

Neutrinoless double beta decay studied with configuration mixing methods

Tomás R. Rodríguez



Outline

1. Introduction

**2. Method:
GCM
+PNAMP**

**3. Results:
GCM+PNAMP**

**4. Summary
and
Conclusions**

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1. Introduction

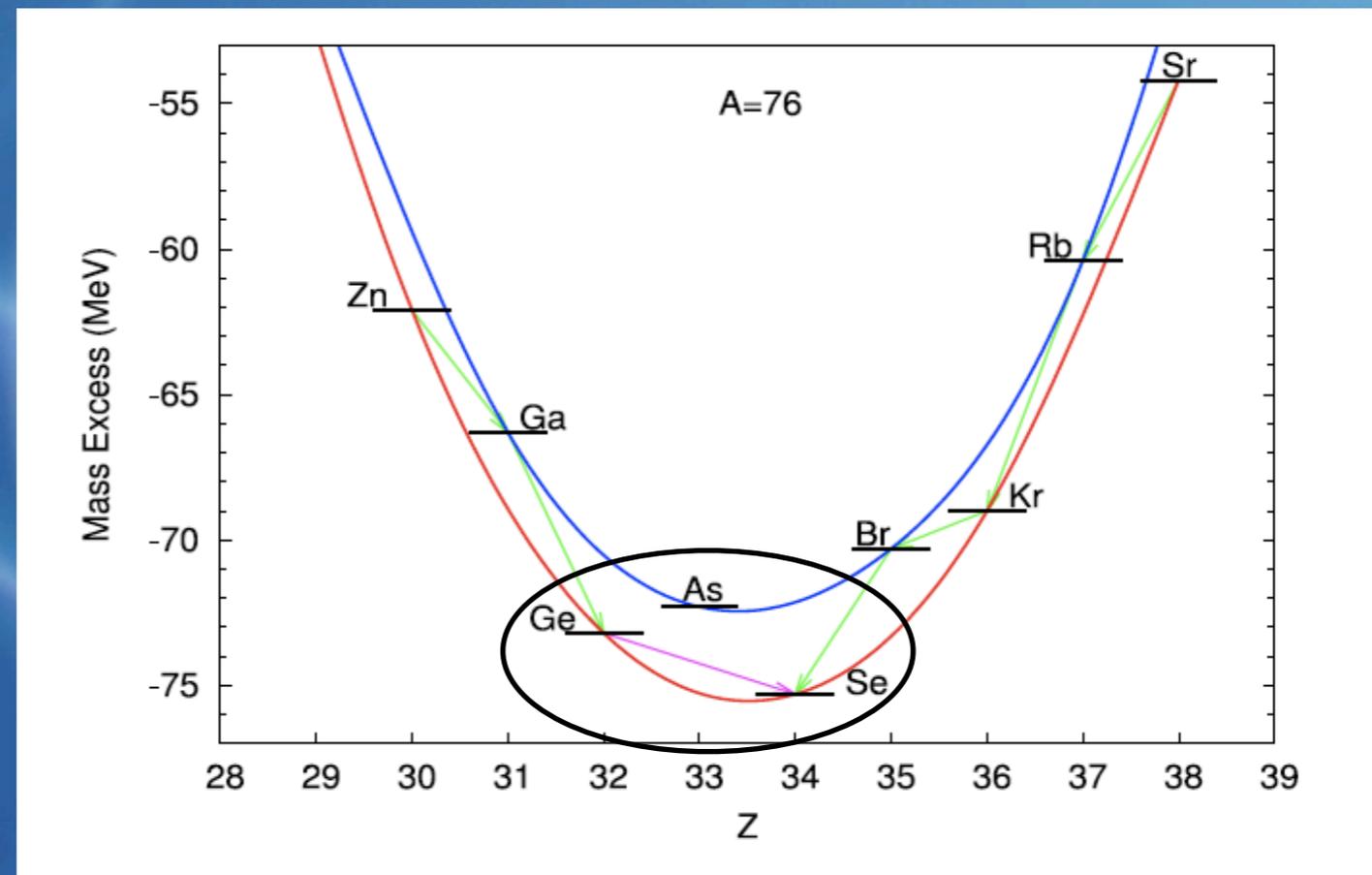
2. Method: GCM
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Introduction

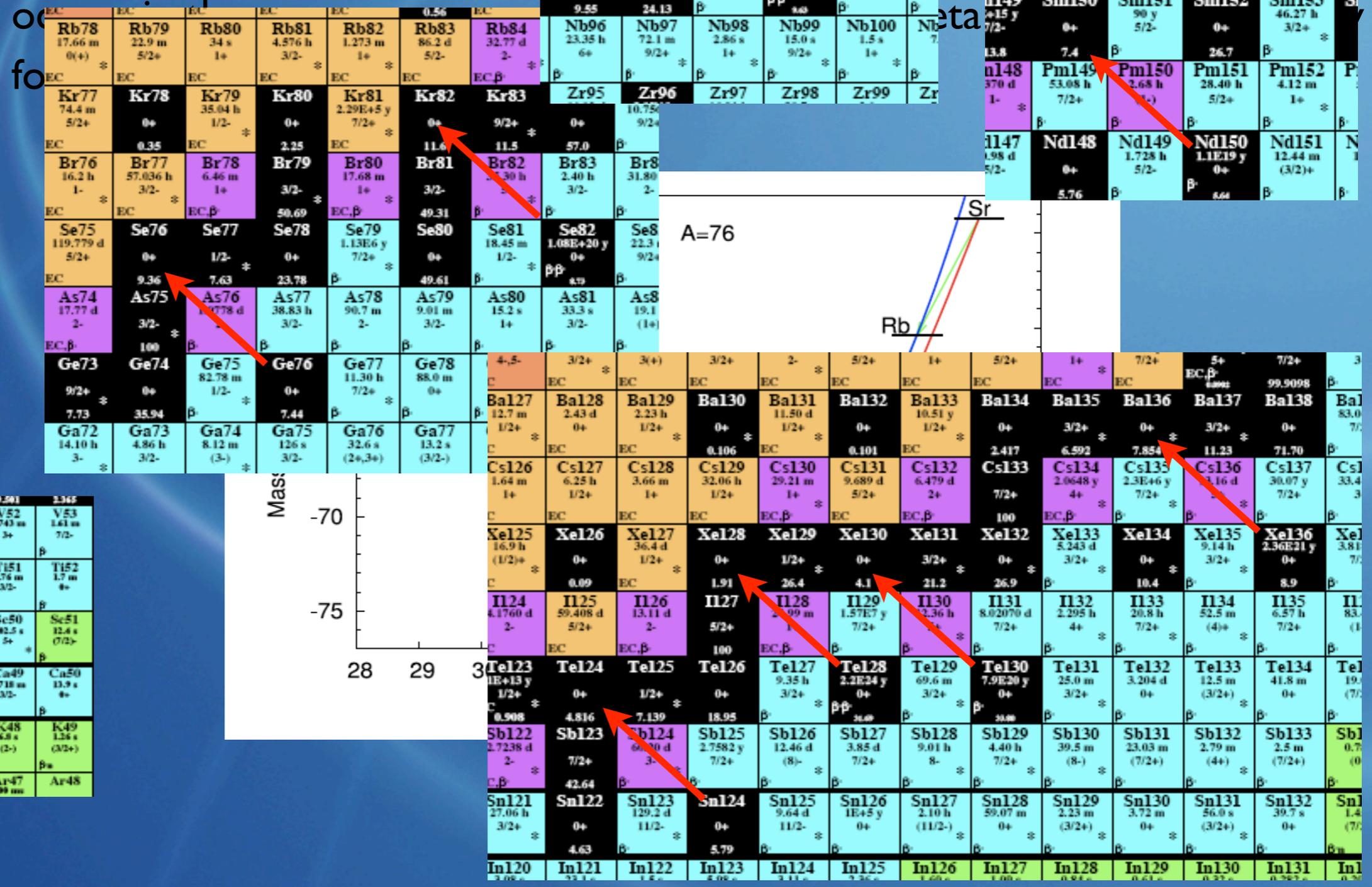
Double beta (-) decay: Process mediated by the weak interaction which occurs in those even-even nuclei where the single beta decay is energetically forbidden.



Introduction

Double beta (-) decay: $0\nu\beta\beta$

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V48	V49	V50	V51	V52	V53
15.9735 d	338 d	1.48E+17 y	70-	3.743 m	1.61 m
4+	7/2-	6+	7/2-	3+	7/2-
EC		EC,β		β	β
Ti47	Ti48	Ti49	Ti50	Ti51	Ti52
7.3	7.3	5.5	5.4	β	β
5/2-	0+	7/2-	0+	3/2-	0+
β	β	β	β	β	β
Se46	Se47	Se48	Se49	Se50	Se51
83.79 d	3.3492 d	43.67 h	57.2 m	182.5 s	12.4 s
4+	7/2-	6+	7/2-	5+	(7/2)-
β	β	β	β	β	β
Cu45	Cu46	Cu47	Cu48	Cu49	Cu50
162.61 d	0+	4.534 d	68+18 y	8.718 m	13.9 s
7/2-	0+	7/2-	0+	3/2-	0+
β	β	β	β,ββ	β	β
K44	K45	K46	K47	K48	K49
22.13 m	17.3 m	185 s	17.58 s	4.8 s	1.24 s
2-	3/2+	(2-)	3/2+	(2-)	(3/2+)
β	β	β	β	β	β
Ar43	Ar44	Ar45	Ar46	Ar47	Ar48
5.37 m	11.87 m	21.48 s	8.4 s	798 m	

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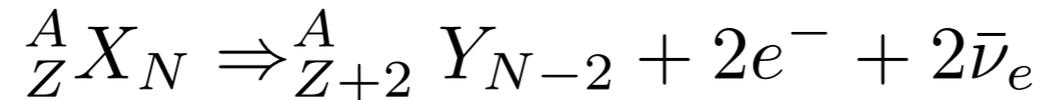
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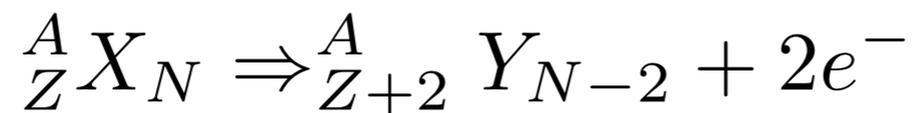
Introduction

Two neutrino double beta decay $2\nu\beta\beta$



- Conserves the leptonic number
- Compatible with massive or massless Dirac/Majorana neutrinos
- Experimentally observed ($T_{1/2} \sim 10^{19-21}$ y)
- Within the Standard Model

Neutrinoless double beta decay $0\nu\beta\beta$



- Violates the leptonic number conservation
- Neutrinos are massive Majorana particles
- Except one controversial claim (Klapdor-Kleingrothaus et al. PLB 586, 198, 2004) has not been experimentally observed ($T_{1/2} \sim 10^{25}$ y)
- Beyond the Standard Model

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Relevance of neutrinoless double beta decay

- Neutrino oscillation observations (solar, atmospheric and reactors) establish that the neutrinos have a finite mass \longrightarrow NEW PHYSICS BEYOND THE SM.
- From neutrino oscillation the absolute mass scale cannot be measured (only differences and mixing angles)
- Neutrinoless double beta decay rates depend directly on the effective neutrino mass so there are several experiments running or projected devoted to search for this process

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Table 1

Limits on neutrino-less double β^- decays. $Q_{\beta\beta}$: Q value for the $0^+ \rightarrow 0^+$ ground state transition. $G^{0\nu}$: kinematical factor (phase space volume) in units of 10^{-14} y^{-1} , $T_{1/2}^{0\nu}$: half-life limits in units of 10^{24} y and $\langle m_\nu \rangle$: limit on the effective ν mass in units of eV.

Isotope	$Q_{\beta\beta}$ (MeV)	$G^{0\nu}$	$T_{1/2}^{0\nu}$ (10^{24})	$\langle m_\nu \rangle$ (eV)	Future experiments
^{48}Ca	4.276	4.46	>0.014	$<7.2\text{--}45$	CANDLES
^{76}Ge	2.039	0.44	$>19(22)$	$<0.35(0.32)$	GERDA
^{76}Ge	2.039	0.44	>16	$<0.33\text{--}1.35$	MAJORANA
^{82}Se	2.992	1.89	>0.36	$<0.9\text{--}1.6$	S-NEMO MOON
^{100}Mo	3.034	3.17	>1.1	$<0.45\text{--}0.93$	MOON CaMoO_4
^{116}Cd	2.804	3.24	>0.17	<1.7	COBRA CdWO_4
^{130}Te	2.529	2.86	>3	<0.46	CUORE
^{136}Xe	2.467	3.03	>0.44	$<1.8\text{--}5.2$	EXO KamLAND BOREXINO
^{150}Nd	3.368	13.4	>0.018	$<1.7\text{--}2.4$	S-NEMO SNO+ DCBA

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Half-life neutrinoless double beta decay (Doi et al (1985))

$$\left(T_{1/2}^{0\nu\beta\beta} (0^+ \rightarrow 0^+) \right)^{-1} = G_{01} \left| M^{0\nu\beta\beta} \right|^2 \left(\frac{\langle m_\nu \rangle}{m_e} \right)^2$$

light-neutrino exchange mechanism

- Kinematic phase space factor:

$$G_{01} = \frac{(Gg_A(0))^4 m_e^4}{64\pi^5 \ln 2} \int F_0(Z, \varepsilon_1) F_0(Z, \varepsilon_2) \times p_1 p_2 \delta(\varepsilon_1 + \varepsilon_2 - E_f - E_i) d\varepsilon_1 d\varepsilon_2 d(\hat{p}_1 \cdot \hat{p}_1)$$

- Effective neutrino mass:

$$\langle m_\nu \rangle = \sum_j U_{ej}^2 m_j$$

- Nuclear Matrix Element (NME):

$$M^{0\nu\beta\beta} = - \left(\frac{g_V(0)}{g_A(0)} \right)^2 M_F^{0\nu\beta\beta} + M_{GT}^{0\nu\beta\beta} - M_T^{0\nu\beta\beta}$$

Fermi

Gamow-Teller

Tensor

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Nuclear Matrix Elements

$$M^{0\nu\beta\beta} = - \left(\frac{g_V(0)}{g_A(0)} \right)^2 M_F^{0\nu\beta\beta} + M_{GT}^{0\nu\beta\beta} - M_T^{0\nu\beta\beta}$$

- Each term can be written as the expectation value of a transition operator acting on the ground states of the mother and granddaughter nuclei:

$$M_{\xi}^{0\nu\beta\beta} = \langle 0_f^+ | \hat{O}_{\xi}^{0\nu\beta\beta} | 0_i^+ \rangle$$

- Nuclear structure methods for calculating these NME:
 - Quasiparticle Random Phase Approximation in different versions: QRPA, RQRPA, SRQRPA. (Tübingen group, Jyväskylä group)
 - Interacting Shell Model -ISM- (Strasbourg-Madrid collaboration)
 - Interacting Boson Model -IBM- (Yale group)
 - Projected Hartree-Fock-Bogoliubov -PHFB- (Lucknow-UNAM group)

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- Nuclear structure methods for calculating these NME:

Different ways to deal with:

- Quasiparticle Random Phase Approximation in different versions:
 - Finding the best initial and final ground states.
 - Handling the transition operator (inclusion of most relevant terms, corrections, approximations, etc.).
- Interacting Shell Model -ISM- (Strasbourg-Madrid collaboration)

Some remarks about these methods:

- Interacting Boson Model -IBM- (Yale group)
 - Calculations with limited single particle bases.
- Projected Hartree-Fock-Bogoliubov -PHFB- (Lucknow-UNAM group)
 - Interactions fitted to the specific region (ISM) or to each nucleus individually (rest).
 - Difficulties to include collective degrees of freedom.
 - Problems with particle number conservation.

