Hadron properties in AdS/QCD

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based on
PRD 80 (2009) 055014
PRD 82 (2010) 074022
PRD 83 (2011) 036001

Erice School, 16-24 September 2011

Introduction

- Holographic QCD (HQCD) – approximation to QCD:
  Hadron Physics in terms of fields/strings living in extra dimensions (AdS space)

- Motivation: AdS/CFT correspondence 1998 (Maldacena, Polyakov, Witten et al)

  Gauge/Gravity Duality: Dynamics of the superstring theory in AdS$_{d+1}$ background is encoded in d conformal field theory living on the AdS boundary.

- AdS metric $ds^2 = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$ Poincaré form

  z is extra dimensional (holographic) coordinate; $z = 0$ is UV boundary

AdS/CFT dictionary

Gauge	Gravity
Operator $\hat{\mathcal{O}}$	Bulk field $\Phi(x,z)$
Δ — scaling dimension of $\hat{\mathcal{O}}$	m — mass of $\Phi(x,z)$
Source of $\hat{\mathcal{O}}$	Non-normalizable bulk profile near $z = 0$
$\langle\hat{\mathcal{O}}\rangle$	Normalizable bulk profile near $z = 0$

Introduction

- Towards to QCD:
  – Break conformal invariance and generate mass gap
  – Tower of normalized bulk fields (Kaluza-Klein modes) $\leftrightarrow$ Hadron wave functions
  – Spectrum of Kaluza-Klein modes $\leftrightarrow$ Hadrons spectrum

- HQCD: Description of low-energy QCD

- Bottom-up HQCD: hard-wall and soft-wall models

- Hard-wall:
  AdS geometry is cutted by two branes UV $(z = \epsilon \to 0)$ and IR $(z = z_{\rm IR})$
  Analogue of quark bag model, linear dependence on $J(L)$ of hadron masses

- Soft-wall:
  Soft cuttoff of AdS space by dilaton field $e^{-\varphi(z)}$
  Analytical solution of EOM, Regge behavior $M^2 \sim J(L)$

- Objective:
  SW holographic approach for mesons, baryons, exotic states with any n, J, L, S

Approach: Fields propagating in AdS

- Conformal group contains 15 generators:

  10 Poincaré (translations P_μ, Lorentz transformations $M_{\mu\nu}$),
  5 conformal (conformal boosts K_μ, dilatation D):

$M_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu)$	rotational symmetry
$D = i(x\,\partial)$	energy
$P_\mu = i\partial_\mu$	raising energy
$K_\mu = 2ix_\mu(x\,\partial) - ix^2\partial_\mu$	lowering energy

- Isomorphic to $SO(4,2)$ – the isometry group of AdS$_5$ space

- Fields in AdS$_5$ are classified by unitary, irreducible representations of $SO(4,2)$

Approach: Scalar Field

- Action for scalar field

$$S = \frac{1}{2}\int d^d x dz\, \sqrt{g}\, e^{-\varphi(z)} \left(\partial_N \Phi \partial^N \Phi - (m^2 + U(z))\, \Phi^2\right)$$

- dilaton $\varphi(z) = \kappa^2 z^2$ (Regge behavior of hadron masses)
- metric $g_{MN}(z) = \epsilon^a_M(z)\epsilon^b_N(z)\eta_{ab}, \quad g = |\mathrm{det} g_{MN}|$
- vielbein $\epsilon^a_M(z) = e^{A(z)}\delta^a_M, \quad A(z) = \log(R/z)$ (conformal)
- interval $ds^2 = g_{MN} dx^M dx^N = e^{2A(z)}(g_{\mu\nu}dx^\mu dx^\nu - dz^2)$
- potential $U(z) = e^{-2A(z)}[\varphi''(z) + (d-1)\varphi'(z)A'(z)]$
- UV asymptotics $\Phi(x,z)\Big|_{z\to 0} \to z^{d-\Delta}\Big[\Phi_0(x) + O(z^2)\Big] + z^\Delta\Big[\Phi_{\rm ph}(x) + O(z^2)\Big]$

  $\Phi_0(x)$ is source of the CFT operator $\hat{\mathcal{O}}$, $\quad \Phi_{\rm ph}(x) \sim \langle\hat{\mathcal{O}}\rangle$ is physical fluctuation

- Towards to QCD Brodsky, Téramond

  $\Delta \equiv \tau = 2 + L$ scaling dimension of two-parton state with $L = \max|L_z| = 0, 1$.

