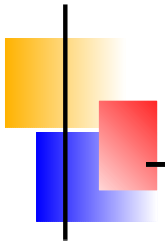


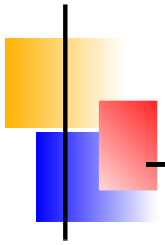
From confinement to new states of dense QCD matter

From Quarks and Gluons to Hadrons and Nuclei, Erice, Sicily, 17 Sept2011

Kurt Langfeld

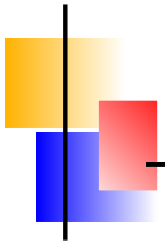
School of Comp. and Mathematics and The HPCC, Univ. of Plymouth, UK

Andreas Wipf
TPI, University of Jena, Germany



Introduction:

- The empty vacuum of Yang-Mills theory
  $\Rightarrow$ centre sectors



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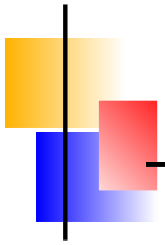

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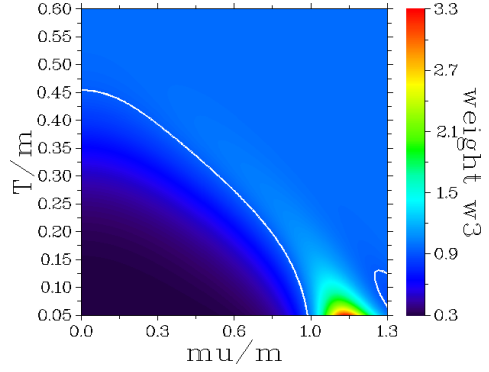

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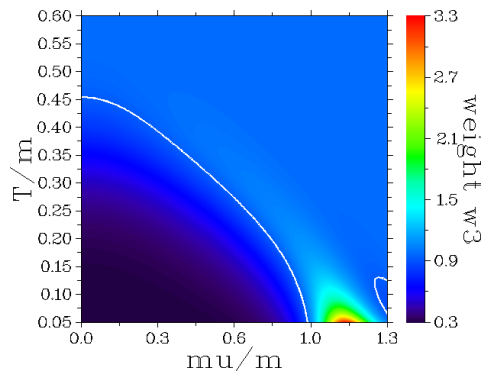
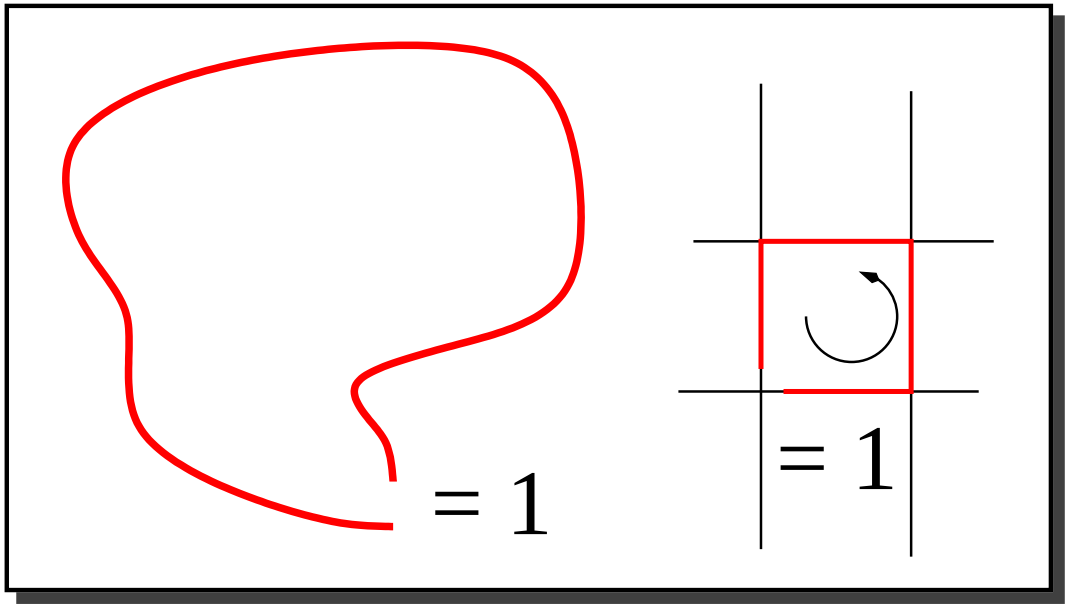


Yang-Mills moduli

- My name is vacuum - `the vacuum:`

  (pert.) vacuum $\leftrightarrow$ all contractible loops are 1



$$\exp\{i \int A_\mu \, dx^\mu\} = 1$$

example:

$A_\mu(x) = 0$ or

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more vacua?

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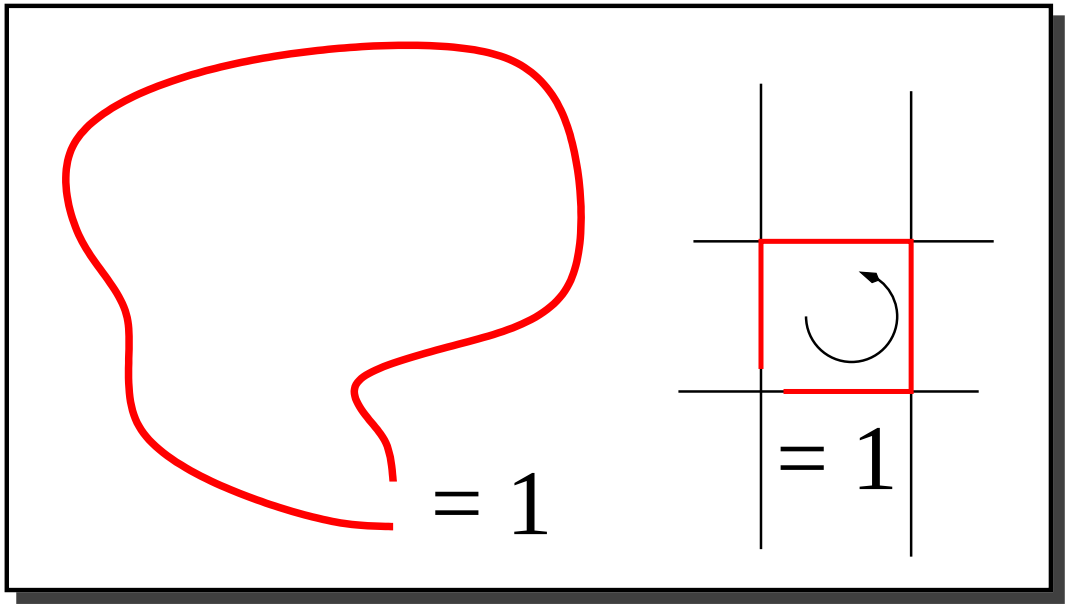

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more vacua?

- constructing the moduli space

  $\Rightarrow$ need to "devide out" the gauge transformations

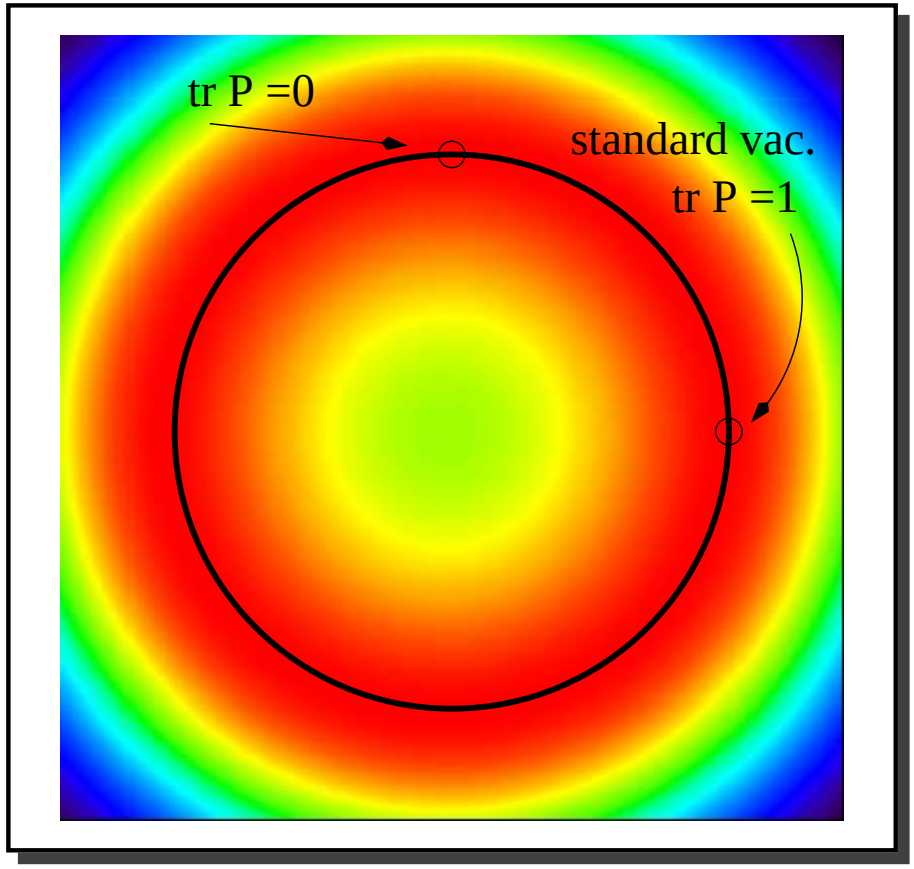
`[Keurentjes, Rosly, Smilga, PRD 58 (1998) 081701]`

`[Schaden, PRD 71 (2005) 105012]`

`[Langfeld, Lages, Reinhardt, PoS LAT2005:201,2006.]`

Yang-Mills moduli

- There is a $U(1)^{4(N_c-1)}$ manifold of gauge in-equivalent vacua $\Rightarrow$ flat directions



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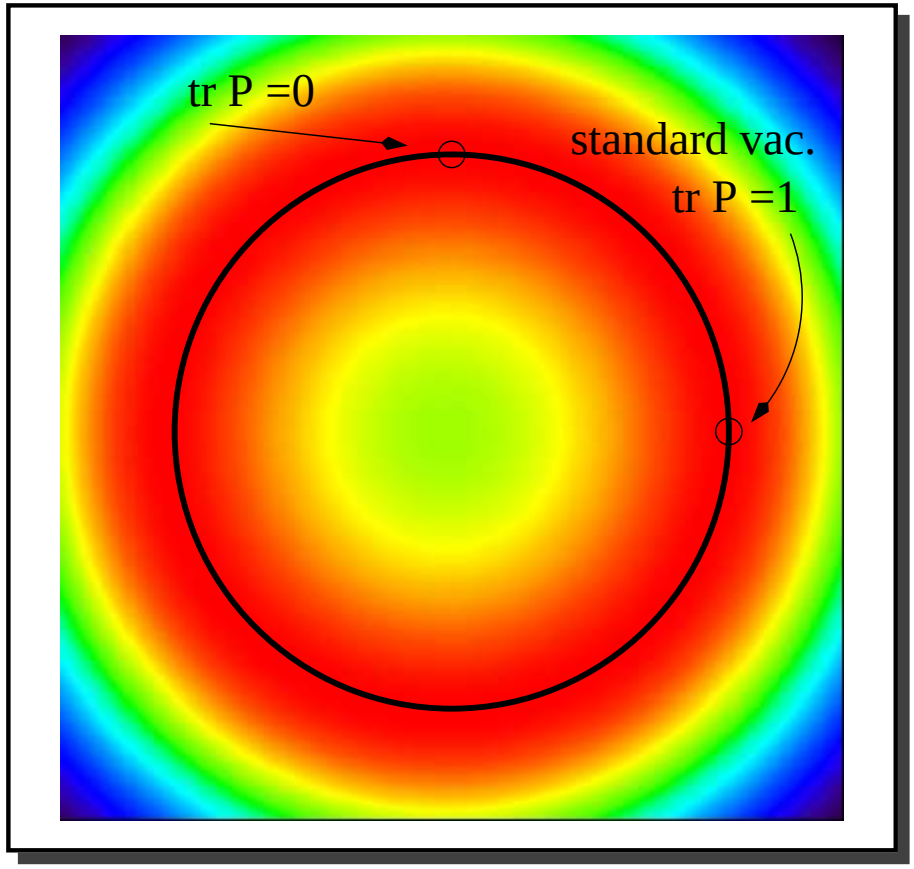