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Nuclear Matrix Elements

$$M^{0\nu\beta\beta} = - \left(\frac{g_V(0)}{g_A(0)} \right)^2 M_F^{0\nu\beta\beta} + M_{GT}^{0\nu\beta\beta} - \cancel{M_T^{0\nu\beta\beta}}$$

- Neglect the tensor term.
- Closure approximation (10% error at most)

$$M_F^{0\nu\beta\beta} = \left(\frac{g_A(0)}{g_V(0)} \right)^2 \langle 0_f^+ | \hat{V}_F(1, 2) \hat{\tau}_-^{(1)} \hat{\tau}_-^{(2)} | 0_i^+ \rangle$$

$$M_{GT}^{0\nu\beta\beta} = \langle 0_f^+ | \hat{V}_{GT}(1, 2) \hat{\tau}_-^{(1)} \hat{\tau}_-^{(2)} | 0_i^+ \rangle$$

$$\langle \vec{r}_1 \vec{r}_2 | \hat{V}_F(1, 2) | \vec{r}'_1 \vec{r}'_2 \rangle = v_F (|\vec{r}_1 - \vec{r}_2|) \delta(\vec{r}_1 - \vec{r}'_1) \delta(\vec{r}_2 - \vec{r}'_2)$$

$$\langle \vec{r}_1 \vec{r}_2 | \hat{V}_{GT}(1, 2) | \vec{r}'_1 \vec{r}'_2 \rangle = v_{GT} (|\vec{r}_1 - \vec{r}_2|) \delta(\vec{r}_1 - \vec{r}'_1) \delta(\vec{r}_2 - \vec{r}'_2) \hat{\sigma}^{(1)} \cdot \hat{\sigma}^{(2)}$$

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Nuclear Matrix Elements

$$M^{0\nu\beta\beta} = - \left(\frac{g_V(0)}{g_A(0)} \right)^2 M_F^{0\nu\beta\beta} + M_{GT}^{0\nu\beta\beta} - \cancel{M_T^{0\nu\beta\beta}}$$

- Neglect the tensor term.
- Closure approximation (10% error at most)

$$M_F^{0\nu\beta\beta} = \left(\frac{g_A(0)}{g_V(0)} \right)^2 \langle 0_f^+ | \hat{V}_F(1, 2) \hat{\tau}_-^{(1)} \hat{\tau}_-^{(2)} | 0_i^+ \rangle$$

$$M_{GT}^{0\nu\beta\beta} = \langle 0_f^+ | \hat{V}_{GT}(1, 2) \hat{\tau}_-^{(1)} \hat{\tau}_-^{(2)} | 0_i^+ \rangle$$

$$\begin{aligned} \langle \vec{r}_1 \vec{r}_2 | \hat{V}_F(1, 2) | \vec{r}'_1 \vec{r}'_2 \rangle &= v_F(|\vec{r}_1 - \vec{r}_2|) \delta(\vec{r}_1 - \vec{r}'_1) \delta(\vec{r}_2 - \vec{r}'_2) \\ \langle \vec{r}_1 \vec{r}_2 | \hat{V}_{GT}(1, 2) | \vec{r}'_1 \vec{r}'_2 \rangle &= v_{GT}(|\vec{r}_1 - \vec{r}_2|) \delta(\vec{r}_1 - \vec{r}'_1) \delta(\vec{r}_2 - \vec{r}'_2) \hat{\sigma}^{(1)} \cdot \hat{\sigma}^{(2)} \end{aligned}$$

Neutrino potentials

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Neutrino potentials

Starting from the weak lagrangian that describes the process some approximations are made:

1. Non-relativistic approach in the hadronic part.
2. Closure approximation in the virtual intermediate state
3. Nucleon form factors taken in the dipolar approximation.
4. Tensor contribution is neglected.
5. High order currents are included (HOC).
6. Short range correlations are included with an UCOM correlator.

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6. Short range correlations are included with an UCOM correlator.

- Find the initial and final 0^+ states within the GCM+PNAMP method (axial calculations)
- Evaluate the transition operators between these states

Method: GCM+PNAMP

See also P.-H. Heenen's talk (Sunday afternoon)

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🌀 **Effective nucleon-nucleon interaction (Density Dependent): Gogny force (D1S)** that is able to describe properly many phenomena along the whole nuclear chart.

$$\begin{aligned}
 V(1,2) = & \sum_{i=1}^2 e^{-(\vec{r}_1 - \vec{r}_2)^2 / \mu_i^2} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau) \\
 & + iW_0 (\sigma_1 + \sigma_2) \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2) \vec{k} + t_3 (1 + x_0 P^\sigma) \delta(\vec{r}_1 - \vec{r}_2) \rho^\alpha ((\vec{r}_1 + \vec{r}_2)/2) \\
 & + V_{\text{Coulomb}}(\vec{r}_1, \vec{r}_2)
 \end{aligned}$$

🌀 **Method of solving the many-body problem:**

First step: Particle Number Projection (before the variation) of HFB-type wave functions.

Second step: Simultaneous Particle Number and Angular Momentum Projection (after the variation).

Third step: Configuration mixing within the framework of the **Generator Coordinate Method (GCM)**.

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Method: GCM+PNAMP

Determination of mother and granddaughter states (I)

Intrinsic state: Solve the PN-VAP equations with the Gogny DIS interaction

$$|\bar{\Phi}\rangle \text{ HFB states} \longrightarrow \delta (E^{N,Z} [|\bar{\Phi}(q)\rangle])_{|\bar{\Phi}\rangle=|\Phi\rangle} = 0$$

$$E^{N,Z} [|\Phi\rangle] = \frac{\langle \Phi | \hat{H} \hat{P}^N \hat{P}^Z | \Phi \rangle}{\langle \Phi | \hat{P}^N \hat{P}^Z | \Phi \rangle} + \varepsilon_{DD}^{N,Z} (|\Phi\rangle) - \lambda_q \langle \Phi | \hat{Q} | \Phi \rangle$$

Particle number and angular momentum projected state:

$$|IMK; NZ; q\rangle = \frac{2I+1}{8\pi^2} \int \mathcal{D}_{MK}^{I*}(\Omega) \hat{R}(\Omega) \hat{P}^N \hat{P}^Z |\Phi(q)\rangle d\Omega$$

General form (GCM state):

$$|IM; NZ\sigma\rangle = \sum_{Kq} f_{Kq}^{I;NZ,\sigma} |IMK; NZ; q\rangle$$

Hill-Wheeler-Griffin equation (GCM)

$$\sum_{K'q'} \left(\mathcal{H}_{KqK'q'}^{I;NZ} - E^{I;NZ;\sigma} \mathcal{N}_{KqK'q'}^{I;NZ} \right) f_{K'q'}^{I;NZ;\sigma} = 0$$

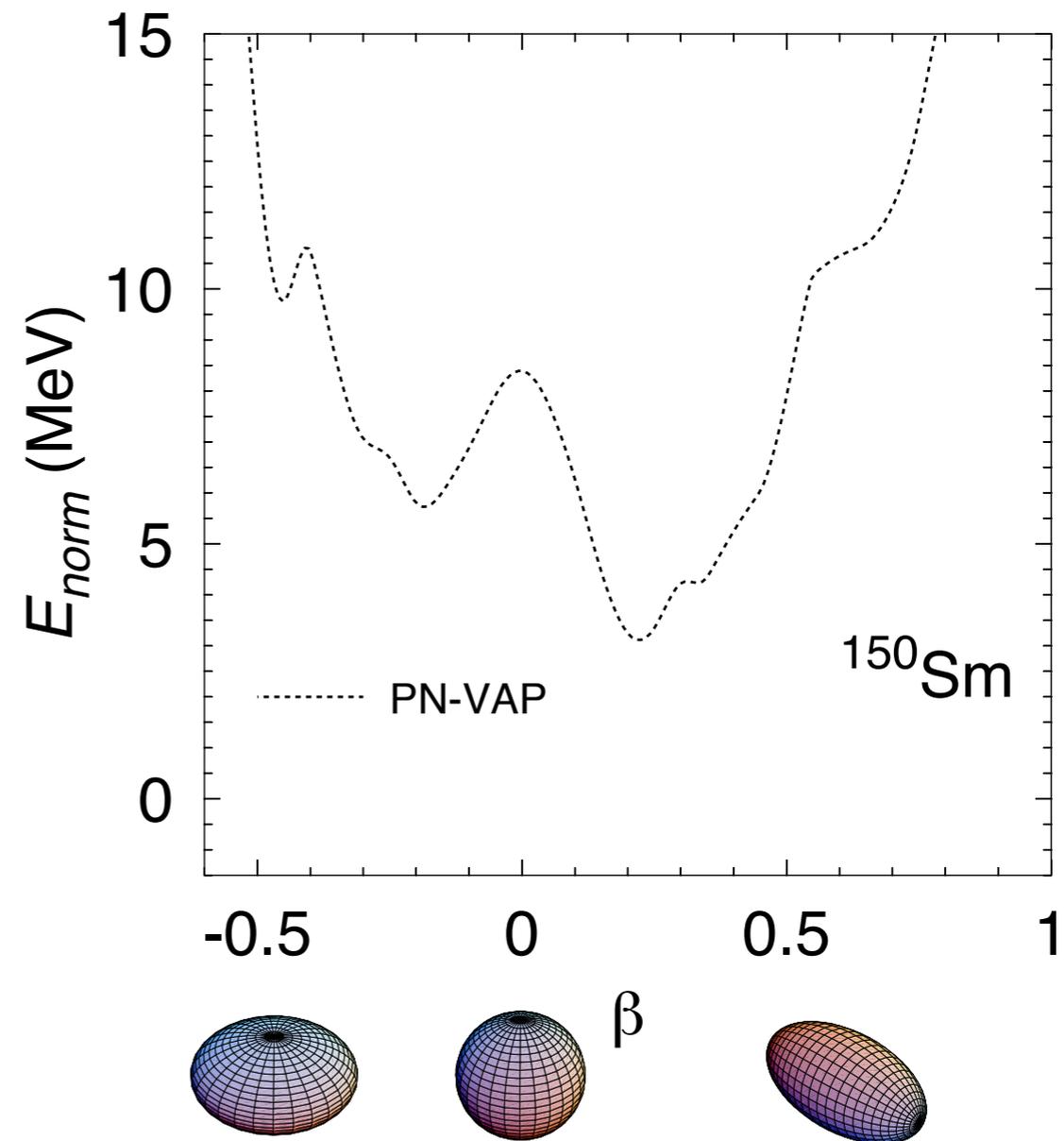
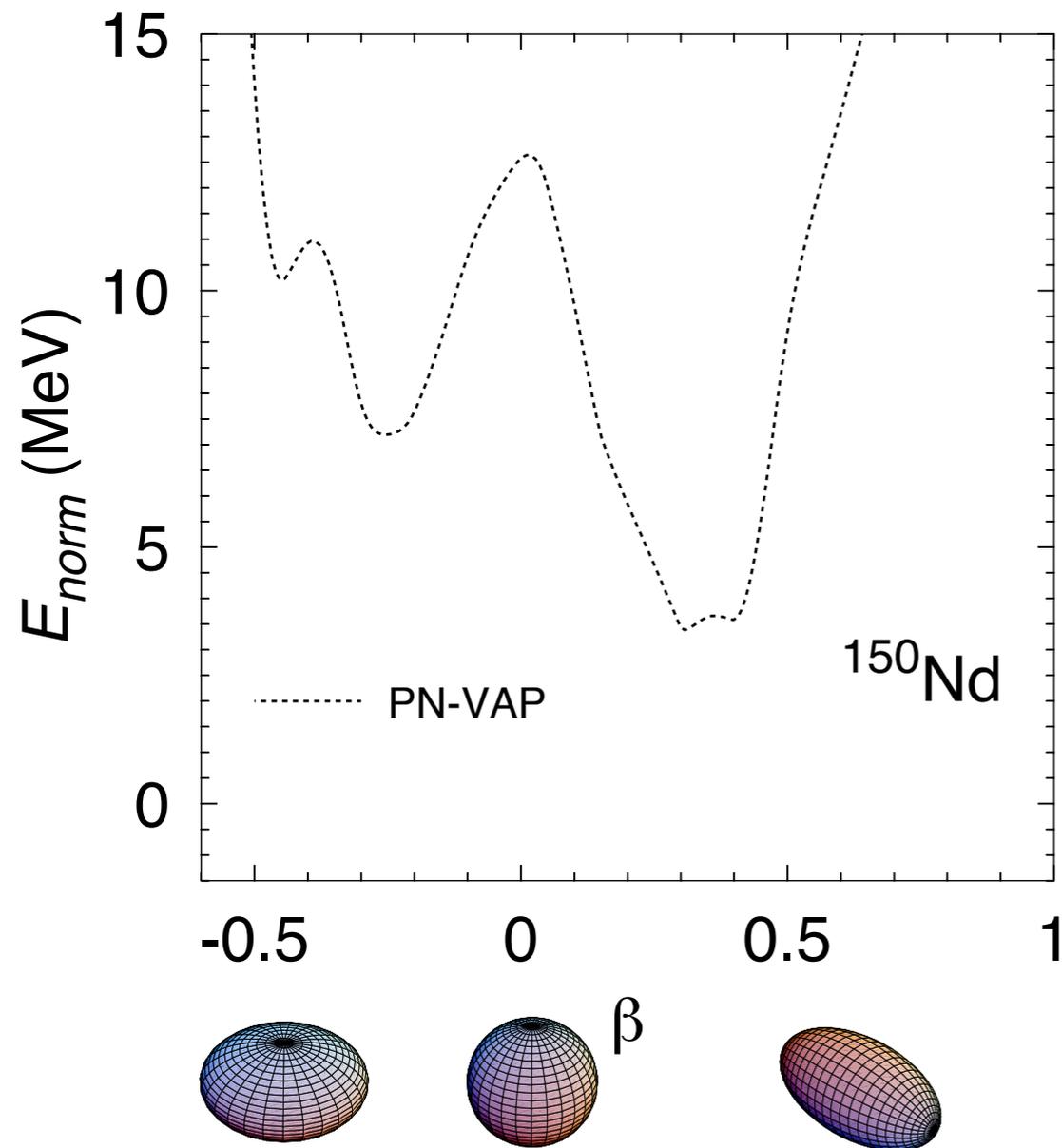
$$\mathcal{N}_{KqK'q'}^{I;NZ} \equiv \langle IMK; NZ; q | IMK'; NZ; q' \rangle$$

$$\mathcal{H}_{KqK'q'}^{I;NZ} \equiv \langle IMK; NZ; q | \hat{H} | IMK'; NZ; q' \rangle + \varepsilon_{DD}^{IKK';NZ} [|\Phi(q)\rangle, |\Phi(q')\rangle]$$

→ generalized eigenvalue problem

Method: GCM+PNAMP

Determination of mother and granddaughter states (I)



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Method: GCM+PNAMP

Determination of mother and granddaughter states (II)

Intrinsic state: Solve the VAP-PN equations with the Gogny DIS interaction

$$|\Phi\rangle \text{ HFB states} \longrightarrow \delta (E^{N,Z} [|\bar{\Phi}(q)\rangle])_{|\bar{\Phi}\rangle=|\Phi\rangle} = 0$$

$$E^{N,Z} [|\Phi\rangle] = \frac{\langle \Phi | \hat{H} \hat{P}^N \hat{P}^Z | \Phi \rangle}{\langle \Phi | \hat{P}^N \hat{P}^Z | \Phi \rangle} + \varepsilon_{DD}^{N,Z} (|\Phi\rangle) - \lambda_q \langle \Phi | \hat{Q} | \Phi \rangle$$

Particle number and angular momentum projected state:

