Quark-lepton complementarity in an $S_4$ Pati Salam inspired scenario

Based on work in progress with Federica Bazzocchi and Luca Merlo, hep-ph/0910.xxxx

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Outline

1. Introduction: family physics
2. TriBiMaximal versus BiMaximal mixing
3. The model
4. Results and Conclusions
Family physics

- Family physics aims to explain the apparent structure in the fermion mass sector
  - Hierarchical masses
  - Quarks: moderate Cabibbo-mixing
  - Leptons: strong mixing
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![Diagram of fermion masses](image)
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\[
| (V_{CKM})_{ij} | = \\
\begin{pmatrix}
0.97 & 0.23 & 0.0039 \\
0.23 & 1.0 & 0.041 \\
0.0081 & 0.038 & 1?
\end{pmatrix}
\]

Family physics

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\[
\begin{vmatrix}
(U_{PMNS})_{ij}\end{vmatrix} = \\
\begin{pmatrix}
0.81 & 0.56 & <0.22 \\
0.39 & 0.59 & 0.68 \\
0.38 & 0.55 & 0.70 \\
\end{pmatrix}
\]
Family physics

- We charge the families under a symmetry group.
- To make the Lagrangian invariant, we need to add Higgs-like fields 'flavons'.
- Structure in the VEVs of the flavons leads to structure in the fermion masses. A very good introduction: G. Altarelli, Models of neutrino masses and mixings; hep-ph/0611117
Family Physics and Grand Unification

- Family symmetries unify the three families.
- Grand Unified Theories unify
  - The gauge couplings
  - The SM particles in fewer representations
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\[
\begin{align*}
\left[ (u_R \ u_G \ u_B) + (u^c_R \ u^c_G \ u^c_B) + (d_R \ d_G \ d_B) + \left( \nu \ e \ (e^c) + (\nu^c) \right) \right]_{\text{SM}} \rightarrow \\
\left[ (u_R \ u_G \ u_B \ d_R \ d_G \ d_B \ u^c_R \ u^c_G \ u^c_B \ d^c_R \ d^c_G \ d^c_B \ \nu \ e \ e^c \ \nu^c) \right]_{\text{SO}(10)}
\end{align*}
\]
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We like to build models that combine Family Physics and Grand Unification.

$$\begin{array}{cccccc}
  u^c & v_1^c & c^c & v_2^c & t^c & v_3^c \\
  d^c & e^c & s^c & \mu^c & b^c & \tau^c
\end{array} \rightarrow \begin{array}{cccccc}
  u^c & v_1^c & c^c & v_2^c & t^c & v_3^c \\
  d^c & e^c & s^c & \mu^c & b^c & \tau^c
\end{array}$$
Pati-Salam

- We use Pati and Salam’s $SU(4)_c \times SU(2)_L \times SU(2)_R$
  - Gauge: not quite unifying
  - All SM particles in two representations.
  - The type II seesaw can be dominant

This prevents unwanted correlations between quarks and neutrinos

![Graph showing the relationship between $\log_{10} (\mu/\text{GeV})$ and $\Delta$](image)
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\[
\begin{pmatrix}
(u_R & u_G & u_B)

\end{pmatrix}
\begin{pmatrix}
(\nu)

\end{pmatrix}
\quad \& \quad
\begin{pmatrix}
(u_R^c & u_G^c & u_B^c)

d_R^c & d_G^c & d_B^c

\end{pmatrix}
\begin{pmatrix}
(\nu^c)

e^c

\end{pmatrix}
\]
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$$m^I_\nu = -m^T_{\text{Dir}} \frac{1}{M_{RR}} m_{\text{Dir}} \propto M_R^{-1}$$

$$m^{II}_\nu = M_{LL} \propto M_{3L}^{-1}$$
TriBiMaximal mixing


\[
U_{\text{TBM}} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\end{pmatrix}
\]

\[\theta_{12} = 35^\circ, \quad \theta_{13} = 0^\circ, \quad \theta_{23} = 45^\circ.\]

- This describes the data really well ...
- TBM mixing might describe the neutrino data a bit too well.
  - Most models describe the quark mixing poorly at Leading Order*.
  - The Cabibbo angle arises only via NLO effects.
  - Often these NLO effects influence the PMNS as well.

* LO is defined as having the lowest number of flavons possible; NLO has one extra flavon.
BiMaximal mixing

- An alternative is BiMaximal mixing
- At leading order, two angles are maximal

\[ U_{BM} = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}
\end{pmatrix} \]

- And we can write \( U_{BM} = \) as the product of two maximal rotations

\[ U_{BM} = R_{23}(\frac{-\pi}{4}) \times R_{12}(\frac{\pi}{4}) \]
The leading order does not explain the data well.