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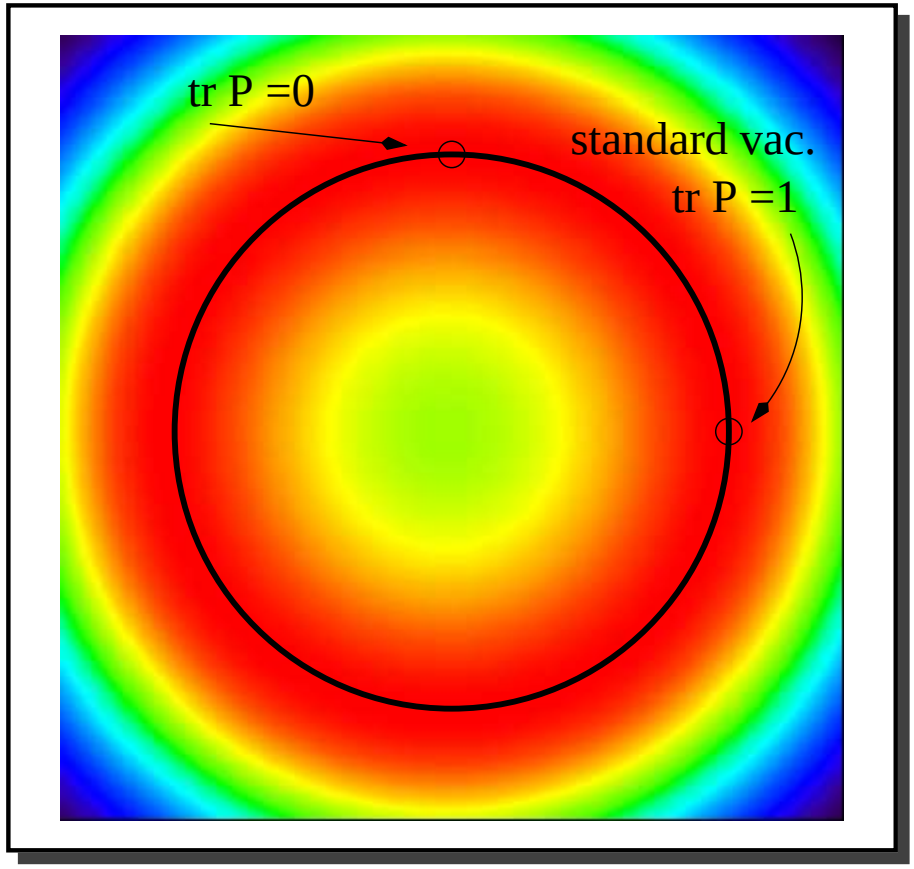
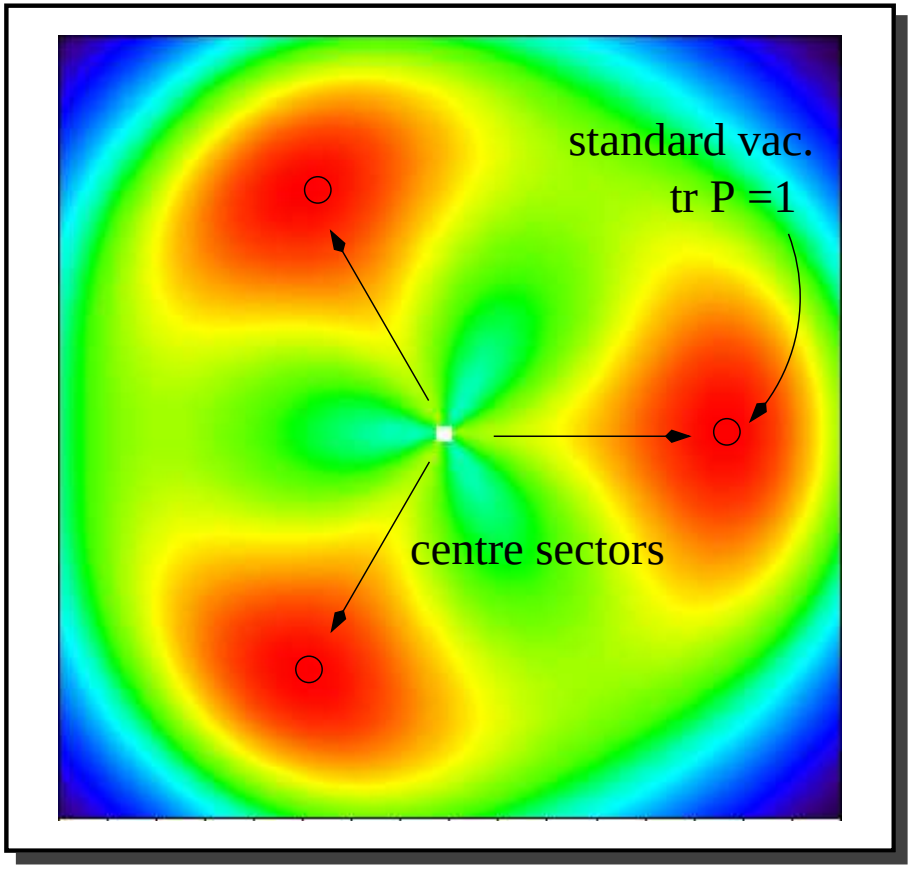


- Polyakov line P is sensitive to the vacuum
  $\Rightarrow$ Litmus paper
- *none* of the vacuum states confines quarks
  (trivial potential)

Yang-Mills moduli: SU(3)

- quantum fluctuations lift the flat directions



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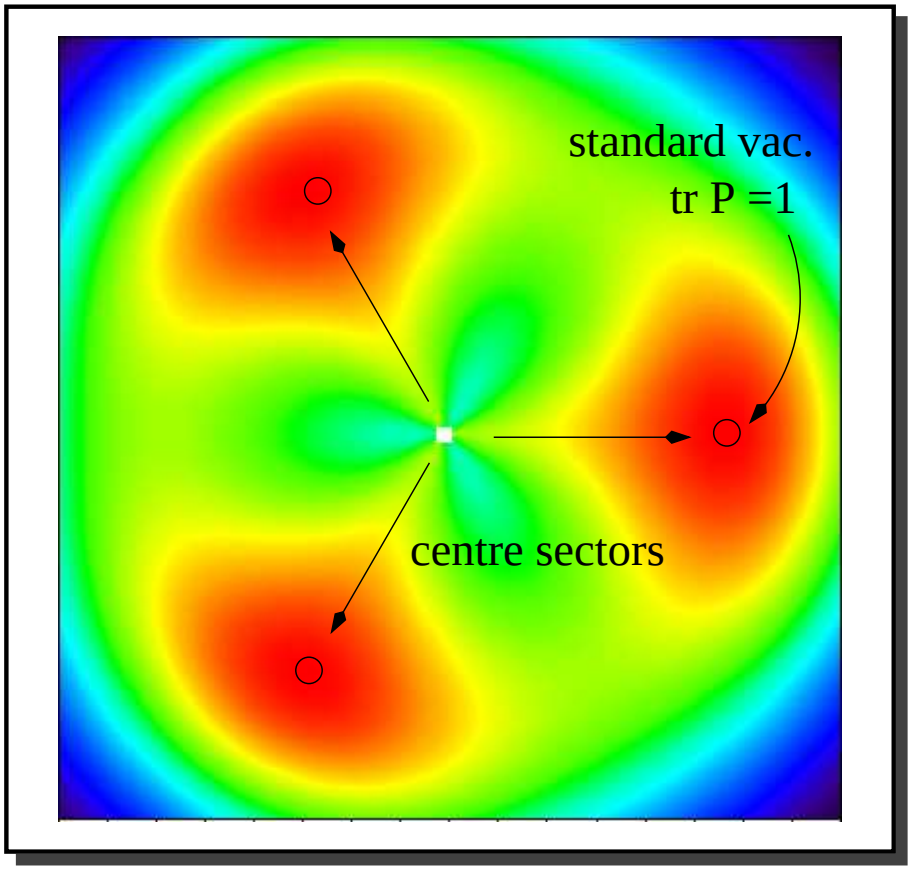

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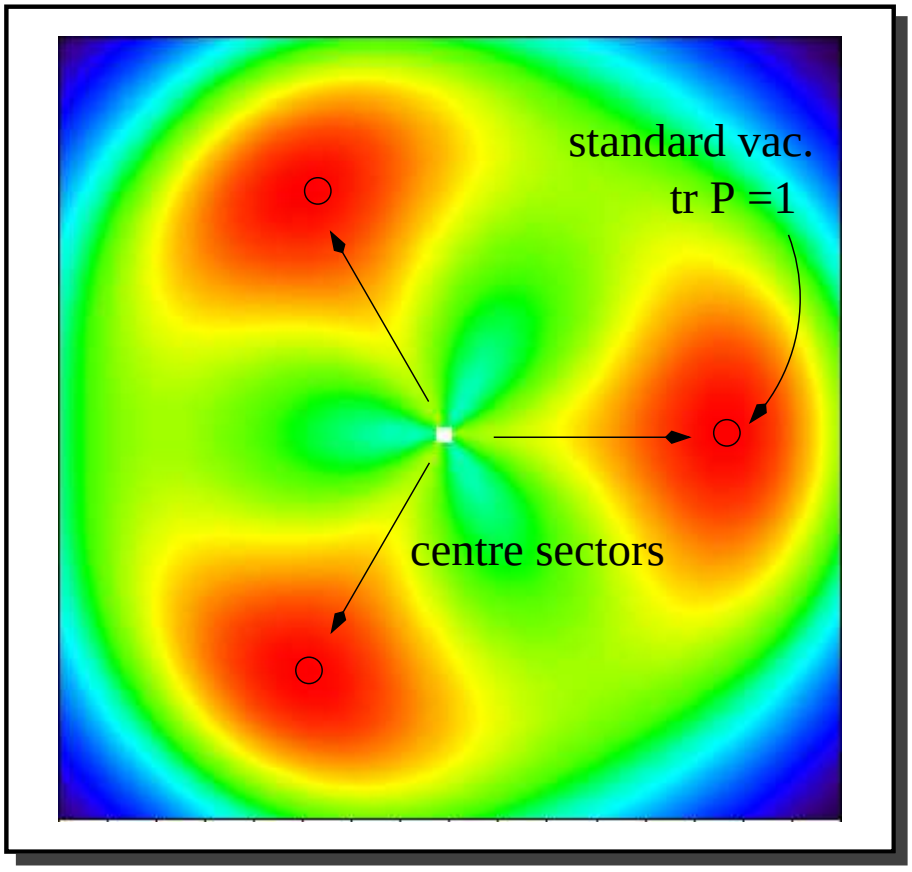

