The Spin Structure of the Nucleon: a phenomenological introduction

Franco Bradamante
Trieste University and INFN
• the QCD structure
  
  Longitudinal
  Transverse

• the experiments

• results on transversity
  
  Sivers effects

• outlook
Magnetic moments

The nucleon is not a Dirac particle \((point-like\) particle) \(\mu = \frac{e\hbar}{2mc}\)

\[\mu_p = +2.79 \mu_N\] Frisch and Stern (1933)

\[\mu_n = -1.91 \mu_N\] should be 0

\(\rightarrow\) per-se indication of internal structure
THE QUARK MODEL
Major Breakthrough

hadron spectroscopy $\iff$ the QUARK MODEL (1964)

$|p\rangle = |uud\rangle$  $|n\rangle = |udd\rangle$  SU(6)

magnetic moments:

$$\mu_p = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d$$

$$\mu_n = \frac{4}{3} \mu_d - \frac{1}{3} \mu_u$$
Major Breakthrough

hadron spectoscopy $\iff$ the QUARK MODEL (1964)

$|p\rangle = |uud\rangle \quad |n\rangle = |udd\rangle$  

SU(6)

magnetic moments:

$$\mu_p = \frac{4}{3} \mu_u - \frac{1}{3} \mu_d \quad \mu_n = \frac{4}{3} \mu_d - \frac{1}{3} \mu_u$$

assuming $u$ and $d$ Dirac particles with

$$m \approx \frac{1}{3} M_N$$

$$\mu_u = \frac{q \hbar}{2m_u c} = 2\mu_N \quad \mu_d = -\mu_N$$

$$\mu_p = 3\mu_N \quad \mu_n = -2\mu_N$$

similar agreement for all baryons
The Constituent Quark Model

in this model the **spin of the nucleon** is given by the spin of the quarks

probability of finding a quark in a given state of polarization

\[
\begin{align*}
\uparrow u &= \frac{5}{3} & \uparrow d &= \frac{1}{3} \\
\downarrow u &= \frac{1}{3} & \downarrow d &= \frac{2}{3} \\
\Delta u &= u - u = \frac{4}{3} & \Delta d &= d - d = -\frac{1}{3}
\end{align*}
\]

\[
\Delta \Sigma = \Delta u + \Delta d = 1
\]
The Constituent Quark Model

In this model, the spin of the nucleon is given by the spin of the quarks.

The existence of quarks and their properties firmly established in DEEP INELASTIC SCATTERING.

\[ \frac{\Delta u}{u} = \frac{5}{3}, \quad \frac{\Delta d}{d} = \frac{1}{3}, \quad \Delta u = u - u = \frac{4}{3}, \quad \Delta d = d - d = -\frac{1}{3}, \]

\[ \Delta \Sigma = \Delta u + \Delta d = 1 \]

SLAC
Friedmann and Kendell (1969)
Bjorken, Feynman
The Quark Contribution to the Nucleon Spin

EMC 1988

$$\Gamma_1^p = 0.123 \pm 0.013 \pm 0.019$$

$$\Delta \Sigma = 0.12 \pm 0.17$$

⇒ SPIN CRISIS
HOW IS $\Delta \Sigma$ MEASURED?
Deep Inelastic Scattering

key role in the study of the partonic structure of the nucleon

\[ Q^2 = -q^2 > 0 \]
\[ x = Q^2 / 2Mv \]
\[ v = E - E' \]
\[ y = v/E \]
\[ \gamma = \sqrt{Q^2 / v} \]

Inclusive DIS: only the incident and scattered leptons are measured

Semi-Inclusive DIS: the incident and scattered leptons, and at least one final state hadron are measured
Deep Inelastic Scattering

key role in the study of the partonic structure of the nucleon

valence quarks
sea quarks
 gluons

\[ Q^2 = -q^2 > 0 \]
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Inclusive DIS: only the incident and scattered leptons are measured
Semi-Inclusive DIS: the incident and scattered leptons, and at least one final state hadron are measured

