Neutrino mass in tritium and rhenium single beta decay

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Neutrinos in Cosmology, in Astro-, Particle- and Nuclear Physics, Erice, Sicily, Sept. 16-24, 2009

Outlook

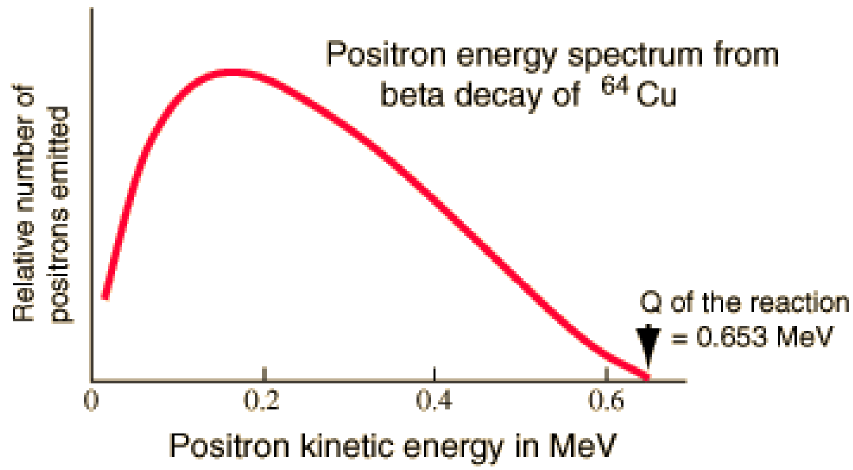
- Introduction
- Tritium beta decay within standard approach
- Exact relativistic treatment of ^{3}H decay
- First unique forbidden decay of ^{187}Re
- Comparison of Kurie plots for ^{3}H & ^{187}Re decays
- Relic neutrinos
- Summary

Neutrino

Neutrino was suggested in y. 1930 by Pauli to explain the continuity of β spectrum as a spin 1/2 particle obeying Fermi-Dirac statistics

I have done a terrible thing I invented a particle that cannot be detected

W. Pauli



4th December 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

Tübingen

Neutrino oscillations

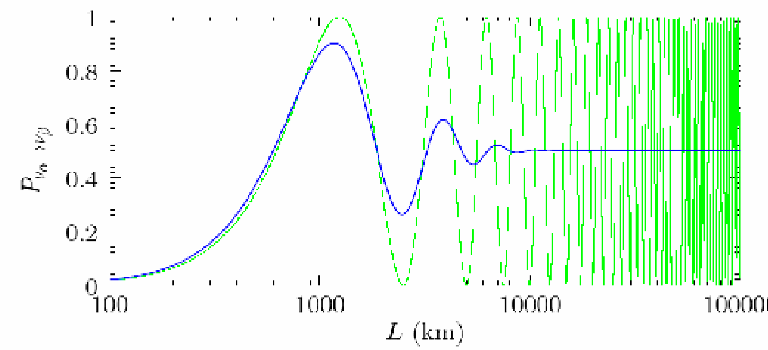
Pontecorvo
-Maki-Nakagawa-Sakata
matrix

Zh.Eksp.Teor .Fiz.,32(1957)

Maki,Nakagawa,Sakata. Prog.Theor.Phys.28(1962)870

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

oscillations $\Rightarrow$ massive neutrinos

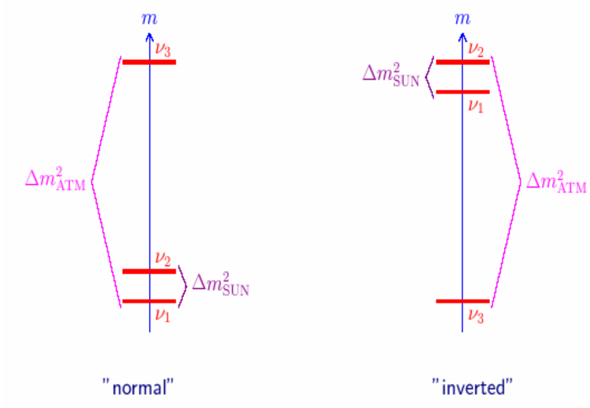


Flavor eigenstates

Mass eigenstates

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\vartheta \sin^2 (\frac{\Delta m^2}{4Et})$$

Absolute mass scale of neutrinos ?