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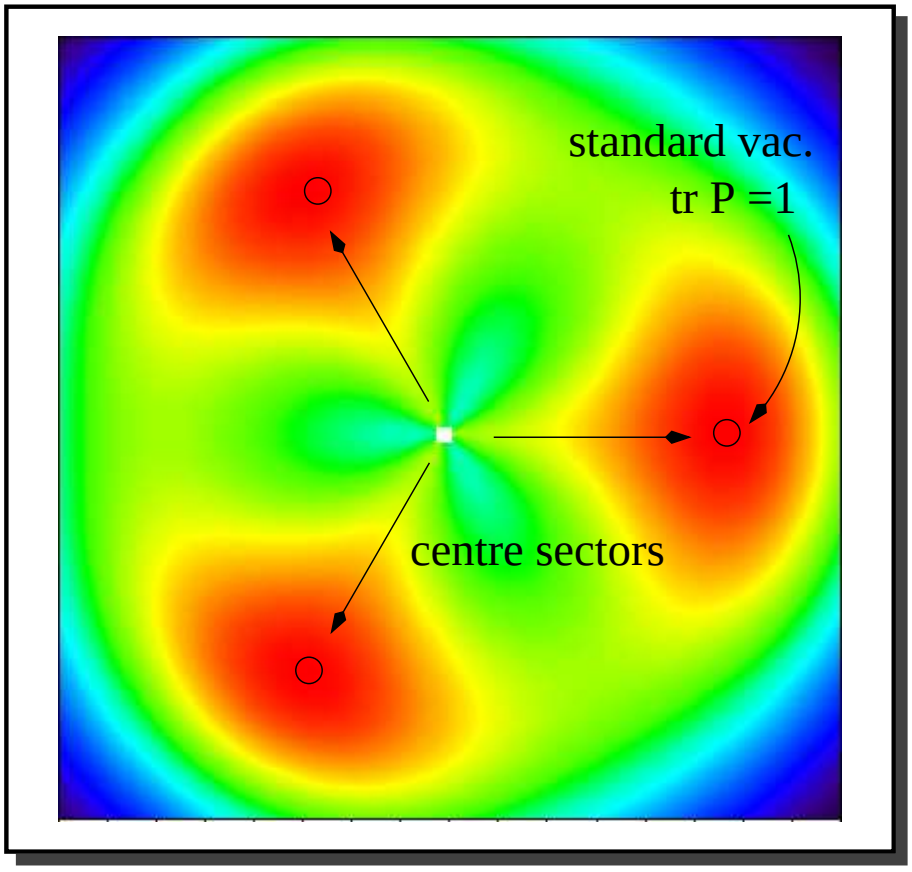

- remanent Z_3 symmetry
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- confinement phase:
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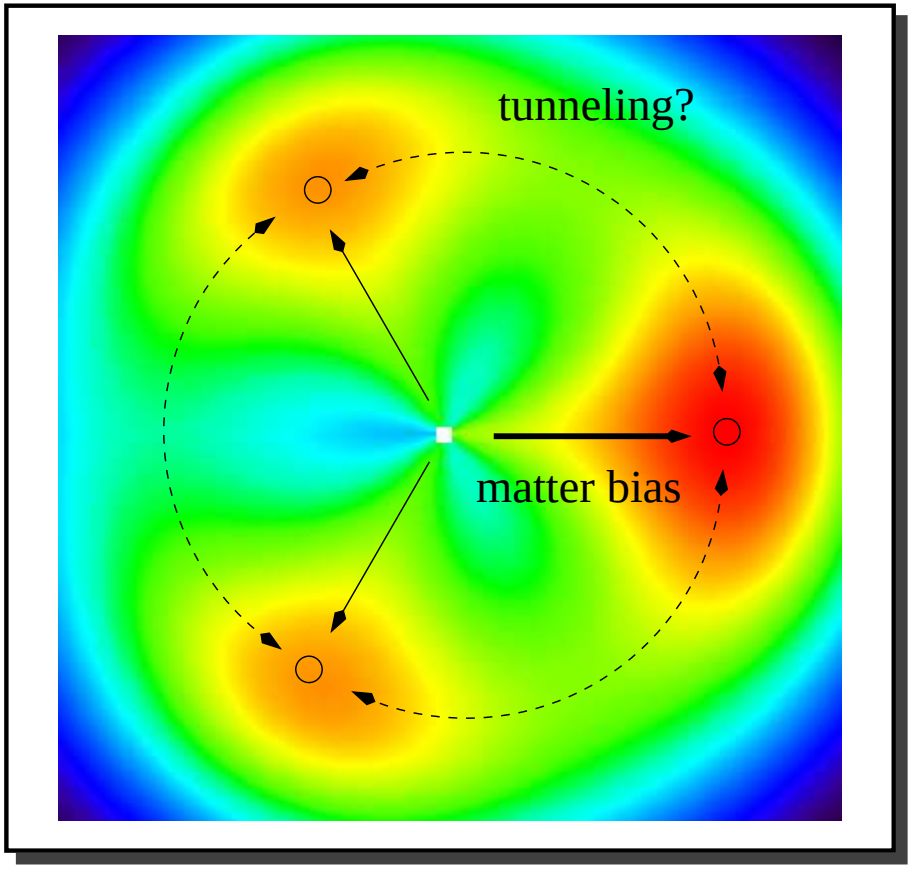
- high temperature phase:
  frozen centre sector (SSB)
  $\Rightarrow$ deconfinement

Yang-Mills moduli: SU(3)

- including dynamical matter (e.g. quarks, Higgs, ...)



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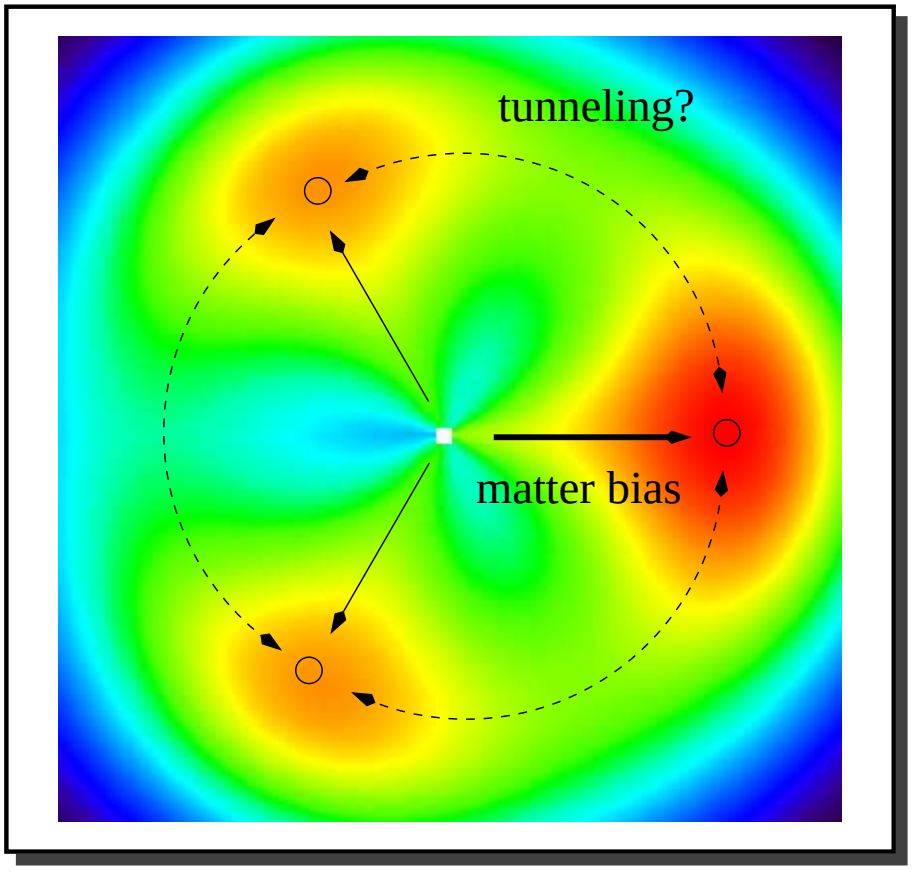

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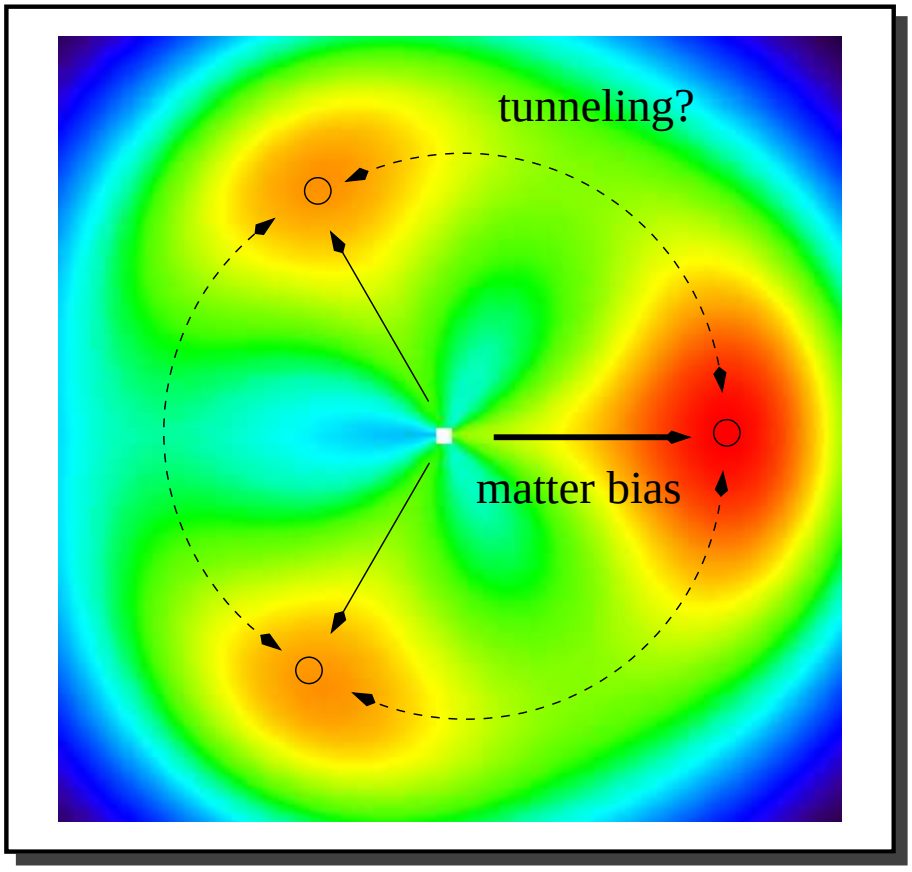

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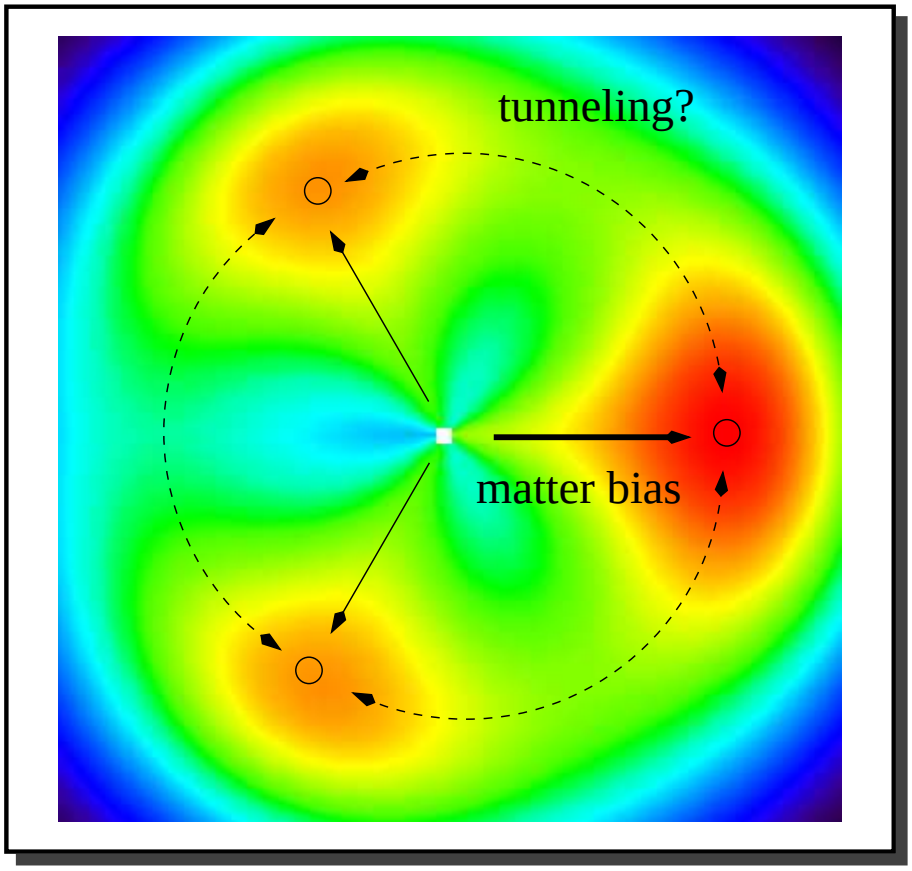


