

# Quark-lepton complementarity in an $S_4$ Pati Salam inspired scenario

Based on work in progress with Federica Bazzocchi and Luca Merlo, hep-ph/0910.xxxx



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# Outline

- 1 Introduction: family physics
- 2 TriBiMaximal versus BiMaximal mixing
- 3 The model
- 4 Results and Conclusions

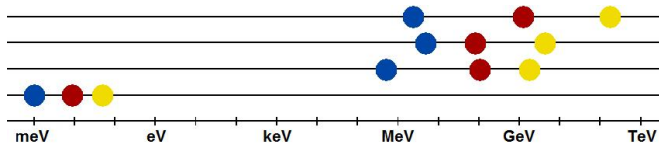
# Family physics

- Family physics aims to explain the apparent structure in the fermion mass sector
  - Hierarchical masses
  - Quarks: moderate Cabibbo-mixing
  - Leptons: strong mixing



# Family physics

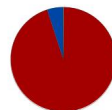
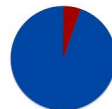
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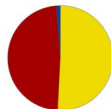
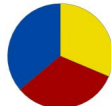
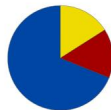
$$|(V_{CKM})_{ij}| = \begin{pmatrix} 0.97 & 0.23 & 0.0039 \\ 0.23 & 1.0 & 0.041 \\ 0.0081 & 0.038 & 1? \end{pmatrix}$$



## Family physics

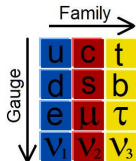
- Family physics aims to explain the apparent structure in the fermion mass sector
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$$|(U_{PMNS})_{ij}| = \begin{pmatrix} 0.81 & 0.56 & <0.22 \\ 0.39 & 0.59 & 0.68 \\ 0.38 & 0.55 & 0.70 \end{pmatrix}$$



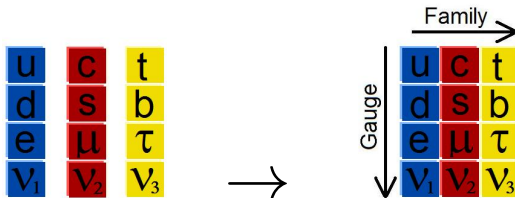
## Family physics

- We charge the families under a symmetry group.
- To make the Lagrangian invariant, we need to add Higgs-like fields 'flavons'.
- Structure in the VEVs of the flavons leads to structure in the fermion masses. [A very good introduction: G. Altarelli, Models of neutrino masses and mixings; hep-ph/0611117](#)



# Family Physics and Grand Unification

- Family symmetries unify the three families.
- Grand Unified Theories unify
  - The gauge couplings
  - The SM particles in fewer representations



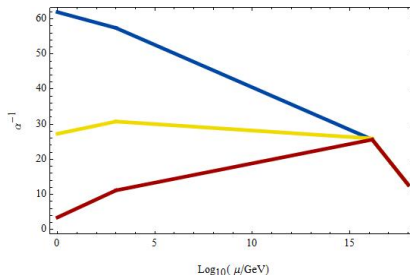


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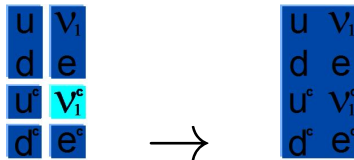
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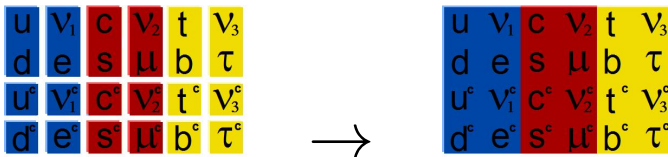


$$\left[ \begin{pmatrix} u_R & u_G & u_B \\ d_R & d_G & d_B \end{pmatrix} + (u_R^c \quad u_G^c \quad u_B^c) + (d_R^c \quad d_G^c \quad d_B^c) + \begin{pmatrix} \nu \\ e \end{pmatrix} + (e^c) + (\nu^c) \right]_{\text{SM}} \rightarrow$$

$$\left[ (u_R \quad u_G \quad u_B \quad d_R \quad d_G \quad d_B \quad u_R^c \quad u_G^c \quad u_B^c \quad d_R^c \quad d_G^c \quad d_B^c \quad \nu \quad e \quad e^c \quad \nu^c) \right]_{\text{SO}(10)}$$

# Family Physics and Grand Unification

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- Grand Unified Theories unify
  - The gauge couplings
  - The SM particles in fewer representations
- We like to build models that combine Family Physics and Grand Unification.



