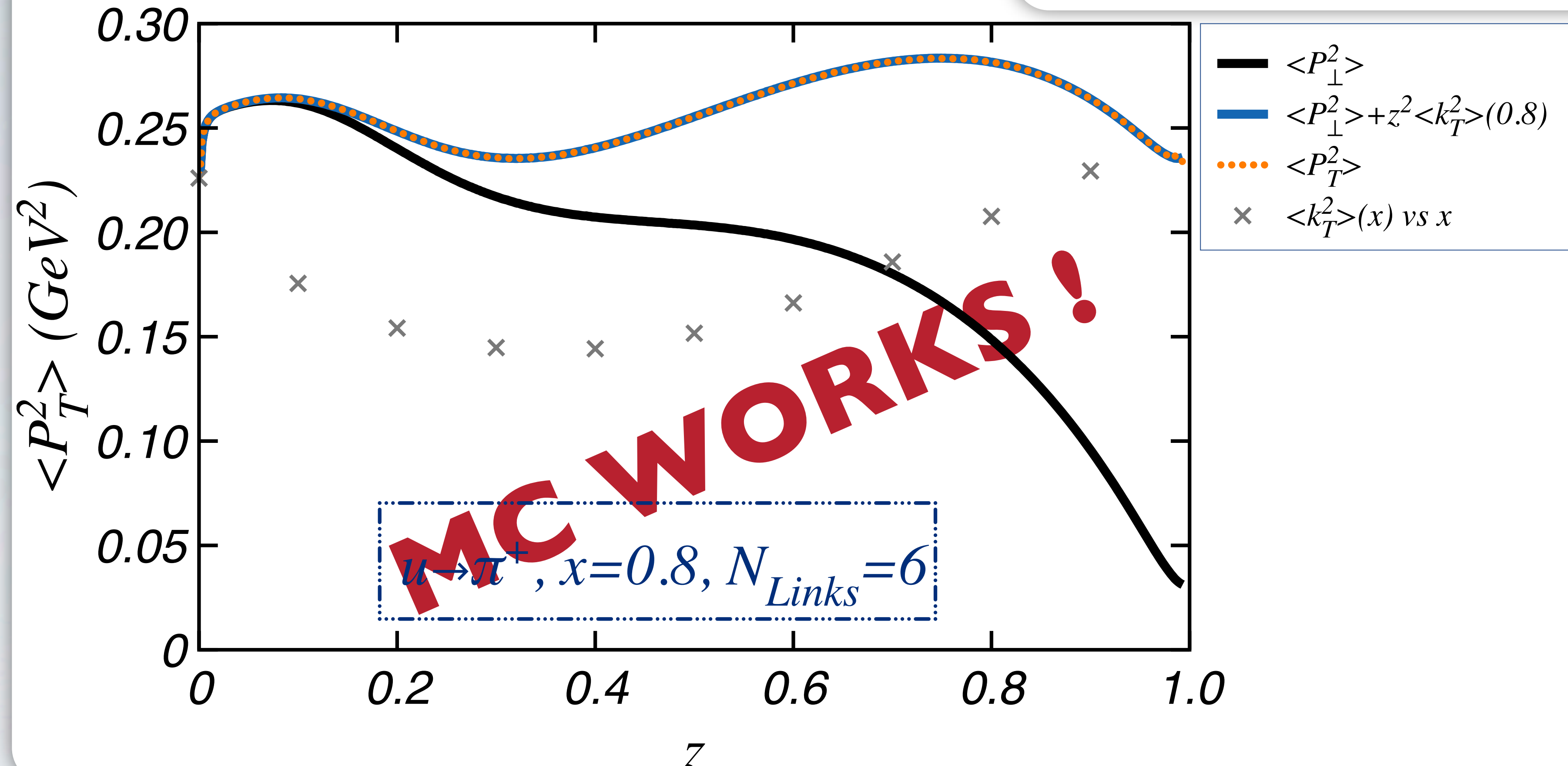


Input: $\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}$

Output: $\langle P_T^2 \rangle = \langle P_\perp^2 \rangle + z^2 \langle k_T \rangle$

