The Cosmological QCD Phase Transition Revisited

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Work done with Tillmann Boeckel

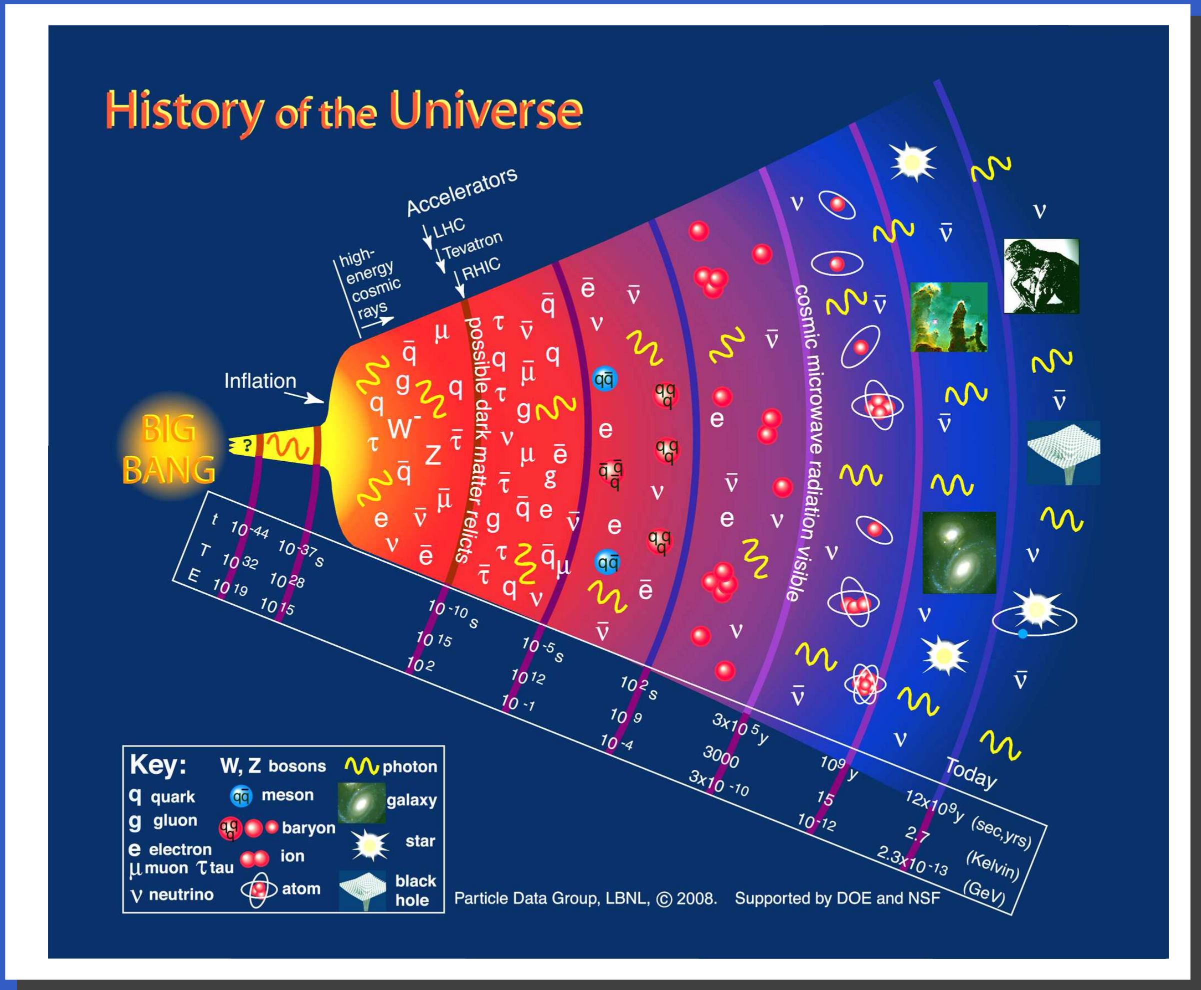
Boeckel and JSB, PRL 105 (2010) 041301
Boeckel, Hempel, Sagert, Pagliara, Sa'd, JSB,
JPG 37 (2010) 094005

and Simon Schettler
(to be published)

Outline

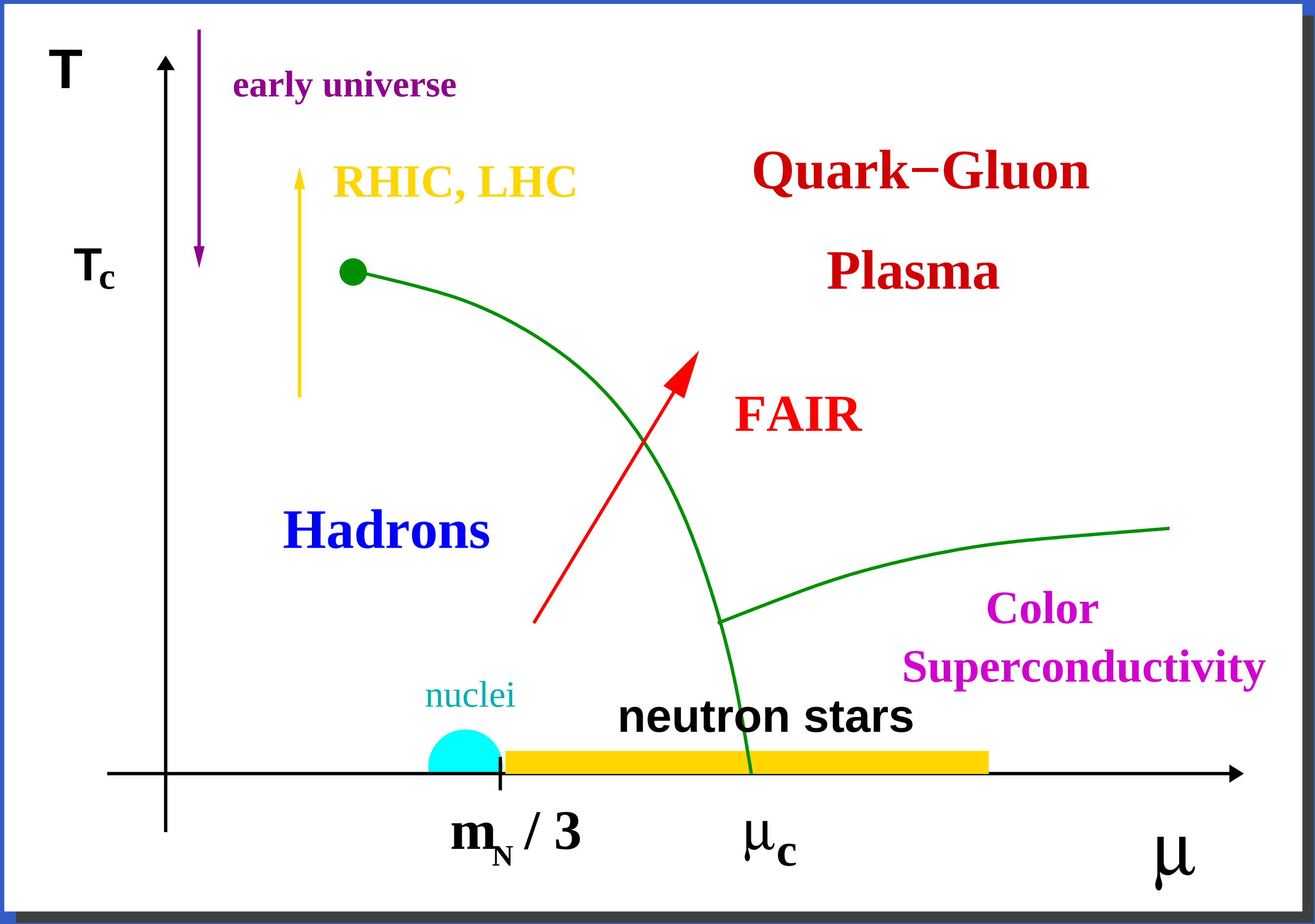
- Standard cosmology
- QCD phase transition with a little inflation
- Ingredients: Affleck-Dine baryogenesis, metastable vacuum, bubble nucleation
- Implications and possible signals:
  - large-scale structure
  - WIMPs and mini black holes
  - cosmological magnetic fields
  - gravitational wave background
- Summary