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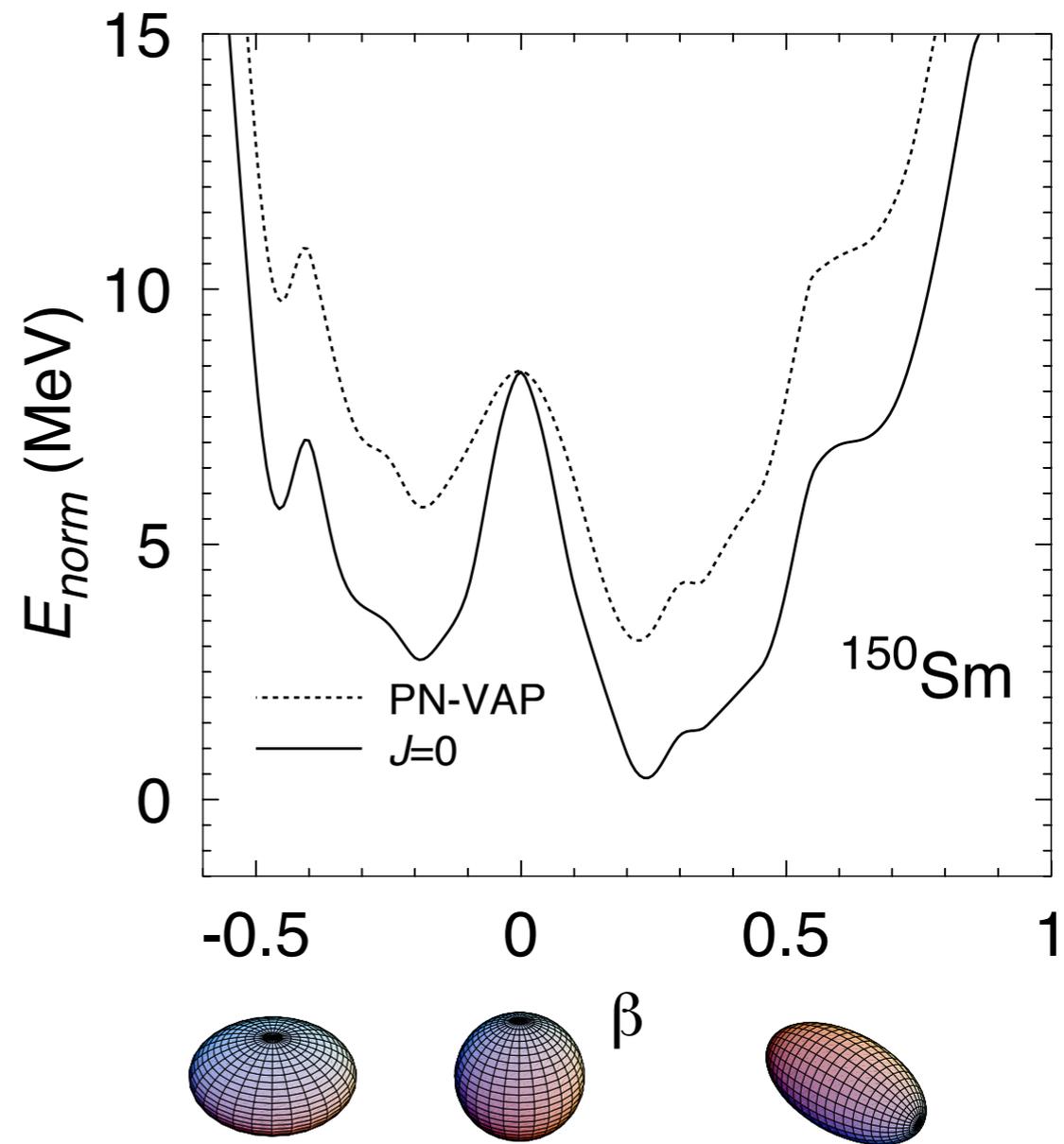
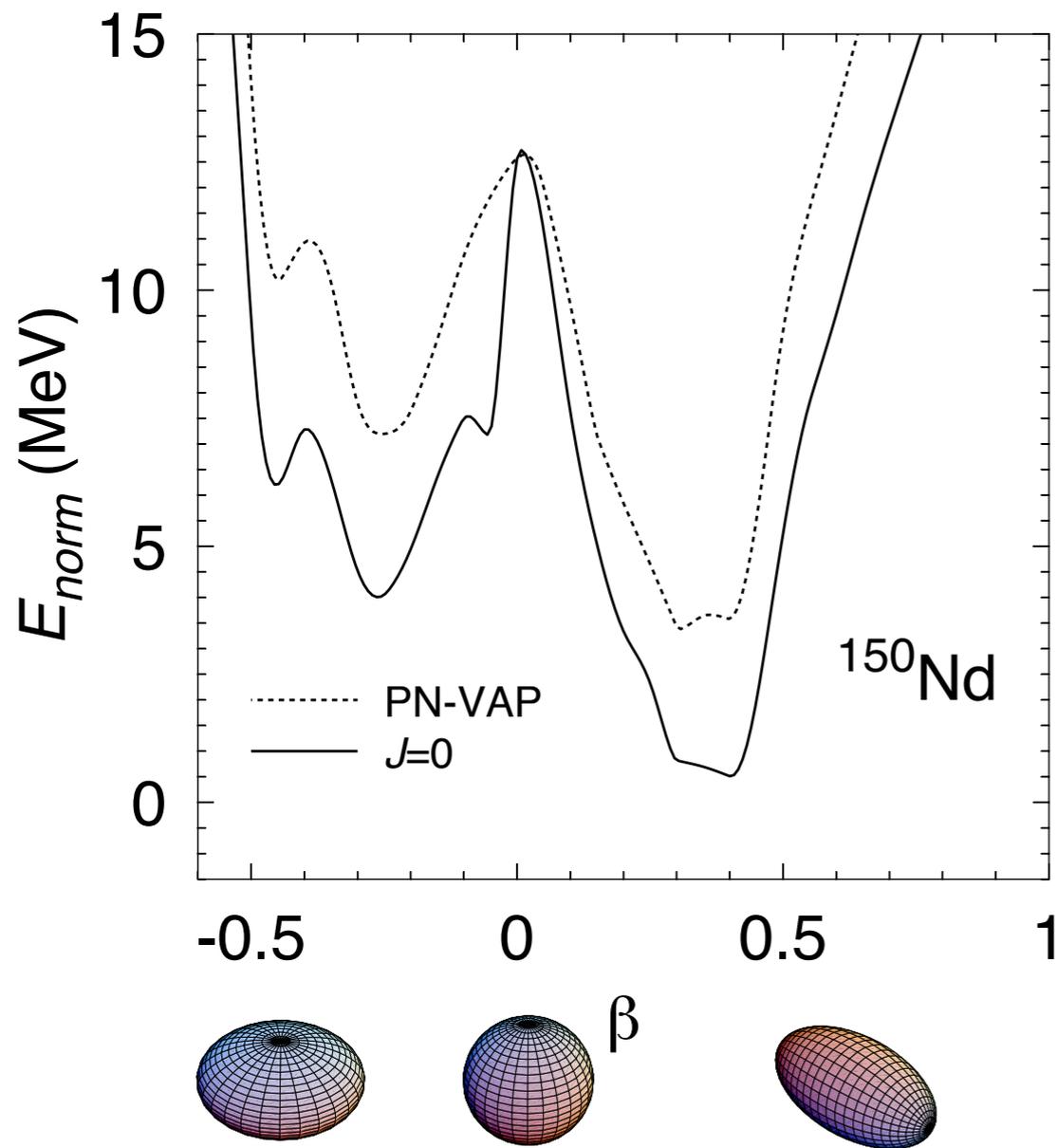
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→ generalized eigenvalue problem

Method: GCM+PNAMP

Determination of mother and granddaughter states (II)



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Determination of mother and granddaughter states (III)

Intrinsic state: Solve the VAP-PN equations with the Gogny DIS interaction

$$|\Phi\rangle \text{ HFB states} \longrightarrow \delta (E^{N,Z} [|\bar{\Phi}(q)\rangle])_{|\bar{\Phi}\rangle=|\Phi\rangle} = 0$$

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Particle number and angular momentum projected state:

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Solving HWG equation:

1. Diagonalization of the norm overlap:

$$\sum_{K'q'} \mathcal{N}_{KqK'q'}^{I;NZ} u_{K'q';\Lambda}^{I;NZ} = n_{\Lambda}^{I;NZ} u_{Kq;\Lambda}^{I;NZ}$$
2. Natural basis:

$$|\Lambda^{IM;NZ}\rangle = \sum_{Kq} \frac{u_{Kq;\Lambda}^{I;NZ}}{\sqrt{n_{\Lambda}^{I;NZ}}} |IMK; NZ; q\rangle ; n_{\Lambda}^{I;NZ} / n_{max}^{I;NZ} > \zeta$$
3. Normal eigenvalue problem:

$$\sum_{\Lambda'} \langle \Lambda^{I;NZ} | \hat{H} | \Lambda'^{I;NZ} \rangle G_{\Lambda'}^{I;NZ;\sigma} = E^{I;NZ;\sigma} G_{\Lambda}^{I;NZ;\sigma}$$

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$$|IM; NZ\sigma\rangle = \sum_{\Lambda} G_{\Lambda}^{I;NZ;\sigma} |\Lambda^{I;NZ}\rangle$$

Solving HWG equation:

1. Diagonalization of the norm overlap:

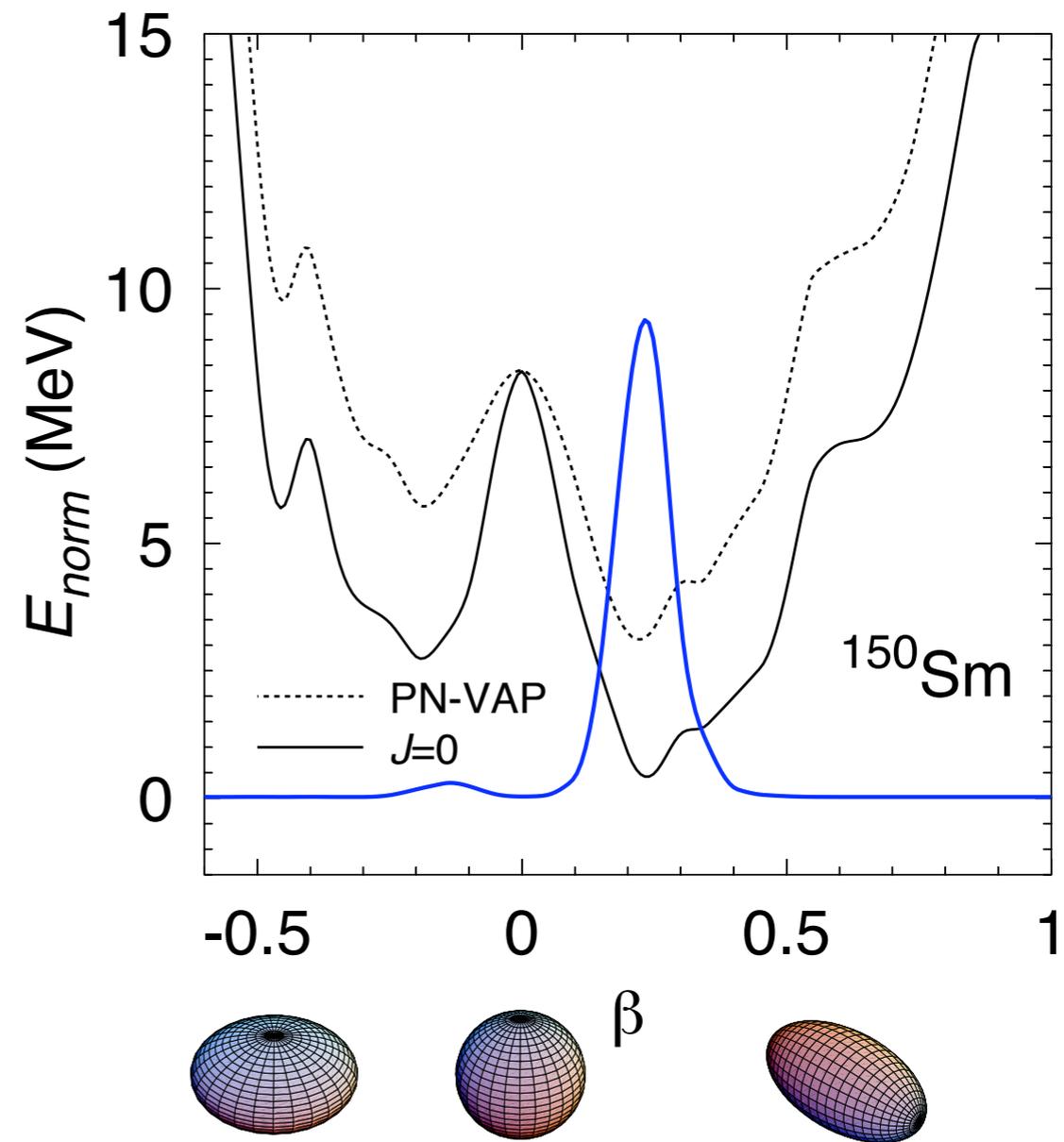
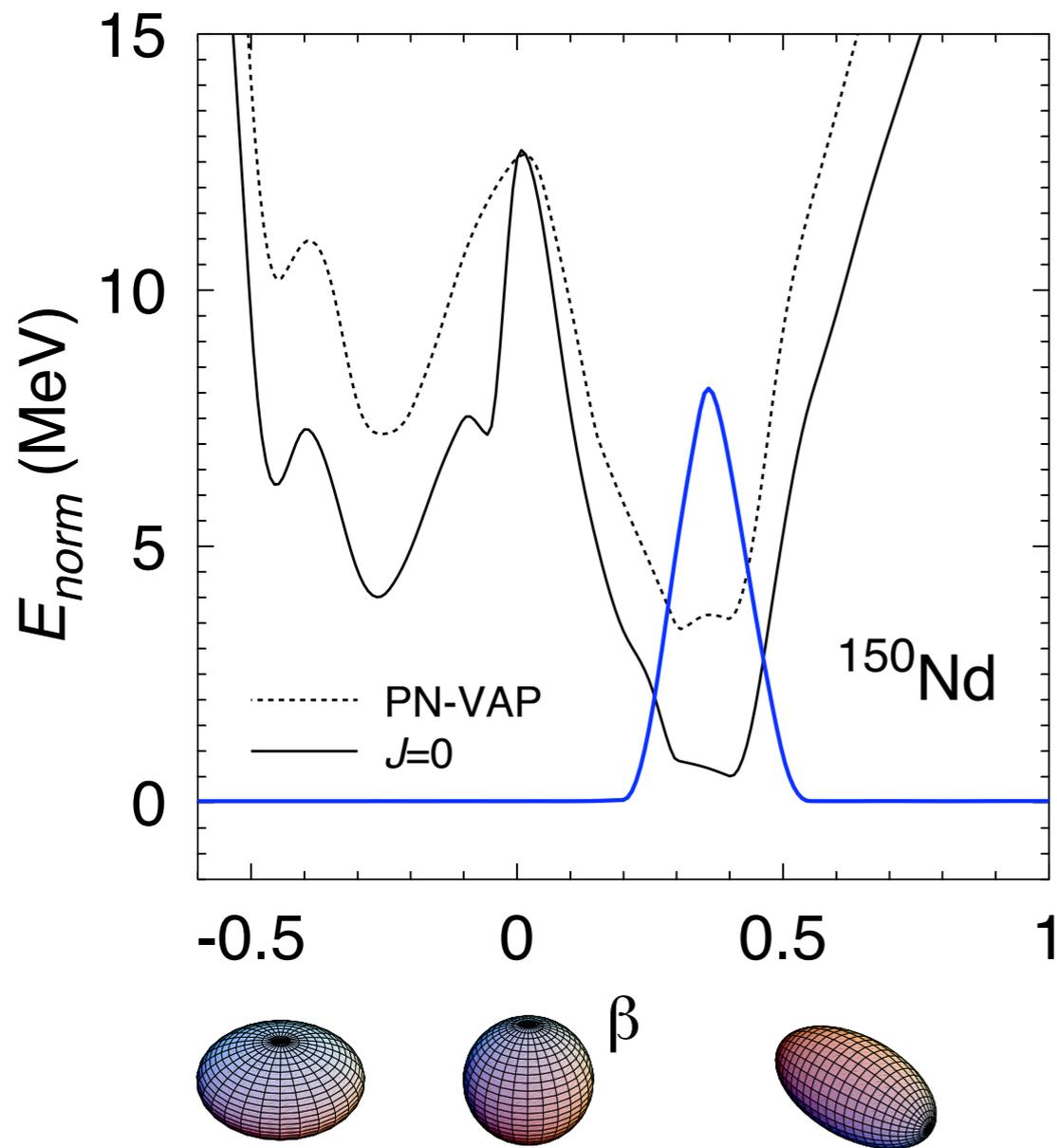
$$\sum_{K'q'} \mathcal{N}_{KqK'q'}^{I;NZ} u_{K'q';\Lambda}^{I;NZ} = n_{\Lambda}^{I;NZ} u_{Kq;\Lambda}^{I;NZ}$$
2. Natural basis:

$$|\Lambda^{IM;NZ}\rangle = \sum_{Kq} \frac{u_{Kq;\Lambda}^{I;NZ}}{\sqrt{n_{\Lambda}^{I;NZ}}} |IMK; NZ; q\rangle ; n_{\Lambda}^{I;NZ} / n_{max}^{I;NZ} > \zeta$$
3. Normal eigenvalue problem:

$$\sum_{\Lambda'} \langle \Lambda^{I;NZ} | \hat{H} | \Lambda'^{I;NZ} \rangle G_{\Lambda'}^{I;NZ;\sigma} = E^{I;NZ;\sigma} G_{\Lambda}^{I;NZ;\sigma}$$