Approach: Scalar Field

- Kaluza-Klein expansion $\Phi(x,z) = \sum\limits_n S_n(x)\,\Phi_n(z)$

- Substitution $\Phi_n(z) = e^{-A(z)(d-1)/2+\varphi(z)/2}\phi_n(z)$

- Schrödinger-type EOM for $\phi_n(z)$:

  $$\left[-\frac{d^2}{dz^2} + \frac{4L^2-1}{4z^2} + \kappa^4 z^2 - 2\kappa^2\right]\phi_n(z) = M_n^2\phi_n(z)$$

- $\phi_n(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+1)}}\,\kappa^{L+1}\,z^{L+1/2}\,e^{-\kappa^2 z^2/2}\,L_n^L(\kappa^2 z^2)$

- $M_n^2 = 4\kappa^2\left(n + \frac{L}{2}\right) = 4\kappa^2\left(n + \frac{\tau}{2} - 1\right)$

- Laguerre-Gaussian laser beams: Siegman, "Lasers", 1986

- Massless pion $M_\pi^2 = 0$ for $n = L = 0$ (Brodsky, Téramond)

- $\Phi_n(z) = e^{\kappa^2 z^2/2}\,z^{3/2}\,\phi_n(z) \sim z^{2+L}$ (at small z) , $\Phi_n(z) \to 0$ (at large z)

- $S = \frac{1}{2}\sum\limits_n \int d^dx\left[\partial_\mu S_n(x)\,\partial^\mu S_n(x) - M_n^2 S_n^2(x)\right]$

Approach: Higher J boson fields

- Fradkin, Vasiliev, Metsaev, Buchbinder et al, Karch et al, $\cdots$

$$S = \frac{1}{2}\int d^d x dz \sqrt{g}\, e^{-\varphi(z)} \Big(\nabla_N \Phi_{M_1 \dots M_J} \nabla^N \Phi^{M_1 \dots M_J} - \left(\mu_J^2 + U_J\right) \Phi_{M_1 \dots M_J} \Phi^{M_1 \dots M_J} \Big) + \cdots$$

- $\nabla_N \Phi_{M_1 \dots M_J} = \partial_N \Phi_{M_1 \dots M_J} - \Gamma^K_{N M_1} \Phi_{K M_2 \dots M_J} - \Gamma^K_{N M_J} \Phi_{M_1 \dots M_{J-1} K}$

- Affine connection $\Gamma^K_{MN} = \frac{1}{2} g^{KL} \left(\frac{\partial g_{LM}}{\partial x^N} + \frac{\partial g_{LN}}{\partial x^M} - \frac{\partial g_{MN}}{\partial x^K} \right)$

- Gauge constraints (transversity, traceless)

  $\nabla^{M_1} \Phi_{M_1 M_2 \dots M_J} = 0$ and $g^{M_1 M_2} \Phi_{M_1 M_2 \dots M_J} = 0$

- Effective potential: $U_J(z) = e^{-2A(z)} [\varphi''(z) + (d - 1 - 2J)\, \varphi'(z)\, A'(z)]$

- Bulk mass $\mu_J^2 R^2 = (\Delta - J)(\Delta + J - d) - J$ with $\Delta = 2 + L$

- Extension to multiparton states: $\Delta = N + L$, where N is the number of partons

Approach: Higher J boson fields

- EOM $\left[-\frac{d^2}{dz^2}+\frac{4L^2-1}{4z^2}+\kappa^4z^2+2\kappa^2(J-1)\right]\phi_{nJ}(z)=M^2_{nJ}\phi_{nJ}(z)$
- Solutions at $d=4$:

$$\begin{aligned}\phi_{nJ}(z) &= \sqrt{\frac{2n!}{(n+L)!}}\,\kappa^{1+L}\; z^{1/2+L}\; e^{-\kappa^2z^2/2}L_n^L(\kappa^2z^2)\\ M^2_{nJ} &= 4\kappa^2\left(n+\frac{L+J}{2}\right)\end{aligned}$$

- At $J(L)\to\infty \quad M^2_{nJ}=4\kappa^2(n+J)$
- Scaling $\Phi_{nJ}=z^{3/2}\,\phi_{nJ}\;\sim\; z^\tau,$ twist $\tau=2+L$

