Scaling in the Neutrino Mass Matrix and the See-Saw Mechanism

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\[ m_\nu = m_L - m_D^T M_R^{-1} m_D \]
  [arXiv:0905.2126 [hep-ph]]

  [arXiv:0706.3801 [hep-ph]]

What is Scaling?

- Ansatz for the neutrino mass matrix $m_{\nu}$
- obtainable in many scenarios/models
- leads to inverted hierarchy
- alternative to $L_e - L_\mu - L_\tau$
- alternative to tri-bimaximal, trimaximal, $\mu-\tau$ symmetry etc.
- $|U_{e3}| = 0 \leftrightarrow$ stable under RG
- predictive for low energy phenomenology and for see-saw parameters
Ansatz for $m_\nu$

$m_\nu = \begin{pmatrix} A & B & B/c \\ B & D & D/c \\ B/c & D/c & D/c^2 \end{pmatrix}$

$\text{rank}(m_\nu) = 2 \Rightarrow \text{one eigenvalue zero}$

$m_\nu \begin{pmatrix} 0 \\ -1 \\ c \end{pmatrix} = 0$

so the predictions are \textbf{inverted hierarchy} ($m_3 = 0$) with

$U_{e3} = 0$ and $\tan^2 \theta_{23} = 1/c^2$

in general: $\theta_{23}$ non-maximal!
Predictivity

- 5 physical parameters for $\theta_{12}, \theta_{23}, \Delta m^{2}_\odot, \Delta m^{2}_A, |m_{ee}|$
- only one (Majorana) phase:

$$|m_{ee}| = \sqrt{\Delta m^2_A} \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \alpha}$$
Renormalization

take RG into effect by multiplying \((m_\nu)_{\alpha\beta}\) with

\[(1 + \delta_\alpha)(1 + \delta_\beta)\quad \text{with} \quad \delta_\alpha = C \frac{m_\nu^2}{16\pi v^2} \ln \frac{M_X}{M_Z}\]

with \(m_\nu\) obeying scaling:

\[
m_\nu = I_K \begin{pmatrix} A & B & B/c (1 + \delta_\tau) \\ B & D & D/c (1 + \delta_\tau) \\ B/c (1 + \delta_\tau) & D/c (1 + \delta_\tau) & D/c^2 (1 + \delta_\tau)^2 \end{pmatrix}
\]

\[\begin{pmatrix} \tilde{A} & \tilde{B} & \tilde{B}/\tilde{c} \\ \tilde{B} & \tilde{D} & \tilde{D}/\tilde{c} \\ \tilde{B}/\tilde{c} & \tilde{D}/\tilde{c} & \tilde{D}/\tilde{c}^2 \end{pmatrix}\quad \text{with} \quad \tilde{c} = c (1 + \delta_\tau)
\]

\[
\Rightarrow m_3 = U_{e3} = 0 \quad \text{not modified!}
\]

Comparison with $L_e - L_\mu - L_\tau$


$$m_\nu = \begin{pmatrix} 0 & A & B \\ A & 0 & 0 \\ B & 0 & 0 \end{pmatrix}$$

- also gives $m_3 = U_{e3} = 0$ and $\theta_{23} \neq \pi/4$
- but predicts $\Delta m^2_\odot = 0$ and $\theta_{12} = \pi/4$
- highly tuned perturbations required: $\Delta m^2_\odot / \Delta m^2_A \ll \pi/4 - \theta_{12}$
- perturbations must be of order 30-40 %
- effective mass small: $|m_{ee}| \simeq \cos^2 \theta_{13} \sqrt{\Delta m^2_A} \cos 2\theta_{12}$
Charged Lepton Corrections

\[ U = U^\dagger_\ell U_\nu \]

with \( U_\nu \) from scaling:

- if \( U_\ell \) is only 23-rotation: \( c \rightarrow \tilde{c} \equiv (c \cos \theta_{23} - \sin \theta_{23})/\left(\cos \theta_{23} + c \sin \theta_{23}\right) \)
- from \( \mu-\tau \) symmetry in general:

\[
|U_{e3}| \approx \frac{1}{\sqrt{2}} \left| \sin \theta_{12} - \sin \theta_{13} e^{i\phi_1} \right|
\]

\[\sin^2 \theta_{23} \approx \frac{1}{2} + \sin \theta_{23} \cos \phi_2 - \frac{1}{4} \left( \sin^2 \theta_{12} - \sin^2 \theta_{13} \right) + \frac{1}{2} \cos \phi_1 \sin \theta_{12} \sin \theta_{13} \]

rather tuned to have \( |U_{e3}| = 0 \) and \( \theta_{23} \neq \pi/4 \)
A Model

<table>
<thead>
<tr>
<th>Field</th>
<th>$D_4 \times Z_2$ quantum number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_e$</td>
<td>$1^+_1$</td>
</tr>
<tr>
<td>$e_R$, $N_e$, $\phi_1$</td>
<td>$1^-_1$</td>
</tr>
<tr>
<td>$N_\mu$, $\phi_2$</td>
<td>$1^+_2$</td>
</tr>
<tr>
<td>$N_\tau$</td>
<td>$1^-_2$</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>$1^-_4$</td>
</tr>
<tr>
<td>$\begin{pmatrix} L_\mu \ L_\tau \end{pmatrix}$, $\begin{pmatrix} \phi_4 \ \phi_5 \end{pmatrix}$</td>
<td>$2^+$</td>
</tr>
<tr>
<td>$\begin{pmatrix} \mu_R \ \tau_R \end{pmatrix}$</td>
<td>$2^-$</td>
</tr>
</tbody>
</table>

A Model

charged leptons and heavy neutrinos $M_R$ are diagonal

$$m_D = v \begin{pmatrix} a e^{i\varphi} & 0 & 0 \\ b & d & e \\ 0 & 0 & 0 \end{pmatrix}$$

and low energy mass matrix

$$m_\nu = -\frac{v^2}{M_2} \begin{pmatrix} \frac{M_2}{M_1} a^2 e^{2i\varphi} + b^2 & b d & b e \\ b d & d^2 & d e \\ b e & d e & e^2 \end{pmatrix}$$

gives scaling with $\tan^2 \theta_{23} = e^2/d^2$
A Model

Higgs Potential

\[ V = - \sum_{i=1}^{3} \mu_i^2 \phi_i^\dagger \phi_i - \mu_4 (\phi_4^\dagger \phi_4 + \phi_5^\dagger \phi_5) + \sum_{i=1}^{3} \lambda_i (\phi_i^\dagger \phi_i)^2 + \lambda_4 (\phi_4^\dagger \phi_4 + \phi_5^\dagger \phi_5)^2 + \lambda_{12} (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_{13} (\phi_1^\dagger \phi_1)(\phi_3^\dagger \phi_3) + \lambda_{23} (\phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3) + \sum_{i=1}^{3} \kappa_i (\phi_i^\dagger \phi_i)(\phi_4^\dagger \phi_4 + \phi_5^\dagger \phi_5) + \alpha_1 [(\phi_1^\dagger \phi_2)^2 + h.c.] + \alpha_2 |\phi_1^\dagger \phi_2|^2 + \alpha_3 (\phi_2^\dagger \phi_3)^2 + h.c.] + \alpha_4 |\phi_2^\dagger \phi_3|^2 + \alpha_5 (\phi_4^\dagger \phi_5 + \phi_5^\dagger \phi_4)^2 + \alpha_6 (\phi_4^\dagger \phi_5 - \phi_5^\dagger \phi_4)^2 + \alpha_7 (\phi_4^\dagger \phi_4 - \phi_5^\dagger \phi_5)^2 + \alpha_8 (\phi_4^\dagger \phi_4 - \phi_5^\dagger \phi_5)(\phi_1^\dagger \phi_3 + h.c.) + \alpha_9 (\phi_2^\dagger \phi_4)^2 + (\phi_2^\dagger \phi_5)^2 + h.c.] + \alpha_{10} (|\phi_2^\dagger \phi_4|^2 + |\phi_2^\dagger \phi_5|^2) + \alpha_{11} (\phi_1^\dagger \phi_4)^2 + (\phi_1^\dagger \phi_5)^2 + h.c.] + \alpha_{12} (|\phi_1^\dagger \phi_4|^2 + |\phi_1^\dagger \phi_5|^2) + \alpha_{13} (\phi_3^\dagger \phi_4)^2 + (\phi_3^\dagger \phi_5)^2 + h.c.] + \alpha_{14} (|\phi_3^\dagger \phi_4|^2 + |\phi_3^\dagger \phi_5|^2) + \alpha_{15} (\phi_1^\dagger \phi_4)(\phi_3^\dagger \phi_4) - (\phi_1^\dagger \phi_5)(\phi_3^\dagger \phi_5) + h.c.] + \alpha_{16} (\phi_1^\dagger \phi_4)(\phi_4^\dagger \phi_3) - (\phi_1^\dagger \phi_5)(\phi_5^\dagger \phi_3) + h.c.] + \alpha_{17} (|\phi_1^\dagger \phi_4|^2 + |\phi_1^\dagger \phi_5|^2) + h.c.] + \alpha_{18} (|\phi_3^\dagger \phi_4|^2 + |\phi_3^\dagger \phi_5|^2) + h.c.\]
Higgs Potential