History of the early universe



- Early universe: temperature increases with scale parameter as a^{-1}
- at $t = 1$s to 3 minutes: BBN ($T = 0.1$ to 1 MeV)
- at $t \approx 10^{-5}$s: QCD phase transition ($T \approx 170$ MeV)
- at $t \approx 10^{-10}$s: electroweak phase transition ($T \approx 100$ GeV)

Phase Transitions in QCD



- early universe at small baryon density and high temperature
- neutron star matter at small temperature and high density
- first order phase transition at high density?
- probed by heavy-ion collisions with CBM@FAIR!

Standard cosmology

from microwave background radiation and big bang nucleosynthesis:

$$n_B/s \sim n_B/n_\gamma \sim \mu/T \sim 10^{-9}$$

note: baryon number per entropy is conserved

$\Longrightarrow$ early universe evolves along $\mu/T \sim 10^{-9} \sim 0$

$\Longrightarrow$ crossover transition, nothing spectacular, no cosmological signals

Friedmann equation for radiation dominated universe:

$$H^2 = \frac{8\pi G}{3}\rho \sim g(T)\frac{T^4}{M_p^2}$$

$g(T)$: effective number of relativistic degrees of freedom at T

Hubble time (true time $t = 3t_H$ for radiation dominated universe):

$$t_H = \frac{1}{H} \sim g^{-1/2}\frac{M_P}{T^2} \Longrightarrow \frac{t}{1\text{ sec}} \sim \left(\frac{1\text{ MeV}}{T}\right)^2$$

A little inflation at the QCD phase transition

what happens if the early universe passes through a first order phase transition?

- is this possible? $\Longrightarrow$ Yes! no contradiction with present data
- could this be observable? $\Longrightarrow$ Yes! by gravitational waves

1st order phase transition $\Longrightarrow$ false metastable vacuum
$\Longrightarrow$ de Sitter solution $\Longrightarrow$ (additional small) inflationary period

$$H = \dot{a}/a \sim M_p^{-1} \rho_{\rm v}^{1/2} = H_{\rm v} = {\rm const.} \to a \sim \exp(H_{\rm v} \cdot t)$$

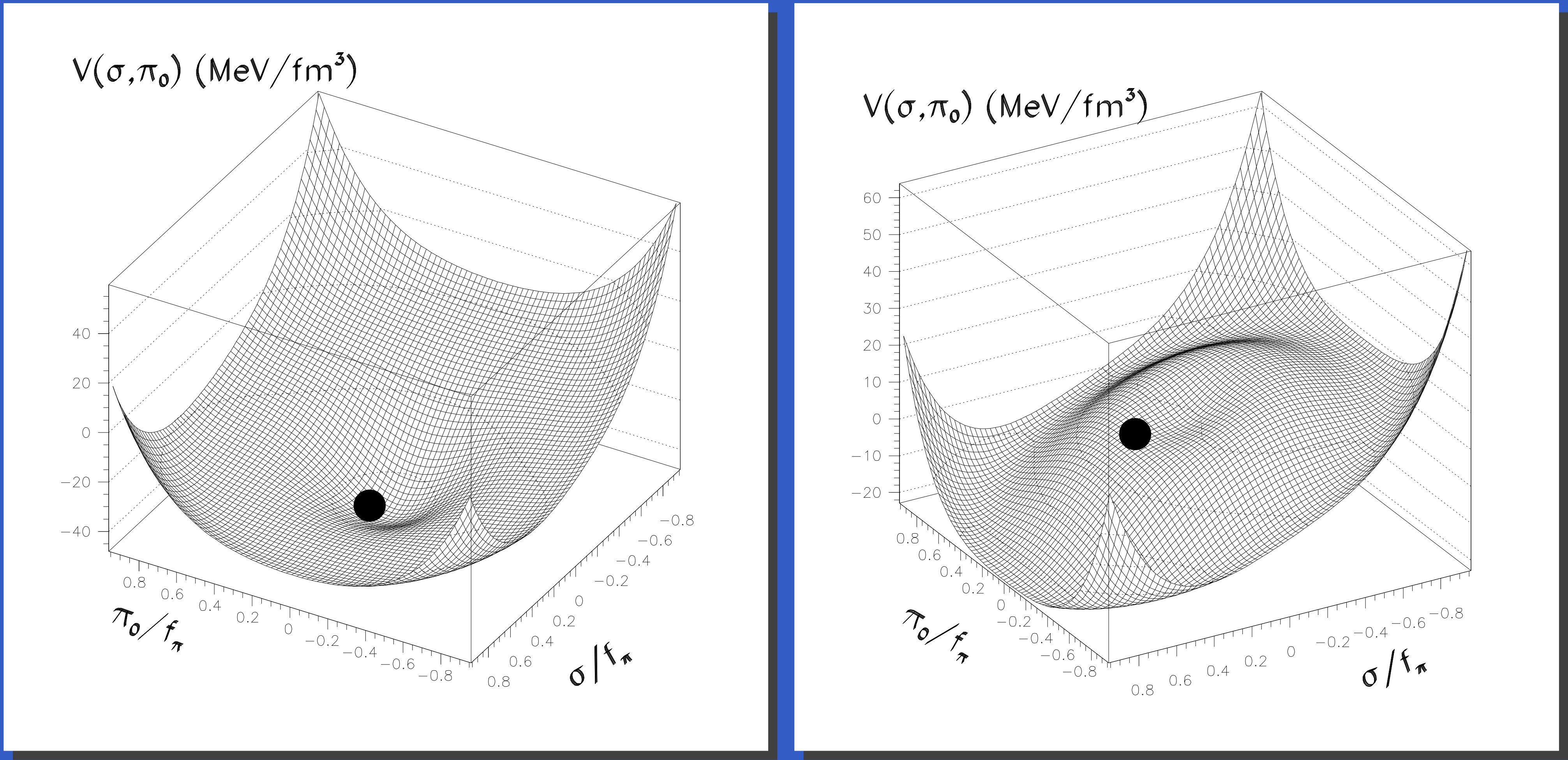
just a few e-folds are enough (standard inflation needs $N \sim 50$):

$$\left(\frac{\mu}{T}\right)_f \approx \left(\frac{a_i}{a_f}\right)^3 \left(\frac{\mu}{T}\right)_i$$