NB: COMPLEMENTARY APPROACH @ RHIC
(will not mention)
Inclusive DIS: unpolarised

\[
\frac{d\sigma}{dx \, dy} = \frac{e^4}{4\pi^2 Q^2} \cdot \left\{ \frac{y}{2} \cdot F_1 + \frac{1}{2xy} \cdot \left( 1 - \frac{y}{2} - \frac{y^2}{4} \cdot \gamma^2 \right) \cdot F_2 \right\}
\]

\[F_2(x) = 2x \cdot F_1(x)\] \hspace{1cm} \text{Callan-Gross}

in the parton model

\[F_1(x) = \frac{1}{2} \sum_q e_q^2 \cdot \begin{pmatrix} q(x) + \bar{q}(x) \end{pmatrix} \]

\[q = u, d, s\]
Inclusive DIS: unpolarised

\[
\frac{d\sigma}{dx\,dy} = \frac{e^4}{4\pi^2Q^2} \cdot \left\{ \frac{y}{2} F_1 + \frac{1}{2xy} \left( 1 - \frac{y^2}{4\,\gamma^2} \right) F_2 \right\}
\]

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in the parton model

\[
F_1(x) = \frac{1}{2} \sum_q e_q^2 \cdot \left[ q(x) + \bar{q}(x) \right]
\]

\(q = u, d, s\)
Structure Functions and PDFs: $q(x)$

Inclusive DIS: unpolarised

$$\frac{d\sigma}{dx\,dy} = \frac{e^4}{4\pi^2 Q^2} \cdot \left\{ \frac{y}{2} \cdot F_1 + \frac{1}{2xy} \cdot \left( 1 - \frac{y}{2} - \frac{y^2}{4} \cdot y^2 \right) \cdot F_2 \right\}$$

measured at CERN, HERA, SLAC

$F_2(x) = 2x \cdot F_1(x)$  \text{Callan-Gross}

in the parton model

$$F_1(x) = \frac{1}{2} \sum_q e_q^2 \cdot \frac{1}{2} (x) + \overline{q}(x)$$

$q = u, d, s$

$q(x)$ from global analysis of DIS and hard scattering data (QCD fits)
Inclusive DIS: unpolarised

\[ \frac{d\sigma}{dx \, dy} = \frac{e^4}{4\pi^2 Q^2} \cdot \left\{ \frac{y}{2} F_1 + \frac{1}{2xy} \left[ \frac{1 - \frac{y}{2} - \frac{y^2}{4} \cdot \gamma^2}{\gamma^2} \right] \cdot F_2 \right\} \]

\[ F_2(x) = 2x \cdot F_1(x) \quad \text{Callan-Gross} \]

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\[ F_1(x) = \frac{1}{2} \sum_q e_q^2 \cdot q(x) + \bar{q}(x) \]

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Parton Distribution Functions

$q(x)$: **number density or unpolarised distribution**

- probability of finding a quark with a fraction $x$ of the longitudinal momentum of the parent nucleon
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- probability of finding a quark with a fraction $x$ of the longitudinal momentum of the parent nucleon

$\Delta q(x) = q^\uparrow - q^\downarrow$: longitudinal polarization or helicity distribution

- in a longitudinally polarised nucleon, probability of finding a quark with a momentum fraction $x$ and spin parallel to that of the parent nucleon
$\Delta q$’s can be extracted from the DIS cross-section asymmetry $\Delta \sigma$ for parallel and antiparallel lepton and nucleon spins
Inclusive DIS: beam and target longitudinally polarized

\[ \frac{d\Delta\sigma}{dx \, dy} = \lambda \cdot \frac{e^4}{4\pi^2 Q^2} \left[ \left( 1 - \frac{y}{2} - \frac{y^2}{4} \cdot \gamma^2 \right) \cdot g_1 - \frac{y}{2} \cdot \gamma^2 \cdot g_2 \right] \]

\[ d\sigma = d\bar{\sigma} \pm d\Delta\sigma \]

beam/target helicity

\( g_1 \) measured at
SLAC, EMC, SMC,
HERMES, COMPASS

\( g_2 \) suppressed by a factor \( \gamma^2 \approx 0.01 \)
at 100 GeV (SMC, SLAC)
Inclusive DIS: beam and target longitudinally polarized

