Effects Of Anisotropy in (2+1)-dimensional *QED*

JAB, C. S. Fischer, R. Williams
Effects of Anisotropy in QED_3 from Dyson–Schwinger equations in a box,
PRB 84, 024520 (2011)

J. A. Bonnet

Outline

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The Problem

Discovery of high-temperature superconductivity in 1986.

Bednorz, Müller, Zeitschrift f. Physik B Condensed Matter **64***, no. 2, 189 (1986).*

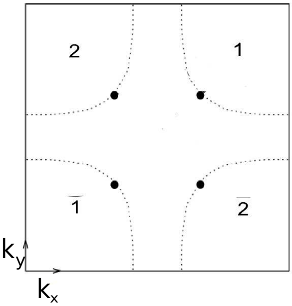
- critical temperature > 77 K
- ceramical compounds
- need 'critical doping'
- non-superconducting phase is insulating anti-ferromagnetic

Look for effective theory to describe phenomenon

The Features

Experiments show:
*Ding, Norman, Campuzano, PRB **54**, R 9678 (1996)*

- energy gap function with "d-wave-symmetry"
  $\rightarrow$ nodal quasiparticles (qp)
- qp: linear energy dispersion relation at the nodes
- vortex-antivortex interactions described by U(1) gauge theory
- qp + gauge fields confined to superconducting plane

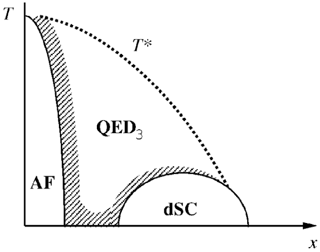


*Herbut, PRB **66**, 094504 (2002).*

$\Rightarrow$We get a hint to QED_3.

Possible Solution: QED_3

Translation of “experimental output” to QED-language?



Franz, Tešanović, Vafek, PRB **66***,054535 (2002).*

phase transition from pseudogap phase to insulating phase

$\leftrightarrow^*$

phase transition from massive to massless QED_3

Reformulate task:

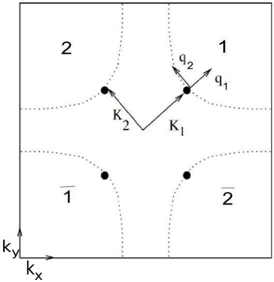
→study order parameter of the transition
→find critical quantities

* *Franz, Tešanović , PRL* **87***, 257003 (2001) & PRL* **84***, 3 (2000); Vafek, Melikyan, Franz, Tešanović, PRB* **63***, 134509; Franz,Tešanović, Vafek, PRB* **66***,054535 (2002); Herbut, PRB* **66***, 094504 (2002).*

Feature: Inherent Anisotropy

Nodes of gap function and inherent anisotropy

define the metric-like quantity...



Herbut, PRB **66**, *094504 (2002).*

$$
\begin{aligned}
\epsilon_{\vec{k}} &= v_f\, q_1 + \mathcal{O}(q^2) \\
\Delta_{\vec{k}} &= v_\Delta\, q_2 + \mathcal{O}(q^2)
\end{aligned}
$$

$$
\begin{aligned}
(g_1^{\mu\nu}) &= \begin{pmatrix} 1 & & \\ & (v_F)^2 & \\ & & (v_\Delta)^2 \end{pmatrix} \\
(g_2^{\mu\nu}) &= \begin{pmatrix} 1 & & \\ & (v_\Delta)^2 & \\ & & (v_F)^2 \end{pmatrix}
\end{aligned}
$$

Outline

The Anisotropic Lagrangian

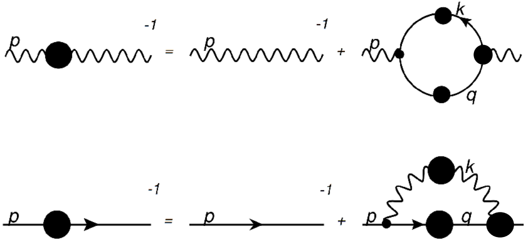
$$\mathcal{L}^{iso} = \sum_{j=1,2} \bar{\Psi}_j \left\{ \sum_{\mu=0}^{2} \gamma_\mu \left(\partial_\mu + \mathrm{i} a_\mu\right) \right\} \Psi_j .$$

$$\Downarrow$$

$$\mathcal{L}^{aniso} = \sum_{j=1,2} \bar{\Psi}_j \left\{ \sum_{\mu=0}^{2} \gamma_\nu \sqrt{g^j}_{\nu\mu} \left(\partial_\mu + \mathrm{i} a_\mu\right) \right\} \Psi_j$$

The Anisotropic Dyson–Schwinger Equations

- Landau gauge



$$S_{Fi}^{-1}(\vec{p}) = S_{0i}^{-1}(\vec{p}) + e^2 \int \frac{d^3q}{(2\pi)^3} \{ \sqrt{g_i}^{\mu\alpha} \gamma_\alpha(\vec{q}) S_{Fi}(\vec{q}) \times$$
$$\times \sqrt{g_i}^{\nu\gamma} \Gamma_\gamma(\vec{q}) D_{\mu\nu}(\vec{p}-\vec{q}) \}$$

How To Solve The DSEs?

We have a rather complex structure of the equations..