Hence $(\mu/T)_i \sim \mathcal{O}(1)$ for just $N = \ln(a_f/a_i) \sim \ln(10^3) \sim 7$ e-folds

(first order phase transition by a large lepton asymmetry: Schwarz, Stuke 2009)

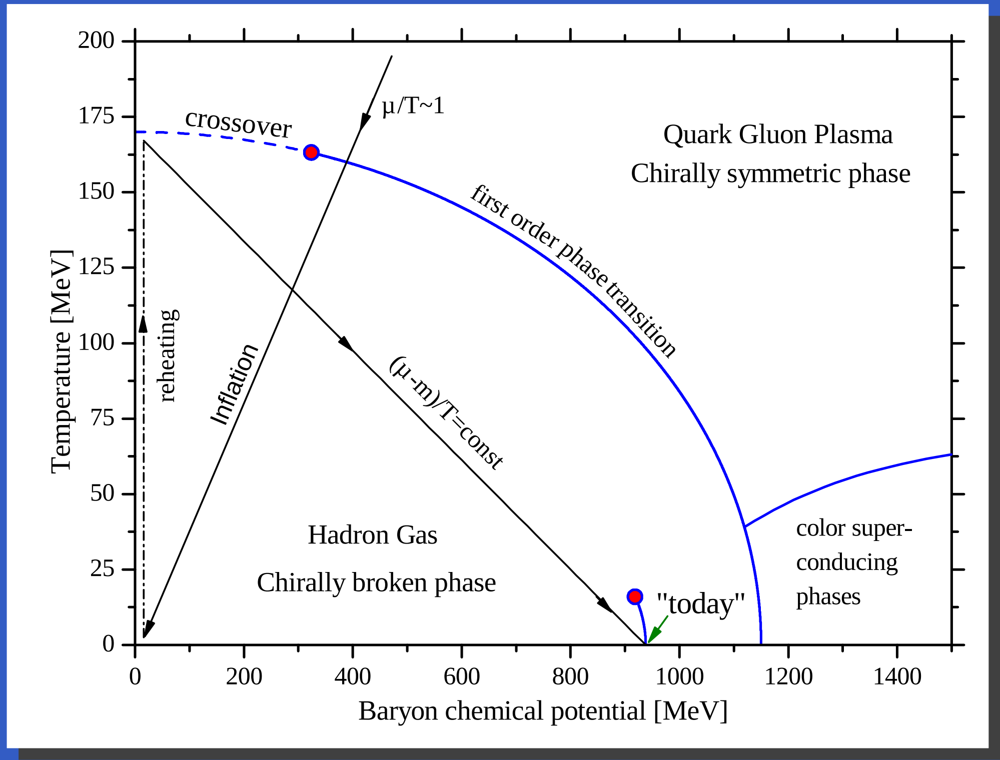
First-order phase transition: linear σ model



(Scavenius, Dumitru 1999)

- potential within the linear σ model at finite temperature
- left plot: high T, right plot: low T, system being trapped in false vacuum state
- possibility of a 'quench' at finite μ, two scalar fields in QCD – hybrid inflation?

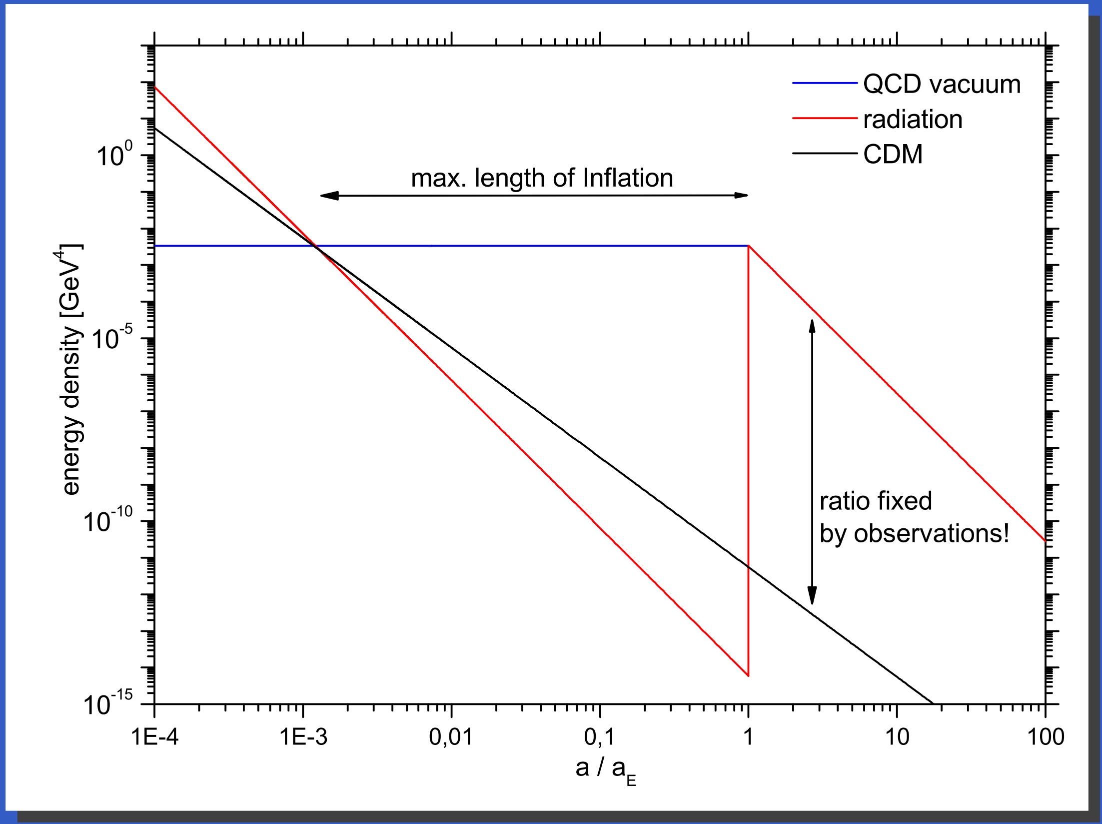
A little inflation in the QCD phase diagram