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\frac{d\Delta \sigma}{dx \ dy} = \lambda \cdot \frac{e^4}{4\pi^2 Q^2} \left[ 1 - \frac{y}{2} - \frac{y^2}{4} \cdot \gamma^2 \right] \cdot \frac{y}{2} \cdot \gamma^2 \cdot g_2
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\[ \frac{d\Delta\sigma}{dx\,dy} = \frac{\lambda}{4\pi^2 Q^2} \cdot \frac{e^4}{2} \cdot \left( \frac{1}{2} - \frac{y^2}{4} \cdot \gamma^2 \right) \cdot \frac{g_1}{2} - \frac{g_2}{\gamma^2} \]

beam/target helicity

\[ d\sigma = d\bar{\sigma} \pm d\Delta\sigma \]

\(g_1\) measured at SLAC, EMC, SMC, HERMES, COMPASS

\(g_2\) suppressed by a factor \(\gamma^2 \approx 0.01\) at 100 GeV (SMC, SLAC)

in the parton model

\[ g_1(x) = \frac{1}{2} \sum_q e_q^2 \cdot q(x) + \Delta\bar{q}(x) \]
The Quark Contribution to the Nucleon Spin

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s \]

\[ \Delta q = \int \Delta q dx = \int \bar{q} \cdot \bar{q} + q \cdot q - \bar{q} \cdot q \cdot dx \]
The Quark Contribution to the Nucleon Spin

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s \]

\[ \Delta q = \int q \cdot q + \bar{q} \cdot \bar{q} \cdot dx \]

in polarised inclusive DIS one measures

\[ g_1 = \frac{1}{2} \sum_q e_q^2 \cdot \Delta q \]

\[ \Gamma_1 = \int g_1 \cdot dx \]

using complementary information from the WEAK DECAY CONSTANTS of the BARYONS

\[ \Delta u - \Delta d = F + D = 1.257 \pm 0.003 \]

\[ \Delta u + \Delta d - 2 \Delta s = 3F - D = \sqrt{3} \cdot 0.34 \pm 0.02 \]

one can get \( \Delta u, \Delta d, \Delta s \) and then \( \Delta \Sigma \)
The Quark Contribution to the Nucleon Spin

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s \]

in polarised inclusive DIS one measures

\[ g_1 \equiv \frac{1}{2} \sum_q e_q^2 \cdot \Delta q \]

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one can get \( \Delta u, \Delta d, \Delta s \) and then \( \Delta \Sigma \)

\( \Delta u, \Delta d, \Delta s \) can also be measured in semi-inclusive DIS

\[ \rightarrow E. \text{Kabuss} \]
Experiments

a worldwide effort since decades
# Experiments

**a worldwide effort since decades**

<table>
<thead>
<tr>
<th>Year</th>
<th>SLAC</th>
<th>CERN</th>
<th>DESY</th>
<th>JLab</th>
<th>RHIC</th>
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<td>2000</td>
<td>E142/3</td>
<td>COMPASS</td>
<td></td>
<td>CLAS/HALL-A</td>
<td>Phenix/Star</td>
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</table>

**Spin Crisis**
The Players, today

- **HERMES** @ DESY  *pure H and D target*
- **COMPASS** @ CERN  *high energy $\mu$-beam*
- **JLAB** Experiments  *very high luminosity*
longitudinally polarised muon beam
longitudinally or transversely polarised target
calorimetry
particle identification

luminosity: $\sim 5 \cdot 10^{32}$ cm$^{-2}$ s$^{-1}$
beam intensity: $2 \cdot 10^8 \mu^+/\text{spill (4.8s/16.2s)}$
beam momentum: 160 GeV/c
solid state target operated in frozen spin mode

2002-2004: $^6\text{LiD}$ (polarised deuteron, L&T)
dilution factor $f = 0.38$
polarization $P_T = 50$

two 60 cm long cells
with opposite polarisation (systematics)

during data taking with transverse polarisation,
polarisation reversal in the cells after $\sim 4$-5 days
solid state target operated in frozen spin mode

2002-2004: $^6$LiD (polarised deuteron, L&T)
- dilution factor $f = 0.38$
- polarization $P_T = 50\%$

- two 60 cm long cells
- with opposite polarisation (systematics)

2006:
- PTM replaced with the large acceptance COMPASS magnet (180 mrad)
- 2 target cells $\rightarrow$ 3 target cells
- $^6$LiD (L)

2007: NH$_3$ (polarised protons, L&T)
- dilution factor $f = 0.14$
- polarization $P_T = 90\%$