Approach: Higher J fermion fields

- $S_\Psi = \int d^d x dz \sqrt{g}\, e^{-\varphi(z)}\, \overline{\Psi}^{M_1 \cdots M_{J-1/2}} \left(\epsilon_a^M \Gamma^a \mathcal{D}_M - \mu_J - \frac{\varphi(z)}{R} \right) \Psi_{M_1 \cdots M_{J-1/2}}$
  $+ \cdots$
- $\mathcal{D}_M = \nabla_M - \frac{1}{8} \omega_M^{ab} [\Gamma_a, \Gamma_b], \quad \omega_M^{ab} = A'(z) \, (\delta_z^a \delta_M^b - \delta_z^b \delta_M^a)$
- Relation of spin and affine connection

  $\omega_M^{ab} = \epsilon_K^a \left(\partial_M \epsilon^{Kb} + \epsilon^{Nb} \Gamma_{MN}^K \right)$
- Gauge constraints (transversity, traceless)

  $\nabla^{M_1} \Psi_{M_1 M_2 \cdots M_{J-1/2}} = 0, \qquad \Gamma^{M_1} \Psi_{M_1 M_2 \cdots M_{J-1/2}} = 0,$

  $g^{M_1 M_2} \Psi_{M_1 M_2 \cdots M_{J-1/2}} = 0$
- Bulk mass $\mu_J R = \Delta_J - d/2$ with $\Delta_J = J + d - 2$ Metsaev
- Toward QCD: $\Delta_J \equiv \tau + 1/2 = 7/2 + L$

  independent on J and gives correct scaling of nucleon FF

Approach: Higher J fermion fields

- EOM $\left[iz\not{\partial} + \gamma^5 z\partial_z - \frac{d}{2}\gamma^5 - \mu R - \varphi(z)\right]\Psi_{a_1\cdots a_{J-1/2}}(x,z) = 0$

- $\Psi(x,z) = \Psi_L(x,z) + \Psi_R(x,z)\,, \quad \Psi_{L/R} = \frac{1\mp\gamma^5}{2}\Psi$

- $\Psi_{L/R}(x,z) = \sum_n \int \frac{d^d p}{(2\pi)^d}\; e^{-ipx}\; \Psi_{L/R}(p)\; F^n_{L/R}(p,z)$

- $F^n_{L/R}(p,z) = e^{-A(z)\cdot d/2}\, f^n_{L/R}(p,z)$

- $\left[-\partial_z^2 + \kappa^4 z^2 + 2\kappa^2\left(\mu R \mp \frac{1}{2}\right) + \frac{\mu R(\mu R \pm 1)}{z^2}\right] f^n_{L/R}(z) = M_n^2\, f^n_{L/R}(z)$

- For $d = 4$ and $\mu R = L + 3/2$

$$f^n_L(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+3)}}\;\kappa^{L+3}\; z^{L+5/2}\; e^{-\kappa^2 z^2/2}\; L_n^{L+2}(\kappa^2 z^2)$$

$$f^n_R(z) = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+2)}}\;\kappa^{L+2}\; z^{L+3/2}\; e^{-\kappa^2 z^2/2}\; L_n^{L+1}(\kappa^2 z^2)$$

$$M_n^2 = 4\kappa^2\Big(n + L + 2\Big) = 4\kappa^2\Big(n + \tau - 1\Big),$$

$$F^n_L(z) \sim z^{\tau+3/2}\,, \qquad F^n_R(z) \sim z^{\tau+1/2}$$

Approach: Hadronic Wave Function

- Brodsky, Téramond
  Correspondence of holographic coordinate z to impact variable ζ in LF

- Two parton case: $q_1 \bar{q}_2$ mesons $z \to \zeta, \qquad \zeta^2 = \mathbf{b}_\perp^2 x(1-x)$

  ζ - impact variable; $\mathbf{b}_\perp$ - impact separation (conjugate to $\mathbf{k}_\perp$)

- Mapping $\Phi_{nJ}(z)$ to the transverse mode of LFWF

- $\psi_{nJ}(x, \zeta, m_1, m_2) = \psi_{\mathrm{T}}(\zeta) \cdot \psi_{\mathrm{L}}(x) \cdot \psi_{\mathrm{A}}(\varphi)$

  $\psi_T = \Phi_{nJ}(\zeta)$ — transverse (from AdS/QCD)
  $\psi_L = f(x, m_1, m_2) = e^{-m_1^2/(2x\lambda^2) - m_2^2/(2(1-x)\lambda^2)}$ — longitudinal
  $\psi_A = e^{im\varphi}$ — angular mode
  λ - additional scale parameter

  $$M_{nJ}^2 = 4\kappa^2\left(n + \frac{L+J}{2}\right) + \int_0^1 dx \left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right) f^2(x, m_1, m_2)$$