Via quark lepton complementarity at NLO it might.


\[ \delta \theta_{12} = -0.7 \theta_C, \quad \delta \theta_{13} = 0.5 \theta_C, \text{ so} \]
\[ \theta_{12} = 36^\circ, \quad \theta_{13} = 7^\circ, \quad \theta_{23} = 45^\circ, \text{ while } \delta = 0. \]
Rotations at leading order

- BiMaximal mixing models have at leading order
  - $V_{CKM} = V_u^t V_d = 1$,
  - $U_{PMNS} = V_e^t V_\nu = R_{23}(-\frac{\pi}{4})R_{12}(\frac{\pi}{4})$

- This can be achieved if
  - $M_\nu^{II}$ is diagonalized by $V = R_{12}(\frac{\pi}{4})$
  - And $M_e^t M_e$, $M_d^t M_d$ and $M_u^t M_u$ by $V_{e,d,u} = R_{23}(\frac{\pi}{4})$
Matter content of the model

- How to get such mass matrices?
  → Model details
  - Our family group is the discrete group $S_4$
  - We put the left-handed PS multiplets $F_L$ in a 3$a$ of $S_4$
  - We put the right-handed PS multiplets $F^{c}_{1,2,3}$ in 1$b$, 1$b$ and 1$a$ of $S_4$
  - We introduce two flavons $(\sigma, \chi)$ in the neutrino sector
  - And two $(\varphi, \varphi')$ for the charged particles
Neutrinos

- In the neutrino sector we introduce a singlet $\sigma$ and a triplet $\chi$

$$\chi \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- This gives the type II seesaw contributions

$$F_L F_L \rightarrow \begin{pmatrix} u_1 & 0 & 0 \\ 0 & u_1 & 0 \\ 0 & 0 & u_1 \end{pmatrix}, \quad F_L F_L \chi \rightarrow \begin{pmatrix} 0 & u_2 & 0 \\ u_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- That is indeed diagonalized by $V = R_{12}(\frac{\pi}{4})$
Charged particles

- For the charged particles we introduce a $3_a$ triplet $\varphi$ and a $3_b$ triplet $\varphi'$

\[
\varphi \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \varphi' \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}
\]

- This leads to masses

\[
F_L F_3 c \varphi \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y \\ 0 & 0 & y \end{pmatrix}, \quad F_L F_2 c \varphi' \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & x & 0 \\ 0 & -x & 0 \end{pmatrix}
\]

- The square is indeed diagonalized by $V = R_{23}(\frac{\pi}{4})$

\[
M M^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y^2 + x^2 & y^2 - x^2 \\ 0 & y^2 - x^2 & y^2 + x^2 \end{pmatrix}
\]
Rotations at leading order

- We indeed have at Leading order.
  - $M^{II}_\nu$ is diagonalized by $V = R_{12}(\frac{\pi}{4})$
  - And $M_eM_e^\dagger$, $M_dM_d^\dagger$ and $M_uM_u^\dagger$ by $V_{e,d,u} = R_{23}(\frac{\pi}{4})$
- So we indeed reproduce BiMaximal mixing.
  - $V_{CKM} = V_u^\dagger V_d = 1,$
  - $U_{PMNS} = V_e^\dagger V_\nu = R_{23}(-\frac{\pi}{4})R_{12}(\frac{\pi}{4})$
Next-to-Leading Order

- At NLO $M_{\nu}^{II}$ is still diagonalized by $V = R_{12}(\frac{\pi}{4})$

\[
M_{\nu}^{II} = \begin{pmatrix}
  u_1 & u_2\lambda & 0 \\
  u_2\lambda & u_1 & 0 \\
  0 & 0 & u_1 + u_3\lambda^2
\end{pmatrix} \quad \lambda \approx 0.2
\]

- The charged particle mass matrices now read

\[
M_x = \begin{pmatrix}
  0 & \tilde{x}\lambda & \tilde{y}\lambda \\
  0 & x & y \\
  0 & -x & y
\end{pmatrix}
\]
Next-to-Leading Order

- $MM^\dagger$-matrices are diagonalized by

$$V_x = R_{23}(\frac{\pi}{4}) \times R_{13}(f\lambda) \times R_{12}(g_x\lambda),$$

where $f = f_u = f_d = f_e$ and $g_u \neq g_d = g_e$ are $O(1).$

- The CKM matrix is

$$V_{CKM} = V_u^\dagger V_d = R_{12}((g_d - g_u)\lambda)$$

- And the PMNS matrix is

$$V_{PMNS} = V_e^\dagger V_\nu = R_{23}(-\frac{\pi}{4}) \times R_{13}(\tilde{f}\lambda) \times R_{12}(\frac{\pi}{4} - \tilde{g}_e\lambda)$$
Some Results

- The angle $\theta_{13}$ is predicted to be rather large and can be measured in near-future experiments.
- The neutrino masses are near-degenerate.
- Neutrinoless double beta decay is in the sensitive area of near-future experiments.

\[
\theta_{13}^{\text{PMNS}} = \tilde{f}\lambda \quad \lesssim 0.18 \\
\theta_{12}^{\text{CKM}} = (g_d - g_u)\lambda \approx 0.23 \\
\theta_{12}^{\text{PMNS}} - \frac{\pi}{4} = -\tilde{g}_e\lambda \quad \approx -0.17
\]
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Family Physics and Grand Unification both give a unified descriptions of the SM fermions.

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We described a Pati-Salam $\times S_4$ model

- $\theta_{13}$ and $0\nu\beta\beta$ are expected to be measured soon.
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\[ \theta_{13} \]

\[ m_e \]

\[ m_\nu \]