2010: NH$_3$ (T)
2011: NH$_3$ (L)
HERMES

27.5 GeV $e^+$

\[ \sqrt{s} = 7 \text{ GeV} \]

particle ID: lepton ID via RICH
hadron contribution via RICH:

the HERMES polarized target

pure hydrogen gas target

flipped at high frequency (60-90 s)

$^1H \rightarrow <|P_t|> \sim 85 \pm 3.8 \%$

$^2H \rightarrow <|P_t|> \sim 84 \pm 3.5 \%$

$^1H \uparrow <|P_t|> \sim 74 \pm 4.2 \%$
JLab experiments

6 GeV polarized electron beam
Pol=85%, 180μA

Will be upgraded to 12 GeV by ~2014

Hall A: two HRS’
Hall B: CLAS
Hall C: HMS+SOS

Erice, 17 September 2011
F. Bradamante
THE QCD STRUCTURE

• LONGITUDINAL

• TRANSVERSE
Parton Distribution Functions

in the collinear case, three distribution functions are necessary to describe the structure of the nucleon at LO:
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$q(x) : \text{number density or unpolarised distribution}$

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- **q(x)**: number density or unpolarised distribution
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- **Δq(x) = q↑↑ - q↓↓**: longitudinal polarization or helicity distribution
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- **Δₜq(x) = q↑↓ - q↓↑**: transverse polarization or transversity distribution
  - in a transversely polarised nucleon, probability of finding a quark with a momentum fraction x and polarisation parallel to that of the parent nucleon

q quark or antiquark with a specific flavor [notation: Barone, Drago, Raftcliffe 2001]
Parton Distribution Functions

In the collinear case, three distribution functions are necessary to describe the structure of the nucleon at LO:

- $q(x)$: number density or unpolarised distribution
  - Probability of finding a quark with a fraction $x$ of the longitudinal momentum of the parent nucleon

- $\Delta q(x) = q_{\uparrow\downarrow} - q_{\downarrow\uparrow}$: longitudinal polarization or helicity distribution
  - In a longitudinally polarised nucleon, probability of finding a quark with a momentum fraction $x$ and spin parallel to that of the parent nucleon

- $\Delta_T q(x) = q_{\uparrow\uparrow} - q_{\downarrow\downarrow}$: transverse polarization or transversity distribution
  - In a transversely polarised nucleon, probability of finding a quark with a momentum fraction $x$ and polarisation parallel to that of the parent nucleon

$q$ quark or antiquark with a specific flavor  [notation: Barone, Drago, Raftcliffe 2001]

ALL OF EQUAL IMPORTANCE!
HELICITY vs TRANSVERSITY

HELICITY and TRANSVERSITY are different
have different properties
are measured in different ways

thus

one has to deal differently the situations when the target spins are

LONGITUDINAL $\rightarrow$ E. Kabuss

and

TRANSVERSE
Transversity can only be accessed in SIDIS from the azimuthal modulation of the final state hadrons with respect to the lepton plane.
Puzzles in hadronic reactions

Transverse Quark Polarization in Large-$p_T$ Reactions, $e^+e^-$ Jets, and Leptoproduction: A Test of Quantum Chromodynamics

G. L. Kane  J. Pumplin  W. Repko

The quantum-chromodynamics prediction is that $P = 0$ in the scaling limit.

\[ A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \frac{m_q}{\sqrt{s}} \]

i.e.

$m_q = 3\text{MeV}, \sqrt{s} = 20\text{ GeV} \quad \Rightarrow \quad A_N \approx 10^{-4}$
In hadronic reactions like $p^+ + p \rightarrow \pi + X$ with a transversely polarized proton, the spin asymmetry in leading twist perturbative QCD is expected to vanish.

