# A chiral quark-soliton model with broken scale invariance for nuclear matter

Valentina Mantovani Sarti, Alessandro Drago

Department of Physics,University of Ferrara and INFN, Ferrara

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# Outline

- Review of Chiral Lagrangians for nuclear matter
  - Linear $\sigma$- model failure at finite density(R.J.Furnstahl, B.D.Serot,H.-B. Tang,Nucl.Phys.A 598 (1996))
  - Non-Linear $\sigma$ model with a scalar field (R.J.Furnstahl, B.D.Serot,H.-B. Tang,Nucl.Phys.A 598 (1996))
- Chiral-Dilaton Model (Carter,Ellis,Rudaz,Heide, PLB 282 (1992) 271,PLB 293 (1992) 870,NPA 571 (1994),NPA 603 (1996),NPA 618 (1997),NPA 628 (1998))
  - breaking of Scale Invariance in QCD
  - from hadronic to quarks degrees of freedom $\rightarrow$ results in vacuum
- Going to finite density
  - building up a soliton Wigner-Seitz lattice $\rightarrow$ results at finite density
  - Periodic Boundary Conditions and Quark Energy Bands
- Conclusions and Outlooks

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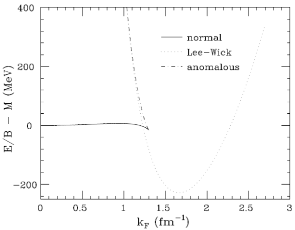
# Failure of Linear-$\sigma$ model at finite density

Linear-$\sigma$ model based on a "Mexican Hat" potential:

$$\mathcal{L} = \bar{\psi}[i\gamma_\mu\partial^\mu - g_v\gamma_\mu V^\mu - g_\pi(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})]\psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}$$
$$- \frac{1}{4}\lambda(\sigma^2 + \boldsymbol{\pi}^2 - \upsilon^2)^2 - \frac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu) + \frac{1}{2}m_v^2 V_\mu V^\mu + \epsilon\sigma$$

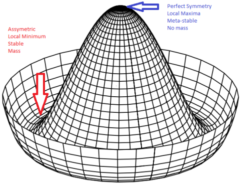
- The
  ground state at high densities is not the normal solution, but the Lee-Wick one, having effective nucleon mass $M^* = 0$
- restauration
  of chiral symmetry already at $\rho \approx \rho_0$




(R.J.Furnstahl, B.D.Serot,H.-B. Tang,Nucl.Phys.A 598 (1996))

# Non-Linear $\sigma$ model with a scalar field

- Furnstahl and Serot (PRC 47 (1993))conclude that the failure of many chiral models is due to the restrictions on the scalar field dynamics imposed by the “Mexican hat“ potential $\rightarrow$ the problem still exists even by introducing scaled versions of the “Mexican hat“




- FST (NPA,598 (1996))use a non-linear realization of chiral symmetry in which a scalar-isoscalar effective field is introduced, as a chiral singlet, to simulate intermediate range attraction and to satisfy broken scale invariance in QCD
- in this way the dynamics of the chiral singlet field is no more regulated by the “Mexican hat“ potential $\rightarrow$ accurate results for finite nuclei

# Breaking of Scale Invariance in QCD (1)

PROBLEM: the linear sigma model fails to yield saturation. It provides chiral symmetry restoration ($m_N = 0$) already at low density due to the form of the meson self-interaction

SOME PHYSICAL INGREDIENT IS MISSING

So, keeping in mind the idea of including in a dynamic way the chiral symmetry in our model, what can we do to get a better description of nuclear matter at finite density?

CHANGE THE POTENTIAL IN THE LAGRANGIAN DENSITY

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# Breaking of Scale Invariance in QCD (2)

- In QCD, scale symmetry is broken by trace anomaly. This mechanism is responsible for the existence of $\Lambda_{QCD}$ parameter, which sets the scale of hadron masses and radii
- Formally the non conservation of the dilatation current is strictly connected to a non vanishing gluon condensate

$$\langle \partial_\mu j^\mu_{QCD} \rangle = \frac{\beta(g)}{2g} \langle F^a_{\mu\nu}(x) F^{a\mu\nu}(x) \rangle$$

- In an effective model, the dynamics of the gluon condensate at mean-field level, is obtained by introducing a scalar field $\phi$, the dilaton field $\phi$ (Schechter 1980), so that the potential is determined by:

$$\Theta^\mu_\mu = 4V(\phi) - \phi \frac{\partial V}{\partial \phi} = 4\epsilon_{vac} \left( \frac{\phi}{\phi_0} \right)^4$$