- 5 Higgs doublets
  - 5 real scalars
  - 4 pseudoscalars
  - 4 charged scalars
- potential can be minimized by choosing

\[
\langle \Phi_{1,2,4,5} \rangle = \begin{pmatrix} 0 \\ v/\sqrt{5} \end{pmatrix} \quad \text{and} \quad \langle \Phi_3 \rangle = \begin{pmatrix} 0 \\ -v/\sqrt{5} \end{pmatrix}
\]

with masses of scalars above experimental limits
<table>
<thead>
<tr>
<th>$m_D$</th>
<th>Leptogenesis</th>
<th>$\tan^2 \theta_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{pmatrix} 0 &amp; 0 &amp; 0 \ a_2 &amp; 0 &amp; 0 \ a_3 e^{i\alpha_3} &amp; b_3 &amp; c_3 \end{pmatrix}$</td>
<td>$\mathcal{I}_{23}^e = a_2^2 a_3^2 \sin 2\alpha_3$</td>
<td>$\frac{c_3^2}{b_3^2}$</td>
</tr>
<tr>
<td>$\begin{pmatrix} 0 &amp; 0 &amp; 0 \ a_2 &amp; b_2 &amp; c_2 \ a_3 e^{i\alpha_3} &amp; 0 &amp; 0 \end{pmatrix}$</td>
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<td>$\mathcal{I}_{12}^e = a_1^2 a_2^2 \sin 2\alpha_2$</td>
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Scaling and See-Saw

\[ m_\nu = -m_D^T M_R^{-1} m_D \]

usually: \[ m_D = i \sqrt{M_R} R \sqrt{m_\nu^{\text{diag}}} U^\dagger \]

infinite number of \( m_D \) are allowed and hence no predictions for LFV or leptogenesis possible

In contrast, scaling constraints \( m_D \) to have the form:

\[
\begin{pmatrix}
a_1 & b & b/c \\
 a_2 & d & d/c \\
 a_3 & e & e/c \\
\end{pmatrix}
\]

no matter what \( M_R \) is!!
Proof

\[ m_\nu |\psi\rangle = 0 \text{ where } |\psi\rangle = \begin{pmatrix} 0 \\ -1 \\ c \end{pmatrix} \]

\[ \det(m_\nu) = 0 \text{ and therefore } (M_R \text{ can’t be singular}) \]

\[ \det(m_D) = 0 \implies m_D |\chi\rangle = 0 \]

for one of its eigenvectors \( |\chi\rangle \). With \( m_\nu = -m_D^T M_R^{-1} m_D \) it follows

\[ m_\nu |\chi\rangle = 0 \]

Hence, \( |\chi\rangle \) is proportional to \( |\psi\rangle \), which means

\[ m_D |\psi\rangle = 0 \]

solution for \( m_D \) is as given above
Scaling and $Z_2$

Suppose $Z_2$ generated by $\nu_L \rightarrow S(\theta) \nu_L$

\[
S(\theta) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos 2\theta & \sin 2\theta \\
0 & \sin 2\theta & -\cos 2\theta
\end{pmatrix}
\]

acts on low energy mass matrix from

\[
\mathcal{L} = \frac{1}{2} \overline{\nu_L^c} m_\nu \nu_L
\]

this implies $\theta_{23} = \theta$ and $U_{e3} = 0$ “generalized $\mu-\tau$ symmetry”

(Grimus et al., Nucl. Phys. B 713, 152 (2005))