The data strongly contradict this!
Since many years intriguing evidence of large transverse spin effects at high energy

- hyperon polarization
- high $p_t$ effects in hadronic interactions
- asymmetries in hadron production

STAR

$A_N$ vs $x_F$

$p+p \rightarrow \pi^0 + X$ at $\sqrt{s}=200$ GeV

Sivers (HERMES fit)  twist-3

$\langle \eta \rangle = 3.7$

$\langle \eta \rangle = 3.3$

RUN6 PRL 101 (2008) 222001
Since many years intriguing evidence of large transverse spin effects at high energy

- hyperon polarization
- high $p_t$ effects in hadronic interactions
- asymmetries in hadron production

Hope to find solutions at the quark level ($\Delta_T q(x)$ …)
HOW to MEASURE TRANSVERSITY
Δₜq(x) is chiral-odd

→ cannot be measured in inclusive DIS
**HOW to MEASURE TRANSVERSITY**

\[ \Delta_T q(x) \text{ is chiral-odd} \]

\[ \rightarrow \text{cannot be measured in inclusive DIS} \]

---

it can be measured in SIDIS:

the observable is the so-called “Collins asymmetry”,

the convolution of \( \Delta_T q(x) \) with another chiral-odd quantity,

the “Collins” function, which describes a possible left-right asymmetry of the hadrons in the hadronization process

of a transversely polarized quark
Collins asymmetry

in SIDIS off transversity polarised nucleons

amplitude of the \( \sin \Phi_C \) modulation in the azimuthal distribution of the final state hadrons

\[
N_h^\pm \Phi_C \equiv N_h^0 \cdot \left\{ \pm P_T \cdot D_{NN} \cdot A_{Coll} \cdot \sin \Phi_C \right\}
\]

\( \Phi_C = \phi_h + \phi_S - \pi \)

\( \phi_h \) azimuthal angle of the hadron, 
\( \phi_S \) azimuthal angle of the nucleon spin

transversity

“Collins FF”

today the most promising way to access transversity

both unknown!
The conjecture was right !!
The conjecture was right!!

PRL 94 (2005) 012002

2 \langle \sin(\phi + \phi_s) \rangle_U^\pi

0.2

0.1

0

-0.1

-0.2

0.1

0.3

0.5

0.6

\pi^+

\pi^-

PRL 94(2005)202002

A_{\text{Cell}}

all hadrons

leading hadrons

leading hadrons

leading hadrons

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The conjecture was right!!

\[ A_{\text{Coll}} \approx \frac{\sum_q e_q^2 \Delta_T q \otimes \Delta_T^0 D_q^h}{\sum_q e_q^2 q \otimes D_q^h} \]

gives a LR asymmetry in the hadronisation of transversely polarised quarks

- products of Collins FFs can be measured in \( e^+ e^- \rightarrow \pi^+ \pi^- X \)
- first low statistics results from LEP data
- 2005 first data from BELLE
TRANSVERSITY PDFs

Collins asymmetry best fit

PROTON TARGET
TRANSVERSITY PDFs

Collins asymmetry best fit

PROTON TARGET

DEUTERON TARGET

plus BELLE data on $e^+e^- \rightarrow \text{hadrons}$
TRANSVERSITY PDFs

Comparison of extracted transversity distributions with models

\[ x \Delta_T q(x) \]

- Barone, Calarco, Drago PLB 390 287 (97)
- Soffer et al. PRD 65 (02)
- Korotkov et al. EPJC 18 (01)
- Schweitzer et al. PRD 64 (01)
- Wakamatsu, PLB B653 (07)
- Pasquini et al., PRD 72 (05)
- Cloet, Bentz and Thomas PLB 659 (08)
- This analysis.

Anselmino et al., PRD75 (2007)
Collins asymmetry

nice confirmation of the 2007 results,
with better statistics

$\sigma_{\text{syst}} \sim 0.5 \sigma_{\text{stat}}$

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Collins asymmetry

x > 0.032 region - comparison with HERMES results

COMPASS positive hadrons x>0.032 preliminary

HERMES π⁻ PLB 693 (2010) rescaled by 1/D_{NN}

COMPASS negative hadrons x>0.032 preliminary

HERMES π⁻ PLB 693 (2010) rescaled by 1/D_{NN}

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Collins asymmetry

x > 0.032 region - comparison with HERMES results

nice agreement in spite of the different $Q^2$ values
a very important result
The Structure of the Nucleon

new developments
The Structure of the Nucleon

In the collinear case, three distribution functions are necessary to describe the structure of the nucleon at LO.