# The dilatonic potential

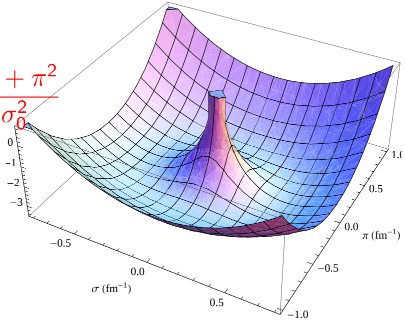
The dilaton field potential:

$$V(\phi,\sigma,\pi) =$$

$$B\phi^4\left(ln\frac{\phi}{\phi_0}-\frac{1}{4}\right)-\frac{1}{2}B\delta\phi^4 ln\frac{\sigma^2+\pi^2}{\sigma_0^2}$$

$$+\frac{1}{2}B\delta\zeta^2\phi^2\left(\sigma^2+\pi^2-\frac{1}{2}\frac{\phi^2}{\zeta^2}\right)$$

$$-\frac{3}{4}\epsilon_1-\frac{1}{4}\epsilon_1\left(\frac{\phi}{\phi_0}\right)^2\left[\frac{4\sigma}{\sigma_0}-2\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right)-\left(\frac{\phi}{\phi_0}\right)^2\right]$$



# The Lagrangian of the CDM with hadronic degrees of freedom

The Lagrangian density becomes:

$$\begin{aligned}\mathcal{L} \quad = \quad & \bar{\psi}\left(i\gamma^\mu\partial_\mu - g_\pi(\sigma + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5) + g_\rho\gamma^\mu\frac{\boldsymbol{\tau}}{2}\cdot(\boldsymbol{\rho}_\mu + \gamma_5\mathbf{A}_\mu) - g_\omega\gamma^\mu\omega_\mu\right)\psi \\ & + \frac{1}{2}(D_\mu\sigma D^\mu\sigma + D_\mu\boldsymbol{\pi}\cdot D^\mu\boldsymbol{\pi}) - \frac{1}{4}(\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \mathbf{A}_{\mu\nu}\cdot\mathbf{A}^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu}) \\ & + \frac{1}{2}m_\rho^2(\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu + \mathbf{A}_\mu\cdot\mathbf{A}^\mu) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - V(\phi,\sigma,\pi)\end{aligned}$$

- in the hadronic sector $\rightarrow$ fermionic fields are *nucleons*;
- chiral fields $(\sigma, \boldsymbol{\pi})$ $\rightarrow$ nuclear physics at low densities (Heide, Rudaz, Ellis, Nucl.Phys.A571, 713 (1994)), restoration of chiral symmetry at quite high densities (Drago, Bonanno, Phys.Rev.C79:045801,2009);

# From nucleons to quarks (1)

MAIN IDEA: use the same nucleon Lagrangian, but now introducing quarks degrees of freedom $\rightarrow$ fermionic fields are quarks

$$\mathcal{L} \quad = \quad \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - g_{\pi}(\sigma + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5)\right)\psi + \frac{1}{2}(\partial_{\mu}\sigma\partial^{\mu}\sigma + \partial_{\mu}\boldsymbol{\pi}\cdot\partial^{\mu}\boldsymbol{\pi}) - V(\phi_0, \sigma, \pi)$$

Keeping the dilaton frozen at its vacuum value $\phi_0$, the potential reads:

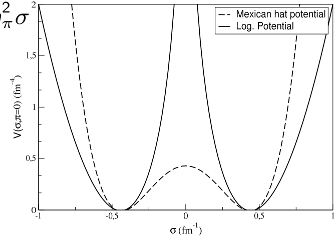
$$V(\sigma,\pi) = \lambda_1^2(\sigma^2 + \pi^2) - \lambda_2^2 \ln(\sigma^2 + \pi^2) - \sigma_0 m_\pi^2 \sigma$$

$$\lambda_1^2 = \frac{1}{4}(m_\sigma^2 + m_\pi^2)$$

$$\lambda_2^2 = \frac{\sigma_0^2}{4}(m_\sigma^2 - m_\pi^2)$$

$\sigma_0 = f_\pi = 93\,MeV\,,\ m_\sigma = 550\,MeV\,,$

$m_\pi = 139.6\,MeV\,,\ g = 5$



# From nucleons to quarks (2)

The nucleons should now emerge as chiral solitons from the dynamics of the quarks.