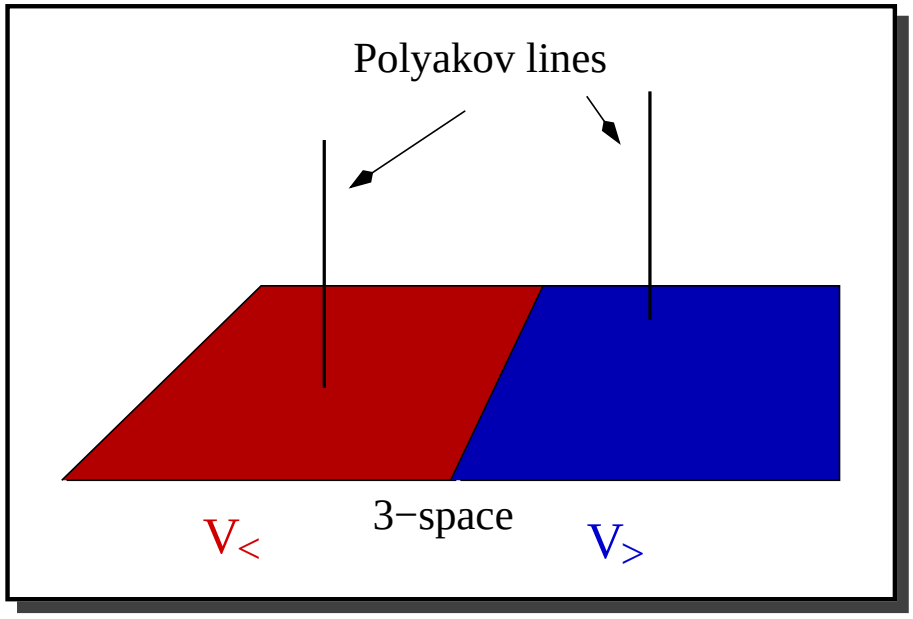
- $\Rightarrow$ bias towards the trivial centre sector
- do *centre sector transitions* still take place ?
- quarks are sensitive to the centre sector phenomenology of the centre sector transitions?

Centre sector transitions:

- Define:

$$P_< = \sum_{V_<} \text{tr} \prod_t U_0(x)$$

$$P_> = \sum_{V_>} \text{tr} \prod_t U_0(x)$$



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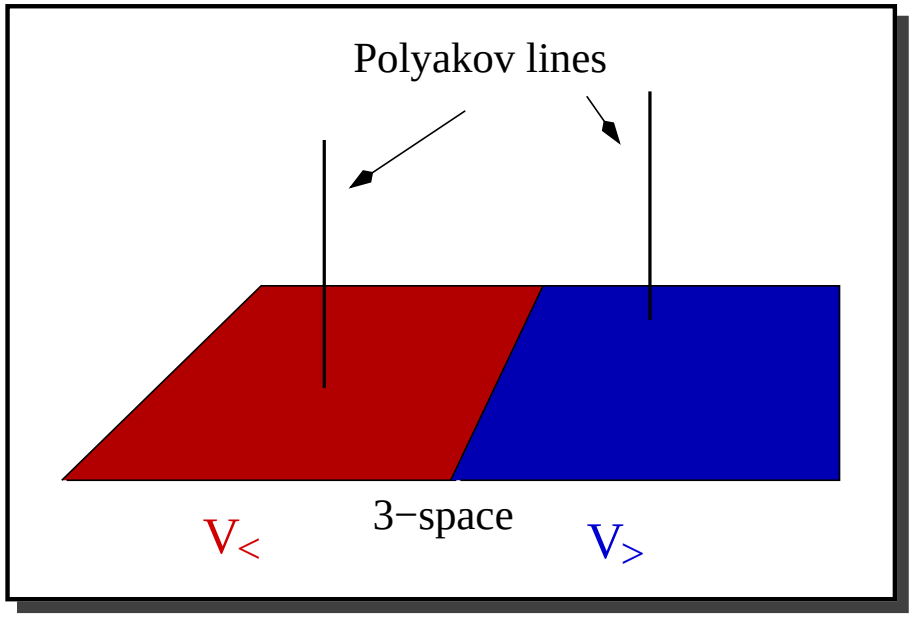
$$C = n: \ P \in \mathbb{C} \to z_n$$



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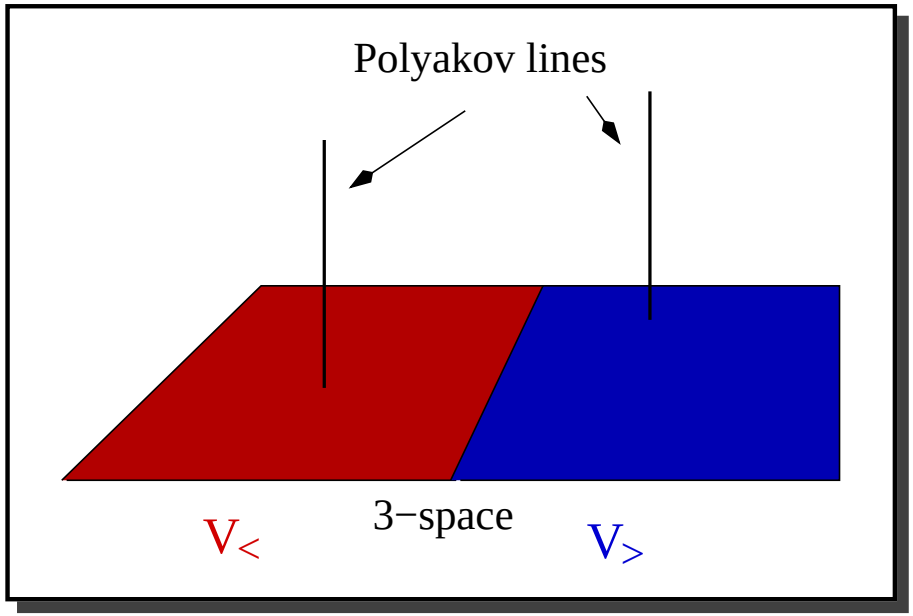

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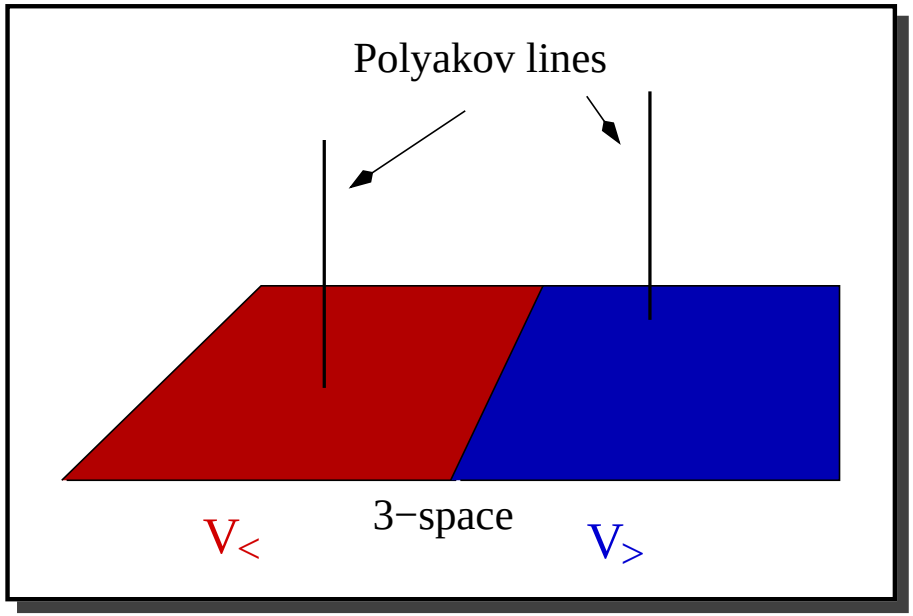

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- here: SU(2) Yang-Mills in comparison to SU(2) Yang-Mills + Higgs

Centre sector transitions:

- tunneling coefficient for SU(2):

$C(P_<)$	-1	-1	$+1$	$+1$
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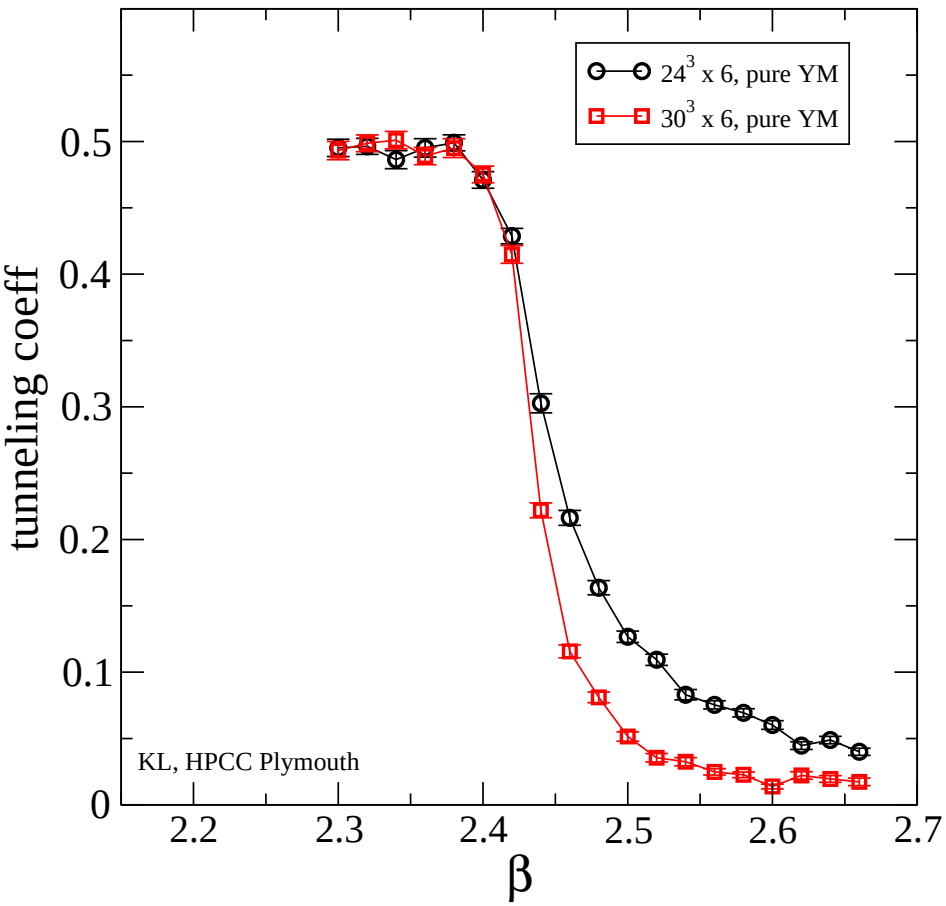