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<thead>
<tr>
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<th>U</th>
<th>L</th>
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<tbody>
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<td>U</td>
<td>( f_1 )</td>
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The Structure of the Nucleon

taking into account the quark intrinsic transverse momentum $k_T$, at leading order 8 PDFs are needed for a full description of the nucleon structure “TMDs”

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<td>Boer Mulders</td>
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<td><strong>of the nucleon</strong></td>
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<td><strong>and the transverse</strong></td>
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<td><strong>momentum</strong></td>
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<tr>
<td><strong>of the quark</strong></td>
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<tr>
<td><strong>sensitive to orbital angular momentum</strong></td>
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Erice, 17 September 2011

F. Bradamante
The Structure of the Nucleon

taking into account the quark intrinsic transverse momentum $k_T$, at leading order 8 PDFs are needed for a full description of the nucleon structure "TMDs"

<table>
<thead>
<tr>
<th>nucleon polarisation</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U</strong></td>
<td>$f_1$</td>
<td>$g_1$</td>
<td>$f_{1T}$</td>
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<tr>
<td><strong>L</strong></td>
<td>$g_1$</td>
<td>$g_{1T}$</td>
<td>$h_{1T}$</td>
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<tr>
<td><strong>T</strong></td>
<td>$h_{1L}$</td>
<td>$h_{1T}$</td>
<td>$h_1$</td>
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</table>

SIDIS give access to all of them

Erice, 17 September 2011  F. Bradamante
when taking into account the intrinsic transverse momentum of the quarks several azimuthal modulations are possible in the SIDIS cross-section

the amplitudes of the modulations are convolutions of the different Transverse Momentum Dependent PDFs e FFs:

\[
\begin{align*}
\sin 2\phi_h \\
\sin (\phi_h + \phi_S) & \rightarrow \text{Transversity PDF x Collins FF} \\
\sin (\phi_h - \phi_S) & \rightarrow \text{Sivers PDF} \\
\sin (3\phi_h - \phi_S) \\
& \ldots
\end{align*}
\]

all these amplitudes can be extracted from the SIDIS data
a long debate

- 1992 introduced by D. Sivers
- 1993 J. Collins demonstrate that it must vanish
- 2002 S. Brodsky et al.: it can be ≠ 0 because of FSI
- 2002 J. Collins: process dependent, change of sign SIDIS ↔ DY

....
the Sivers function

a long debate

- 1992 introduced by D. Sivers
- 1993 J. Collins demonstrate that it must vanish
- 2002 S. Brodsky et al.: it can be $\neq 0$ because of FSI
- 2002 J. Collins: process dependent, change of sign SIDIS $\leftrightarrow$ DY

... 

- 2005 first measurements of the Sivers asymmetry in SIDIS

$$A_{Siv} = \frac{\sum q e^2 f_{IT}^{\perp q} \otimes D_{l}^{q}}{\sum q e^2 f_{l} \otimes D_{l}^{q}} \frac{F_{UT}^{\sin(\phi_{n} - \phi_{S})}}{F_{UU}}$$

strong signal seen by HERMES for $\pi^+$ on protons
no signal seen by COMPASS for $h^+$ and $h^-$ on deuterons
Sivers asymmetry

again, nice agreement with the 2007 results, with better statistics
\[ \sigma_{\text{syst}} \sim 0.5 \sigma_{\text{stat}} \] in 2010
Sivers asymmetry

results from 2010 data vs results from 2007 data
Sivers asymmetry

$x > 0.032$ region 2010 COMPASS data vs HERMES results
Sivers asymmetry

JLab - neutron

Collins

Sivers
CONCLUSIONS
on transverse spin and transverse momentum phenomena

• TRANSVERSITY is being measured

• NEW Properties of matter have been unveiled
  Collins effect     Sivers effect
  OTHER correlations are still possible (Boer-Mulders)

• more precise measurements are needed to compare with calculations (pQCD and Lattice)
  COMPASS     JLab     RHIC     GSI
  and in the long run
  AN ELECTRON-NUCLEON COLLIDER
NEAR FUTURE

**COMPASS**
- further results from
  - 2010: SIDIS off transversely polarized p target (160 GeV)
  - 2011: SIDIS off longitudinally polarized p target (160 GeV)

**HERMES**
- further results on SIDIS and DVCS (28 GeV)

**JLab**
- SIDIS and DVCS (6 GeV)
- upgrade to 12 GeV
**FUTURE**

### COMPASS II proposal

- **submitted to CERN in July 2010**

### approved for 3 years of running

- → *E. Rocco*

<table>
<thead>
<tr>
<th><strong>DVCS &amp; DVMP</strong></th>
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<tbody>
<tr>
<td>Transverse Imaging</td>
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<tr>
<td>Beam Charge &amp; Spin asymmetry</td>
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<tr>
<td>GPD H (later GPD E)</td>
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</tbody>
</table>