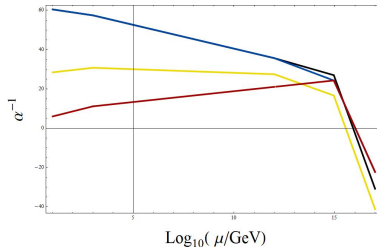
# Pati-Salam



- We use Pati and Salam's  $SU(4)_c \times SU(2)_L \times SU(2)_R$ 
  - Gauge: not quite unifying
  - All SM particles in two representations.
  - The type II seesaw can be dominant



This prevents unwanted correlations between quarks and neutrinos



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$$\left[ \begin{pmatrix} u_R & u_G & u_B \\ u_R & d_G & d_B \end{pmatrix} \quad \begin{pmatrix} \nu \\ e \end{pmatrix} \right] \quad \& \quad \left[ \begin{pmatrix} u_R^c & u_G^c & u_B^c \\ d_R^c & d_G^c & d_B^c \end{pmatrix} \quad \begin{pmatrix} \nu^c \\ e^c \end{pmatrix} \right]$$

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$$m_\nu^I = -m_{\text{Dir}}^T \frac{1}{M_{RR}} m_{\text{Dir}} \propto M_R^{-1}$$

$$m_\nu^{II} = M_{LL} \propto M_{3L}^{-1}$$

## TriBiMaximal mixing

- Many models use tribimaximal mixing to model the neutrino mixing  
 P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B 530 (2002) 167

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12} = 35^\circ, \quad \theta_{13} = 0^\circ, \quad \theta_{23} = 45^\circ.$$

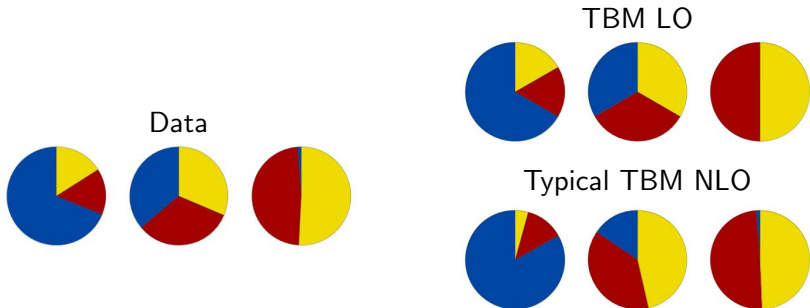
- This describes the data really well ...





- TBM mixing might describe the neutrino data a bit too well.
  - Most models describe the quark mixing poorly at Leading Order\*.
  - The Cabibbo angle arises only via NLO effects.
  - Often these NLO effects influence the PMNS as well.

\* LO is defined as having the lowest number of flavons possible; NLO has one extra flavon



$$\delta\theta_{12} = -0.9\theta_C, \quad \delta\theta_{13} = 0.5\theta_C, \text{ so} \\
\theta_{12} = 23^\circ, \quad \theta_{13} = 7^\circ, \quad \theta_{23} = 45^\circ \text{ while } \delta = 0$$