(Till Boeckel)

- start with $\mu/T \sim 1$ (possible for e.g. Affleck-Dine baryogenesis)
- universe trapped in false vacuum at the transition line
- supercooling and dilution with $\mu/T = \mathrm{const.}$
- decay to the true vacuum state $\rightarrow$ reheating to $T \sim T_c$ so that $\mu/T \sim 10^{-9}$
- then standard cosmological evolution to BBN

A little inflation – evolution of densities



(Boeckel and JSB, arXiv:0906.4520)

- energy density falls as a^{-4} until $\rho \sim \Lambda_{\rm QCD}^4$
- then $\rho = const. \rightarrow$ inflationary period starts
- reheating at the end of inflation
- maximum length of inflation for scale parameter a from CDM density $\sim 10^3$

Power Spectrum of Dark Matter

- dark matter mass within horizon at $T_c \approx 170$ MeV: $10^{-9} M_\odot$
- boosted by little inflation by $(a_f/a_i)^3 \approx 10^9$ so that mass scales of up to $1 M_\odot$ are affected
- additional effect for modes $k_{ph} < H$ at the beginning of inflation
- two scales involved: $H^2 \propto \rho_\mathrm{V} \sim$ const. and $\dot{H} = -4\pi G(\rho + p) = -4\pi G(\rho_{dm} + 4\rho_r/3) \propto (a_i/a)^q$ where $q = 3 \ldots 4$
- three spectral regimes:
  - $(k_{ph}/H)_i > a_f/a_i$: always subhubble
  - $a_f/a_i > (k_{ph}/H)_i > (a_i/a_f)^{q/2}$: intermediate
  - $(k_{ph}/H)_i < (a_i/a_f)^{q/2}$: unaffected
- highest mass scale affected is $M_{max} \sim 10^{-9} M_\odot (a_f/a_i)^{3q/2} \sim (10^6 - 10^8) M_\odot$
- relation to cuspy core, subhalo issues of structure formation?

WIMPs and Black Holes

- freeze-out of weakly interacting massive particles (WIMPs):

$$\Omega_{CDM} \sim \sigma_{\text{weak}}/\sigma_{\text{ann.}}$$

- ρ_{CDM} will be larger by $(a_f/a_i)^3$ during freeze-out *before* inflation
- need substantially reduced annihilation cross section, correspondingly reduced production cross section
- can be checked @LHC! (if SUSY particles are not found)
- primordial black hole production due to collapsing bubbles:

$$M_{bh} \sim M_{hubble} \sim 1M_\odot$$

as the total energy density after inflation is involved

(Jedamzik 1997; Kapusta, Springer 2007)

Tensor perturbations and QCD trace anomaly

- crucial input for tensor perturbations in GR: trace anomaly of QCD!
- EoM for tensor perturbation amplitude $v_k = a \cdot h_k$ in Fourier space (gauge invariant):

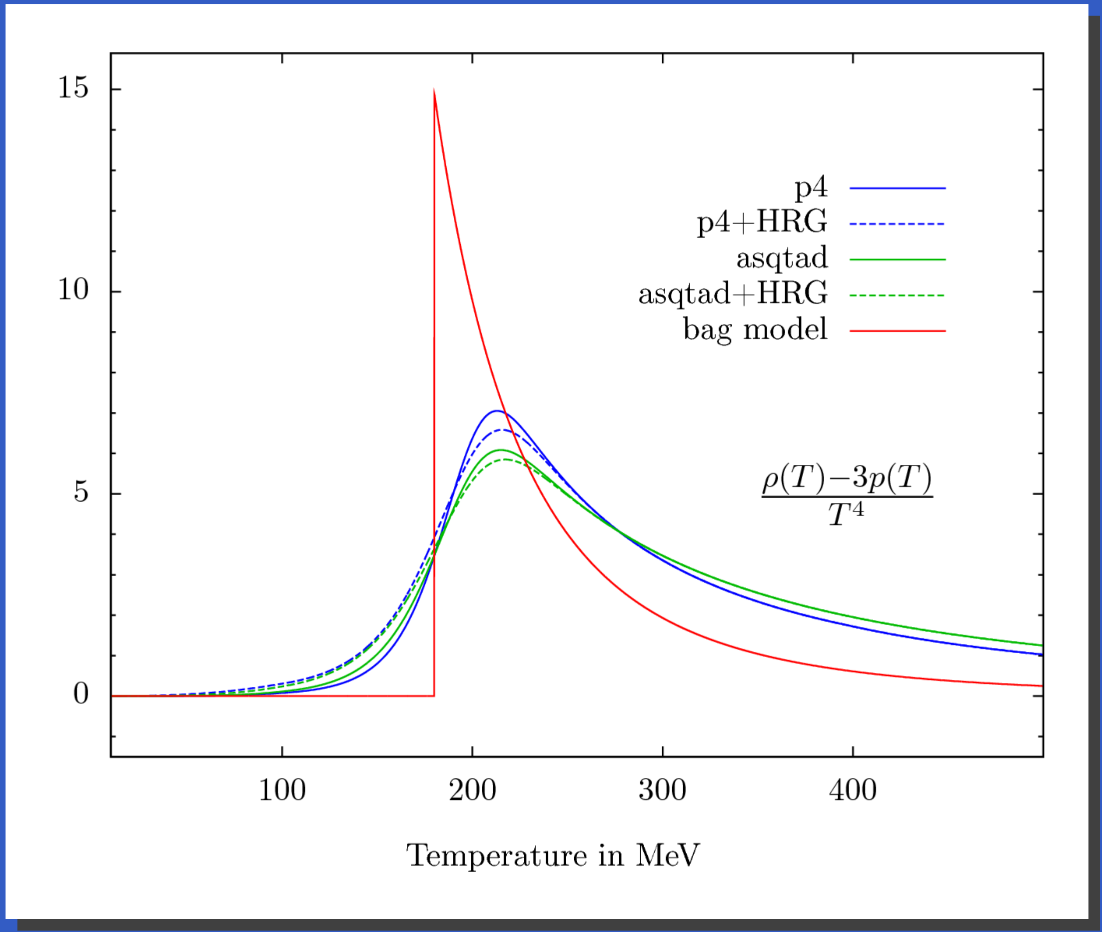
$$v_k''(\eta) + \left(k^2 - \frac{a''}{a}\right) v_k(\eta) = 0$$

where

$$\frac{a''}{a} = \frac{4\pi G a^2}{3}(\rho - 3p)$$

- only input needed: QCD trace anomaly
- use several lattice parameterizations, compare with simple bag model

QCD trace anomaly



(Simon Schettler)