$\Rightarrow$ scaling can be special case of this $S(\theta)$ when $\cos 2\theta = \frac{c^2 - 1}{1 + c^2}$
Scaling and $Z_2$

Now assume see-saw

$$\mathcal{L} = \frac{1}{2} \overline{N_R} M_R N_R^c + \overline{N_R} m_D \nu_L$$

and $Z_2$ with $\nu_L \to S(\theta) \nu_L$ implies that $m_D S(\theta) = m_D$ and thus

$$m_D = \begin{pmatrix} a_1 & b & b/c \\ a_2 & d & d/c \\ a_3 & e & e/c \end{pmatrix}$$

$\Rightarrow$ scaling in $m_\nu$!!
Scaling and Lepton Flavor Violation

SUSY see-saw

\[ \text{BR}(\ell_i \rightarrow \ell_j \gamma) \propto \left| (\tilde{m}^\dagger_D L \tilde{m}_D)_{ij} \right|^2 \text{ with } L_{ij} = \delta_{ij} \log M_i/M_X \]

Note: basis in which \( M_R \) diagonal: \( \tilde{m}_D = V_R^T m_D \)

If \( m_D \) obeys scaling, then

\[
\frac{\left| (\tilde{m}^\dagger_D L \tilde{m}_D)_{12} \right|^2}{\left| (\tilde{m}^\dagger_D L \tilde{m}_D)_{13} \right|^2} = c^2 = \cot^2 \theta_{23}
\]

and \( \tau \rightarrow e\gamma \) too rare to be observable
Lepton Flavor Violation

Note: if $\mu-\tau$ symmetric see-saw

$$m_D = \begin{pmatrix}
a_1 & d_1 & d_1 \\
a_2 & d_2 & d_3 \\
a_2 & d_3 & d_2
\end{pmatrix} \quad \text{and} \quad M_R = \begin{pmatrix}
W & X & X \\
X & Y & Z \\
X & Y & Y
\end{pmatrix}$$

then LFV prediction is

$$\left| (\tilde{m}_D^\dagger \tilde{m}_D)_{12} \right|^2 = 1$$

and logarithmic corrections due to $L$
Scaling and Leptogenesis

if $\mu - \tau$ symmetric see-saw

$$m_D = \begin{pmatrix} a_1 & d_1 & d_1 \\ a_2 & d_2 & d_3 \\ a_2 & d_3 & d_2 \end{pmatrix} \quad \text{and} \quad M_R = \begin{pmatrix} W & X & X \\ X & Y & Z \\ X & Y & Y \end{pmatrix}$$

then

- 2 RH neutrinos: no leptogenesis, neither flavored nor unflavored
- 3 RH neutrinos: unflavored $Y_B \propto \Delta m^2_{\odot}$

Scaling and Leptogenesis

2 RH neutrinos and scaling

\[
m_D = \begin{pmatrix} A_1 & B & B/c \\ A_2 & D & D/c \end{pmatrix}
\]

and

\[
M_R = V_R^* D_R V_R^\dagger
\]

gives decay asymmetry

\[
\varepsilon_1 = -\frac{3}{16\pi v^2} \frac{M_1}{\tilde{m}_1} r \frac{\Delta m^2_{\odot}}{|1 + r e^{i\rho}|^2} \sin \rho
\]

just as for \(\mu-\tau\) symmetry and 3 RH neutrinos!

wash-out parameter:

\[
\tilde{m}_1 = \frac{m_1 + r m_2}{|1 + r e^{i\rho}|} = \mathcal{O} \left( \sqrt{\Delta m^2_{\Lambda}} \right) \Rightarrow \text{strong wash-out}
\]

(flavored leptogenesis: \(\varepsilon_i^\alpha\) determined by \(\Delta m^2_{\Lambda}\))
Scaling and Leptogenesis

add additional $Z_2$

\[ N_R \rightarrow S(\theta) N_R \]

which implies

\[ S(\theta) m_D = m_D \quad \text{and} \quad S(\theta) M_R S(\theta) = M_R \]

This $Z_{2L} \times Z_{2R}$ gives

\[
m_D = \begin{pmatrix}
A_1 & B & B s_{23}/c_{23} \\
A_2 & c_{23} & D s_{23} \\
A_2 s_{23} & D s_{23} & D s_{23}^2/c_{23}
\end{pmatrix}
\quad \text{and} \quad
M_R = \begin{pmatrix}
A & B & B/c \\
B & F(c - 1/c) + G & F \\
B/c & F & G
\end{pmatrix}
\]