0νββ-decay $m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 \; m_i$

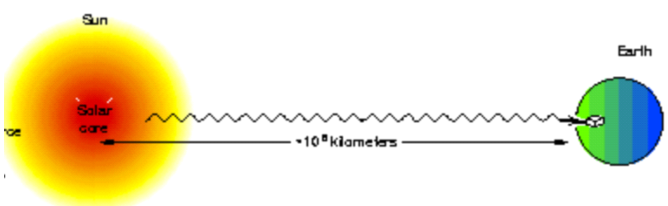
^{3}H decay $m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 \; m_i^2}$

Cosmology $\sum_{i=1}^{3} m_i$

We need 3 mass eigenstates
To explain 2 different Δm^2

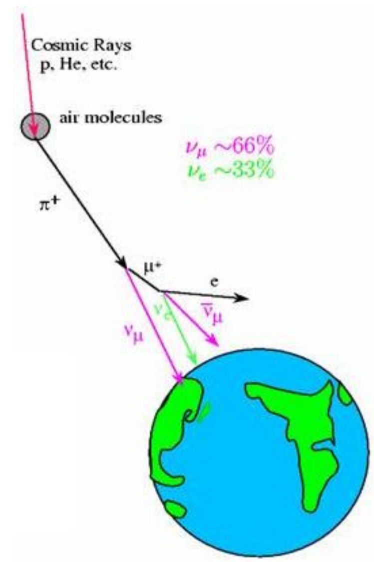
$m_2^2 - m_1^2 = \Delta m_{sol}^2 \approx 3.10^{-5}$ eV^2
$m_3^2 - m_2^2 = \Delta m_{atm}^2 \approx 2.10^{-3}$ eV^2

Solar neutrinos



1968
Homestake

Atmospheric neutrinos



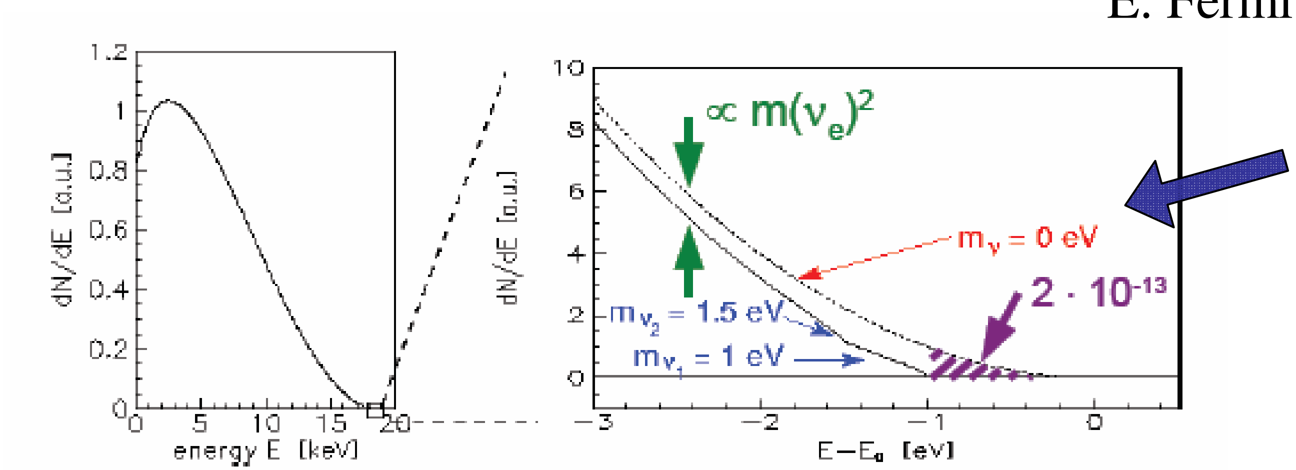
1998
SuperKamiokande

Tritium beta decay

$$^{3}H \rightarrow {}^{3}He + e^{-} + \tilde{\nu}$$

1934 – Fermi pointed out that shape of electron spectrum in beta decay near the endpoint is sensitive to neutrino mass

E. Fermi, Z. Phys. **88**, (1934)



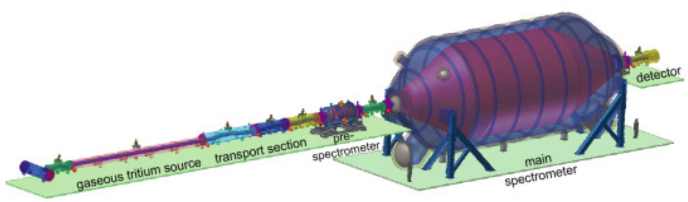
Endpoint beta spectrum

First measured by G. Hanna, B. Pontecorvo: Phys. Rev. **75,** 983 (1940) with estimation m_{ν} ~ 1 keV

Tritium beta decay

- low endpoint – Q=18.6 keV
- super-allowed nuclear transition (Fermi, Gamow-Teller M.E.)
- short half-live $T_{1/2}$ = 12.32 y

KATRIN experiment



Measuring last
30 eV endpoint

Upper limit on nu mass

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2} < 0.2\ eV$$

E.W. Otten, C. Weinheimer:
Rept. Prog. Phys. **71**:
5086201 (2008)

Adequate electron energy description near the endpoint is necessary

Standard approach

Neglecting the recoil and integrating over neutrino momentum conserving the energy in decay. We obey the electron energy spectrum.