<table>
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<tr>
<th><strong>μ p SIDIS</strong></th>
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<tbody>
<tr>
<td>s(x), Kaon FF</td>
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<tr>
<td>Boer Mulders PDFs and $k_T$</td>
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</table>

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<thead>
<tr>
<th><strong>Drell-Yan $\pi p$</strong></th>
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<tr>
<td>Sivers and Boer Mulders PDFs</td>
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<tr>
<td>Test of universality</td>
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<tr>
<th><strong>Primakoff</strong></th>
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<tr>
<td>Chiral Perturbation Theory</td>
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</table>
SPARE SLIDES
$\sigma_{3/2} \sim e_q^2 \cdot q^-$  
\[\sigma_{1/2} \sim e_q^2 \cdot q^+\]

\[
2F_1 \leftrightarrow \sigma_{1/2} + \sigma_{3/2} \sim \sum_q e_q^2 \cdot \begin{array}{l}
\begin{array}{c}
\uparrow
\end{array}
\end{array}
q^+ \leftrightarrow q^- \leftrightarrow
\]

\[
2g_1 \leftrightarrow \sigma_{1/2} - \sigma_{3/2} \sim \sum_q e_q^2 \cdot \begin{array}{l}
\begin{array}{c}
\uparrow
\end{array}
\end{array}
q^+ \leftrightarrow q^- \leftrightarrow
\]

\[
\frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{g_1 \leftrightarrow}{F_1 \leftrightarrow} = A_1
\]

definitions:

\[
\Delta u = \int_0^1 \begin{array}{c}
\begin{array}{c}
\uparrow
\end{array}
\end{array} \leftarrow u^+ \leftrightarrow \begin{array}{c}
\begin{array}{c}
\uparrow
\end{array}
\end{array} \leftarrow u^- \leftrightarrow dx = \int_0^1 \Delta u \leftrightarrow dx
\]

\[
\Delta d = \int_0^1 \begin{array}{c}
\begin{array}{c}
\uparrow
\end{array}
\end{array} \leftarrow d^+ \leftrightarrow \begin{array}{c}
\begin{array}{c}
\uparrow
\end{array}
\end{array} \leftarrow d^- \leftrightarrow dx
\]

\[
\Delta s = \int_0^1 \begin{array}{c}
\begin{array}{c}
\uparrow
\end{array}
\end{array} \leftarrow s^+ \leftrightarrow \begin{array}{c}
\begin{array}{c}
\uparrow
\end{array}
\end{array} \leftarrow s^- \leftrightarrow dx
\]

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F. Bradamante
Measurement of $g_1$ in inclusive DIS

\[ A = \frac{d^2\sigma}{d\Omega dE'} - \frac{d^2\sigma}{d\Omega dE'} + \frac{d^2\sigma}{d\Omega dE'} - \frac{d^2\sigma}{d\Omega dE'} \]

\[ \Delta = \frac{N - N}{N + N} = \frac{P_\mu \cdot P_p \cdot f \cdot A}{P_\mu \cdot P_p \cdot f \cdot A} \]

\[ A = D \cdot A_1 + \eta A_2 \Rightarrow D \cdot A_1 \]

$D$, $\eta$ kinematical quantities

$A_1$ and $A_2$ are the asymmetries in $\gamma^*p$ ($n$) scattering

\[ g_1 = \frac{A_1 F_2}{2x \cdot F_1 + R} \approx A_1 F_1 \]

\[ R = \frac{\sigma_L}{\sigma_T} \]

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F. Bradamante
$\Delta_T q(x), h_1 q(x), \delta q(x), \delta_T q(x)$, $q=u_v, d_v, q_{sea}$

recently much interest!

properties:

- $\Delta_T q(x) \neq \Delta q(x)$

- probes the relativistic nature of quark dynamics

- no contribution from the gluons $\rightarrow$ simple $Q^2$ evolution

- positivity (Soffer) bound

- first moments: tensor charge $\Delta_T q \equiv \int dx \Delta_T q(x)$

- sum rule for transverse spin in Parton Model framework

- it is related to GPD’s

- is chiral-odd: decouples from inclusive DIS

Bakker, Leader, Trueman, PRD 70 (04)
Transversity and TMD PDFs

Three parton distributions describing quark’s transverse momentum and/or transverse spin