One heavy RH neutrino decouples and formulae are as above
Scaling and Non-Standard Neutrino Physics

- **Dirac neutrinos:**
  \[ m_\nu = \begin{pmatrix} a_1 & b & b/c \\ a_2 & d & d/c \\ a_3 & e & e/c \end{pmatrix} \]

- **sterile neutrinos:**
  \[ m_\nu = \begin{pmatrix} a_1 & b_1 & b_1/c & d_1 & e_1 \\ b_1 & b_2 & b_2/c & d_2 & e_2 \\ b_1/c & b_2/c & b_2/c^2 & d_2/c & e_2/c \\ d_1 & d_2 & d_2/c & d_2/c^2 & e_2/c^2 \\ e_1 & e_2 & e_2/c & e_2/c^2 & e_5 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ -1/\sqrt{1+c^2} \\ c/\sqrt{1+c^2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} U_{e3} \\ U_{\mu3} \\ U_{\tau3} \\ U_{s13} \\ U_{s23} \end{pmatrix} \]
Summary

\[ m_\nu = \begin{pmatrix} A & B & B/c \\ B & D & D/c \\ B/c & D/c & D/c^2 \end{pmatrix} \]

- highly predictive and reconstructible scenario
- inverted hierarchy and \( U_{e3} = 0 \), stable under RG
- obtainable in flavor models, see-saw texture analyses, …
- determines \( m_D \) irrespective of \( M_R \)
- interesting differences to \( \mu-\tau \) symmetry
Thanks!!
Scaling and Leptogenesis

\[ \tilde{m}_D \tilde{m}_D^\dagger = \begin{pmatrix} ZZ^\dagger & 0 \\ 0 & 0 \end{pmatrix} \]

and

\[ m_1 e^{i\alpha_1} = -\frac{Z_{11}^2}{M_1} \left( 1 + \frac{Z_{21}^2}{Z_{11}^2} \frac{M_1}{M_2} \right) \equiv -\frac{Z_{11}^2}{M_1} (1 + r e^{i\rho}) \]

\[ m_2 e^{i\alpha_2} = -\frac{Z_{22}^2}{M_2} \left( 1 + \frac{Z_{21}^2}{Z_{11}^2} \frac{M_1}{M_2} \right) \equiv -\frac{Z_{22}^2}{M_2} (1 + r e^{i\rho}) , \]

\[ \frac{Z_{12}}{Z_{22}} = -\frac{Z_{21}}{Z_{11}} \frac{M_1}{M_2} = -\sqrt{r} \sqrt{\frac{M_1}{M_2}} e^{i\rho/2} \]
Scaling and Leptogenesis

The individual flavored decay asymmetries read

\[ \varepsilon_1^\mu = c_{23}^2 (\varepsilon_1 - \varepsilon_1^e) \]
\[ \varepsilon_1^\tau = s_{23}^2 (\varepsilon_1 - \varepsilon_1^e) \]

and

\[ \varepsilon_1^e = -\frac{3 M_1}{16\pi v^2 \tilde{m}_1 |1 + r e^{i\rho}|^2} \left( r \left( m_2^2 s_{12}^2 - m_1^2 c_{12}^2 \right) \sin \rho + c_{12} s_{12} \sqrt{m_1 m_2} r \left( (m_1 - m_2 r) \sin(\alpha_1 - \alpha_2 - 4\beta - \rho)/2 + (m_1 r - m_2) \sin(\alpha_1 - \alpha_2 - 4\beta + \rho)/2 \right) \right) \]
Comparison with $\mu-\tau$ Symmetry

\[
m_{\nu}^{\mu-\tau} = \begin{pmatrix} A & B & B \\ B & D & E \\ B & E & D \end{pmatrix}
\]

obtained from $Z_2$ invariance $\nu_L \rightarrow S_{\mu\tau} \nu_L$ leading to $S_{\mu\tau}^{-1} m_{\nu} S_{\mu\tau} = m_{\nu}$ with

\[
S_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}
\]

(cf. with scaling and $c = 1$)

Broken $\mu-\tau$ symmetry gives in general $O(|\theta_{23} - \pi/4|) = O(|U_{e3}|)$