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}T} = \frac{(\cos\vartheta_C G_\mathrm{F})^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\,(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

NME within spin-isospin symmetry are given $M_F=1$ & $M_{GT}=\sqrt{3}$

$$|M|^2 = g_V^2 |M_F|^2 + g_A^2 |M_{GT}|^2$$

p, E, T – momentum, energy and kinetic energy of electron

Q – maximal kinetic energy of electron in zero neutrino mass case

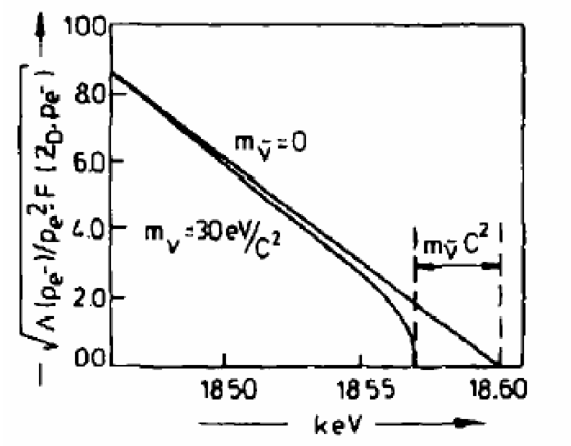
F(E) – Fermi function taking into account the Coulomb interaction between the electron and daughter nucleus

Standard approach

Kurie function

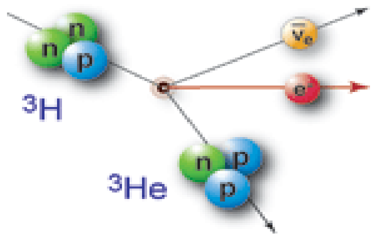
$$K(T)=\sqrt{\frac{\mathrm{d}\Gamma/\mathrm{d}T}{\frac{(\cos\vartheta_C G_{\mathrm{F}})^2}{2\pi^3}\,|\mathcal{M}|^2\,F(E)\,pE}}=\left[(Q-T)\sqrt{(Q-T)^2-m_{\nu_e}^2}\right]^{1/2}$$

The advantage of Kurie plot is that nonlinearity implies nonzero neutrino mass.



Heyde, Basic ideas & concepts in nuclear physics

Relativistic approach to ^{3}H decay



Relativistic description of 3 body decay within Elementary Particle Treatment (EPT - Kim & Primakoff, Phys.Rev. 139, B 1447(1965))

EPT – spin & isospin properties of tritium decay are identical with the decay of free neutron

$$^3H \rightarrow {}^3He + e^- + \tilde{\nu}$$

$$n \rightarrow p + e^- + \tilde{\nu}$$

Spin & parity of
^{3}H (n) and ^{3}He(p)

$$1/2^+ \rightarrow 1/2^+$$

Relativistic approach to ^{3}H decay

We consider recoil momentum in the phase space

$$d\Gamma = \left(\frac{1}{2} \sum_{spins} |M|^2 \delta^{(4)}(P_i - P_f - P_\nu - P_e) \frac{d^3 p_\nu}{E_\nu} \frac{d^3 p_f}{E_f} \right)$$

$$\times \frac{1}{16(2\pi)^5 M_i} F(Z, E_e) \frac{d^3 p_e}{E_e}$$

Exact averaged amplitude of 4 free spin ½ particles within the Fermi V-A contact interaction

$$\frac{1}{2} \sum_{spins} |M|^2 = 16(G_F \cos\theta_c)^2$$
$$\times \left[(g_V + g_A)^2 (P_e \cdot P_f)(P_\nu \cdot P_i) \right.$$
$$+ (g_V - g_A)^2 (P_e \cdot P_i)(P_\nu \cdot P_f)$$
$$\left. (-g_V^2 + g_A^2) M_i M_f (P_e \cdot P_\nu) \right].$$

Relativistic approach to ^{3}H decay

Performing the integration over neutrino and recoil momentum we get the exact relativistic electron energy spectrum for the ^{3}H beta decay

$$y = E_e^{\max} - E_e$$

$$(m_{12})^2 = M_i^2 - 2M_i E_e + m_e^2$$

$$E_e^{\max} = \frac{1}{2M_f}\left[M_i^2 + m_e^2 - \left(M_f^2 - m_\nu^2\right)\right]$$