- parameterization of lattice data with improved staggered fermion actions (asqtad and p4) and physical strange quark masses, with and without a hadron resonance gas (HRG)

  (Bazavov et al., Bielefeld-BNL/RIKEN-Columbia collaboration 2009)

Gravitational wave background from QCD phase transition

- energy density in gravitational wave background: $\Omega_g(k) = \frac{1}{\rho_c}\frac{d\rho_g}{d\ln k}$
- mode h_k is damped by $1/a$ after horizon entry
- entropy conservation: $ga^3T^3 = const. \rightarrow H \sim T^2 g^{1/2} \sim g^{-1/6}a^{-2}$
- as $\Omega_g \sim k^2 a_{in}^2 = H_{in}^2 a_{in}^4 \sim g_k^{-1/3}$, so

$$\frac{\Omega_g(\nu \gg \nu^*)}{\Omega_g(\nu \ll \nu^*)} = \left(\frac{g_f}{g_i}\right)^{1/3} \sim 0.7$$

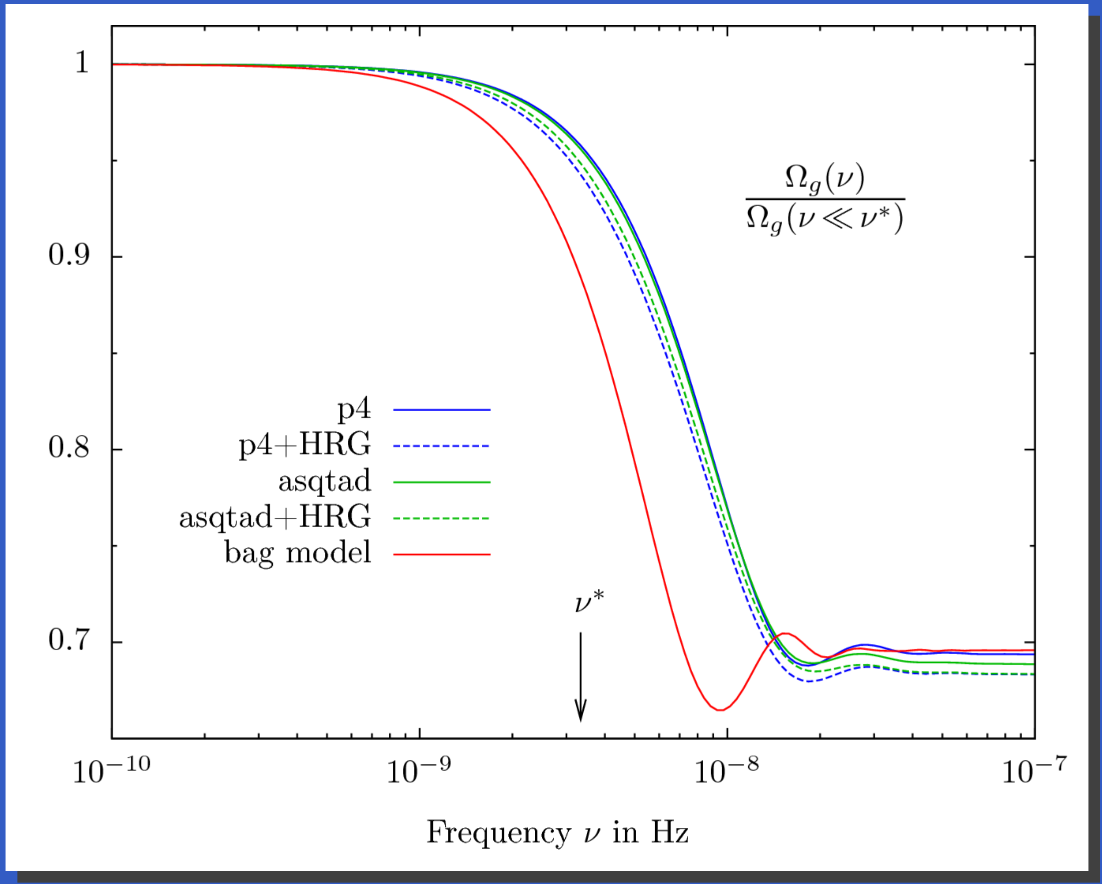
(Schwarz 1998)

- step in amplitude at frequency scale given by (redshifted) horizon scale at the transition point

$$\nu_{\rm peak} \sim H_c \cdot T_{\gamma,0}/T_c \sim T_c/M_p \cdot T_{\gamma,0} \sim 10^{-7}\ \text{Hz}$$

- maximum amplitude $h \sim a/a_0 \sim 10^{-12}$

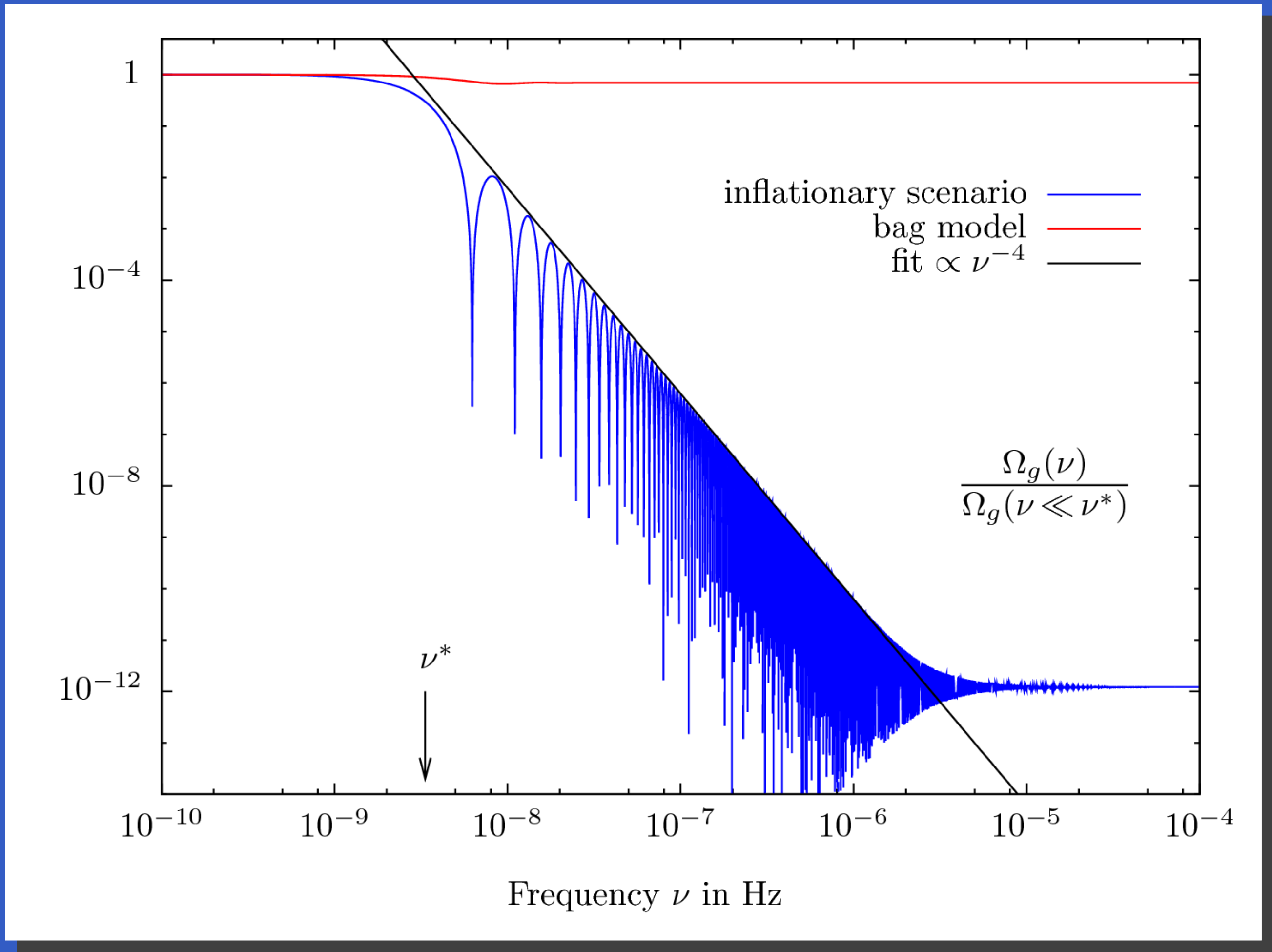
A step in the gravitational wave background