Method: GCM+PNAMP

Determination of mother and granddaughter states (III)



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Transitions

1. Axial states $K = 0$
2. Angular momentum $I = 0$
3. Ground states $\sigma = 0$
4. Quadrupole deformations $q = q_{20}$



$$|0; N_i Z_i; \sigma\rangle = \sum_{\Lambda_i} G_{\Lambda_i}^{0; N_i Z_i; \sigma} |\Lambda_i^{0; N_i Z_i}\rangle$$

$$|0; N_f Z_f; \sigma\rangle = \sum_{\Lambda_f} G_{\Lambda_f}^{0; N_f Z_f; \sigma} |\Lambda_f^{0; N_f Z_f}\rangle$$

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 \end{array}
 \longrightarrow
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 \end{array}$$

TRANSITIONS:

$$\begin{aligned}
 M_{\xi}^{0\nu\beta\beta} &= \langle 0_f^+ | \hat{O}_{\xi}^{0\nu\beta\beta} | 0_i^+ \rangle = \langle 0; N_f Z_f | \hat{O}_{\xi}^{0\nu\beta\beta} | 0; N_i Z_i \rangle = \\
 &= \sum_{\Lambda_f \Lambda_i} \left(G_{\Lambda_f}^{0; N_f Z_f} \right)^* \langle \Lambda_f^{0; N_f Z_f} | \hat{O}_{\xi}^{0\nu\beta\beta} | \Lambda_i^{0; N_i Z_i} \rangle G_{\Lambda_i}^{0; N_i Z_i} = \sum_{q_i q_f; \Lambda_f \Lambda_i} \\
 &= \left(\frac{u_{q_f, \Lambda_f}^{0; N_f Z_f}}{\sqrt{n_{\Lambda_f}^{0; N_f Z_f}}} \right)^* \left(G_{\Lambda_f}^{0; N_f Z_f} \right)^* \langle 0; N_f Z_f; q_f | \hat{O}_{\xi}^{0\nu\beta\beta} | 0; N_i Z_i; q_i \rangle \left(G_{\Lambda_i}^{0; N_i Z_i} \right) \left(\frac{u_{q_i, \Lambda_i}^{0; N_i Z_i}}{\sqrt{n_{\Lambda_i}^{0; N_i Z_i}}} \right)
 \end{aligned}$$

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TRANSITIONS:

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Matrix elements of the double beta transition operators between particle number and angular momentum projected states

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 &= \left(\frac{u_{q_f, \Lambda_f}^{0; N_f Z_f}}{\sqrt{n_{\Lambda_f}^{0; N_f Z_f}}} \right)^* \left(G_{\Lambda_f}^{0; N_f Z_f} \right)^* \langle 0; N_f Z_f; q_f | \hat{O}_{\xi}^{0\nu\beta\beta} | 0; N_i Z_i; q_i \rangle \left(G_{\Lambda_i}^{0; N_i Z_i} \right) \left(\frac{u_{q_i, \Lambda_i}^{0; N_i Z_i}}{\sqrt{n_{\Lambda_i}^{0; N_i Z_i}}} \right)
 \end{aligned}$$

Results: GCM+PNAMP

T.R.R., G. Martinez-Pinedo, arXiv:1008.5260

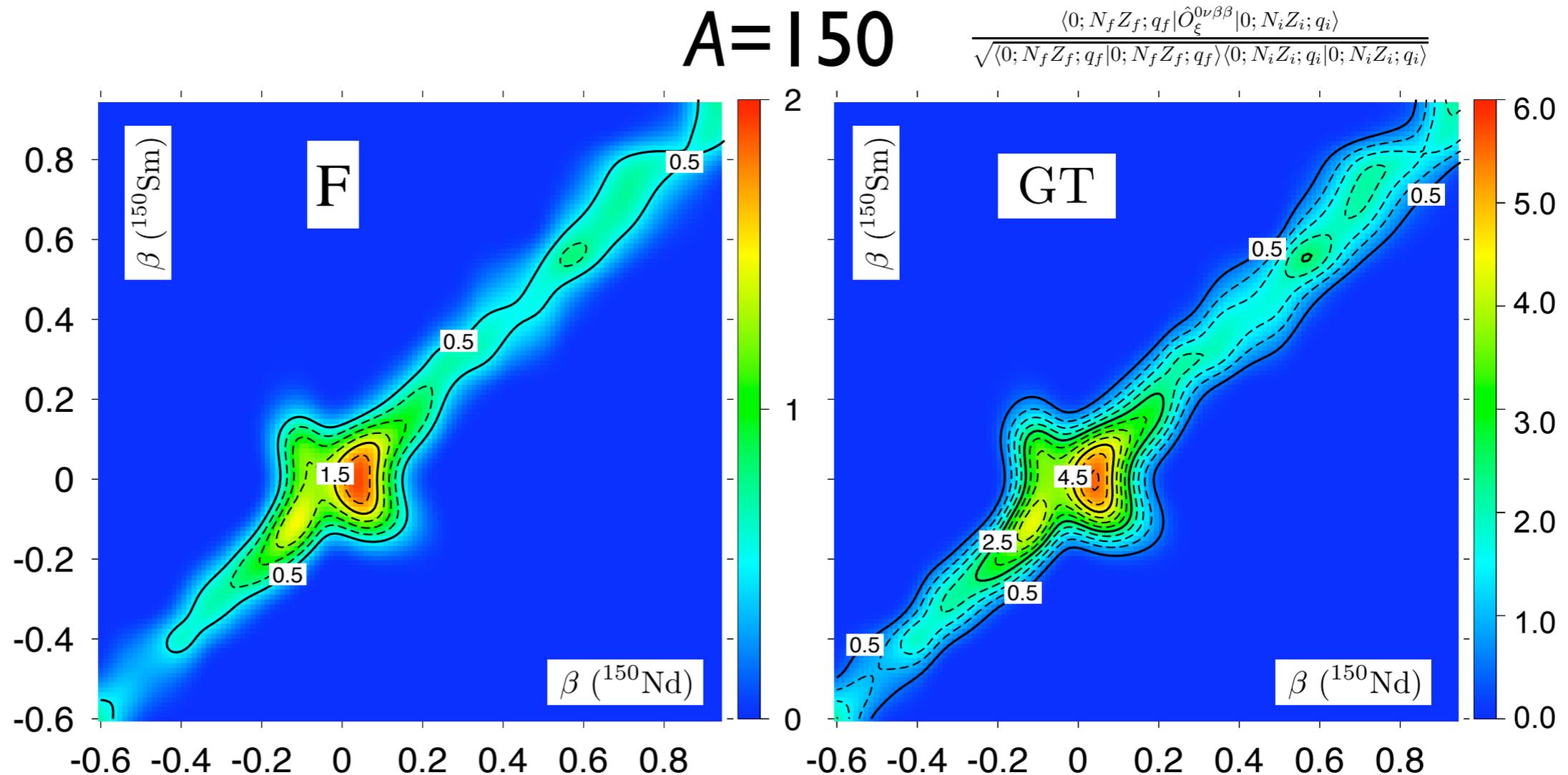
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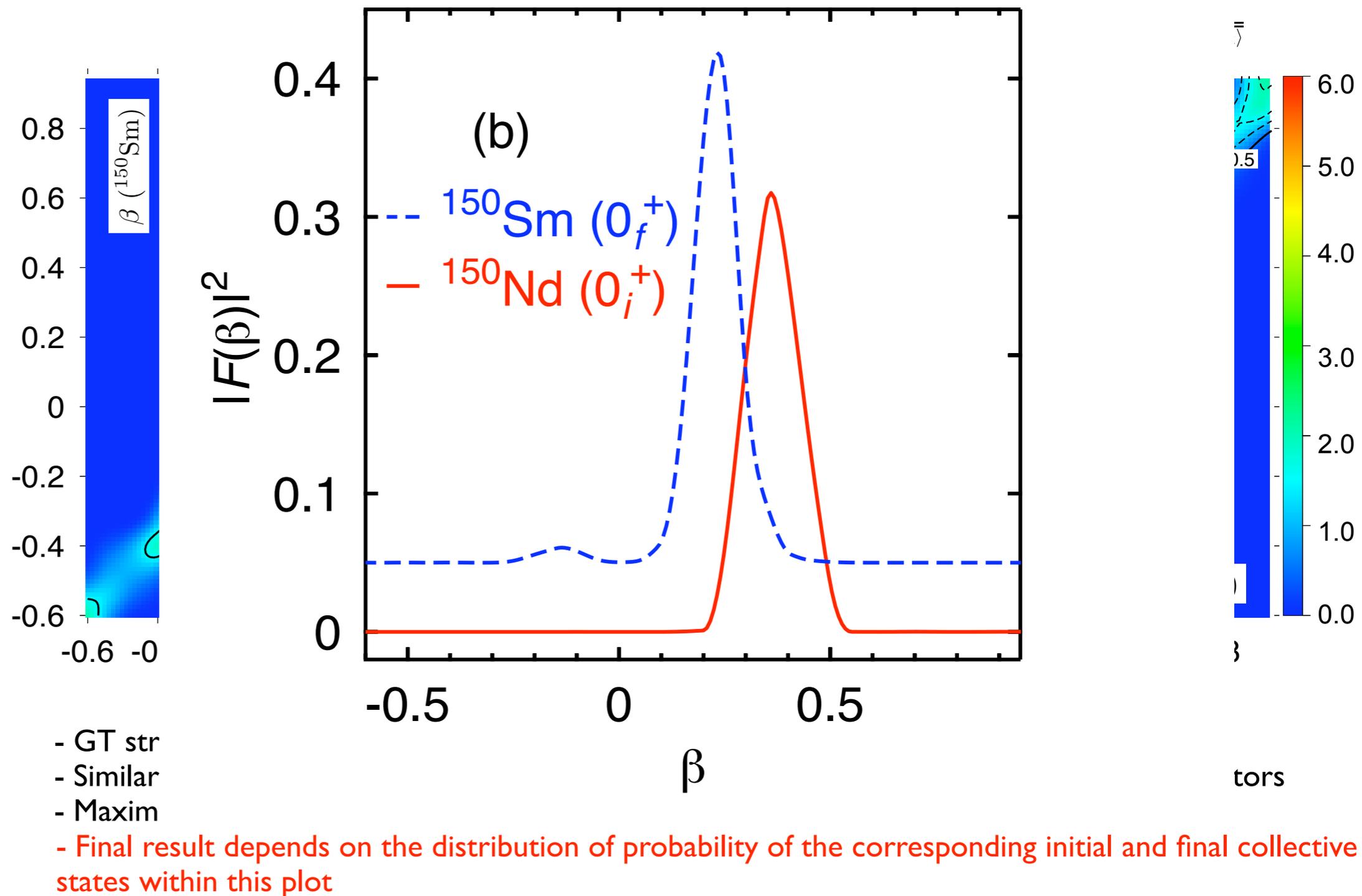
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- Maxima are found close to sphericity although some other local maxima are found

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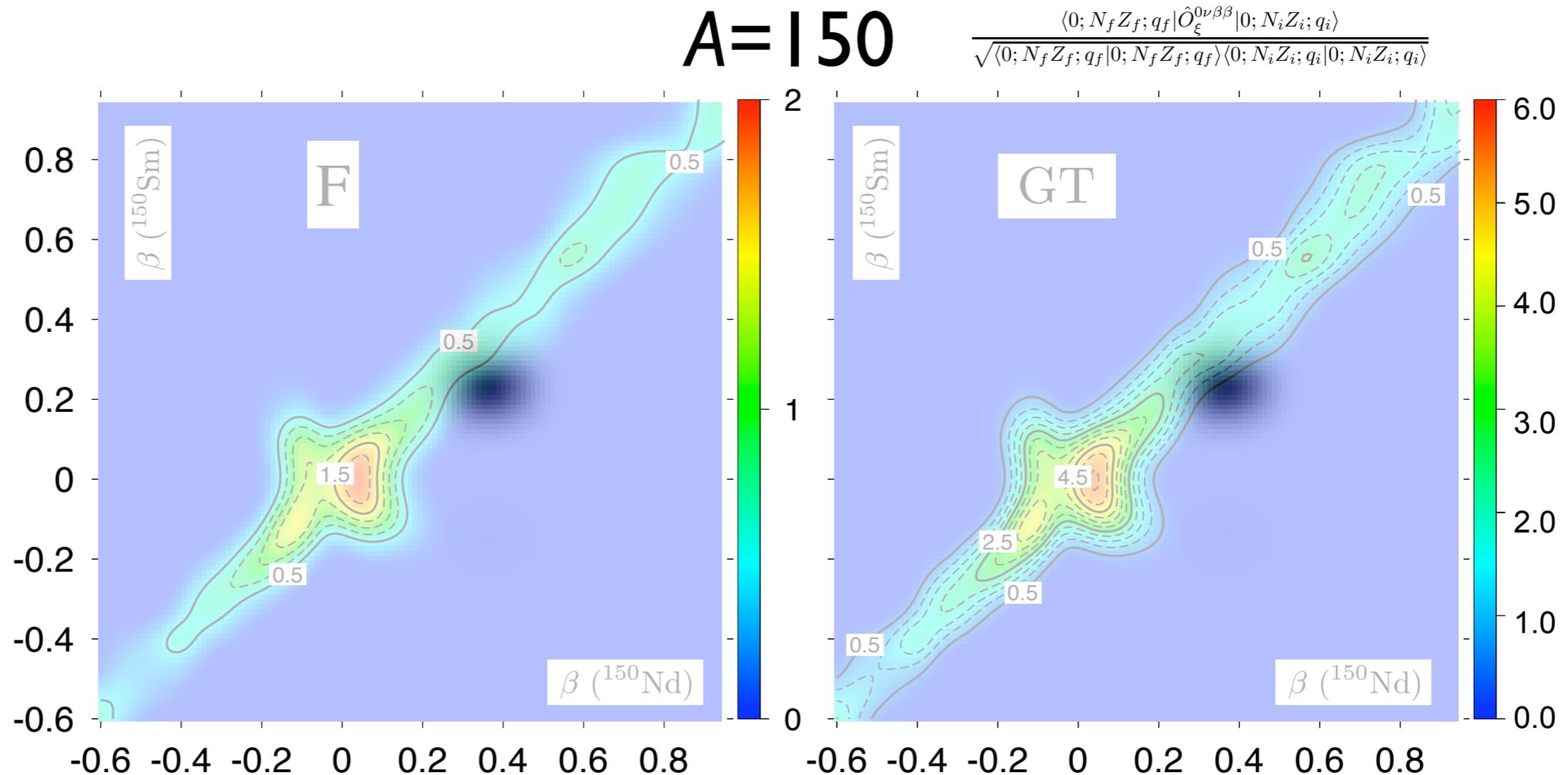
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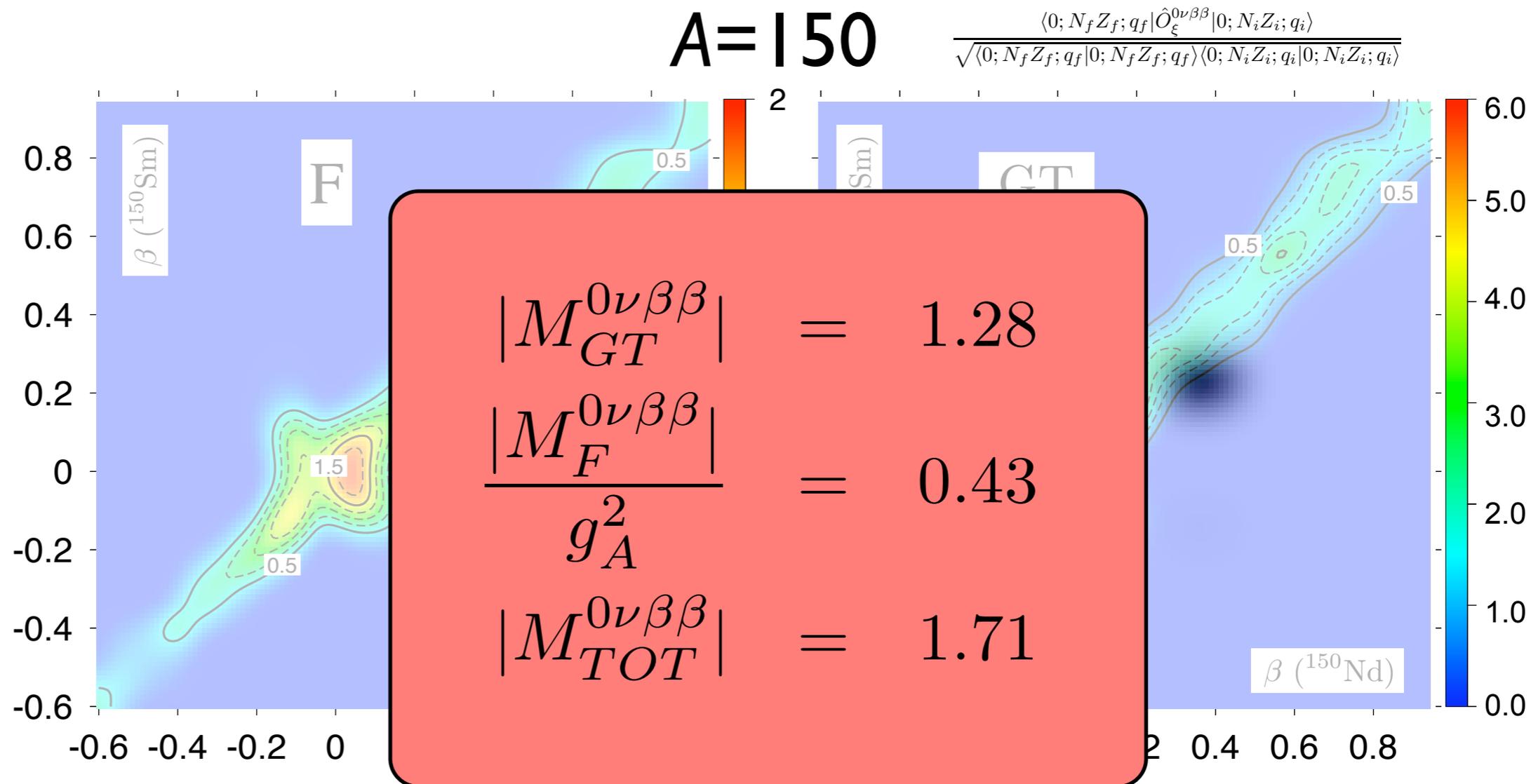
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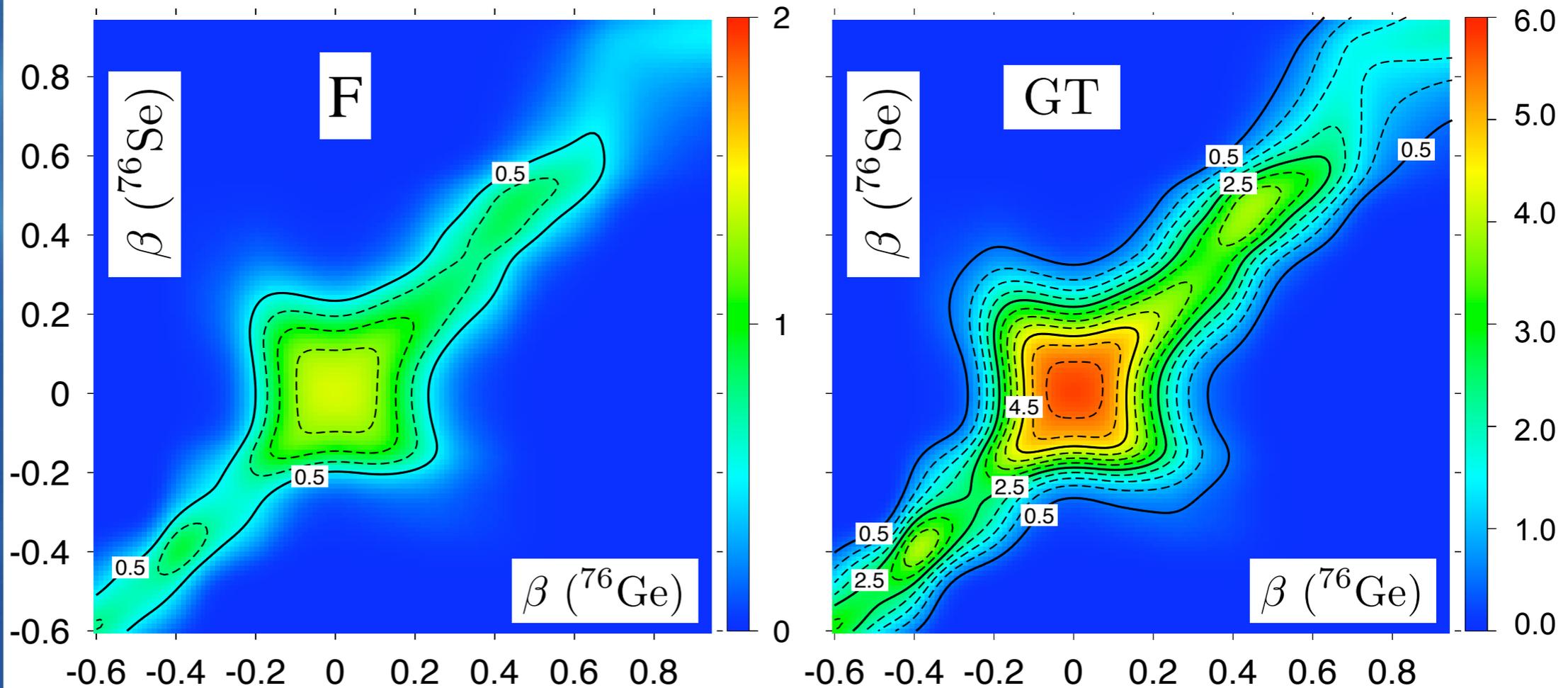
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A=76