Maximal e$^-$ energy about 3.4 eV lower than standard value

$$E_e^{\max} = M_i - M_f - m_\nu$$

$$
\begin{aligned}
\frac{d\Gamma}{dE_e} = {} & \frac{1}{(\pi)^3}(G_F\cos\theta_c)^2 F(Z,E_e)p_e \\
& \times \frac{M_i^2}{(m_{12})^2}\sqrt{y\left(y + 2m_\nu\frac{M_f}{M_i}\right)} \\
& \times \Bigg[(g_V+g_A)^2 y\left(y + m_\nu\frac{M_f}{M_i}\right)\frac{M_i^2(E_e^2 - m_e^2)}{3(m_{12})^4} \\
& (g_V+g_A)^2\left(y + m_\nu\frac{M_f+m_\nu}{M_i}\right)\frac{(M_iE_e - m_e^2)}{m_{12}^2} \\
& \times\left(y + M_f\frac{M_f+m_\nu}{M_i}\right)\frac{(M_i^2 - M_iE_e)}{m_{12}^2} \\
& -(g_V^2 - g_A^2)M_f\left(y + m_\nu\frac{(M_f+M_\nu)}{M_i}\right) \\
& \times\frac{(M_iE_e - m_e^2)}{(m_{12})^2} \\
& +(g_V-g_A)^2E_e\left(y + m_\nu\frac{M_f}{M_i}\right)\Bigg]
\end{aligned}
$$

S. S. Masood et al.: PRC 76,045501 (2007)

Šimkovic, Dvornický, Fäßler: PRC 77,055502(2008)

Relativistic approach to ^{3}H decay

In order to verify the result we can perform non-relativistic limit of the electron energy spectrum.
Keeping only dominant terms near the endpoint we get:

$$\frac{d\Gamma}{dE_e} \simeq \frac{1}{2\pi^3}(G_F V_{ud})^2 F(Z,E_e) p_e E_e (g_V^2 + 3g_A^2) \times \sqrt{y\,(y+2m_\nu)}\,(y+m_\nu)\,.$$

Remind: No NME, no.
1 & 3 appear naturally.

Assuming g_V=1 the axial coupling can be fixed from known half-live

$$T_{1/2}^{\exp} = 12.32\,y \Rightarrow g_A = 1.247$$

Bare nucleon value

$$g_A^{bare} = 1.2695$$

PDG W.M. Yao et al.: J. Phys. G 33, 1 (2006)

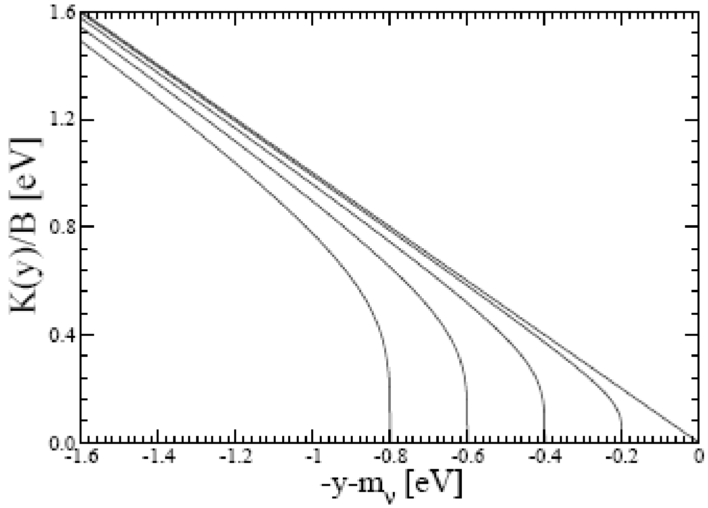
Relativistic approach to ^{3}H decay

We define a Kurie function

$$K(y) = B_T\left(\sqrt{y(y+2m_\nu)}(y+m_\nu)\right)^{1/2}$$

with

$$B_T = \frac{G_F V_{ud}}{\sqrt{2\pi^3}}\sqrt{g_V^2 + 3g_A^2}$$



The ratio K(y)/B$_T$ is free of coupling constants

$$K(y)/B_T = \left(\sqrt{y(y+2m_\nu)}(y+m_\nu)\right)^{1/2}$$

Structure in agreement with ref.:

Šimkovic, Dvornický, Fässler: PRC 77,055502(2008)