- look for CPU friendly environment
- ⇒evaluation on 3 dimensional torus

  Fischer, Alkofer, Reinhardt, PRD **65***:094008 (2002)*
- search for self-consistent solutions

How do we formulate the equations on a torus?

The Torus

- periodic boundary conditions for bosons
- antiperiodic boundary conditions for fermions

$\Rightarrow$ discretized momentum space

$$\int \frac{d^3q}{(2\pi)^3} \cdots \to \frac{1}{L^3} \sum_{\text{all momenta}} \cdots$$

The relevant parameters:

- the box size Le^2 in coordinate space
- the number of lattice points in momentum space N

The Task

Task

study dynamical generation of mass

$\rightarrow$Know: depends on number of fermion flavours
$\rightarrow$Effects of anisotropy?
$\rightarrow$Look at B_{max} depending on v_f, v_Δ

Probe the anisotropic plane for N_f^{crit}.

Outline

Large-N_f Approximation

- expansion in e^2 keeping coupling $\alpha = \frac{N_f e^2}{8}$ fixed
- vacuum polarization given by:

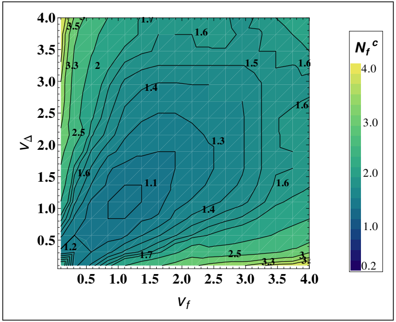
$$\Pi^{\mu\nu}(\vec{p}) = \frac{N_f\, e^2}{16 v_F v_\Delta |\bar{p}|} \sum_i \left(\bar{p}_i^2 g_i^{\mu\nu} - g_i^{\mu\alpha} p_\alpha g_i^{\nu\delta} p_\delta \right)$$

isotropic limit:

$$\Pi^{\mu\nu}(p^2) = \frac{N_f e^2}{8p} \left(p^2 \delta^{\mu\nu} - p^\mu p^\nu \right)$$

Large-N_f Approximation

The phase diagram in velocity space for a torus of 40^3 points and $Le^2 = 600$:



JAB, Fischer, Williams, PRB ***84***,*024520 (2011)*

- N_f^C is strongly volume dependent

  Goecke, Fischer, Williams, PRB ***79***, *064513 (2009).*

- continuum limit can be obtained by extrapolation

$\Rightarrow$ Increasing N_f^C away from plateau around $v_f = v_\Delta = 1$.

Improved Photon And Vertex Ansatz

- Anomalous dimension κ of fermion vector dressing and vacuum polarization in IR

Fischer, Alkofer, Dahm, Maris, PRD **70***:073007(2004).*

- Ansatz for vacuum polarization generalized to anisotropic spacetime

$$\Pi_i\left(\vec{p}\,\right) = \frac{e^2 N_f}{16\, v_F v_\Delta}\left(\frac{1}{\sqrt{\overline{p}_i^2}}\frac{\overline{p}_i^2}{\overline{p}_i^2 + e^2} + \frac{1}{\overline{p}_i^{1+2\kappa}}\frac{e^2}{\overline{p}_i^{\,2} + e^2}\right)$$

JAB, Fischer, Williams, PRB **84***,024520 (2011)*

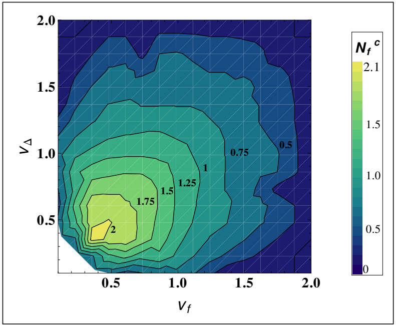
- insert minimal Ball-Chiu vertex

$$\Gamma_i^\beta(\vec{p},\vec{q}) = \gamma^\beta \frac{A_i^\beta(\vec{p}) + A_i^\beta(\vec{q})}{2}$$

Ball, Chiu, PRD **22** *2542 (1980).*

Improved Photon And Vertex Ansatz

The phase diagram in velocity space for a torus of 40^3 points and $Le^2 = 600$:



JAB, Fischer, Williams, PRB **84***,024520 (2011)*

- $\kappa = 0.0358$ fixed in isotropic limit
- agreement with results of One-Photon-Exchange-model: *Concha, Stanev, Tešanović: arXiv:0906.1103v1.*
- agreement with lattice calculations: *Hands, Thomas, PRB* **72***, 054526 (2005); Thomas, Hands, PRB* **75***, 134516 (2007)*

Decreasing N_f^c as a function of v_f and v_Δ away from maximum around $v_f = v_\Delta = 0.4$.

Outline

Conclusion

Summary

- QED_3 is potential effective low-energy theory for high temperature superconductors
- Changes between isotropic and anisotropic QED_3
- Dyson-Schwinger equations in anisotropic space-time
- Results in large-N_f approximation
- Improved results within more sophisticated truncation scheme

What is left to do ...

- influence of finite temperatures (*JAB, Fischer, Williams:* in preparation)
- extrapolation to infinite volume
- solve photon equation explicitly

Thank you for your attention!

Questions??