AVERAGE TRANSVERSE MOMENTA

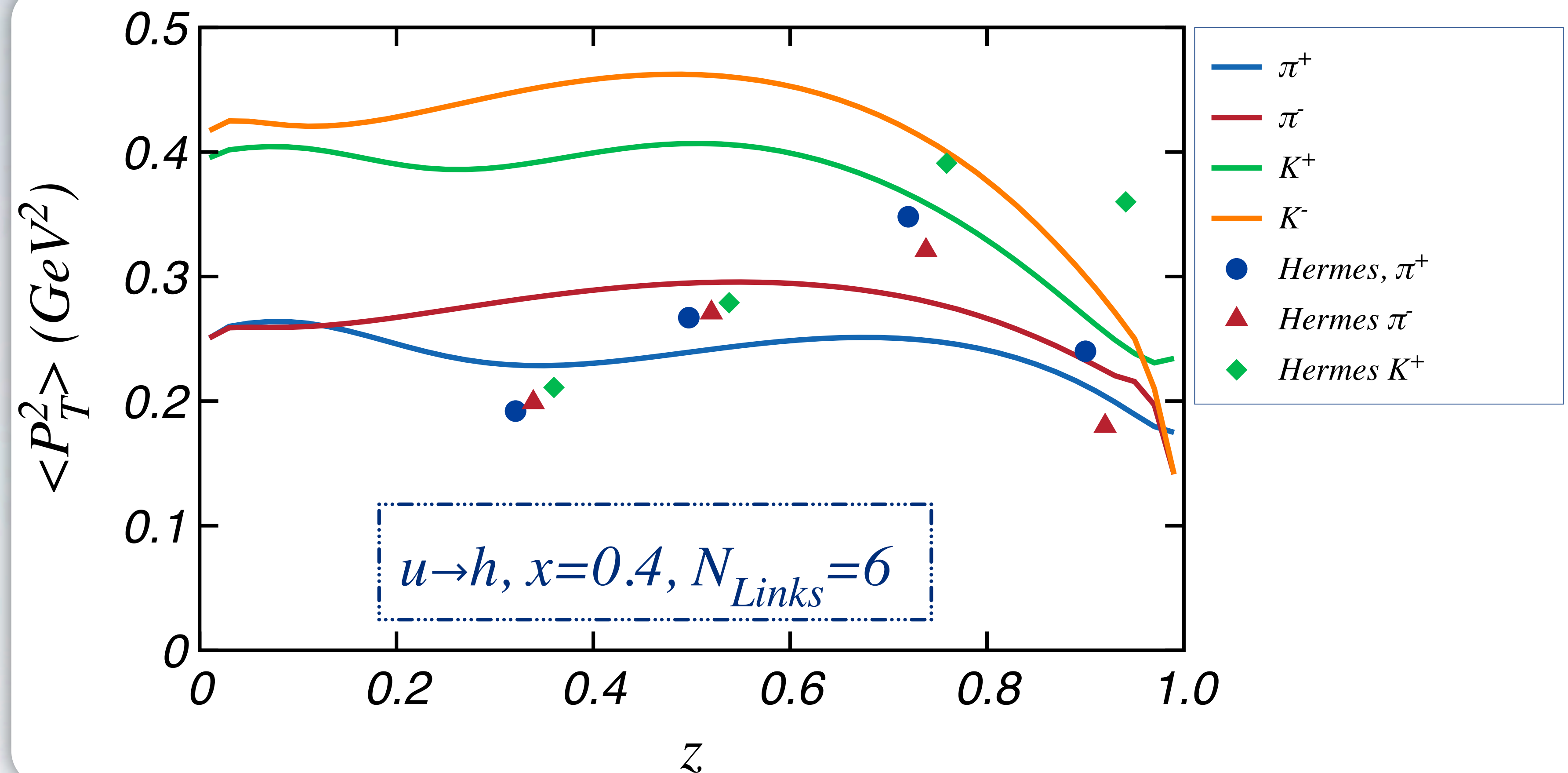
$$\langle P_T^2 \rangle \equiv \frac{\int d^2\mathbf{P_T}\ P_T^2 \tilde{D}(z, P_T^2)}{\int d^2\mathbf{P_T}\ \tilde{D}(z, P_T^2)}$$



Input: $\mathbf{P_T} = \mathbf{P}_\perp + z\mathbf{k_T}$

Output: $\langle P_T^2 \rangle = \langle P_\perp^2 \rangle + z^2 \langle k_T \rangle$

NAIVE COMPARISON WITH EXPERIMENT



A. Airapetian et al. (HERMES Collaboration), Phys.Lett. B684, 114 (2010).

D target, Integration over Q^2 and x .