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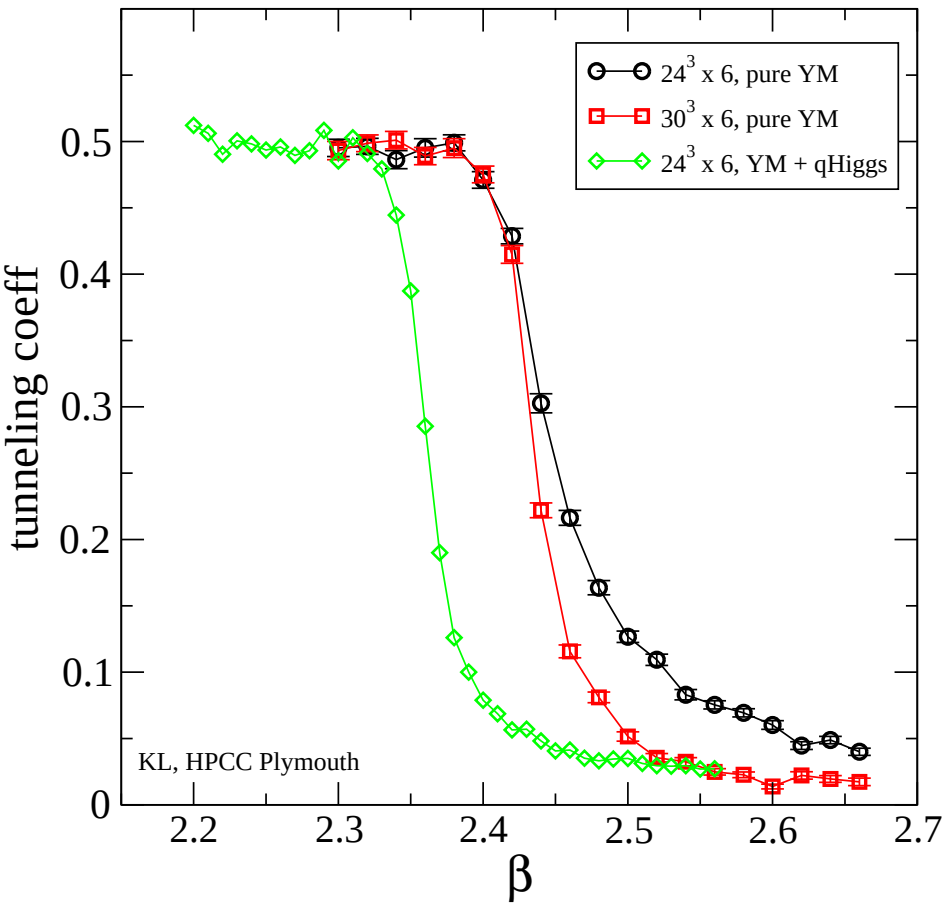
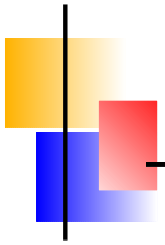


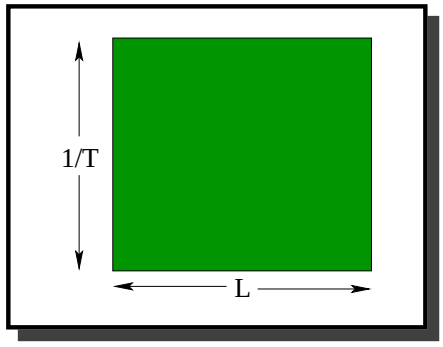


Centre sector transitions:

phenomenological impact ?

The Schwinger model:

- U(1) gauge theory
  massless fermion
  1+1 dimensions, torus



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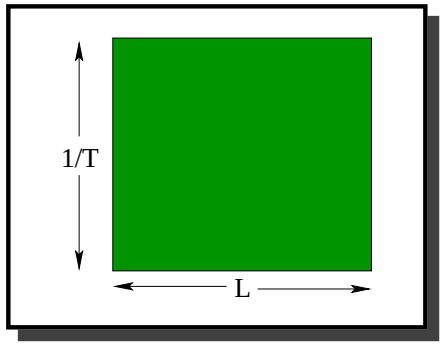

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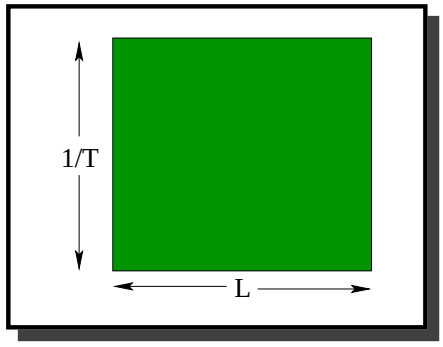

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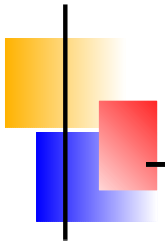
- physical states: *only mesons*
  add quark chemical potential μ
  $\Rightarrow$ partition function is independent of μ
  silver blaze problem



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- partition function factorises:

$$Z(\beta, L, \mu) = (2\pi)^2 \sqrt{\frac{\det'(-\triangle)}{\det'(-\triangle + m_\gamma^2)}} \int_0^1 dh_0 \, dh_1 \, \det(\mathrm{i}\not{\partial}_{h,\mu}) \, ,$$

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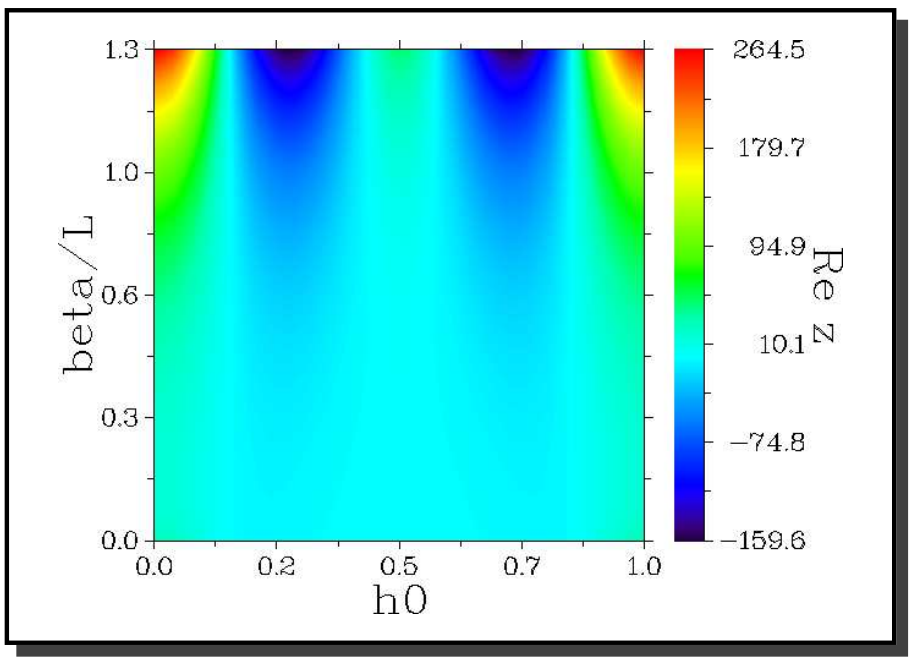

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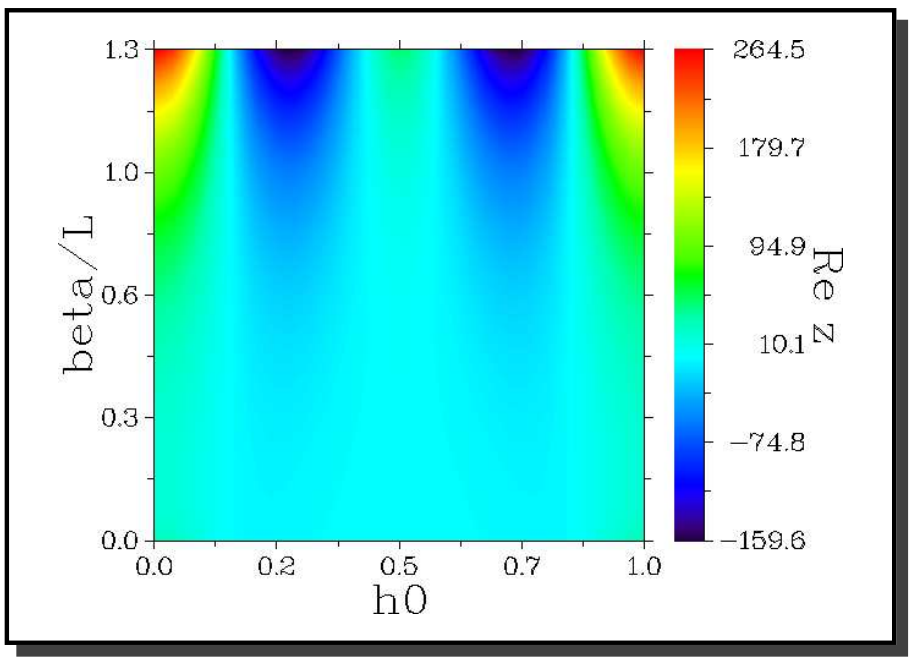

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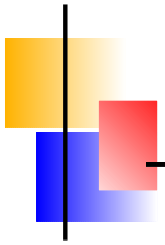
- *overlap problem* for MC simulations
- $\rho_B(h_0) \neq 0$

wrong physics!



The Schwinger model:

- baryon density: $[z = \exp\{-2\pi \mathrm{i}\, h_0\}]$

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- *with* centre sector transitions:
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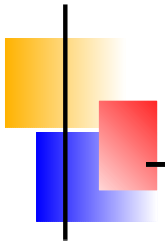
- μ independent!

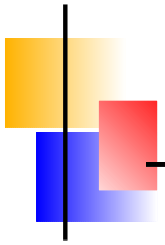
solves the silver blaze problem

[K. Langfeld, A. Wipf, arXiv:1109.0502]



Physics of centre sector transitions:

What is *Fermi Einstein condensation* ?



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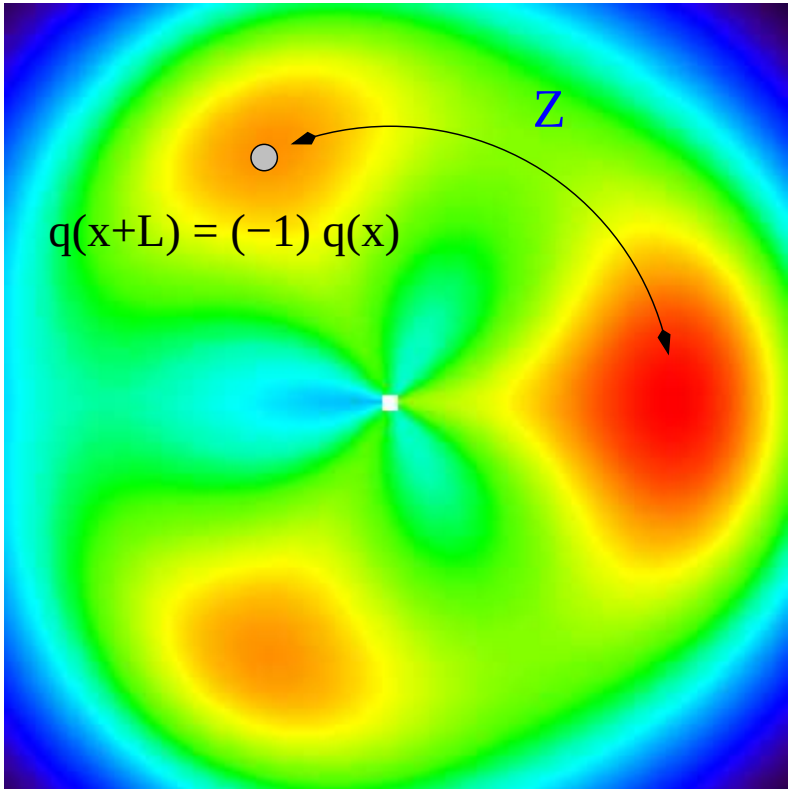
- generic for $SU(N_c)$ (at least for N_c even)!