$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}_\xi^{0\nu\beta\beta} | 0; N_i Z_i; q_i \rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f \rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i \rangle}}$$

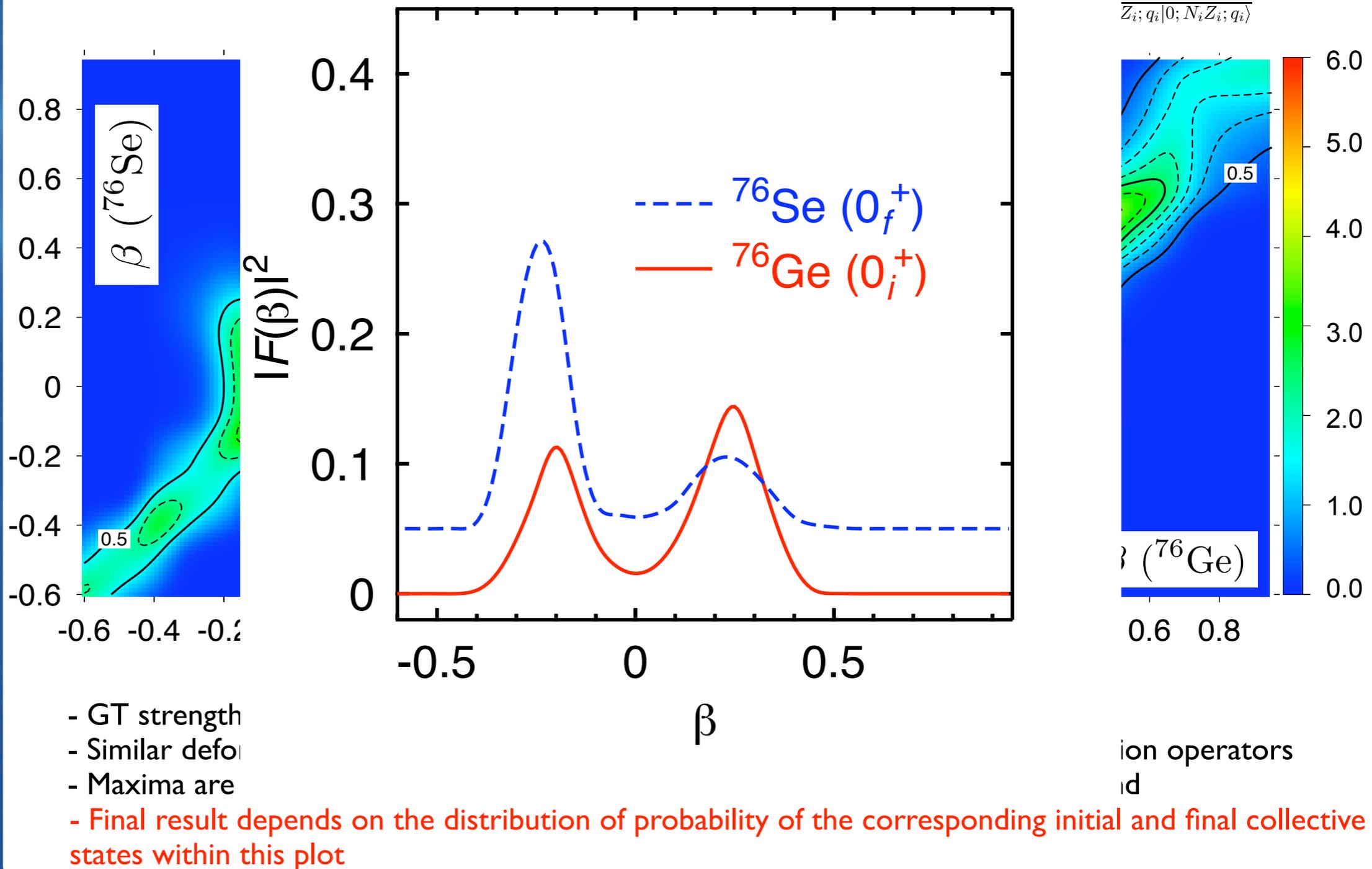


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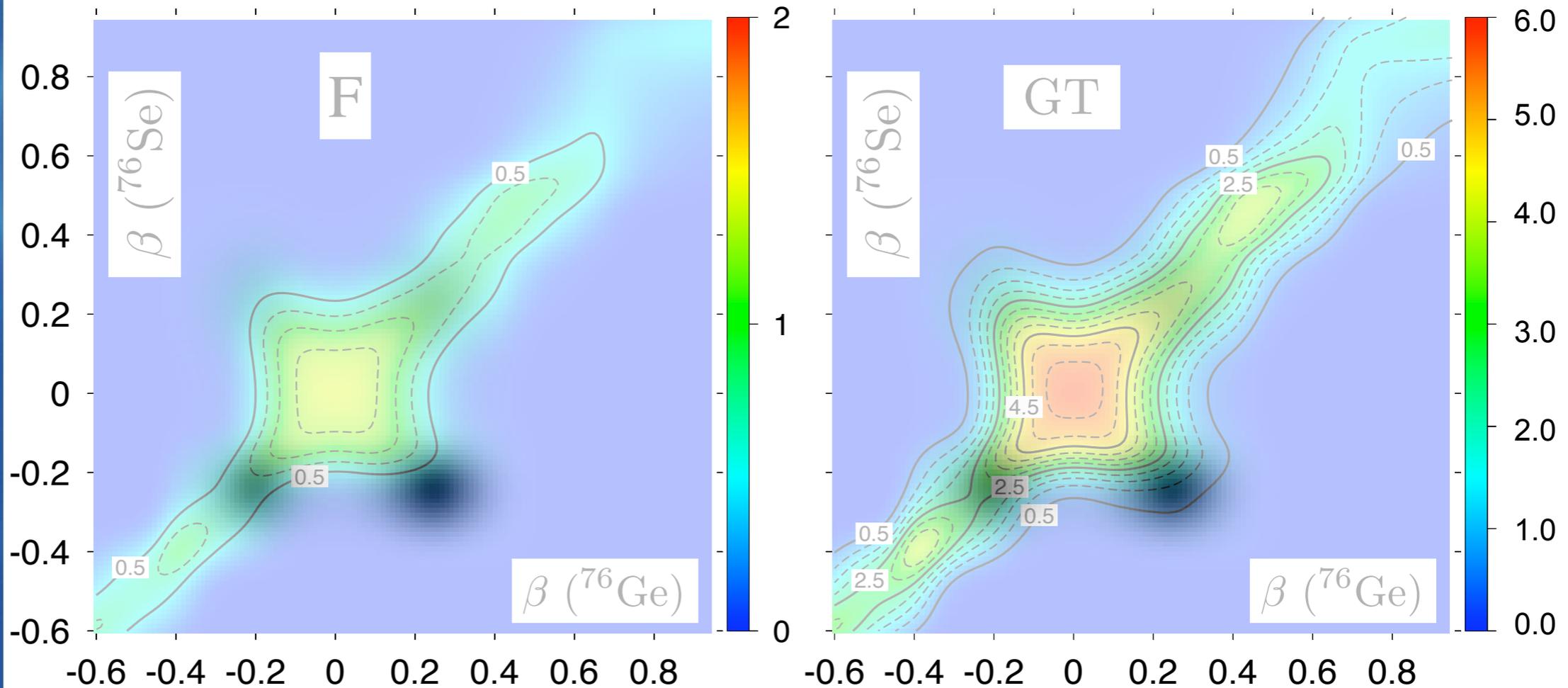
2. Method: GCM+PNAMP

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A=76

$$\frac{\langle 0; N_f Z_f; q_f | \hat{O}_\xi^{0\nu\beta\beta} | 0; N_i Z_i; q_i \rangle}{\sqrt{\langle 0; N_f Z_f; q_f | 0; N_f Z_f; q_f \rangle \langle 0; N_i Z_i; q_i | 0; N_i Z_i; q_i \rangle}}$$