Approach: Choice of parameters

- Constituent quark massses:
  $m = 420\ \mathrm{MeV}\,,\quad m_s = 570\ \mathrm{MeV}\,,\quad m_c = 1.6\ \mathrm{GeV}\,,\quad m_b = 4.8\ \mathrm{GeV}$

- dilaton parameter $\kappa = 550$ MeV

- Dimensional parameters λ in the longitudinal WF are fitted as:

$$\lambda_{qq} = 0.63\ \mathrm{GeV}\,,\ \lambda_{qs} = 1.20\ \mathrm{GeV}\,,\ \lambda_{ss} = 1.68\ \mathrm{GeV}\,,\ \lambda_{qc} = 2.50\ \mathrm{GeV}\,, \lambda_{sc} = 3.00\ \mathrm{GeV}$$
$$\lambda_{qb} = 3.89\ \mathrm{GeV}\,,\ \lambda_{sb} = 4.18\ \mathrm{GeV}\,,\ \lambda_{cc} = 4.04\ \mathrm{GeV}\,,\ \lambda_{cb} = 4.82\ \mathrm{GeV}\,,\ \lambda_{bb} = 6.77\ \mathrm{GeV}$$

- Scaling of dimensional parameters: $\kappa = \mathcal{O}(1), \qquad \lambda_{qQ} = \mathcal{O}(\sqrt{m_Q})$

Approach: HQET Constraints

- Heavy–light mesons

$$
\begin{aligned}
M_{qQ}^2 &= 4\kappa^2\left(n+\frac{L+J}{2}\right)+\int_0^1 dx\left(\frac{m_q^2}{x}+\frac{m_Q^2}{1-x}\right)f^2(x,m_q,m_Q) \\
&= \left(m_Q+\bar\Lambda+\mathcal{O}(1/m_Q)\right)^2
\end{aligned}
$$

- V-P meson mass splitting: $\Delta M_{qQ} = \dfrac{2\kappa^2}{M_{qQ}^V + M_{qQ}^P} \sim \frac{1}{m_Q}$

- Leptonic decay constants

$$
f_P = f_V = \kappa\frac{\sqrt{6}}{\pi}\int_0^1 dx\sqrt{x(1-x)}\,f(x,m_1,m_2) \sim \frac{1}{\sqrt{m_Q}}
$$

- Heavy quarkonia

$$
M_{Q_1\bar{Q}_2} = m_{Q_1} + m_{Q_2} + E + \mathcal{O}(1/m_{Q_{1,2}})
$$

Results: Mass spectrum

Masses of light mesons

Meson	n	L	S	Mass [MeV]			
π	0	0,1,2,3	0	140	1355	1777	2099
π	0,1,2,3	0	0	140	1355	1777	2099
K	0	0,1,2,3	0	496	1505	1901	2207
η	0,1,2,3	0	0	544	1552	1946	2248
$f_0[\bar{n}n]$	0,1,2,3	1	1	1114	1600	1952	2244
$f_0[\bar{s}s]$	0,1,2,3	1	1	1304	1762	2093	2372
$a_0(980)$	0,1,2,3	1	1	1114	1600	1952	2372
$\rho(770)$	0,1,2,3	0	1	804	1565	1942	2240
$\rho(770)$	0	0,1,2,3	1	804	1565	1942	2240
$\omega(782)$	0,1,2,3	0	1	804	1565	1942	2240
$\omega(782)$	0	0,1,2,3	1	804	1565	1942	2240
$\phi(1020)$	0,1,2,3	0	1	1019	1818	2170	2447
$a_1(1260)$	0,1,2,3	1	1	1358	1779	2101	2375

Results: Mass spectrum

Masses of heavy-light mesons

Meson	J^{P}	n	L	S	Mass [MeV]			
$D(1870)$	0^-	0	0,1,2,3	0	1857	2435	2696	2905
$D^*(2010)$	1^-	0	0,1,2,3	1	2015	2547	2797	3000
$D_s(1969)$	0^-	0	0,1,2,3	0	1963	2621	2883	3085
$D_s^*(2107)$	1^-	0	0,1,2,3	1	2113	2725	2977	3173
$B(5279)$	0^-	0	0,1,2,3	0	5279	5791	5964	6089
$B^*(5325)$	1^-	0	0,1,2,3	1	5336	5843	6015	6139
$B_s(5366)$	0^-	0	0,1,2,3	0	5360	5941	6124	6250
$B_s^*(5413)$	1^-	0	0,1,2,3	1	5416	5992	6173	6298

Results: Mass spectrum

Masses of heavy quarkonia $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$