S. S. Masood et al.: PRC 76,045501 (2007)

Relativistic approach to ^{3}H decay

When replacing $y = E_0 - E_e - m_\nu$

We get from rel. Kurie function the standard Kurie function assuming $M_F = 1$ and $M_{GT} = \sqrt{3}$

$$K(y)/B_T = \left(\sqrt{y(y+2m_\nu)}(y+m_\nu)\right)^{1/2}$$

$$B_T = \frac{G_F V_{ud}}{\sqrt{2\pi^3}}\sqrt{g_V^2 + 3g_A^2}$$

Standard non-relativistic Kurie function

$$K(T) = \sqrt{\frac{\mathrm{d}\Gamma/\mathrm{d}T}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3}|\mathcal{M}|^2 F(E)\, pE}} = \left[(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}\right]^{1/2}$$

EPT rel. approach verifies the standard Kurie form near the endpoint

Exotic interactions in ^3H decay

Assume the general form of the weak beta decay Hamiltonian

$$H_\beta = H_{V,A} + H_{S,P} + H_T$$

The terms are given

$$H_{V,A} = \bar{e}\gamma^\mu (C_V - C'_V\,\gamma_5)\nu \bar{p}\gamma_\mu n$$
$$+\bar{e}\gamma^\mu\gamma_5 (C_A - C'_A\,\gamma_5)\nu \bar{p}\gamma_\mu\gamma_5 n + h.c.$$

$$H_{S,P} = \bar{e}(C_S - C'_S\,\gamma_5)\nu \bar{p} n$$
$$+\bar{e}\gamma_5 (C_P - C'_P\,\gamma_5)\nu \bar{p}\gamma_5 n + h.c.$$

$$H_T = \bar{e}\frac{\sigma^{\lambda\mu}}{\sqrt{2}}(C_T - C'_T\,\gamma_5)\nu \bar{p}\frac{\sigma^{\lambda\mu}}{\sqrt{2}} n + h.c.$$

N. Severijns et al.: Rev. Mod. Phys. 78: 991 (2006)

Exotic interactions in ^{3}H decay

EPT is a tool for studies of new interactions in tritium beta decay

Standard V-A plus tensor forces

$$\frac{1}{2}\sum_{spins}|M|^2 =$$
$$32[(C_A^2 - C_V^2)M_iM_f(P_e.P_\nu)$$
$$+((C_A + C_V)^2 + 4(C_T'^2 + C_T^2))(P_e.P_f)(P_i.P_\nu)$$
$$+((C_A - C_V)^2 + 4(C_T'^2 + C_T^2))(P_e.P_i)(P_f.P_\nu)$$
$$+6(C_T'^2 - C_T^2)M_iM_fm_em_\nu$$
$$-2(C_T'^2 + C_T^2)(P_e.P_\nu)(P_i.P_f)]$$

Standard V-A plus pseudo/scalar forces

$$\frac{1}{2}\sum_{spins}|M|^2 =$$
$$= 8[4(C_A^2 - C_V^2)M_iM_f(P_e.P_\nu)$$
$$+4(C_A + C_V)^2(P_e.P_f)(P_i.P_\nu)$$
$$+4(C_A - C_V)^2(P_e.P_i)(P_f.P_\nu)$$
$$+(C_S^2 + C_S'^2 + C_P^2 + C_P'^2)(P_e.P_\nu)(P_i.P_f)$$
$$+(C_S^2 + C_S'^2 - C_P^2 - C_P'^2)M_iM_f(P_e.P_\nu)$$
$$+(-C_S^2 + C_S'^2 + C_P^2 - C_P'^2)m_em_\nu(P_i.P_f)$$
$$+(-C_S^2 + C_S'^2 - C_P^2 + C_P'^2)M_iM_fm_em_\nu$$
$$+2((C_S - C_S')C_V + (C_P - C_P')C_A)m_eM_f(P_i.P_\nu)$$
$$+2((C_S - C_S')C_V - (C_P - C_P')C_A)m_eM_i(P_f.P_\nu)$$
$$-2((C_S + C_S')C_V - (C_P + C_P')C_A)m_\nu M_f(P_e.P_i)$$
$$-2((C_S + C_S')C_V + (C_P + C_P')C_A)m_\nu M_i(P_e.P_f)]$$