[K. Langfeld, A. Wipf, PRD 81 (2010) 114502

Physics of centre sector transitions:

- Consider now SU(3) with matter:



Roberge-Weisz transformation

Physics of centre sector transitions:

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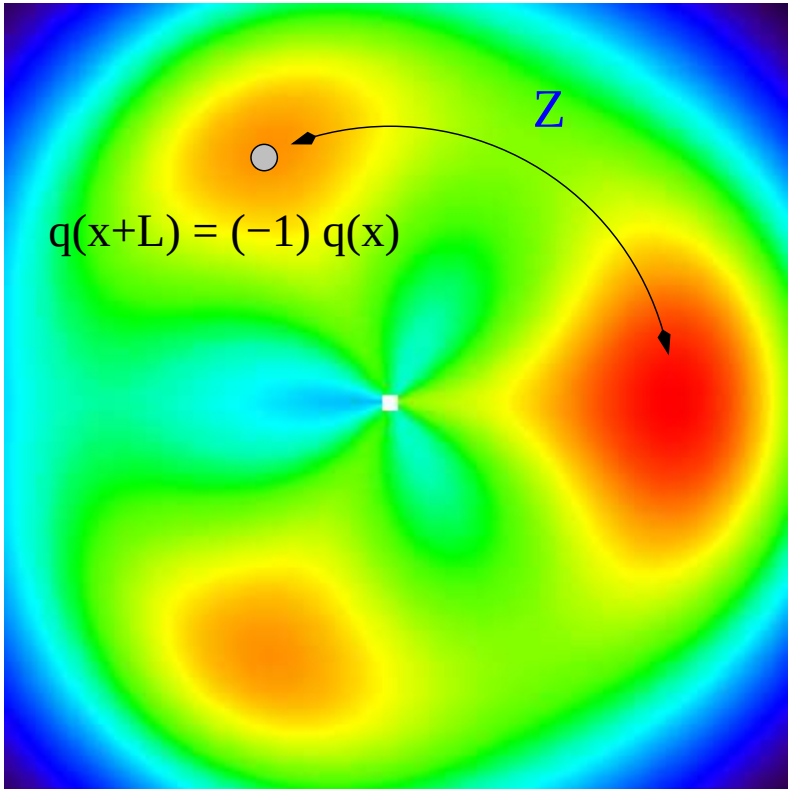
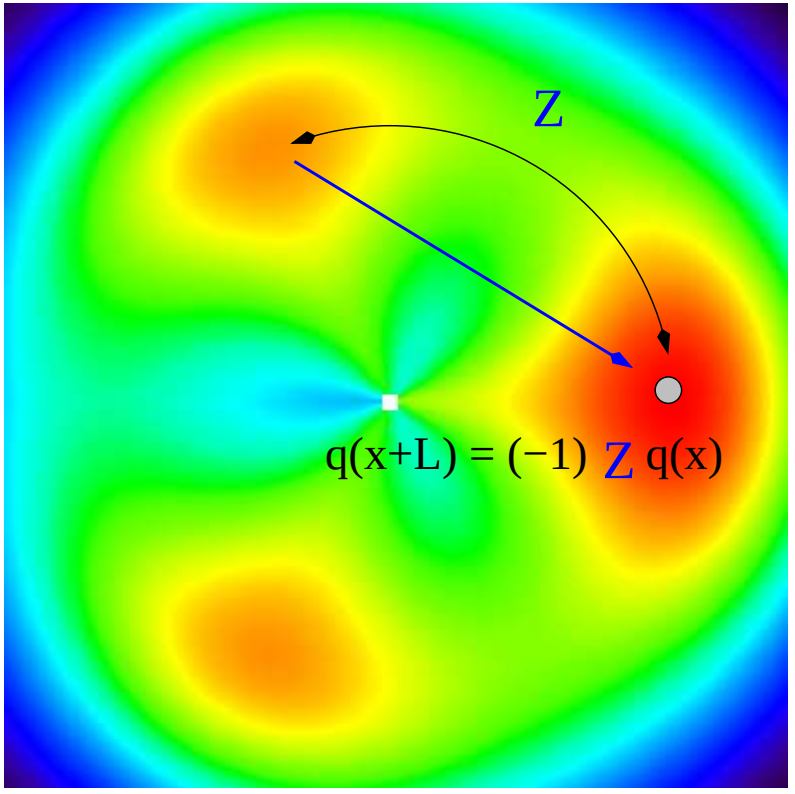
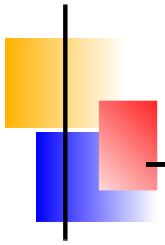




Roberge-Weisz transformation



Physics of centre sector transitions:

- SU(N_c even): Z = -1

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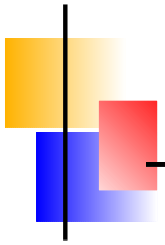

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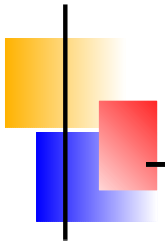
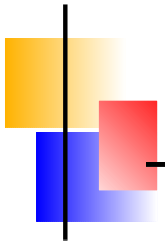
- quark-gluon-plasma phase:
  spontaneous breaking of centre symmetry
  $Z = -1$ sector does not occur $\Rightarrow$ Fermi statistics only
  [K. Langfeld, A. Wipf, PRD 81 (2010) 114502]



SU(3) Fermi gas with confinement

- FEC in SU(3) QCD-like theories ?

  $\Rightarrow \quad q(x+L) \;=\; [1/2 \pm i\sqrt{3}/2]\; q(x)$



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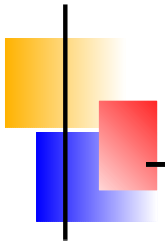

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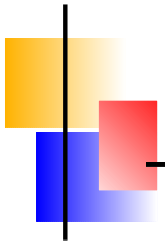
  $\rightarrow$ more realistic $\Rightarrow$ vortex background field



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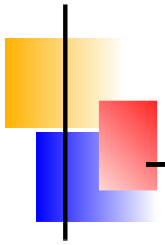
- consider extreme conditions: here T and/or μ

  $\rightarrow$ centre sector freeze out $\Rightarrow$ Fermi gas model



SU(3) Fermi gas with confinement

- Model partition function:

$$Z_Q = \sum_{n=1}^{N_c} \int \mathcal{D}q\mathcal{D}\bar{q}\ \exp\left\{\bar{q}\big(\mathrm{i}\partial\!\!\!/ + (A_0^{(n)} + \mathrm{i}\mu)\gamma^0 + \mathrm{i}m\big)q\right\}$$

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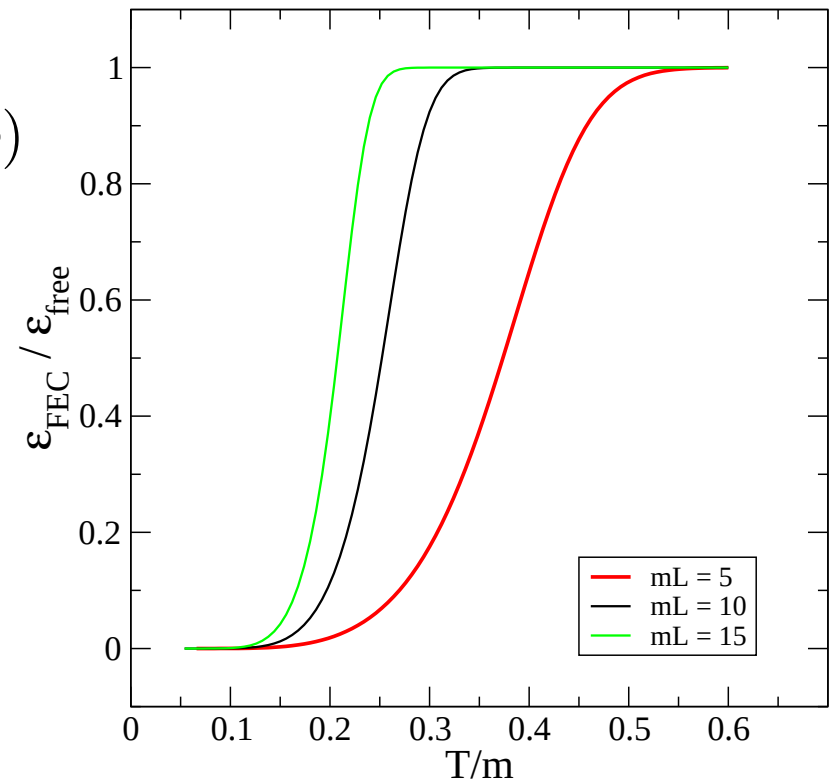
- Thermal energy density:

$$E_{\mathrm{therm}}(T) = \sum_n \omega_n \sum_p E(p) \left[\frac{z_n^*}{\mathrm{e}^{\beta E(p)} + z_n^*} + \frac{z_n}{\mathrm{e}^{\beta E(p)} + z_n}\right] .$$

ω_n : sector weights

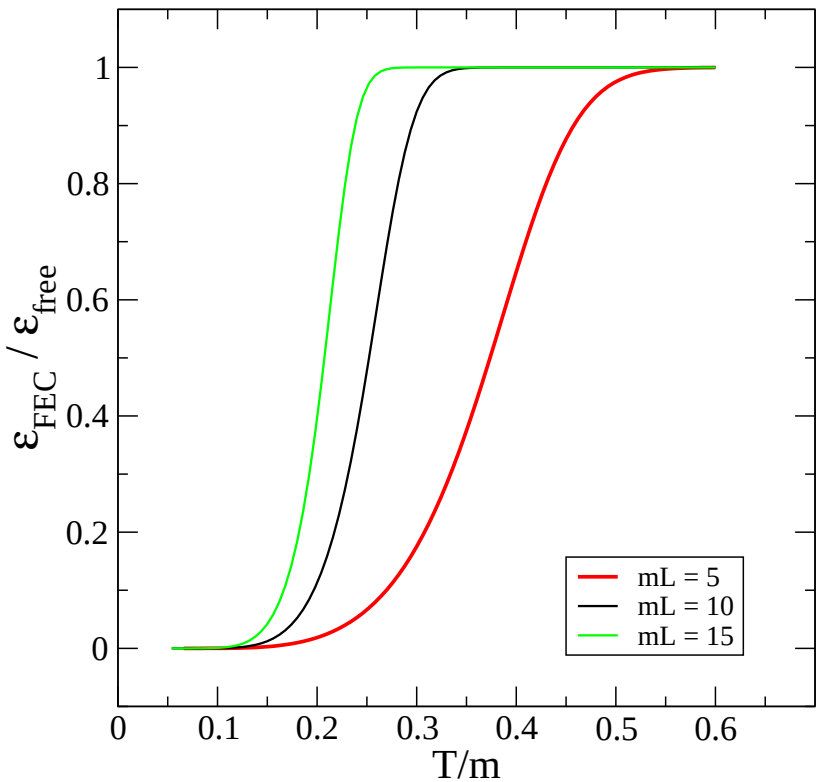
if $\omega_{1,2} = 0,\ \omega_3 = 1,$

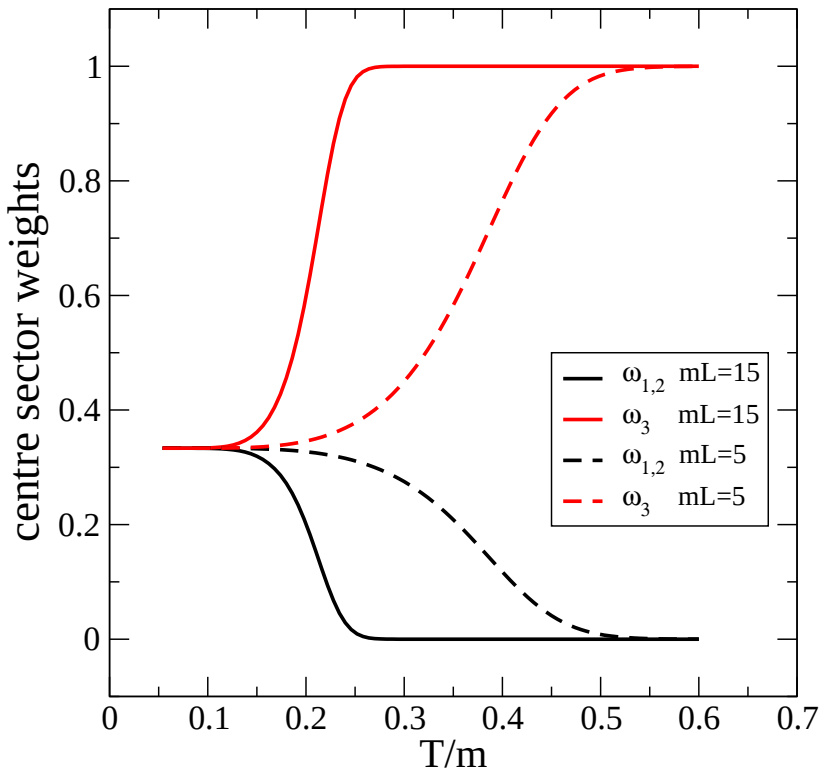
$\Rightarrow$ Fermi-gas model



SU(3) Fermi gas with confinement

- Centre-sector weights and deconfinement:
  $\omega_{1/2}$: non-trivial sectors, ω_3 : perturbative sector