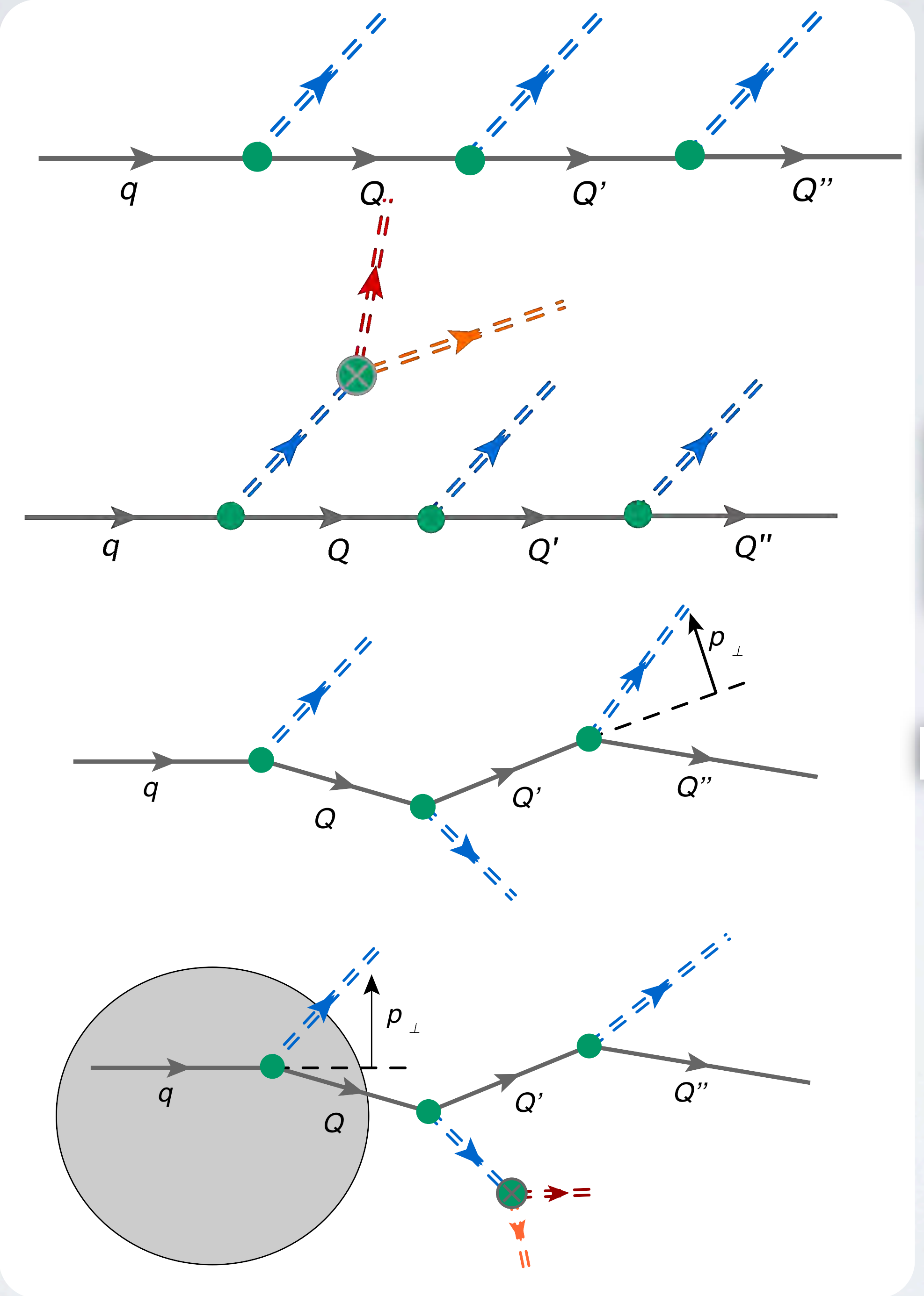
SUMMARY

☑ 2009

☑ 2010

☑ 2011

☐ 2011-2012



Ito et al. Phys.Rev.D80:074008,2009

Matevosyan et al.
Phys.Rev.D83:074003, 2011

Matevosyan et al.
Phys.Rev.D83:114010, 2011

Coming Soon !