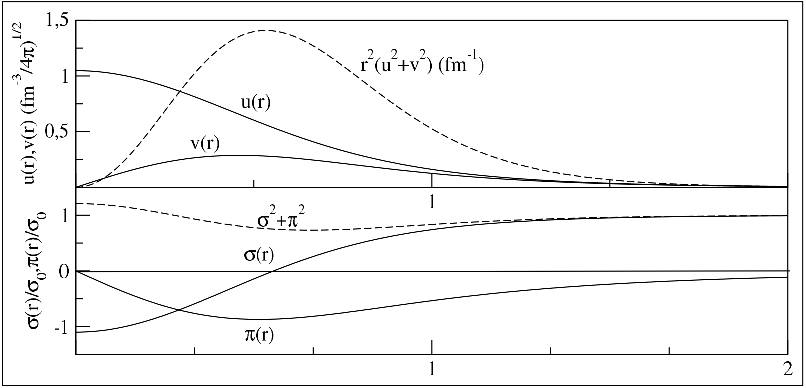
- Does the model provide a reasonable description of the single soliton in vacuum?✔
- Does the model admit stable solitonic solutions at finite density?✔

It has been done before using the $\sigma$-model, but:

- At nucleons level the $\sigma$-model does not provide a good description for nuclear matter, Lee-Wick phase is the ground state already at low densities
- At quarks level (Glendenning PRC34,1072 (1986); McGovern PRC57,3376 (1998))the $\sigma$-model does not provide stable solutions, solutions disappear at low densities

# Single soliton in vacuum

Fields in vacuum for the present model:



# Single soliton in vacuum

| Quantity | Log.Model | $\sigma$ Model | Exp. |
|---|---|---|---|
| $E_{1/2}\,(MeV)$ | 1075.4 | 1002.1 | |
| $M_N\,(MeV)$ | 959.6 | 894.1 | 938 |
| $E_{3/2}\,(MeV)$ | 1140.5 | 1075.2 | |
| $M_\Delta\,(MeV)$ | 1032. | 975.4 | 1232 |
| $\langle r_E^2\rangle_p\,(fm^2)$ | 0.82 | 0.92 | 0.74 |
| $\langle r_E^2\rangle_n\,(fm^2)$ | $-0.03$ | $-0.01$ | $-0.12$ |
| $\langle r_M^2\rangle_p\,(fm^2)$ | 0.82 | 0.87 | 0.74 |
| $\langle r_M^2\rangle_n\,(fm^2)$ | 0.86 | 0.9 | 0.77 |
| $\mu_p\ (\mu_N)$ | 3. | 3.24 | 2.79 |
| $\mu_n\ (\mu_N)$ | $-2.6$ | $-2.5$ | $-1.91$ |
| $g_a$ | 1.52 | 1.1 | 1.26 |
| $\langle N_\pi\rangle_J$ | 1.6, $(J=1/2)$<br>2., $(J=3/2)$ | 1.2, $(J=1/2)$<br>1.6, $(J=3/2)$ | / |

The values of most of the observables in CDM turn out to be slightly closer to the experimental ones than in the $\sigma$ model

# Wigner-Seitz approximation to nuclear matter

- Approximating nuclear matter by a lattice of nucleons $\rightarrow$ we consider the meson fields configuration centered at each lattice point, generating a periodic potential in which the quarks move
- Wigner-Seitz approximation: replace the cubic lattice by a spherical symmetric one $\rightarrow$ each soliton sits on a spherical cell of radius $R$ with specific boundary conditions on the surface of the sphere

The Hamiltonian for a periodic system must obey Bloch's theorem, so the quark spinor must be of the form:

$$\psi_{\mathbf{k}}(r) = e^{i\mathbf{k}\cdot\mathbf{r}}\Phi_{\mathbf{k}}(r), \quad (\mathbf{k} = 0 \text{ for the ground state})$$

The bottom of the band is defined as the state satisfying the following periodic boundary conditions, dictated by symmetry arguments (parity):

$$\begin{aligned} v(R) &= h(R) = 0, \\ u'(R) &= \sigma_h'(R) = 0. \end{aligned}$$

# How to define a band

In our work we use two different methods to estimate the band width:

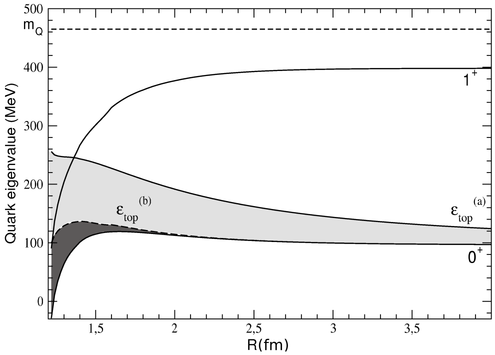
- A (rather crude) approximation to the width of a band can be obtained by using (Glendenning, Banerjee PRC 34(1986)):

$$\begin{aligned} \Delta &= \sqrt{\epsilon_0^2 + \left(\frac{\pi}{2R}\right)^2} - |\epsilon_0|, \\ \epsilon_{top} &= \epsilon_0 + \Delta. \end{aligned}$$

- An alternative approximation is obtained by imposing that the upper Dirac component vanishes at the boundary(U.Weber, J.A.McGovern,PRC 57 (1998))):

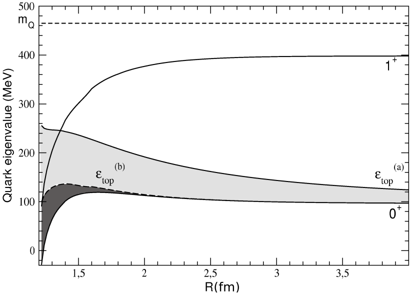
$$u(R) = 0$$

# How to define a band




- As shown in [U.Weber, J.A.McGovern,PRC 57 (1998)], $\epsilon_{top}^{(b)}$ is an upper limit to the top and the true top would be about half way between this upper limit and the bottom of the band,

# How to define a band

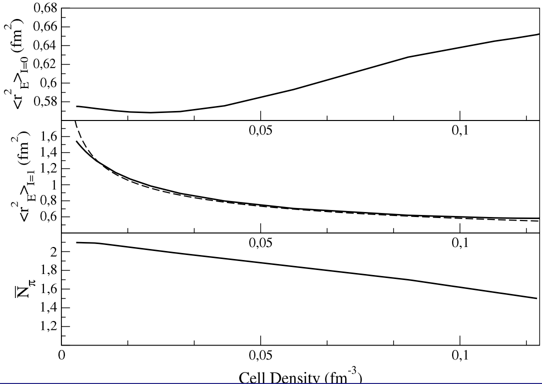



- band narrower at low densities $\rightarrow$ the soliton is localized and quarks are NOT free to move across the lattice (insulator)
- at higher densities the band gets wider $\rightarrow$ crossing between valence band and excited state $\rightarrow$ quarks are free to move from one soliton to the other (conductor) $\rightarrow$ quark deconfinement (Glendenning, Hanh PRC 36 (1987))
- in our case this merging occurs roughly at the same density at which the solution is lost

# Observables at finite density

We approached the question about the modification of the nucleon observables at finite density (EMC, Phys.Lett.B123,275 (1983), Mulders Phys.Rept 185,83 (1990),Rosa-clot, Ericson Phys.Lett.B188,11 (1987)):

- the isoscalar radius shows a slight swelling going at higher densities, trend in agreement with previous results (A.W.Thomas PRC60,068201 (1999), G.A.Miller PRC70,065205, (2004),L.S.Celenza PRL 53, 892 (1984)).



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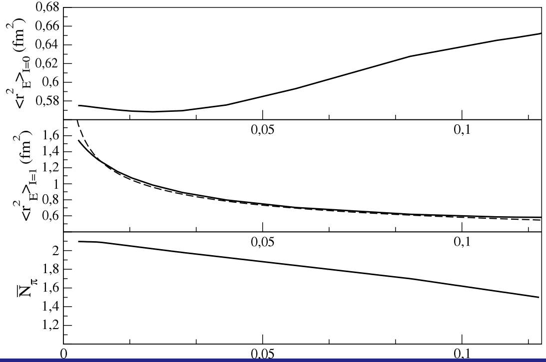
- the average number of pions $\overline{N}_\pi$ shows a decrease as the density raises, in contrast to previous results (Rosa-Clot, Ericson Phys.Lett.B188,11 (1987)).