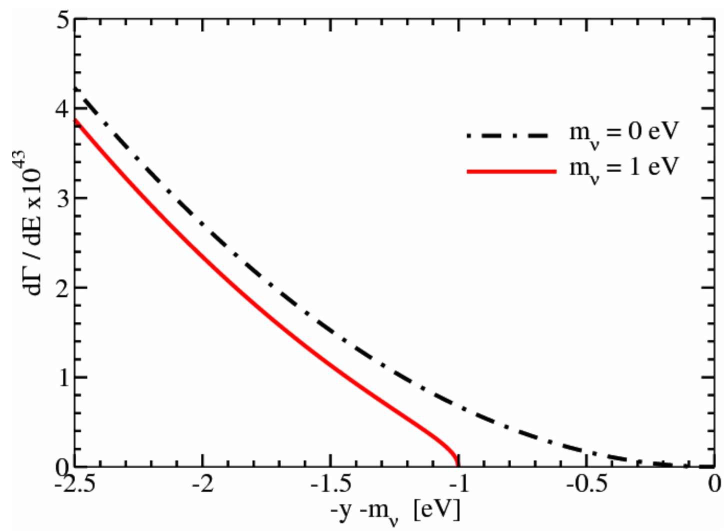
calculations in progress

Rhenium beta decay

- Beta emitter of g.s.→g.s. transition with lowest known Q value (2.47 keV)
- Relative high half-live ($T_{1/2}$=4.35 x 10^{10} y) ~ age of the universe (cosmo – chronometer)
- Natural abundance 63%

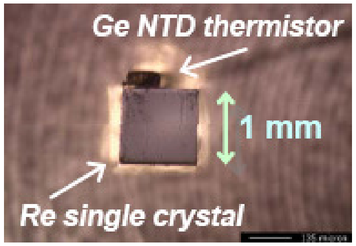
$$^{187}Re \rightarrow {}^{187}Os + e^- + \tilde{\nu}_e$$

Good candidate for the neutrino mass study



Rhenium beta decay

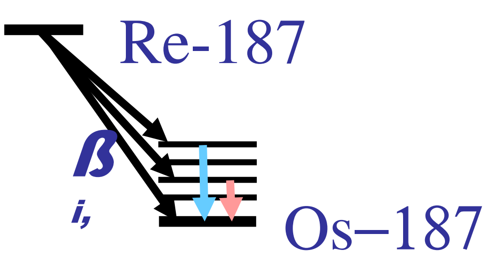
MARE experiment



$T_{1/2}=4.35 \times 10^{10}$ y $\rightarrow$ low radioactivity

bolometer
source=detector

The entire energy is measured in the detector except the neutrino including the molecular & atomic excitations



For more details see talk of E.Fiorini

Rhenium beta decay

The change of the angular momentum and parity between mother and daughter nuclei g.s. $\Rightarrow$ first unique forbidden decay

$$^{187}Re \rightarrow {}^{187}Os + e^- + \tilde{\nu}_e$$

$$5/2^+ \rightarrow 1/2^- \Rightarrow \Delta J^\pi = 2^-$$

Non-vanishing ME we will obey when considering the p-waves of the emitted leptons in the beta decay of ^{187}Re

$$H_\beta = \frac{G_\beta}{\sqrt{2}} \overline{\psi}_e(x)\gamma^\mu(1-\gamma_5)\psi_\nu(x) j_\mu(x) + h.c.$$

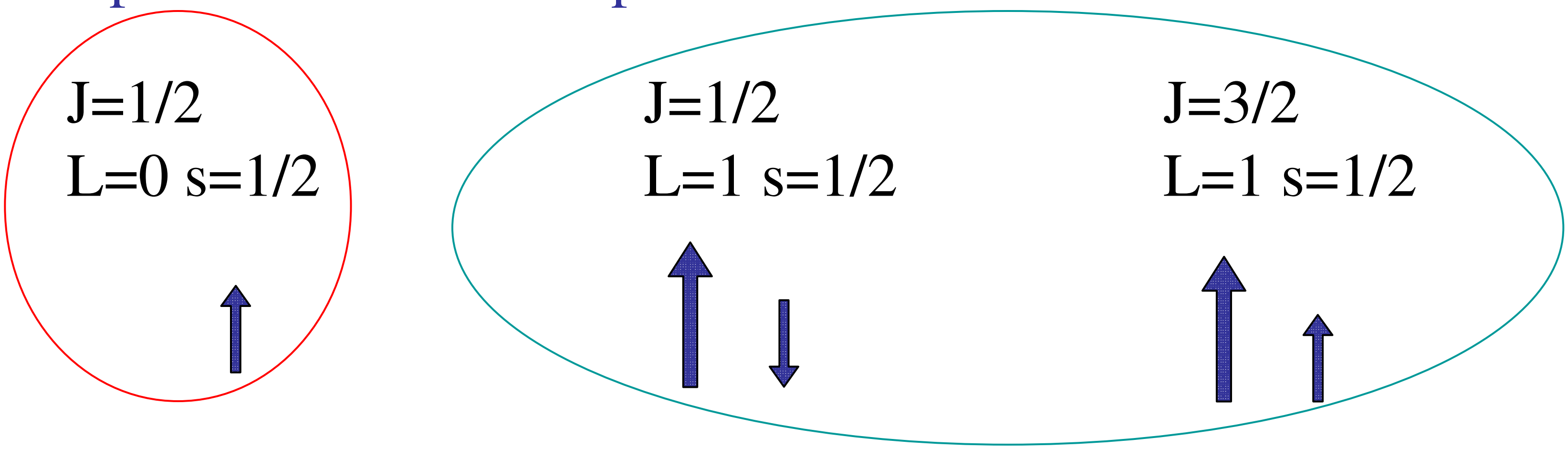
$$\Psi_{leptons} = \Psi_S + \Psi_P + \ldots$$