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Results: GCM+PNAMP

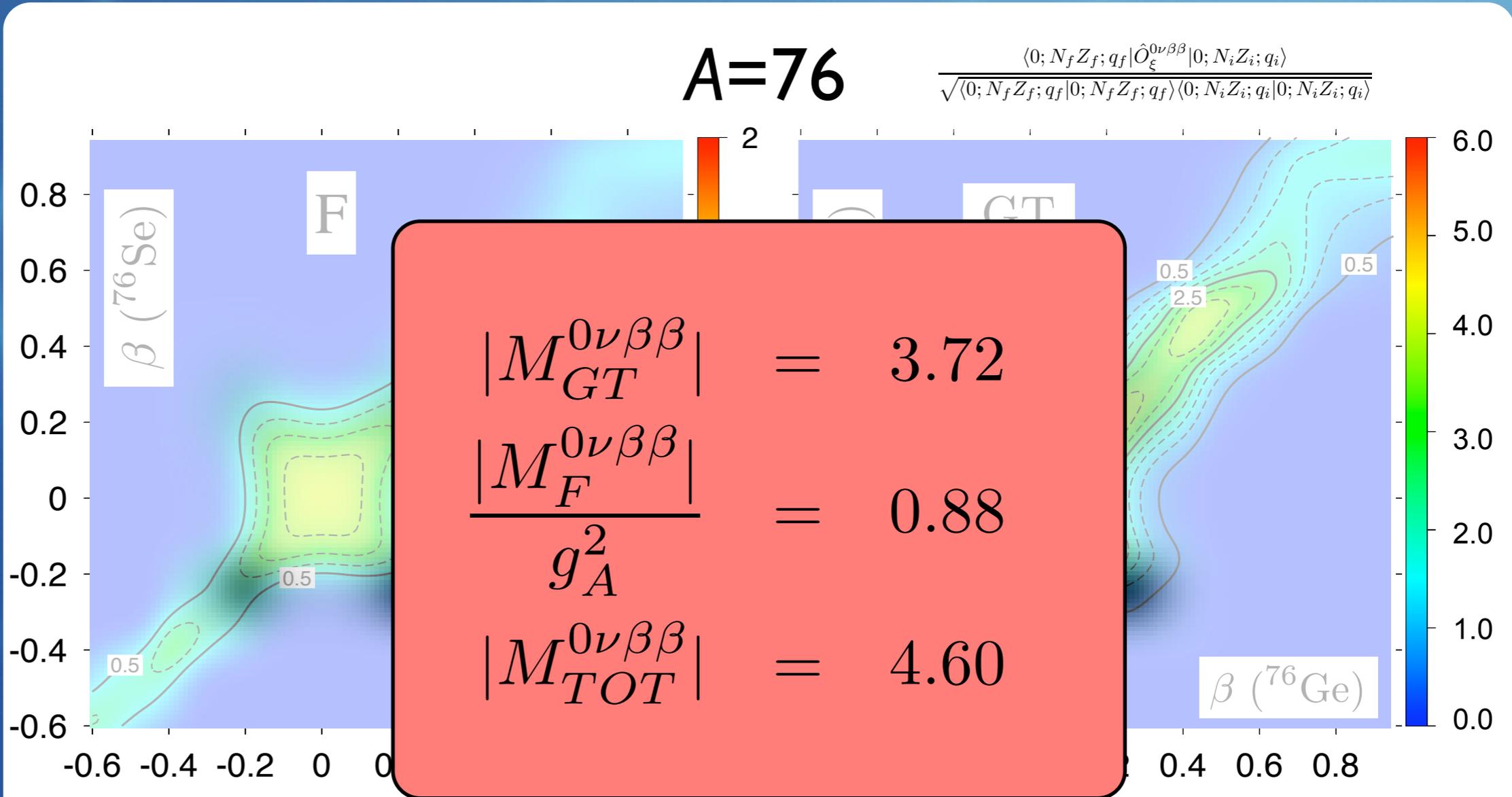
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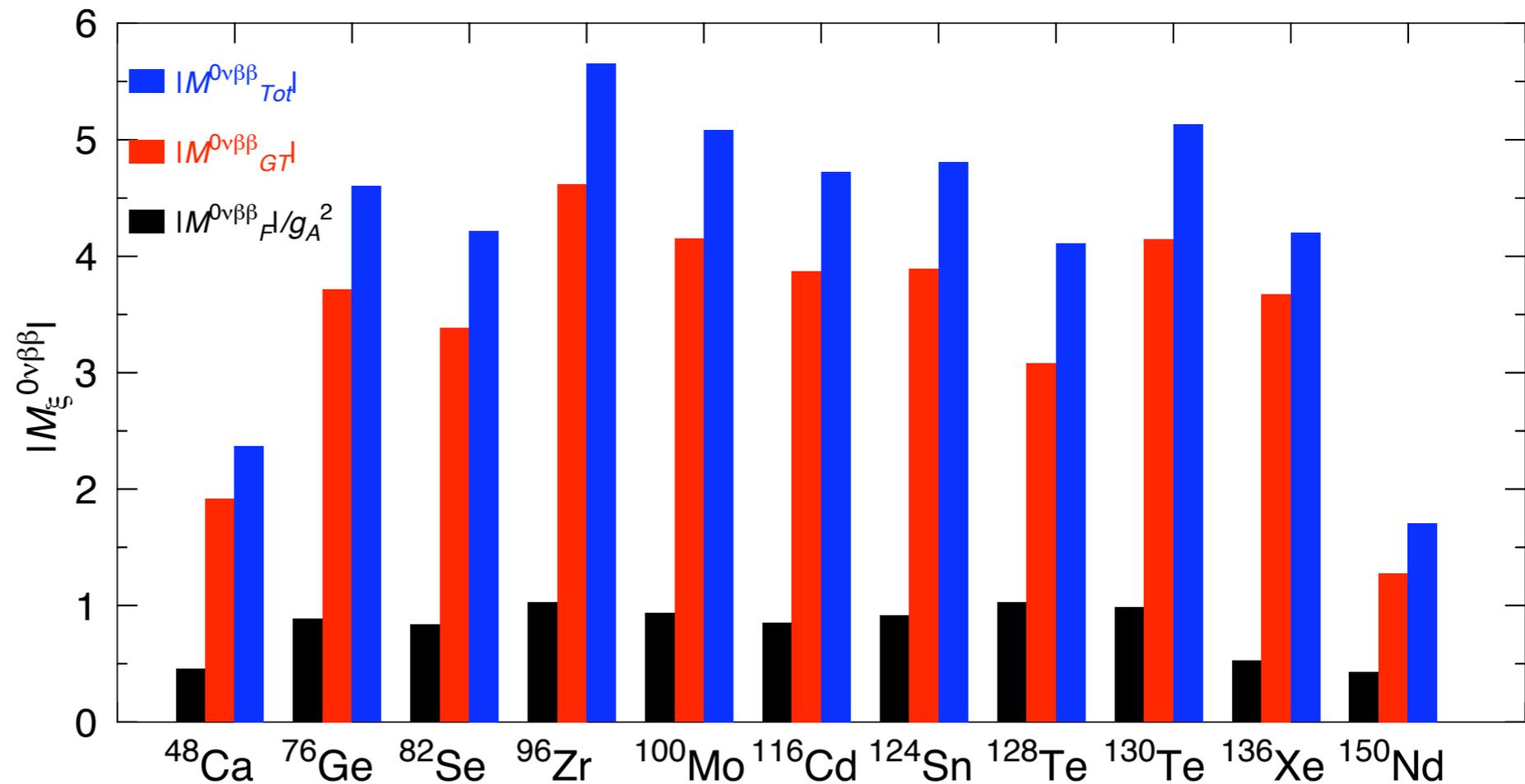
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- Small contribution of Fermi compared to Gamow-Teller.
- Small value for $A=150$ transition due to the difference in deformation.
- Small value for $A=48$ due to the small value of the strength of the operator.

Results: GCM+PNAMP

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TABLE I: Difference between theoretical and experimental Q values, kinematical phase space factors, NME and predicted half-lives for several $0\nu\beta\beta$ decaying nuclei assuming $\langle m_{\beta\beta} \rangle = 0.5$ eV.

Nucleus	$Q_{\text{theo}} - Q_{\text{exp}}$ (MeV)	G_{01} ($\times 10^{-14} \text{ y}^{-1}$)	$M^{0\nu}$	$T_{1/2}$ ($\times 10^{23} \text{ y}$)
^{48}Ca	0.265	6.52	2.37	28.5
^{76}Ge	0.271	0.64	4.60	76.9
^{82}Se	-0.366	2.83	4.22	20.8
^{96}Zr	2.580	5.97	5.65	5.48
^{100}Mo	1.879	4.68	5.08	8.64
^{116}Cd	1.365	5.08	4.72	9.24
^{124}Sn	-0.830	2.79	4.81	16.2
^{128}Te	-0.564	0.18	4.11	343.1
^{130}Te	-0.348	4.49	5.13	8.84
^{136}Xe	-1.027	4.68	4.20	12.7
^{150}Nd	-0.380	21.74	1.71	16.5

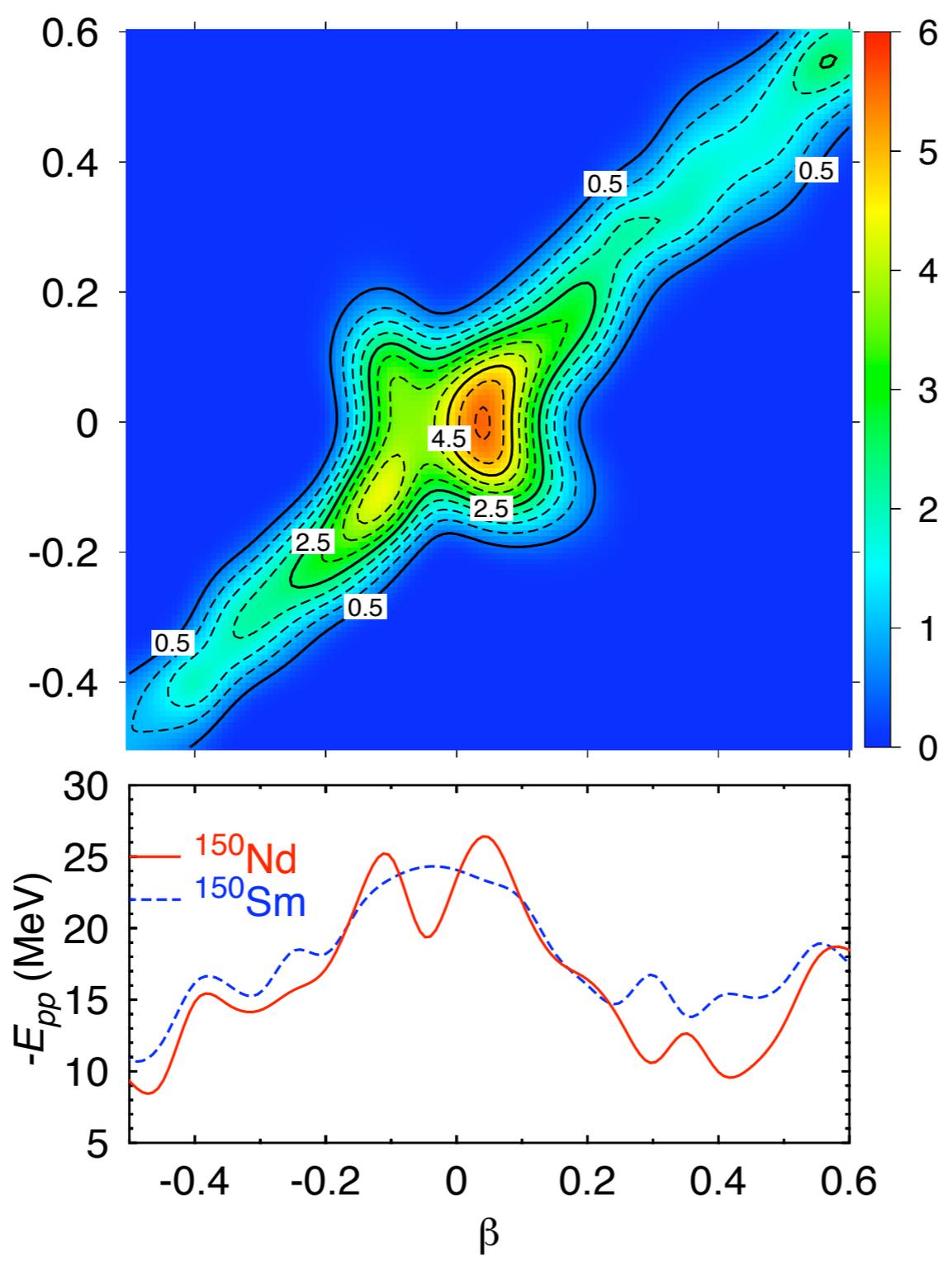
- Good agreement between experimental and theoretical Q -values

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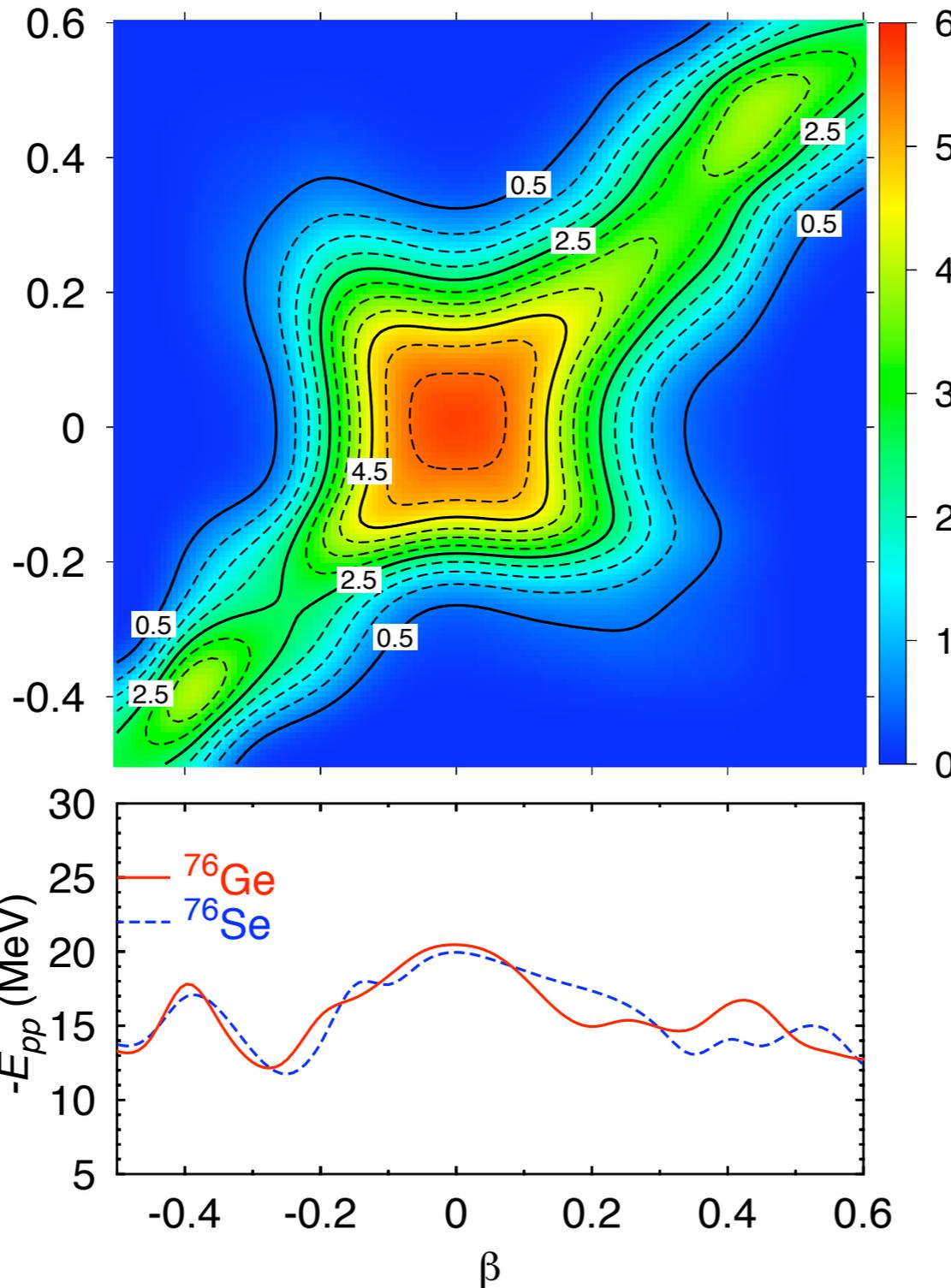
- The structure is related to the pairing energy (particle-particle) of the nuclei involved in the transition
- Maxima of the strength correspond to maxima in pairing energy
- In agreement with seniority arguments (increasing seniority decreases NME)
- Pairing energy and NME involve similar Wick theorem's contractions in this formalism