## BiMaximal mixing

- An alternative is BiMaximal mixing
- At leading order, two angles are maximal

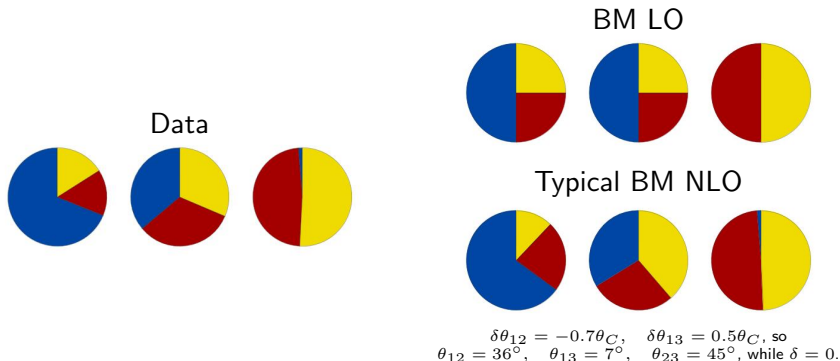
$$U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- And we can write  $U_{\text{BM}} =$  as the product of two maximal rotations

$$U_{\text{BM}} = R_{23}\left(-\frac{\pi}{4}\right) \times R_{12}\left(\frac{\pi}{4}\right)$$

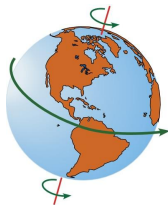
- The leading order does not explain the data well.
- Via quark lepton complementarity at NLO it might.

G. Altarelli, F. Feruglio and L. Merlo, JHEP 0905 (2009) 020



## Rotations at leading order

- BiMaximal mixing models have at leading order
  - $V_{CKM} = V_u^\dagger V_d = \mathbb{1}$ ,
  - $U_{PMNS} = V_e^\dagger V_\nu = R_{23}(-\frac{\pi}{4})R_{12}(\frac{\pi}{4})$
- This can be achieved if
  - $M_\nu^{II}$  is diagonalized by  $V = R_{12}(\frac{\pi}{4})$
  - And  $M_e M_e^\dagger$ ,  $M_d M_d^\dagger$  and  $M_u M_u^\dagger$  by  $V_{e,d,u} = R_{23}(\frac{\pi}{4})$



# Matter content of the model

- How to get such mass matrices?  
→ Model details
  - Our family group is the discrete group  $S_4$
  - We put the lefthanded PS multiplets  $F_L$  in a  $3_a$  of  $S_4$
  - We put the righthanded PS multiplets  $F_{1,2,3}^c$  in  $1_b$ ,  $1_b$  and  $1_a$  of  $S_4$
  - We introduce two flavons ( $\sigma$ ,  $\chi$ ) in the neutrino sector
  - And two ( $\varphi$ ,  $\varphi'$ ) for the charged particles

u	$\nu_1$	c	$\nu_2$	t	$\nu_3$
d	e	s	$\mu$	b	$\tau$

$u^c$	$\nu_1^c$	$c^c$	$\nu_2^c$	$t^c$	$\nu_3^c$
$d^c$	$e^c$	$s^c$	$\mu^c$	$b^c$	$\tau^c$

# Neutrinos

- In the neutrino sector we introduce a singlet  $\sigma$  and a triplet  $\chi$

$$\chi \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- This gives the type II seesaw contributions

$$F_L F_L \rightarrow \begin{pmatrix} u_1 & 0 & 0 \\ 0 & u_1 & 0 \\ 0 & 0 & u_1 \end{pmatrix}, \quad F_L F_L \chi \rightarrow \begin{pmatrix} 0 & u_2 & 0 \\ u_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- That is indeed diagonalized by  $V = R_{12}(\frac{\pi}{4})$

## Charged particles

- For the charged particles we introduce a  $3_a$  triplet  $\varphi$  and a  $3_b$  triplet  $\varphi'$

$$\varphi \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \varphi' \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

- This leads to masses

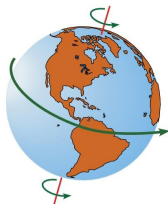
$$F_L F_3^c \varphi \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y \\ 0 & 0 & y \end{pmatrix}, \quad F_L F_2^c \varphi' \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & x & 0 \\ 0 & -x & 0 \end{pmatrix}$$