Meson	J^{P}	n	L	S	Mass [MeV]			
$\eta_c(2986)$	0^-	0,1,2,3	0	0	2997	3717	3962	4141
$\psi(3097))$	1^-	0,1,2,3	0	1	3097	3798	4038	4213
$\chi_{c0}(3414)$	0^+	0,1,2,3	1	1	3635	3885	4067	4226
$\chi_{c1}(3510)$	1^+	0,1,2,3	1	1	3718	3963	4141	4297
$\chi_{c2}(3555)$	2^+	0,1,2,3	1	1	3798	4038	4213	4367
$\eta_b(9300)$	0^-	0,1,2,3	0	0	9428	10190	10372	10473
$\Upsilon(9460)$	1^-	0,1,2,3	0	1	9460	10219	10401	10502
$\chi_{b0}(9860)$	0^+	0,1,2,3	1	1	10160	10343	10444	10521
$\chi_{b1}(9893)$	1^+	0,1,2,3	1	1	10190	10372	10473	10550
$\chi_{b2}(9912)$	2^+	0,1,2,3	1	1	10219	10401	10502	10579
$B_c(6276)$	0^-	0,1,2,3	0	0	6276	6911	7092	7209

Results: Nucleon FFs and GPDs

- Nucleon form factors in AdS/QCD Abidin-Carlson, Brodsky-Teramond

$$
\begin{aligned}
F_1^p(Q^2) &= C_1(Q^2) + \eta_p C_2(Q^2) \sim 1/Q^4 \\
F_2^p(Q^2) &= \eta_p C_3(Q^2) \sim 1/Q^6 \\
F_1^n(Q^2) &= \eta_n C_2(Q^2) \sim 1/Q^4 \\
F_2^n(Q^2) &= \eta_n C_3(Q^2) \sim 1/Q^6, \quad \eta_N = k_N/8
\end{aligned}
$$

- Structure integrals

$$
\begin{aligned}
C_1(Q^2) &= \int dz e^{-\varphi(z)} \frac{V(Q,z)}{2z^4} (F_L^2(z) + F_R^2(z)) = \frac{a+6}{(a+1)(a+2)(a+3)} \\
C_2(Q^2) &= \int dz e^{-\varphi(z)} \frac{\partial_z V(Q,z)}{2z^3} (F_L^2(z) - F_R^2(z)) = \frac{2a(2a-1)}{(a+1)(a+2)(a+3)(a+4)} \\
C_3(Q^2) &= \int dz e^{-\varphi(z)} \frac{2m_N V(Q,z)}{2z^3} F_L(z) F_R(z) = \frac{48}{(a+1)(a+2)(a+3)}
\end{aligned}
$$

$\psi_{L/R}(z)$ — KK modes dual to L/R-handed nucleon fields:
$F_L(z) = \kappa^3 z^{9/2}\,,\ F_R(z) = \kappa^2 z^{7/2}\sqrt{2}\,, \quad a = Q^2/4\kappa^2\,, \quad m_N^2 = 8\kappa^2$

$V(Q,z) = \Gamma(1+a)U(a,0,\kappa^2 z^2)$ bulk-to-boundary propagator of the vector field (holograhic analogue of EM current)

Results: Nucleon FFs and GPDs

- EM radii

$$
\begin{aligned}
\langle r_E^2\rangle^p &= \frac{147}{64\kappa^2}\left(1+\frac{13}{147}\mu_p\right) = 0.910\ \mathrm{fm}^2\ (\mathrm{our})\,, \quad 0.766\ \mathrm{fm}^2\ (\mathrm{data})\\
\langle r_E^2\rangle^n &= \frac{13}{64\kappa^2}\mu_n = -0.123\ \mathrm{fm}^2\ (\mathrm{our})\,, \quad -0.116\ \mathrm{fm}^2\ (\mathrm{data})\\
\langle r_M^2\rangle^p &= \frac{177}{64\kappa^2}\left(1-\frac{17}{177\mu_p}\right) = 0.849\ \mathrm{fm}^2\ (\mathrm{our})\,, \quad 0.731\ \mathrm{fm}^2\ (\mathrm{data})\\
\langle r_M^2\rangle^n &= \frac{177}{64\kappa^2} = 0.879\ \mathrm{fm}^2\ (\mathrm{our})\,, \quad 0.762\ \mathrm{fm}^2\ (\mathrm{data})
\end{aligned}
$$