(Simon Schettler)

- step in gravitational wave background around $\nu \sim 10^{-8}$ Hz
- step in spectrum of about $(g_f/g_i)^{1/3} \sim 0.7$
- rather insensitive to details of the phase transition (Schwarz 1998)

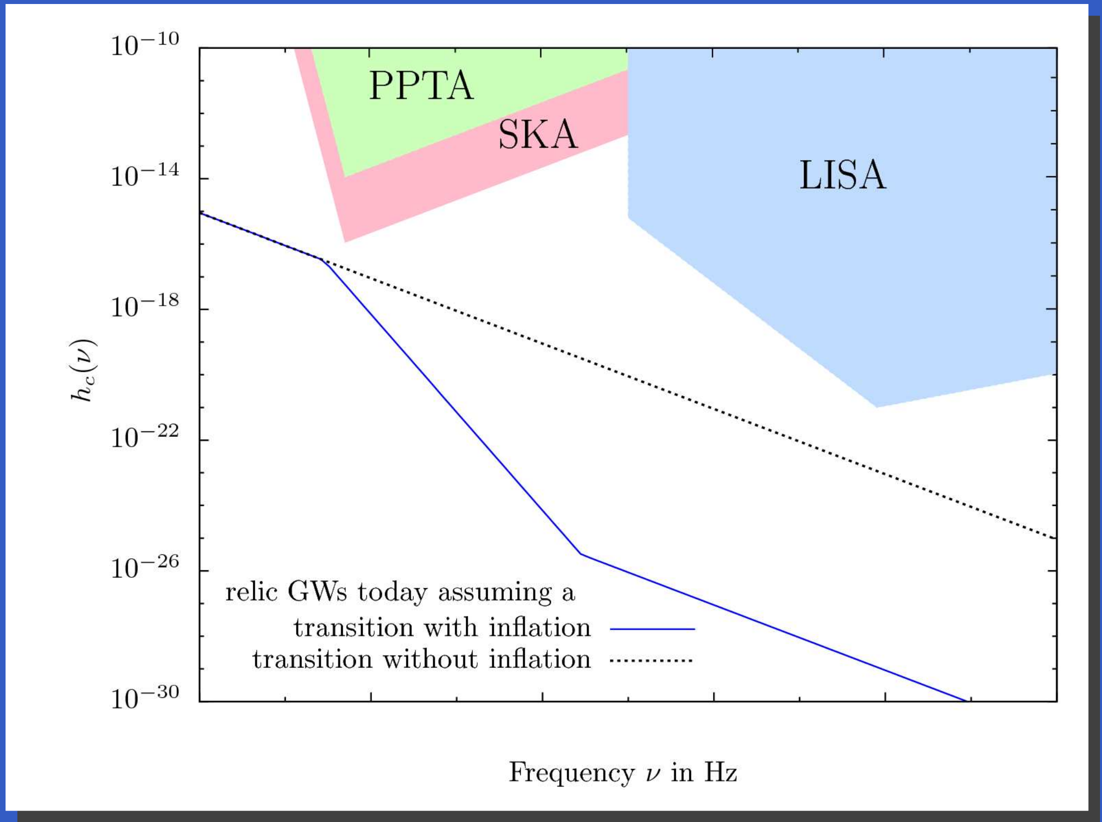
Spektrum of gravitational waves for a little inflation



(Simon Schettler)

- amplitudes are exponentially suppressed during inflation as $h \sim 1/a \sim \exp(H \cdot t)$
- gravitational wave background drops as ν^{-4}