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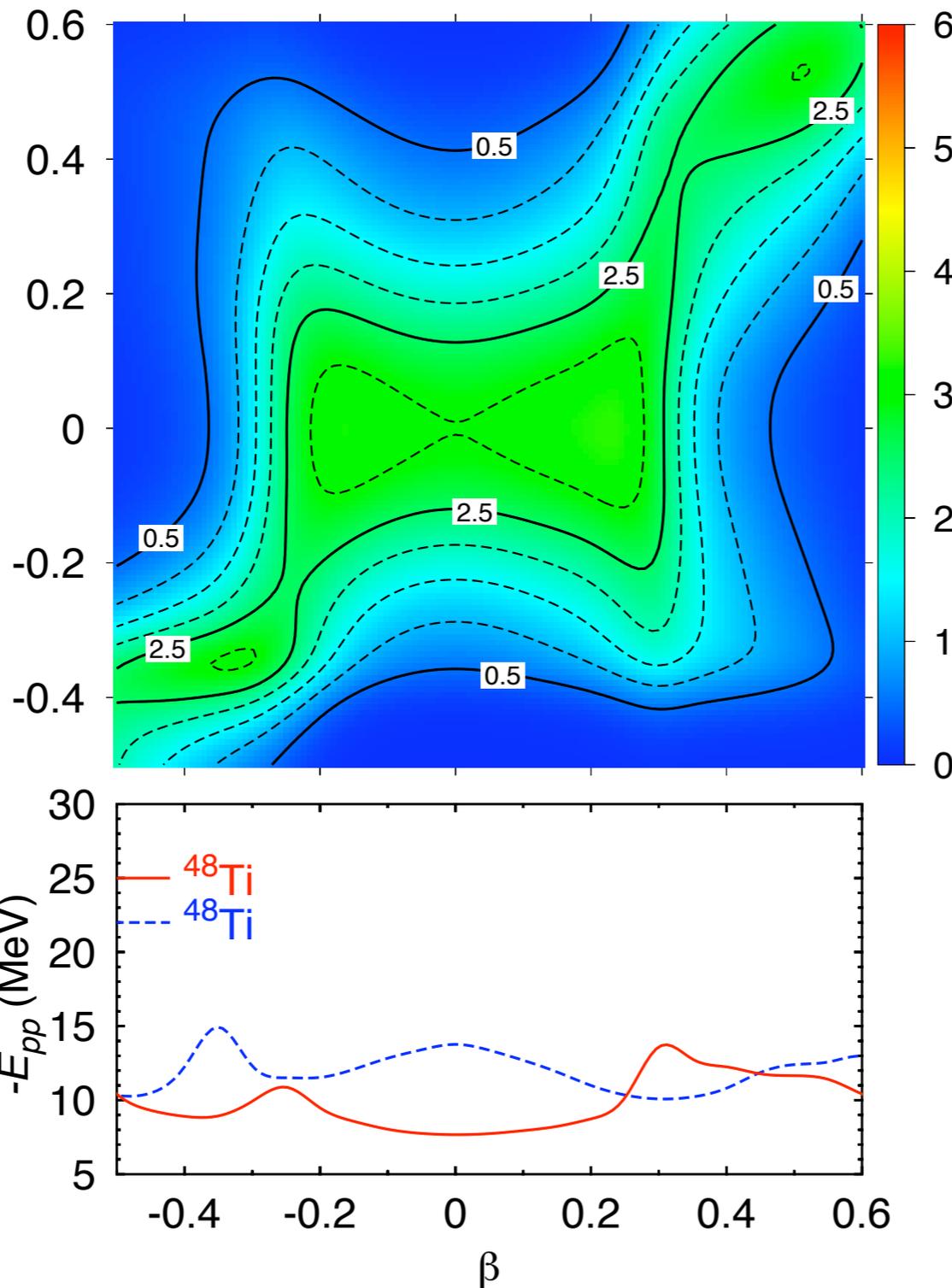
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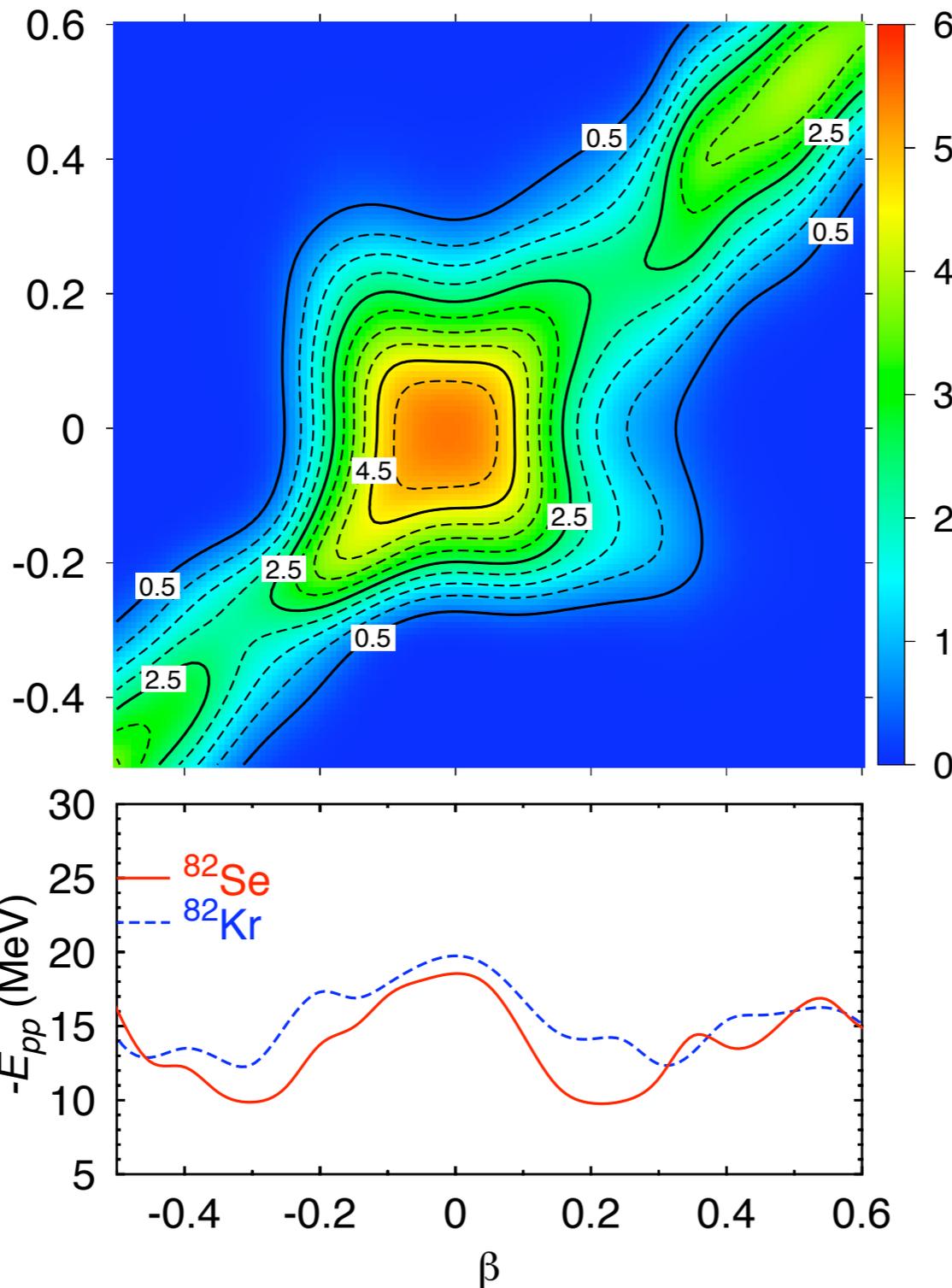
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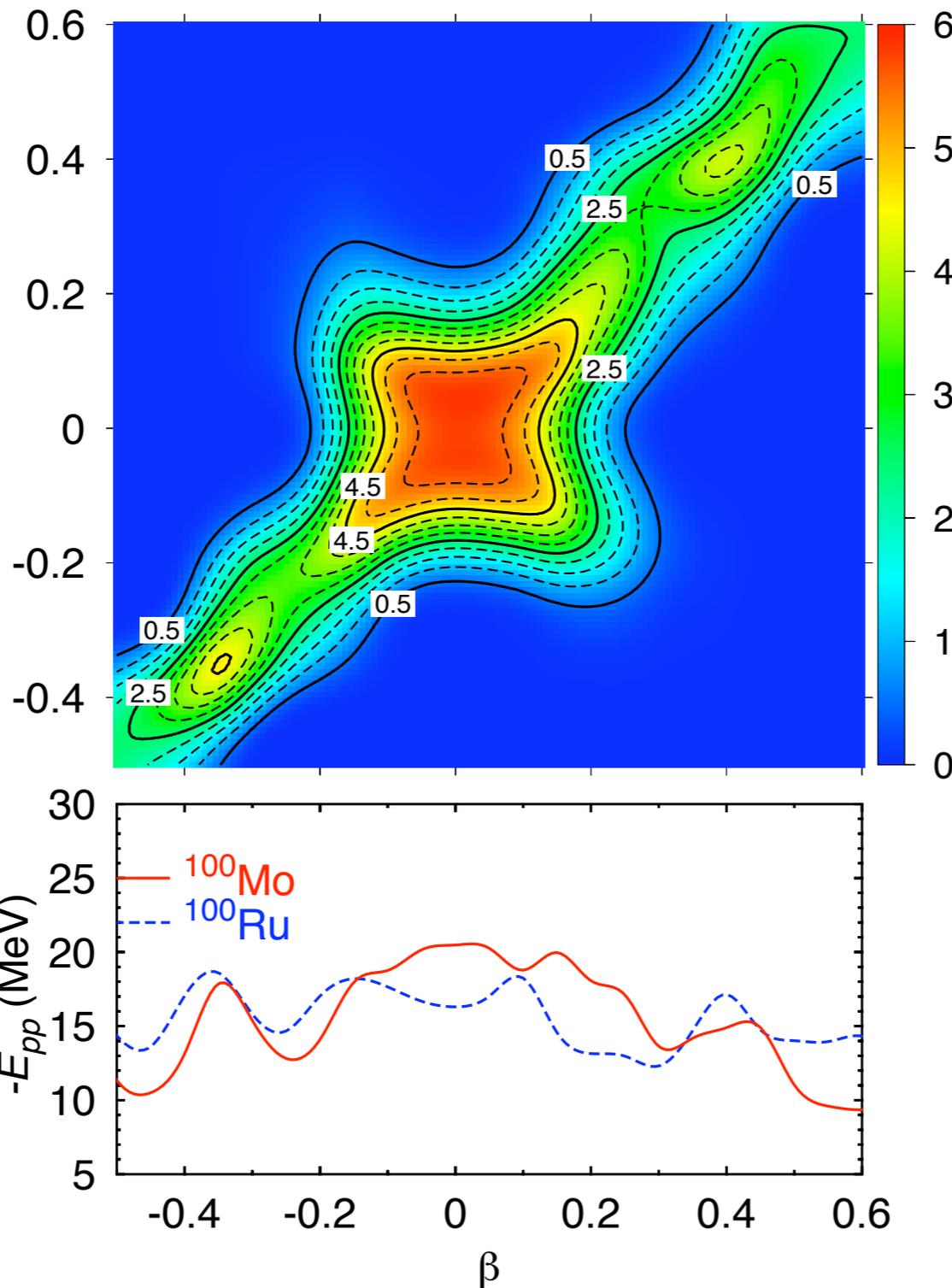
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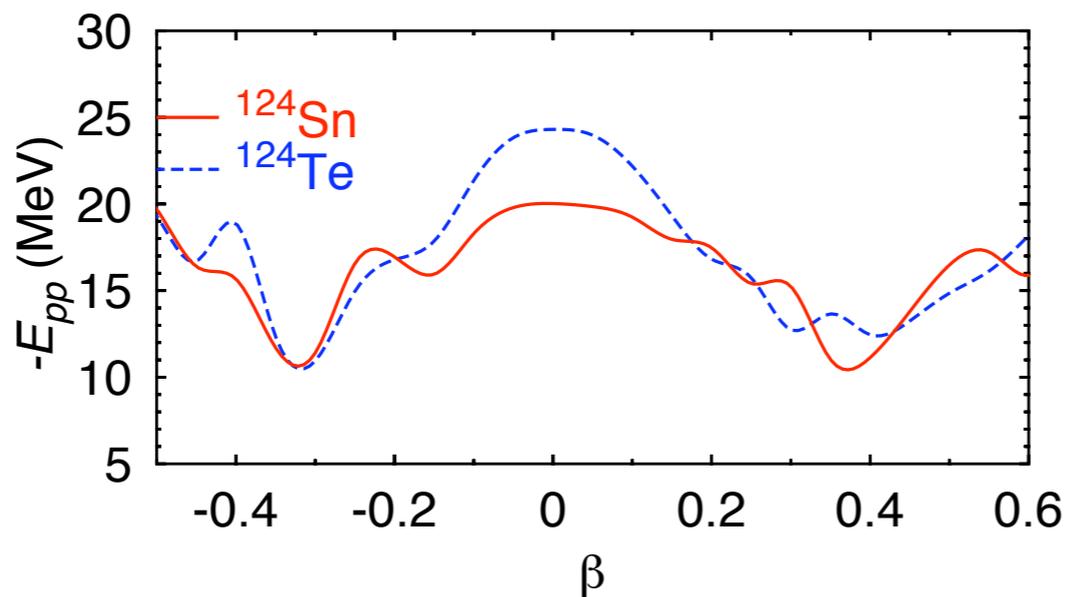
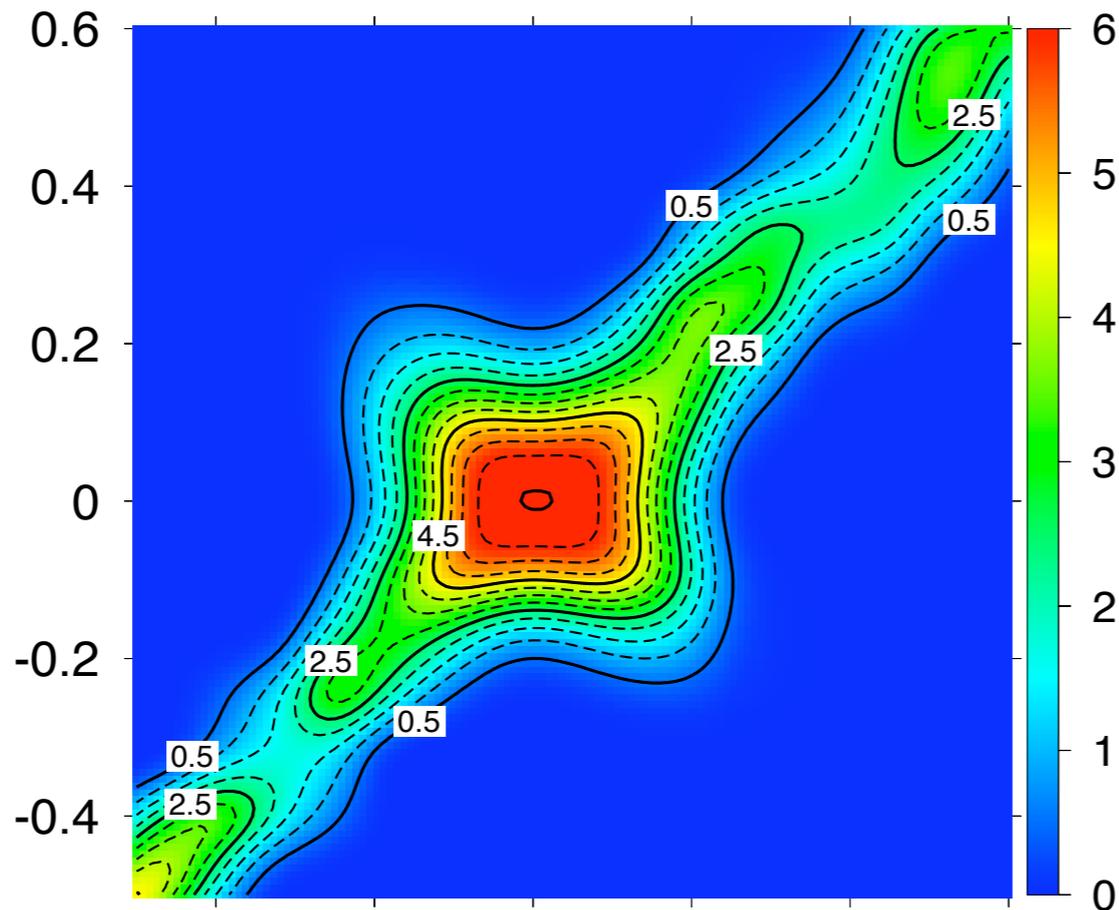
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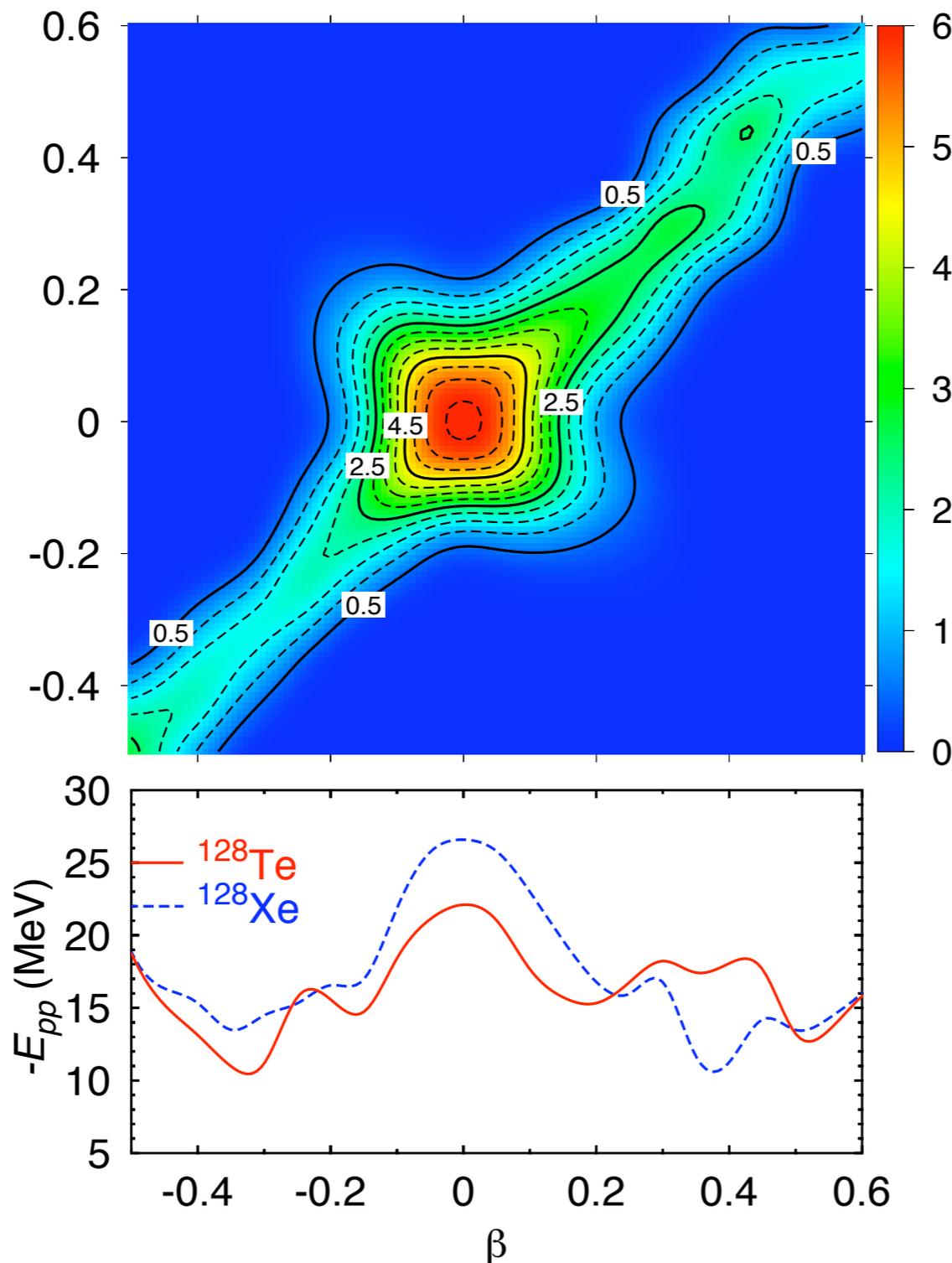
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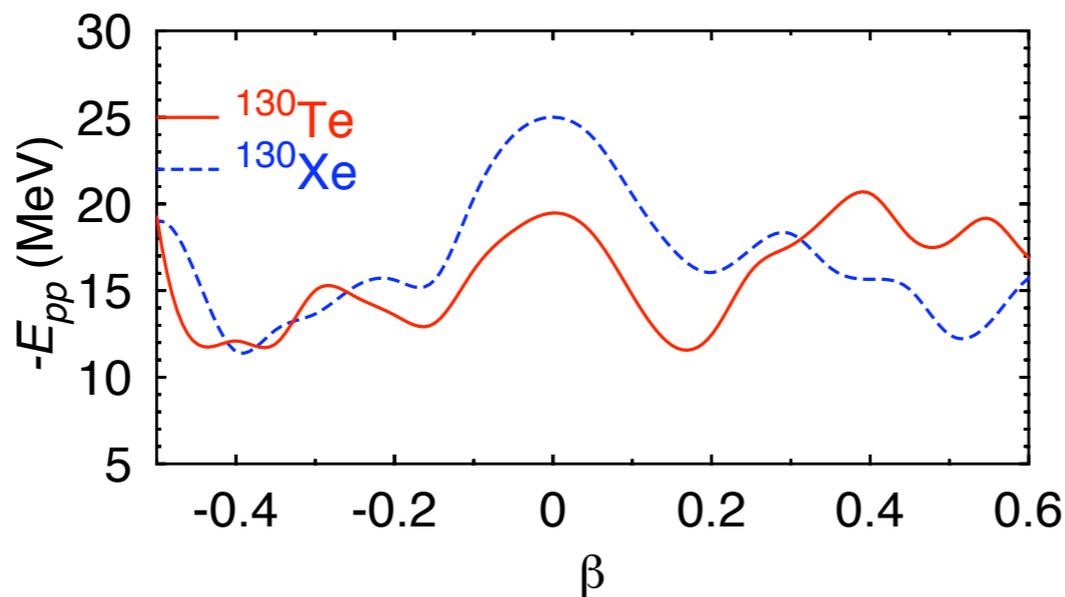
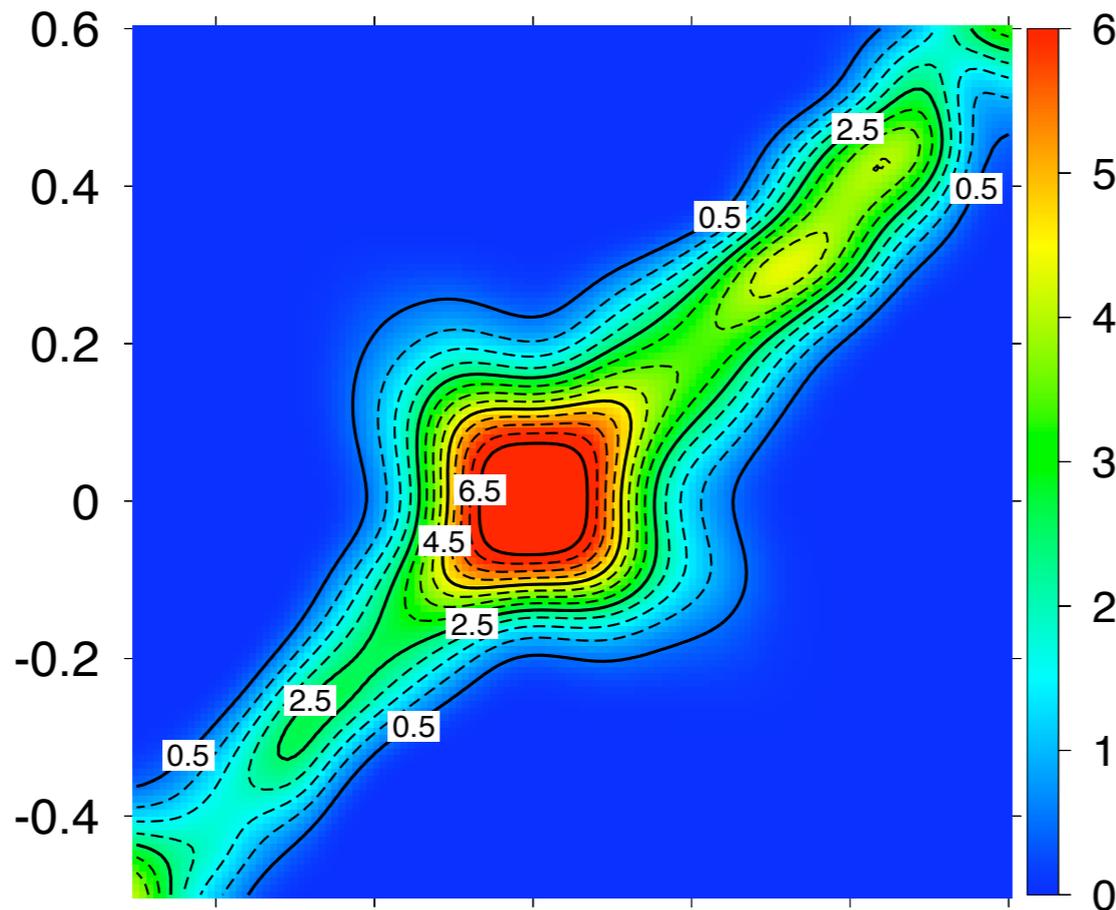
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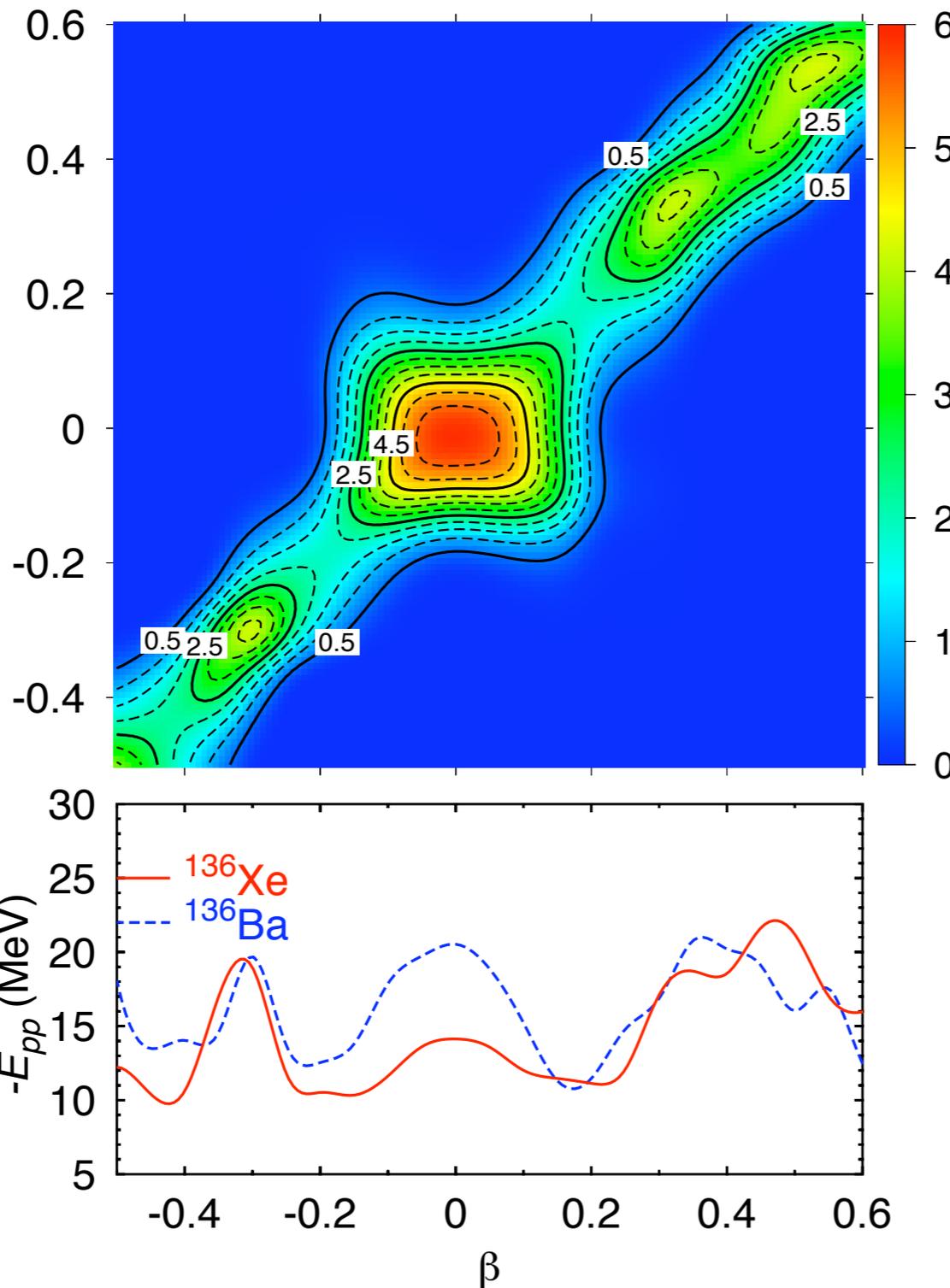
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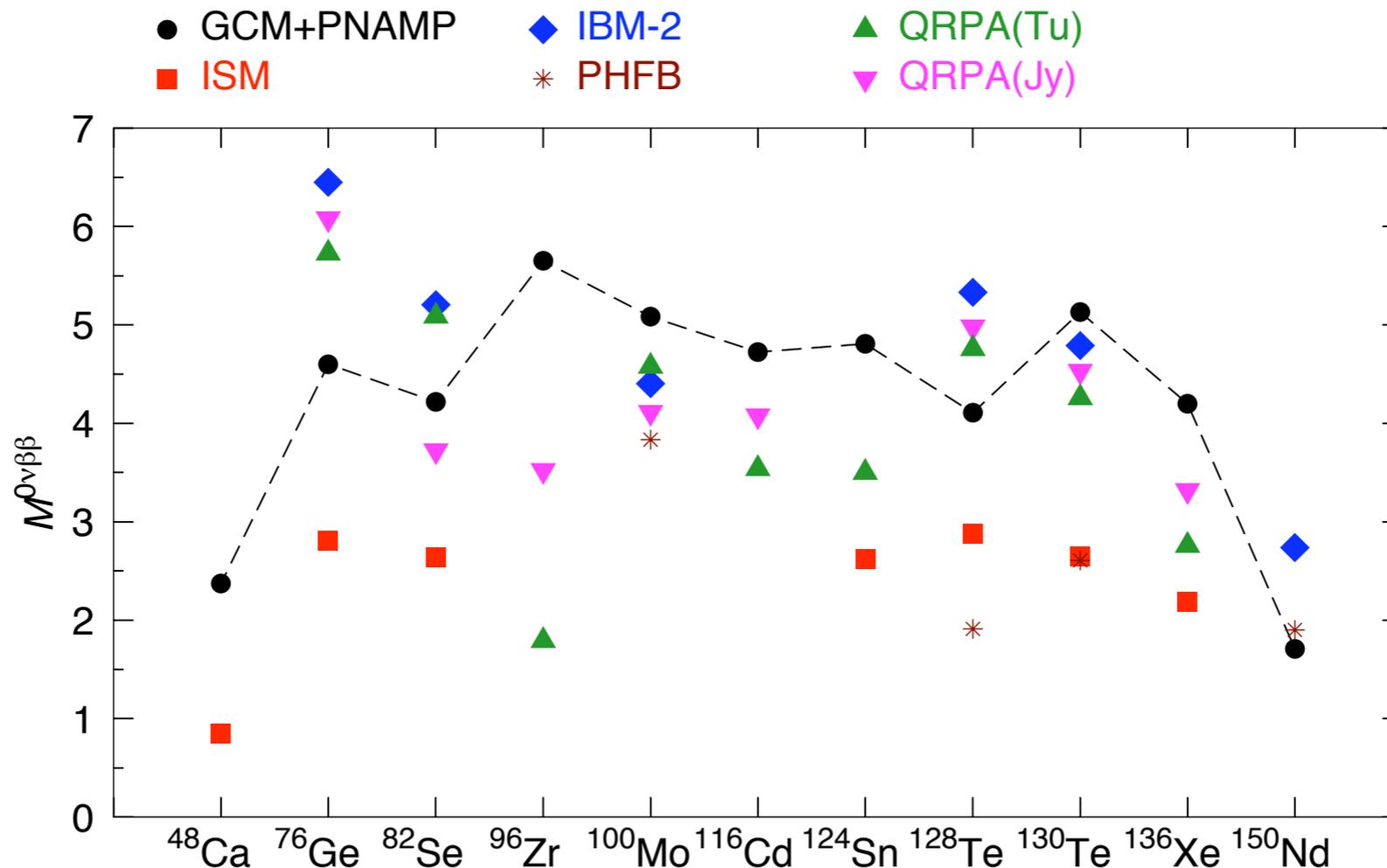
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QRPA (Jy): M. Kortelainen, J. Suhonen, PRC 75, 051303(R) (2007) and PRC 76, 024315 (2007)