Results: Nucleon FFs and GPDs

- Sum rules relating EM FF and GPDs Ji, Radyushkin

$$
\begin{aligned}
F_1^p(t) &= \int_0^1 dx \left(\frac{2}{3} H_v^u(x,t) - \frac{1}{3} H_v^d(x,t)\right) \\
F_1^n(t) &= \int_0^1 dx \left(\frac{2}{3} H_v^d(x,t) - \frac{1}{3} H_v^u(x,t)\right)
\end{aligned}
$$

$$
\begin{aligned}
F_2^p(t) &= \int_0^1 dx \left(\frac{2}{3} E_v^u(x,t) - \frac{1}{3} E_v^d(x,t)\right) \\
F_2^n(t) &= \int_0^1 dx \left(\frac{2}{3} E_v^d(x,t) - \frac{1}{3} E_v^u(x,t)\right)
\end{aligned}
$$

- Grigoryan-Radyushkin integral representation for bulk-to-boundary propagator

$$
V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\frac{x}{1-x}\kappa^2 z^2}
$$

- LF mapping (Brodsky-Teramond): x is equivalent to LC momentum fraction

Results: Nucleon FFs and GPDs

- GPDs $H_v^q(x,Q^2) = q(x)\,x^{\frac{Q^2}{4\kappa^2}}\,, \quad E_v^q(x,Q^2) = e^q(x)\,x^{\frac{Q^2}{4\kappa^2}}$
- Distribution functions $q(x)$ and $e^q(x)$

$$q(x) = \alpha^q \gamma_1(x) + \beta^q \gamma_2(x)\,, \quad e^q(x) = \beta^q \gamma_3(x)$$

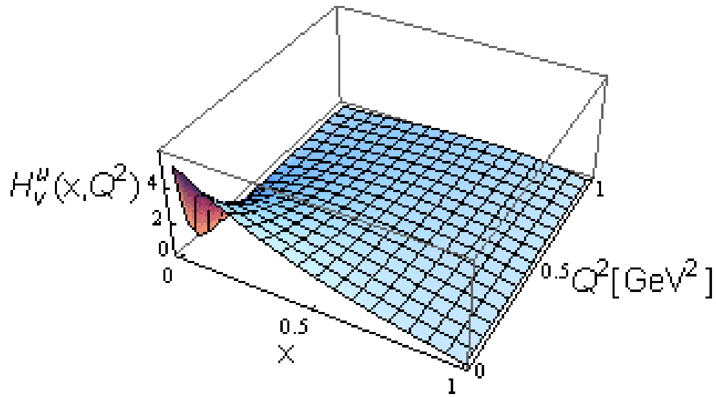
Flavor couplings α^q, β^q and functions $\gamma_i(x)$ are written as

$$\alpha^u = 2\,,\ \alpha^d = 1\,,\ \beta^u = 2\eta_p + \eta_n\,,\ \beta^d = \eta_p + 2\eta_n$$

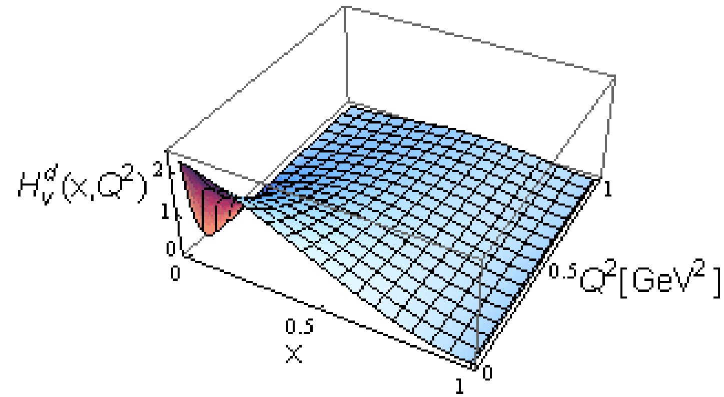
and

$$\begin{aligned}
\gamma_1(x) &= \frac{1}{2}(5 - 8x + 3x^2) \\
\gamma_2(x) &= 1 - 10x + 21x^2 - 12x^3 \\
\gamma_3(x) &= 24(1-x)^2
\end{aligned}$$