- The square is indeed diagonalized by  $V = R_{23}(\frac{\pi}{4})$

$$MM^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y^2 + x^2 & y^2 - x^2 \\ 0 & y^2 - x^2 & y^2 + x^2 \end{pmatrix}$$

## Rotations at leading order

- We indeed have at Leading order.
  - $M_\nu^{II}$  is diagonalized by  $V = R_{12}(\frac{\pi}{4})$
  - And  $M_e M_e^\dagger$ ,  $M_d M_d^\dagger$  and  $M_u M_u^\dagger$  by  $V_{e,d,u} = R_{23}(\frac{\pi}{4})$
- So we indeed reproduce BiMaximal mixing.
  - $V_{CKM} = V_u^\dagger V_d = \mathbb{1}$ ,
  - $U_{PMNS} = V_e^\dagger V_\nu = R_{23}(-\frac{\pi}{4}) R_{12}(\frac{\pi}{4})$





## Next-to-Leading Order



- At NLO  $M_\nu^{II}$  is still diagonalized by  $V = R_{12}(\frac{\pi}{4})$

$$M_\nu^{II} = \begin{pmatrix} u_1 & u_2\lambda & 0 \\ u_2\lambda & u_1 & 0 \\ 0 & 0 & u_1 + u_3\lambda^2 \end{pmatrix} \quad \lambda \approx 0.2$$

- The charged particle mass matrices now read

$$M_x = \begin{pmatrix} 0 & \tilde{x}\lambda & \tilde{y}\lambda \\ 0 & x & y \\ 0 & -x & y \end{pmatrix}$$

## Next-to-Leading Order

- $MM^\dagger$ -matrices are diagonalized by

$$V_x = R_{23}\left(\frac{\pi}{4}\right) \times R_{13}(f\lambda) \times R_{12}(g_x\lambda),$$

where  $f = f_u = f_d = f_e$  and  $g_u \neq g_d = g_e$  are  $\mathcal{O}(1)$ .

- The CKM matrix is

$$V_{CKM} = V_u^\dagger V_d = R_{12}((g_d - g_u)\lambda)$$

- And the PMNS matrix is

$$V_{PMNS} = V_e^\dagger V_\nu = R_{23}\left(-\frac{\pi}{4}\right) \times R_{13}(\tilde{f}\lambda) \times R_{12}\left(\frac{\pi}{4} - \tilde{g}_e\lambda\right)$$

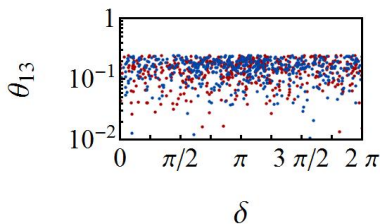
## Some Results

- The angle  $\theta_{13}$  is predicted to be rather large and can be measured in near-future experiments.
- The neutrino masses are near-degenerate
- Neutrinoless double beta decay is in the sensitive area of near-future experiments

$$\theta_{13}^{\text{PMNS}} = \tilde{f}\lambda \quad \lesssim 0.18$$

$$\theta_{12}^{\text{CKM}} = (g_d - g_u)\lambda \approx 0.23$$

$$\theta_{12}^{\text{PMNS}} - \frac{\pi}{4} = -\tilde{g}_e\lambda \quad \approx -0.17$$



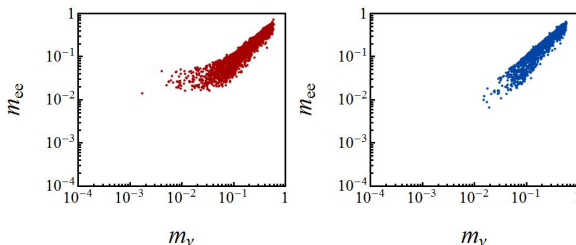
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# Conclusions

- Family Physics and Grand Unification both give a unified descriptions of the SM fermions.
- TriBiMaximal mixing and BiMaximal mixing patterns are both interesting.
- We described a Pati-Salam  $\times S_4$  model
  - $\theta_{13}$  and  $0\nu\beta\beta$  are expected to be measured soon.

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