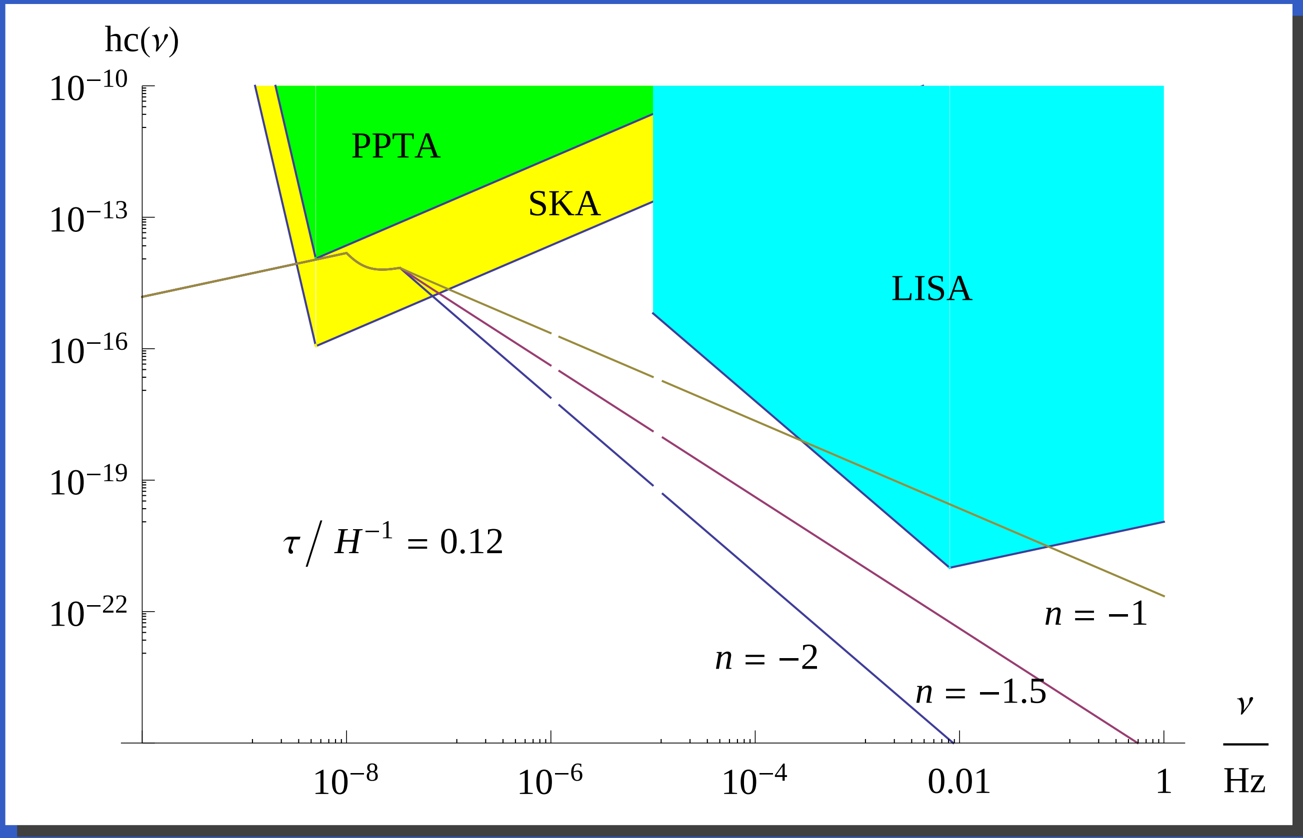
Observations of gravitational wave background



(Simon Schettler and Till Boeckel)

- gravitational wave amplitude versus frequency
- gravitational waves measurable with pulsar timing (PPTA and SKA) or space-based interferometers (LISA), limits from Jenet et al. 2006
- step frequency in the amplitude close to highest sensitivity for pulsar timing

Gravitational waves from bubble collisions



(Boeckel et al. JPG 37 (2010) 094005)

- first order transition produces tensor perturbations $\to$ gravitational waves
- amplitude scales as $h(\nu) \propto \nu^{-1/2}$ for $\nu < H$ (white noise)
  and as $h(\nu) \propto \nu^{-2\dots-1}$ for $\nu > H$ (multi bubble collisions)
  (Kamionkowski, Kosowsky, Turner 1994; Huber, Konstandin 2008)

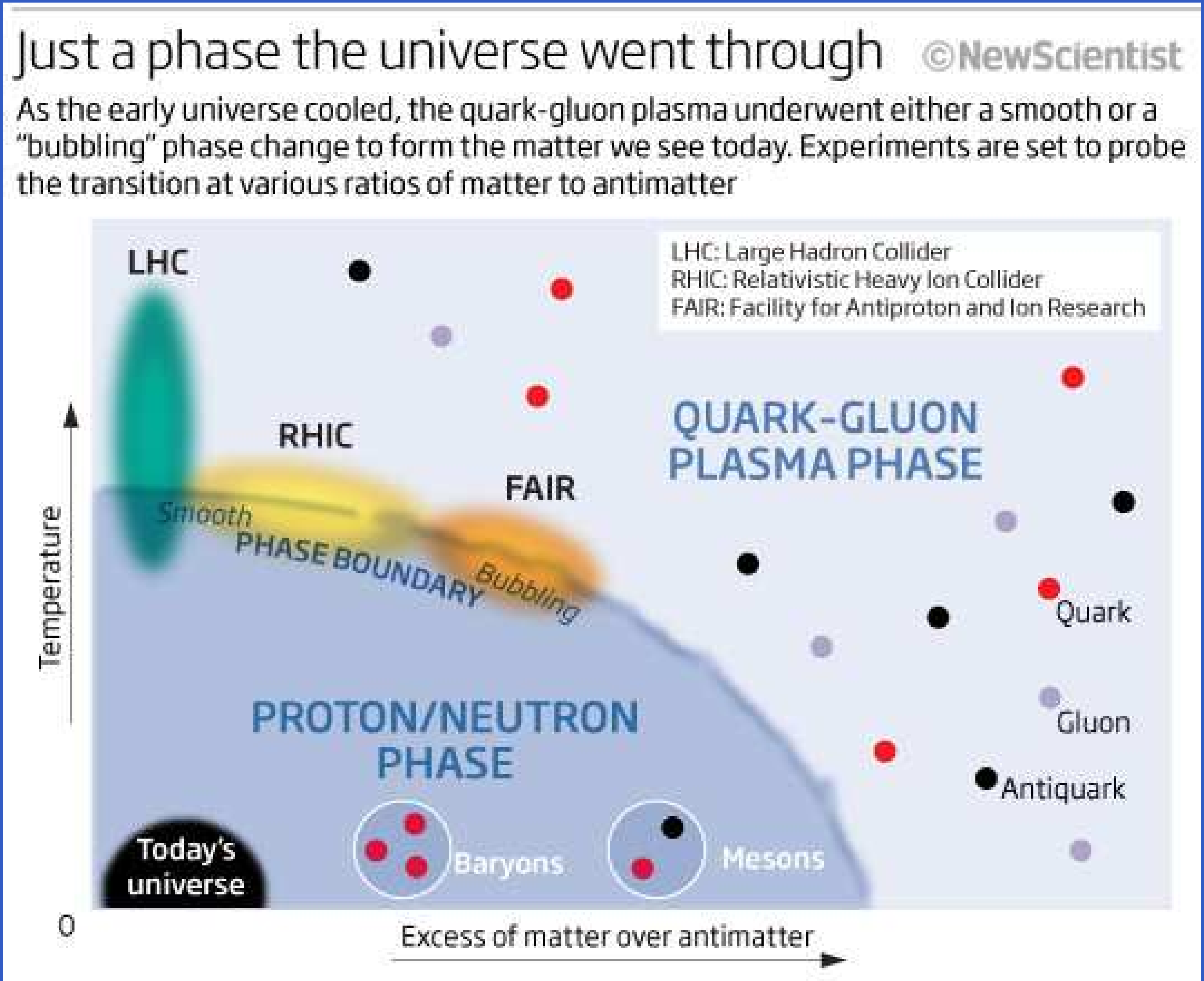
Summary

- first order transition could have happened in the early universe
- need large initial μ/T and a metastable false vacuum state
- large-scale structure modified up to $M \sim 10^9 M_\odot$
  (without QCD inflation only up the horizon mass $\sim 10^{-9} M_\odot$)
- cold dark matter density is diluted by 10^{-9}
  $\rightarrow$ need different WIMP annihilation cross section as
  $\Omega_{\rm CDM} \sim \sigma_{\rm weak}/\sigma_{\rm ann}$ or larger WIMP mass (probed by LHC!)
- generation of the seeds of (extra)galactic magnetic fields:
  $\rightarrow$ possible within the standard model again
- modified gravitational wave background:
  observable with pulsar timing and LISA