Rhenium beta decay

First unique forbidden transition $\Delta J^{\pi} = 2^{-}$

Plane wave expansion for ν $\psi_{\nu}(\vec{r}) = (1 + i\vec{k}.\vec{r})\mathrm{v}(k)$

The electron is emitted in the presence of the Coulombic field of the daughter nucleus therefore the wave function is expressed in terms of spherical waves



S wave P wave

Rhenium beta decay

We neglect higher waves due to centrifugal suppression

$$\Psi_e = \Psi_{S1/2} + \Psi_{P1/2} + \Psi_{P3/2}$$

$$\Psi_S = \begin{pmatrix} \tilde{g}_{-1}\chi_s \\ (\vec{\sigma}.\hat{p})\tilde{f}_1\chi_s \end{pmatrix}$$

J=1/2
L=0 s=1/2

$$\overline{\psi}_{S1/2}(\vec{r}) = \overline{u}(p)\sqrt{F_0(Z,E)}$$

$$\Psi_{P1/2} = i\begin{pmatrix} \tilde{g}_1(\vec{\sigma}.\hat{r})(\vec{\sigma}.\hat{p})\chi_s \\ -\tilde{f}_1(\vec{\sigma}.\hat{r})\chi_s \end{pmatrix}$$

J=1/2
L=1 s=1/2

$$\overline{\psi}_{P1/2}(\vec{r}) = \overline{u}(p)\sqrt{F_0(Z,E)}(-i)\frac{\alpha Z}{2}\gamma^0\vec{\gamma}.\hat{\vec{r}}$$

$$\Psi_{P3/2} = i\begin{pmatrix} \tilde{g}_{-2}[3(\hat{r}.\hat{p}) - (\vec{\sigma}.\hat{r})(\vec{\sigma}.\hat{p})\chi_s \\ \tilde{f}_2[3(\hat{r}.\hat{p})(\vec{\sigma}.\hat{p}) - (\vec{\sigma}.\hat{r})]\chi_s \end{pmatrix}$$

J=3/2
L=1 s=1/2

$$\overline{\psi}_{P3/2}(\vec{r}) = \overline{u}(p)\sqrt{F_1(Z,E)}(-i)(\vec{p}.\vec{r} + \frac{1}{3}\vec{\gamma}.\vec{p}\vec{\gamma}.\vec{r})$$

Doi, Kotani, Takasugi,
PTPS No. **83**,(1985)

F_k are Fermi functions for the spherical waves of electron

Rhenium beta decay

$\Delta J^{\pi} = 2^{-}$ $\Rightarrow$ Emitted leptons have to care the angular momentum L=2

Therefore the construction of amplitude for the beta decay process of ^{187}Re

$$\text{Amplitude} = e\,(s_{1/2})\ \&\ \nu(p_{3/2}) + e(p_{3/2})\ \&\ \nu(s_{1/2})$$

Rhenium beta decay

After performing the calculation we finally obey for the electron energy spectrum

$$\frac{d\Gamma}{dE} = \frac{G_F^2 V_{ud}^2}{2\pi^3} |M|^2 pE(E_0 - E)\sqrt{(E_0 - E)^2 - m_\nu^2}$$

$$\times \frac{1}{3} R^2 \left(p^2 F_1(Z,E) + k^2 F_0(Z,E) \right)$$

Electron in the $p_{3/2}$ state

Electron in the $s_{1/2}$ state

$$\frac{d\Gamma}{dE} = \frac{d\Gamma_P}{dE} + \frac{d\Gamma_S}{dE}$$

$$k = \sqrt{(E_0 - E)^2 - m_\nu^2}$$

Remind: no interference terms due to physically different final states of emitted leptons

Rhenium beta decay

There is only one NME due to the fact of first unique forbidden decay

$$\left|M\right|^2 = \frac{g_A^2}{2J_i+1}\left|< {}^{187}Os \,||\, \sqrt{\frac{4\pi}{3}}\sum_n \tau_n^+ \frac{r_n}{R}\{\sigma_1 \otimes Y_1\}_2 \,||\, {}^{187}Re >\right|^2$$