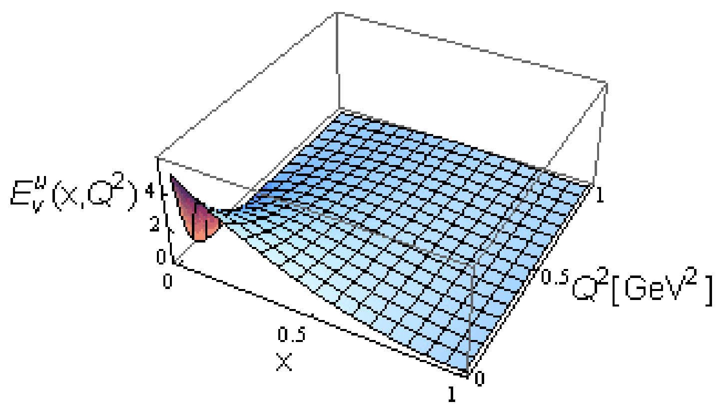
Results: Nucleon FFs and GPDs



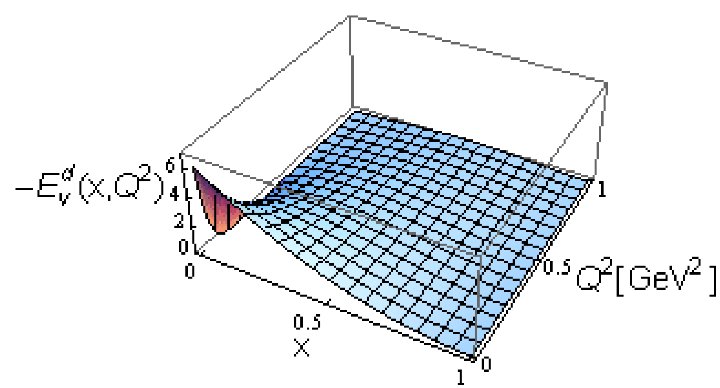
$H_v^u(x,Q^2)$



$H_v^d(x,Q^2)$



$E_v^u(x,Q^2)$



$E_v^d(x,Q^2)$

Results: Nucleon FFs and GPDs

- Nucleon GPDs in impact space Burkardt, Miller, Diehl, Kroll et al

$$
\begin{aligned}
q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} H_q(x,\mathbf{k}_\perp^2) e^{-i\mathbf{b}_\perp\mathbf{k}_\perp} = q(x)\frac{\kappa^2}{\pi\log(1/x)} e^{-\frac{\mathbf{b}_\perp^2\kappa^2}{\log(1/x)}} \\
e^q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} E_q(x,\mathbf{k}_\perp^2) e^{-i\mathbf{b}_\perp\mathbf{k}_\perp} = e^q(x)\frac{\kappa^2}{\pi\log(1/x)} e^{-\frac{\mathbf{b}_\perp^2\kappa^2}{\log(1/x)}}
\end{aligned}
$$

- Parton charge and magnetization densities in transverse impact space

$$
\begin{aligned}
\rho_E^N(\mathbf{b}_\perp) &= \sum_q e_q^N \int_0^1 dx q(x,\mathbf{b}_\perp) = \frac{\kappa^2}{\pi}\sum_q e_q^N \int_0^1 \frac{dx}{\log(1/x)} q(x) e^{-\frac{\mathbf{b}_\perp^2\kappa^2}{\log(1/x)}} \\
\rho_M^N(\mathbf{b}_\perp) &= \sum_q e_q^N \int_0^1 dx e^q(x,\mathbf{b}_\perp) = \frac{\kappa^2}{\pi}\sum_q e_q^N \int_0^1 \frac{dx}{\log(1/x)} e^q(x) e^{-\frac{\mathbf{b}_\perp^2\kappa^2}{\log(1/x)}}
\end{aligned}
$$