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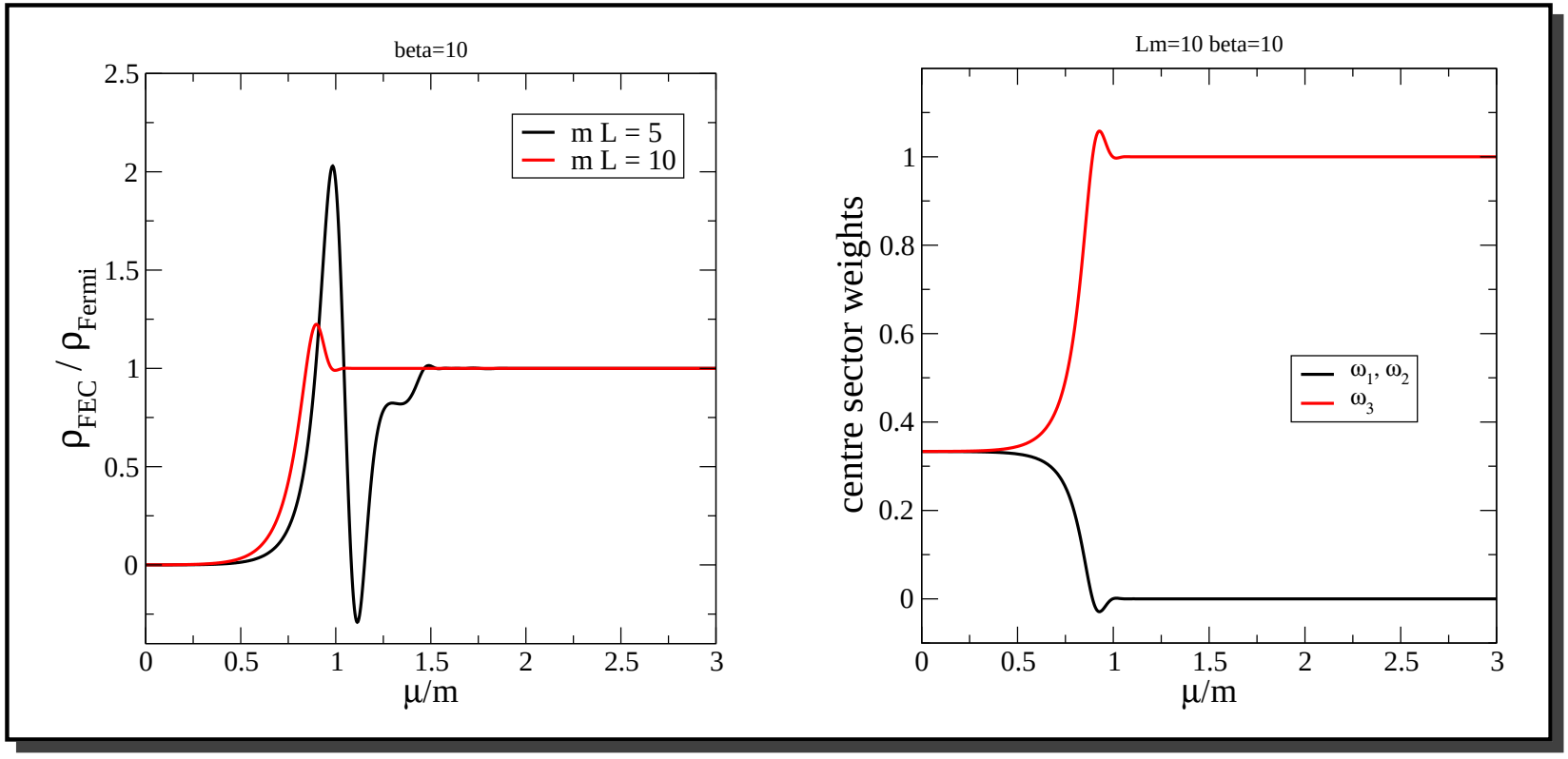

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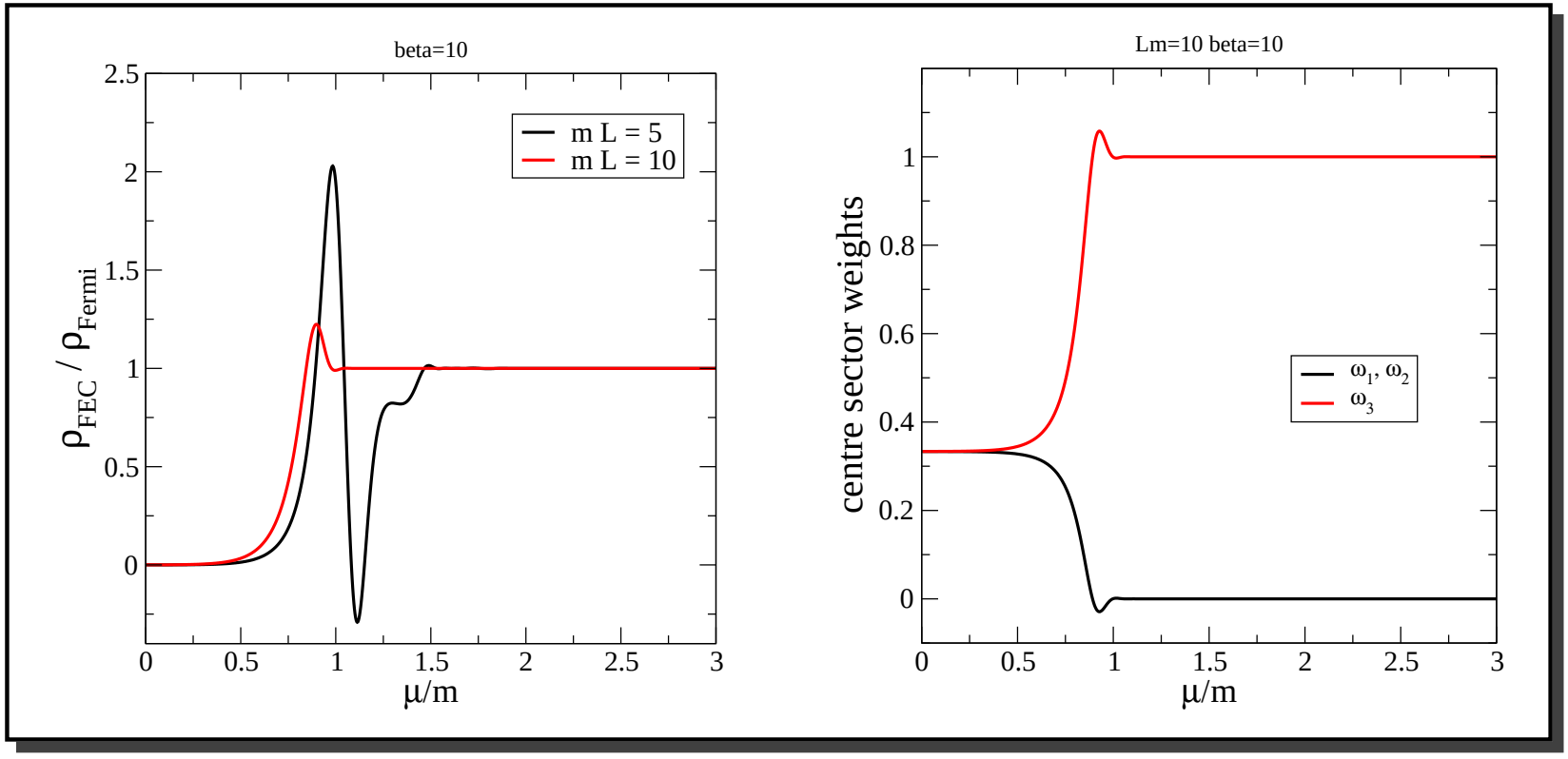

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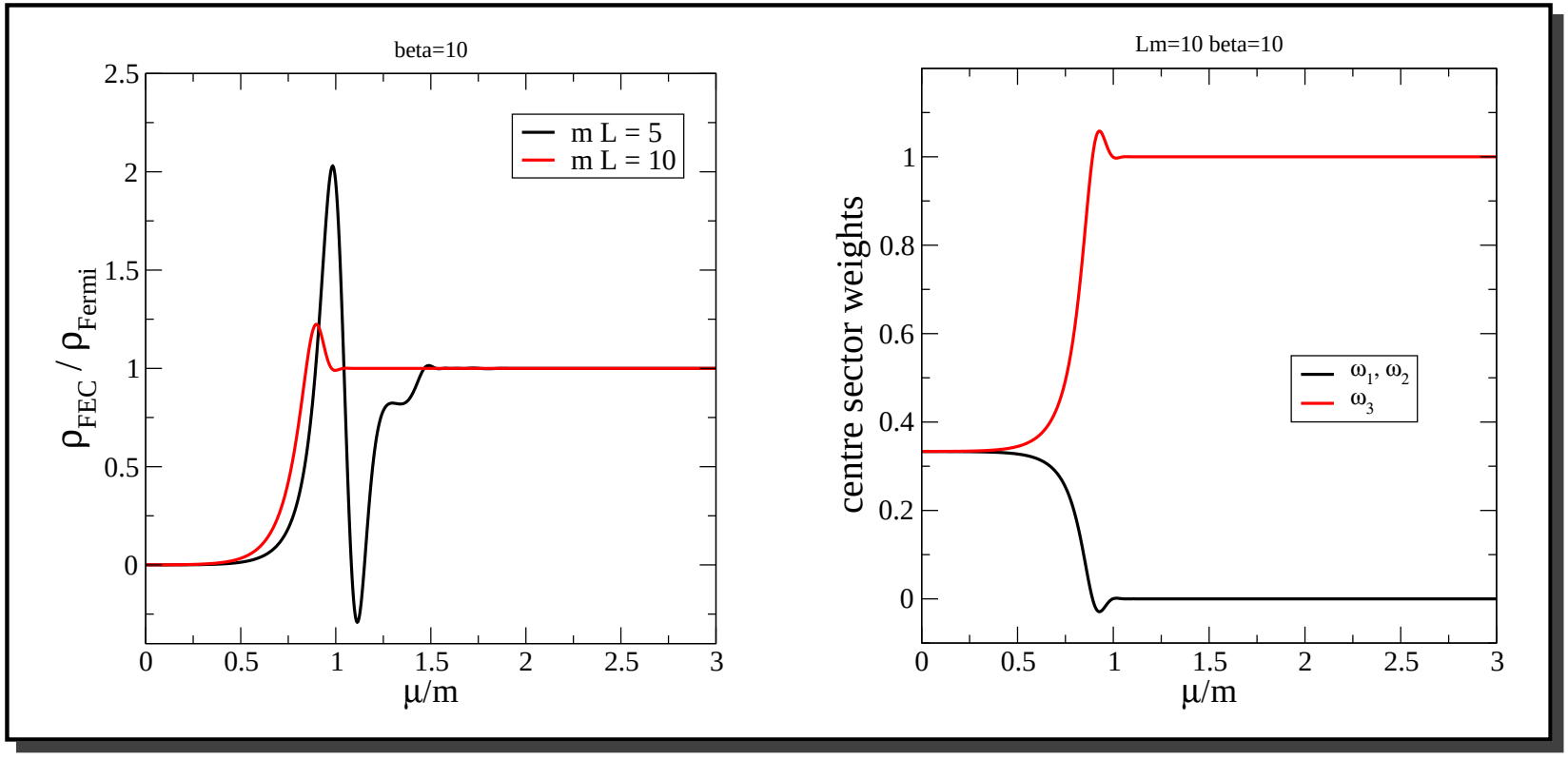
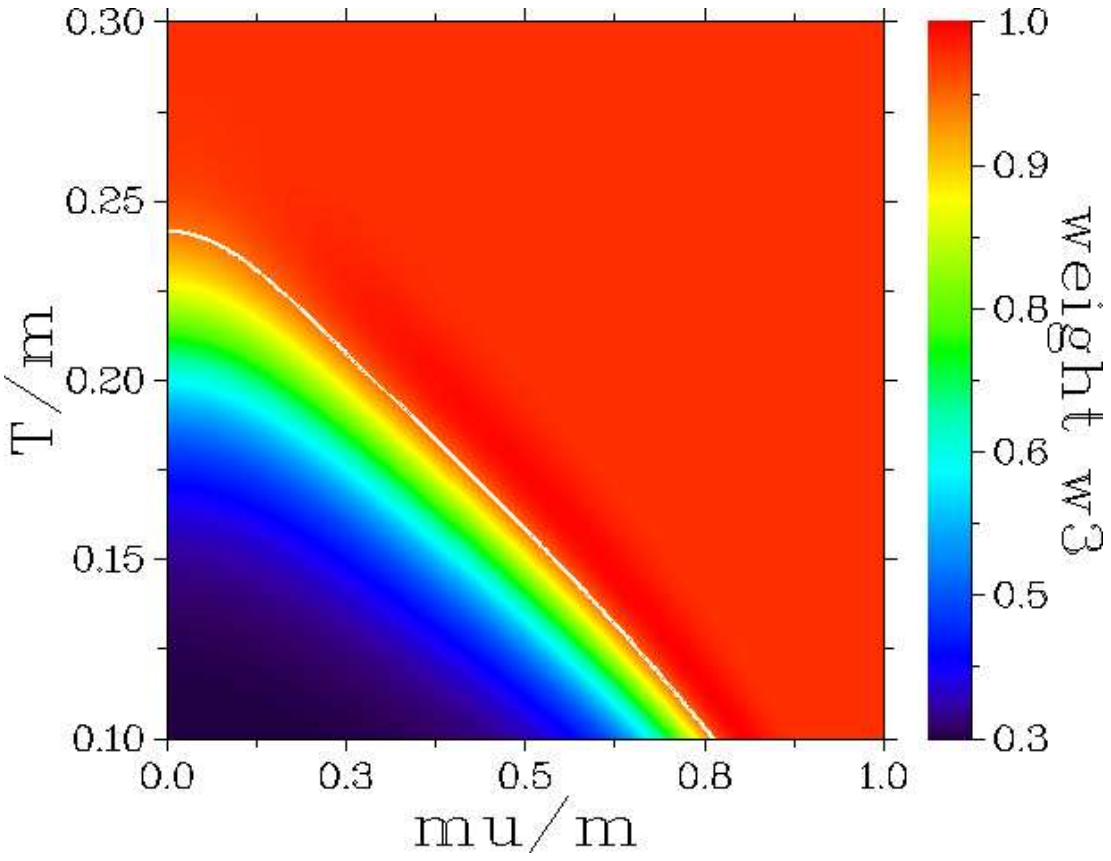