# Conclusions and Outlooks

We used a Lagrangian with quarks degrees of freedom based on chiral and scale invariance to study:

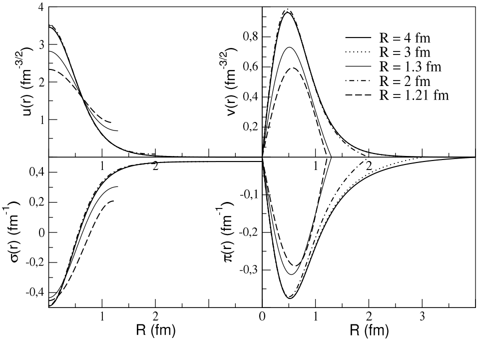
- how the soliton behaves in vacuum
  - the interplay between quarks and chiral fields leads to values of the static observables of the nucleon closer to the experimental ones
  - moreover most of the quantities are slightly better than the ones obtained in the $\sigma$ model
- the single soliton at finite density using the Wigner-Seitz approximation:
  - we showed that the new potential, including the scale invariance, provides more stable solitonic solutions than the $\sigma$ model at higher densities
  - we showed that nucleon observables are modified by finite density effects, at least at subnuclear densities $\rightarrow$ swelling of isoscalar radius, decrease of $\overline{N}_\pi$

# Conclusions and Outlooks

In the future:

- at the moment the model lacks of the necessary repulsion, given by the vector mesons, that could provide saturation at large densities $\rightarrow$ INCLUDING VECTOR MESONS
- improve the calculation of the band effects by using a more sophisticated approach (U.Weber, J.A.McGovern,PRC 57 (1998))and go beyond WS approximation (collaboration with Prof. V.Vento and Prof.B.Y.Park)
- include the dynamics of the dilaton field and study the model also at finite temperature $\rightarrow$ it should lead to a scenario similar to the one proposed by McLerran and Pisarski (Nucl.Phys.A796,83 (2007)) and already obtained in the hadronic sector (Drago, Bonanno, Phys.Rev.C79:045801,2009)but by starting from fundamental ingredients.

# Fields and total energy at finite density




- Down to $R = 2$ fm the solutions do not change significantly, as $R$ shrinks to lower values all the fields are deeply modified by finite density effects

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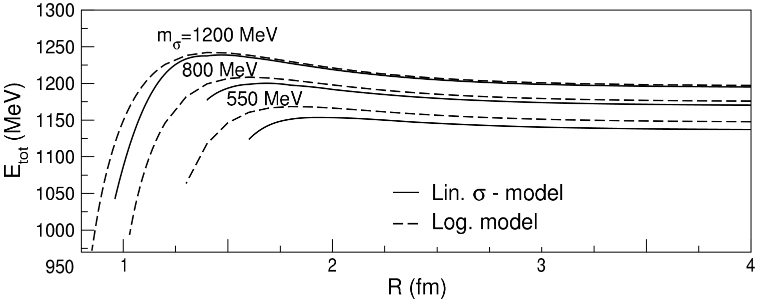
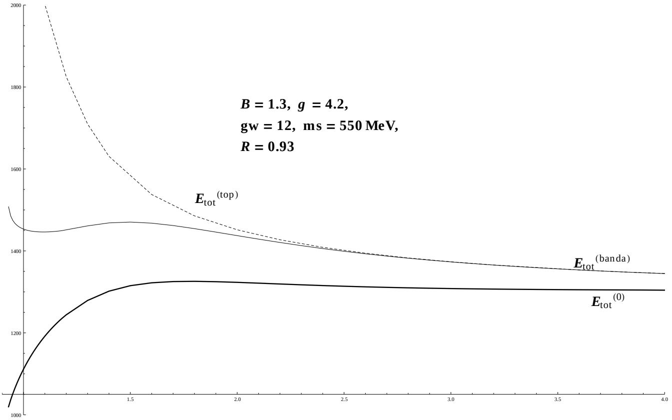



- For a fixed value of $m_\sigma$, the CDM allows the system to reach higher densities

# CDM with Vector Mesons: very preliminary results



# Phases of dense quarks at large $N_c$ (Pisarski, McLerran, Nucl.Phys.A796:83-100,2007.)

In the limit of large number of colors $N_c$:

- based
  on a chiral model (Skyrme crystal)
  $\rightarrow$ splitting between deconfinement
  and chiral symmetry restoration
- essentially they find two phases
  - deconfined phase at $T > T_d \rightarrow$
  - confined phase at $T < T_d \rightarrow$
    chiral symmetry restoration at $\mu \gg m_q$