where $e_u^p = e_d^n = 2/3$ and $e_u^n = e_d^p = -1/3$

Results: Nucleon FFs and GPDs

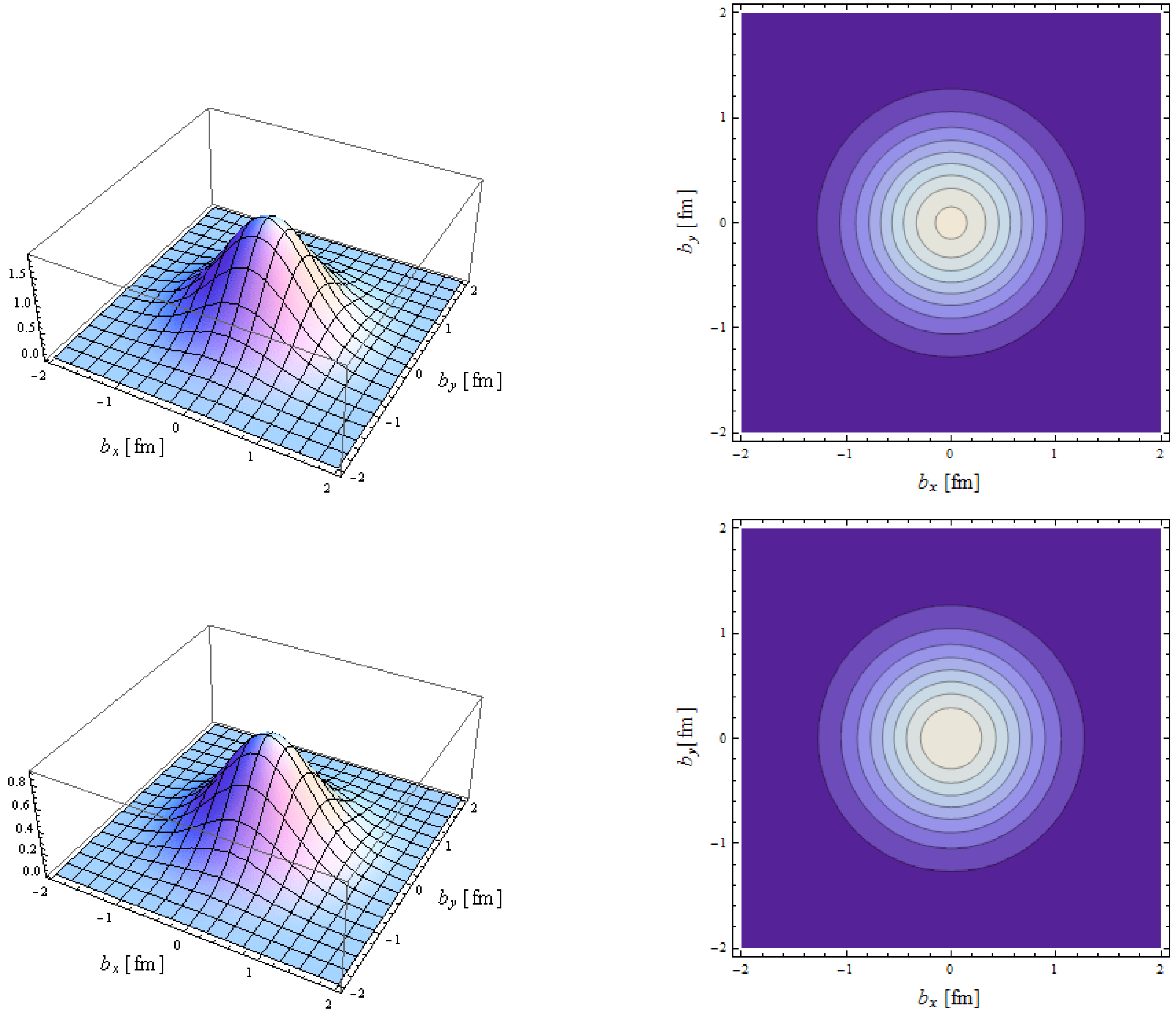
- Transverse width of impact parameter dependent GPD

$$\langle R_\perp^2(x)\rangle_q = \frac{\int d^2\mathbf{b}_\perp \mathbf{b}_\perp^2\, q(x,\mathbf{b}_\perp)}{\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)} = -4\frac{\partial \log H_v^q(x,Q^2)}{\partial Q^2}\bigg|_{Q^2=0} = \frac{\log(1/x)}{\kappa^2}$$

- Transverse rms radius

$$\langle R_\perp^2\rangle_q = \frac{\int d^2\mathbf{b}_\perp \mathbf{b}_\perp^2 \int\limits_0^1 dx q(x,\mathbf{b}_\perp)}{\int d^2\mathbf{b}_\perp \int\limits_0^1 dx q(x,\mathbf{b}_\perp)} = \frac{1}{\kappa^2}\left(\frac{5}{3}+\frac{\beta^q}{12\alpha^q}\right) \simeq 0.527\ \mathrm{fm}^2$$

Results: Nucleon FFs and GPDs



Plots for $q(x,\mathbf{b}_\perp)$ for $x=0.1$: $u(x,\mathbf{b}_\perp)$ - upper pannels, $d(x,\mathbf{b}_\perp)$ - lower pannels

Summary

- Soft–wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances

- Mass spectrum, decay constansts, form factors, GPDs

- Current and Future work:
  – GPDs and Deeply Virtual Exclusive Processes
  – Baryon excitation spectrum and form factors
  – Mesons and baryons: including multiparton states
  – Hybrid and exotic states