QRPA(Tu): F. Simkovic et al., PRC 77, 045503 (2008)

ISM: J. Menendez et al., PRL 100, 52503 (2008)

IBM-2: J. Barea, F. Iachello, PRC 77, 045503 (2008)

PHFB: K. Chaturvedi et al. PRC 78, 054302 (2008)

- Higher values than the ones predicted by ISM calculations (larger valence space, lower seniority components).
- For $A=76, 82, 128, 150$ we predict smaller values than the ones given by QRPA and/or IBM while for $A=96, 100, 116, 124, 130, 136$ larger values are obtained.
- Consistent results with the rest of the models. Notice that we are using the same interaction for all the nuclei.
- Further studies are needed to understand what is missing in the different models.

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Summary and Conclusions (I)

- First calculations of neutrinoless double beta decay using GCM+PNAMP with the Gogny DIS interaction.
- First calculations with Particle Number Projection for different number of particles in bra and ket states.
- Formalism equivalent to a pairing term in the multi-reference EDF formalism.
- Explicit inclusion of deformation and shape mixing.
- Axial calculations with parity and time reversal conservation.

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Summary and Conclusions (II)

- Transition operators favor similar deformation for mother and granddaughter nuclei.
- Fermi contributions are smaller than Gamow-Teller.
- The structure can be understood studying the p-p channel of the nuclei involved in the transition.
- For $A = 76, 82, 96, 100, 116, 124, 128, 130, 136$ values between 4.1-5.6 are obtained
- For $A = 150$ the difference between deformation of the initial and final states lowers the value of the NME.
- For $A = 48$ the small pairing correlations in Ca and Ti produces a small NME.
- Results of the same order of magnitude than ISM, QRPA and IBM are obtained.
- Similarities and differences between different models will be investigated

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J. L. Egido (Universidad Autónoma de Madrid)

K. Langanke (GSI)

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