Three transverse quantities:
1) Nucleon transverse spin  \( \vec{S}_N^T \)
2) Quark transverse spin  \( \vec{S}_q^T \)
3) Quark transverse momentum  \( \vec{k}_q^T \)

\( \Rightarrow \) Three different correlations

1) Transversity
   \[ h_{1T} = \]
   
   Correlation between \( \vec{s}_q^T \) and \( \vec{S}_N^T \)

2) Sivers function
   \[ f_{1T} = \]
   
   Correlation between \( \vec{s}_L^q \) and \( \vec{k}_q^T \)

3) Boer-Mulders function
   \[ h_{1} = \]
   
   Correlation between \( \vec{s}_q^T \) and \( \vec{k}_q^T \)
Relativistic Heavy Ion Collider

RHIC accelerates heavy ions to 100 GeV/A and polarized protons to 250 GeV

$\mathbf{L}_{\text{max}} = 2 \times 10^{32} \text{s}^{-1} \text{cm}^{-2}$

70% polarized

$50 < \sqrt{s} < 500 \text{ GeV}$

$2 \times 10^{11}$ pol. protons / bunch

Pol. Proton Source
500 mA, 300 ms

RHIC pC Polarimeters

BRAHMS & PP2PP (p)

Absolute Polarimeter
(H jet)

Spin Rotators

Partial Siberian Snake

200 MeV Polarimeter

AGS Internal Polarimeter

RF Dipoles

RHIC accelerates heavy ions to 100 GeV/A and polarized protons to 250 GeV
TMD PDFs and SIDIS cross-section

\[ d^6 \sigma = \frac{4\pi \alpha_s^2 s \bar{s}x}{Q^4} \]

\[ f_1 = \]

\[ \left\{ [1 + (1 - y)^2] \sum e_q^2 f_1^q(x)D_1^q(z, P_{h\perp}^2) \right\} \]

Boer-Mulders \( h_1^L = \)

\[ + (1 - y) \frac{P_{h\perp}^2}{4z^2 M_N M_h} \cos(2\phi_h^L) \sum_{q, \bar{q}} e_q^2 h_1^{(1) q}(x) H_1^{1 q}(z, P_{h\perp}^2) \]

Transversity \( h_{1T} = \)

\[ - \left| S_L \right| (1 - y) \frac{P_{h\perp}^2}{4z^2 M_N M_h} \sin(2\phi_h^L) \sum_{q, \bar{q}} e_q^2 h_1^{(1) q}(x) H_1^{1 q}(z, P_{h\perp}^2) \]

Sivers \( f_{1T} = \)

\[ + \left| S_T \right| (1 - y + \frac{1}{2} y^2) \frac{P_{h\perp}}{z M_h} \sin(\phi_h^L - \phi_S^L) \sum_{q, \bar{q}} e_q^2 f_{1T}^{(1) q}(x) D_1^q(z, P_{h\perp}^2) \]

\[ h_{1T} = \]

\[ + \left| S_T \right| (1 - y) \frac{P_{h\perp}^3}{6z^3 M_N^2 M_h} \sin(3\phi_h^L - \phi_S^L) \sum_{q, \bar{q}} e_q^2 h_1^{(2) q}(x) H_1^{1 q}(z, P_{h\perp}^2) \]

\[ g_{1L} = \]

\[ + \lambda_e \left| S_L \right| y(1 - \frac{1}{2} y) \sum_{q, \bar{q}} e_q^2 g_1^q(x) D_1^q(z, P_{h\perp}^2) \]

\[ g_{1T} = \]

\[ + \lambda_e \left| S_T \right| y(1 - \frac{1}{2} y) \frac{P_{h\perp}}{z M_N} \cos(\phi_h^T - \phi_S^T) \sum_{q, \bar{q}} e_q^2 g_{1T}^{(1) q}(x) D_1^q(z, P_{h\perp}^2) \]

\( S_L \) and \( S_T \): L/T target polarizations; \( \lambda_e \): beam L polarization

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