- transition to Fermi gas at μ_c
- excess of density close to μ_c $\Rightarrow$ FEC

SU(3) Fermi gas with confinement

- $\mu - T$ phase diagram: `large volumes:` $mL = 15$

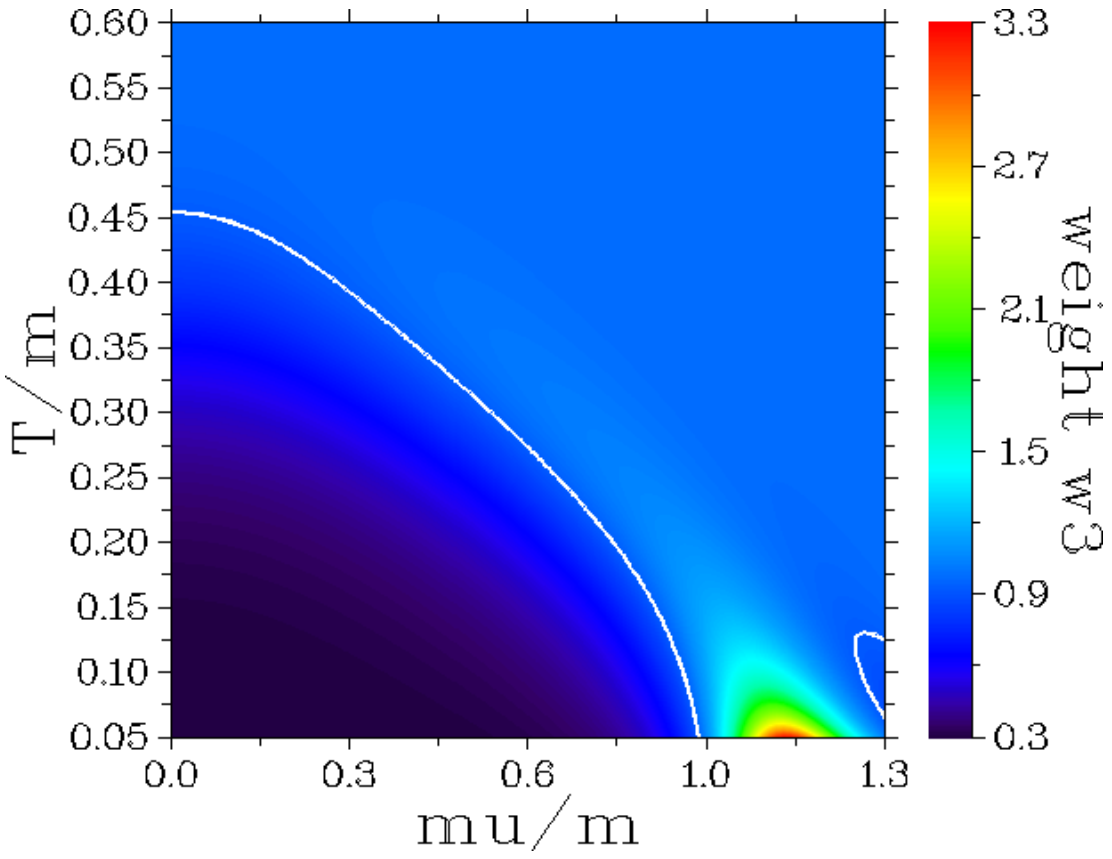


(use weight $|\omega_3|$)

($\omega_3 \in \mathbb{C}$ for $\mu \neq 0$)

SU(3) Fermi gas with confinement

- $\mu - T$ phase diagram: small volumes: $mL = 5$



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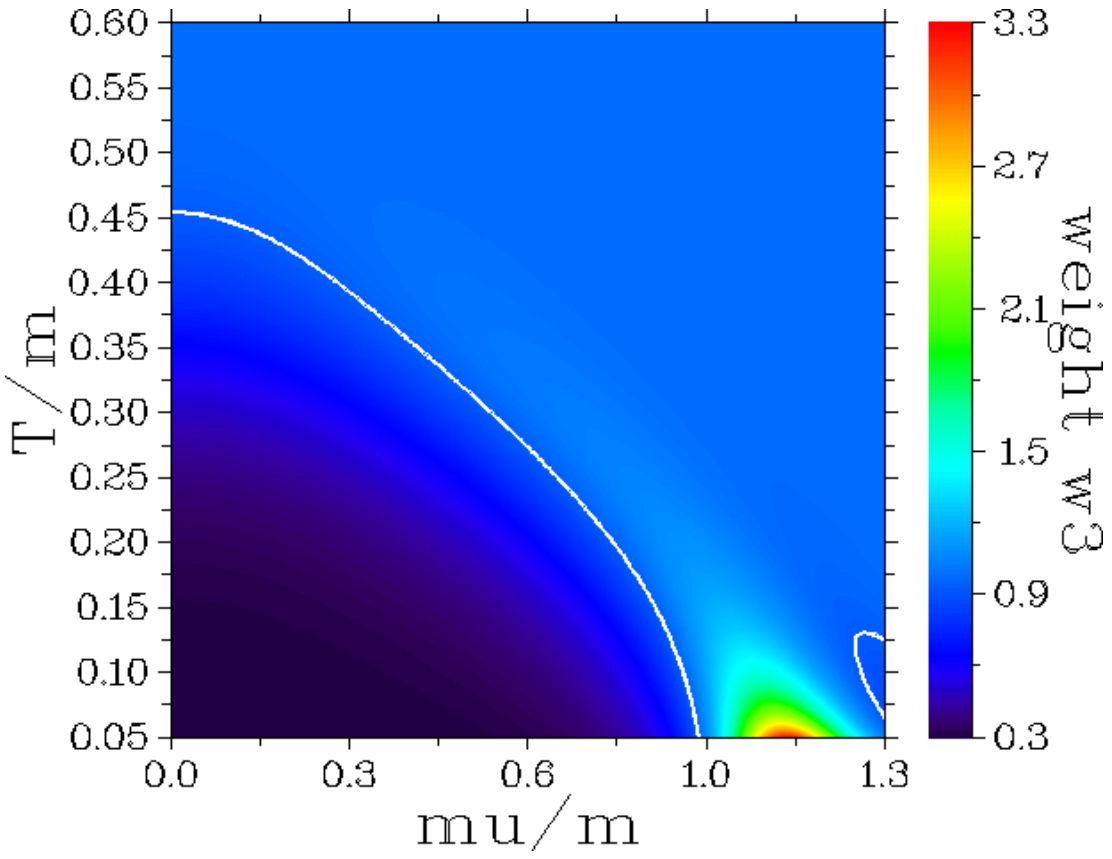

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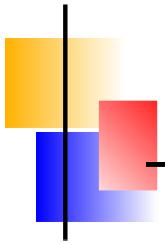
need small volumes (pressure) for FEC!



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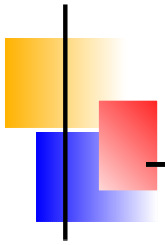

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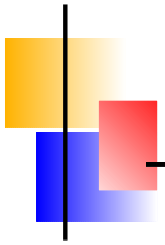

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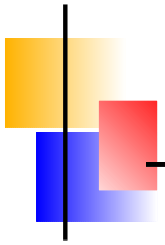
- Fermi Einstein condensation:
  centre dressed quarks acquire Bose-type statistics
  condensation: BEC $\leftrightarrow$ FEC
  (confinement phase only)



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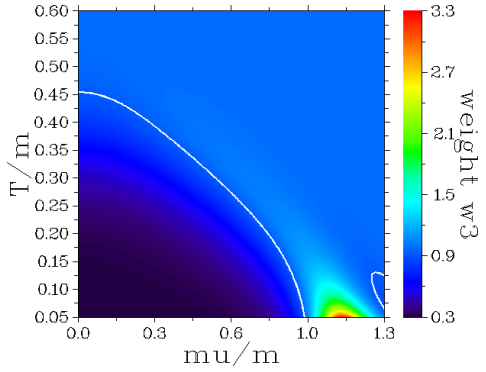

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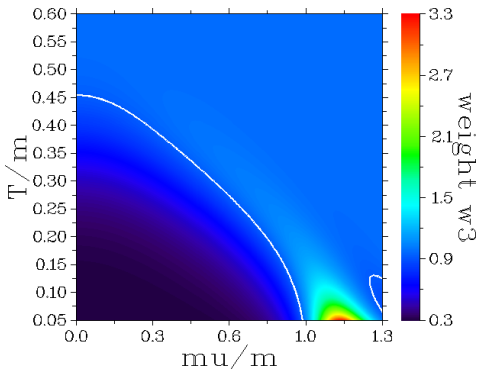

- outlook:

  more realistic QM:

  including chiral SSB, pions,...

  centre sectors $\leftrightarrow$ volumes spanning centre vortices

  FEC $\leftrightarrow$ statistical model?