Epilogue: the bubbling universe

Rachel Courtland on "Big Bang Part II: The Big Boil"
New Scientist, May 2010

Big bang II: the 'new scientist picture'



End of phase transition by bubble nucleation

bubble of new phase grows if they exceed a critical bubble size

free energy:

$$\Delta F = -\frac{4\pi}{3} R^3 \Delta p + 4\pi R^2 \sigma$$

with a critical bubble size of $R_c = 2\sigma/\Delta p$, nucleation rate:

$$\Gamma = P_0 \exp\left(-\Delta F/T\right) \text{ with } P_0 \sim T^4$$

depends crucially on surface tension σ and pressure difference Δp:

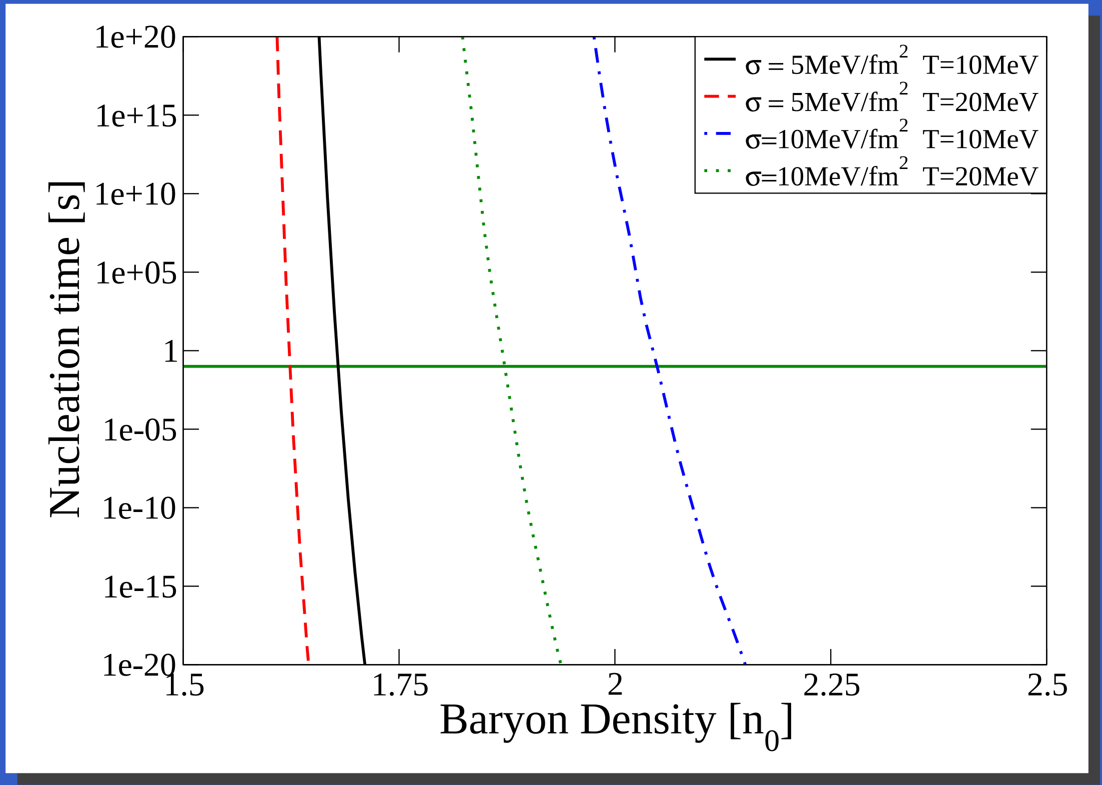
$$\frac{\Delta F_c}{T} = \frac{16\pi\sigma^3}{3T(\Delta p)^2} = \frac{16\pi}{3}\left(\frac{\sigma}{200\text{MeV fm}^{-2}}\right)^3 \left(\frac{200\text{MeV}}{T}\right)\left(\frac{200\text{MeV fm}^{-3}}{\Delta p}\right)$$

exponential suppression!

in general $\sigma = \sigma(T)$ as the barrier vanishes for low T

(ensures a graceful exit!)

Bubble nucleation timescales and surface tension



(Mintz, Fraga, Pagliara, JSB 2009)

failure to nucleate $\tau_{\rm nucl} > t_{\rm hubble}$ for $\sigma > 120$ MeV/fm^2

(MIT bag model with $B^{1/4} = 145$ MeV)

(Jenkovszky, Sysoev, Kämpfer 1990; Csernai, Kapusta 1992; Mintz, Fraga, Pagliara, JSB 2010)

surface tension in QCD: $\sigma = 50 - 150$MeV/fm^2 or smaller or larger . . .

(Voskresensky, Yasuhira, Tatsumi 2003; Palhares, Fraga 2010)

Seeds for magnetic fields

- primordial magnetic fields produced by bubble collisions in first order phase transition (Cheng, Olinto 1994)
- charge dipole layer at surface, high baryon density contrast
- magnetic field can be $B_{QCD} \sim 10^8 - 10^{10}$ G
- amplified by MHD turbulence to equipartition value $B_{eq} = \sqrt{8\pi T^4 v_f^2} \sim 10^{12}$ G (Sigl, Olinto, Jedamzik 1997)
- little inflation scenario boosts magnetic fields by higher density and larger baryon diffusion length
- can explain presently observed (extra)galactic magnetic field strength $B_{obs} \sim 0.1 - 1\mu$G
  works for GUT and QCD phase transition

  (Caprini, Durrer, Fenn 2009)

Producing gravitational waves with bubbles

energy emitted in gravitational waves (quadrupole formula):

$$E_{GW} \sim G\, \dddot{Q}^2\, \tau$$

with duration of collision τ and separation of bubbles $d \sim \tau$

$$\dddot{Q} \sim \frac{\rho_{\rm v} \cdot d^3 \cdot \tau^2}{\tau^3} \sim \rho_{\rm v} \tau^2$$

energy relative to total energy:

$$\frac{E_{\rm GW}}{E_{\rm v}} \sim \frac{G \rho_{\rm v}^2 \tau^2}{\rho_{\rm v} \tau^3} \sim G \rho_{\rm v} \tau^2 \sim \left(\frac{\tau}{H^{-1}} \right)^2$$

limit from Parkes Pulsar Timing Array PPTA: $\tau / H^{-1} < 0.12$

will be improved by full PPTA data set and by Square Kilometre Array SKA in the future