Within the treatment of rel. electron wave function the momentum and position decouple and ME is independent of energy

From the known experimental half-live we can deduce ME value

$$T_{1/2}^{\exp} = 4.35\times 10^{10}\, y \Rightarrow \left|M\right|^2 = 3.573\times 10^{-4}$$

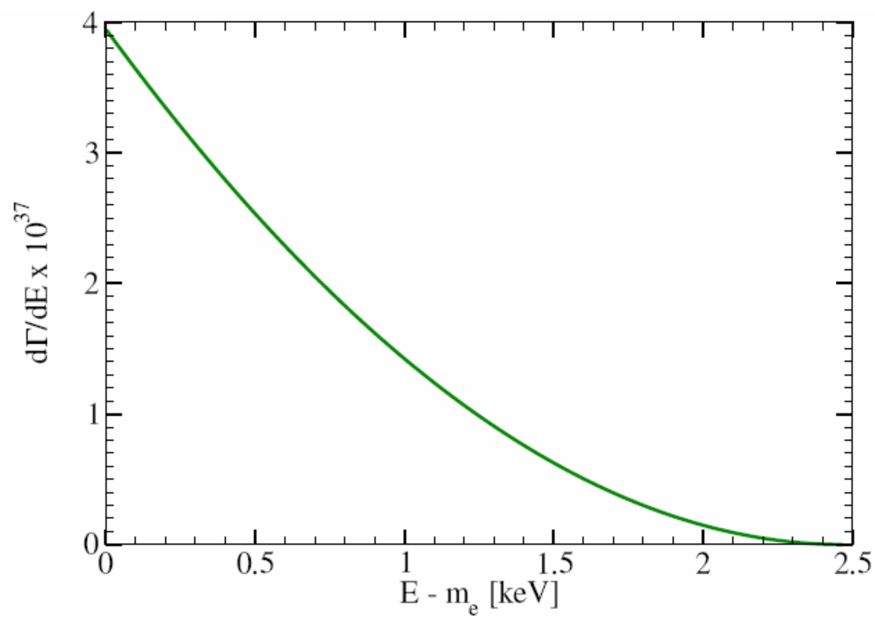
Rhenium beta decay

With beta strength fixed to the experimental value of half-live we can plot the electron energy spectrum

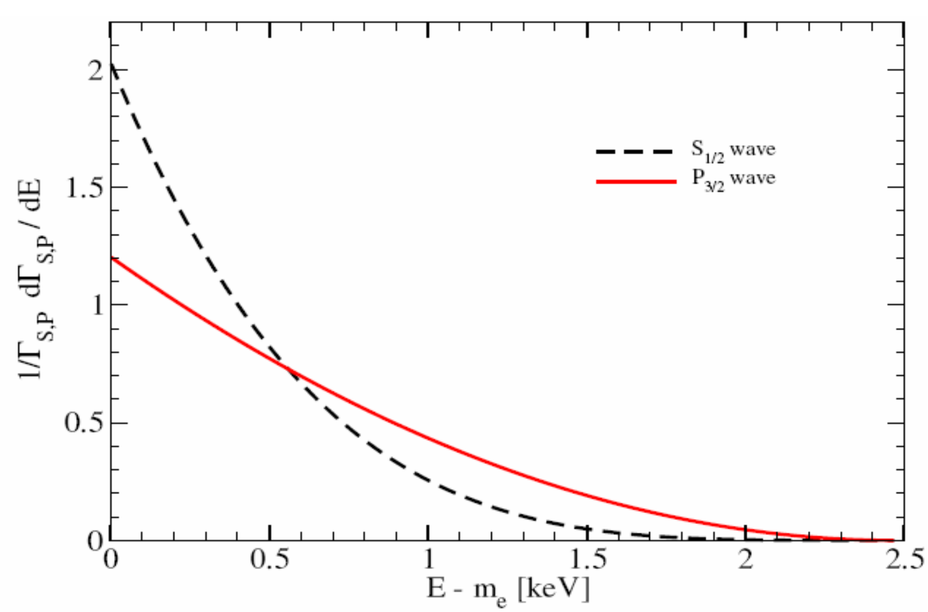
$$\frac{d\Gamma}{dE} = \frac{G_F^2 V_{ud}^2}{2\pi^3} |M|^2 \, pE(E_0 - E)\sqrt{(E_0 - E)^2 - m_\nu^2}$$

$$\times \frac{1}{3} R^2 \left(p^2 F_1(Z,E) + k^2 F_0(Z,E) \right)$$

Norm. to $T_{1/2}^{exp}$



Norm. to unity



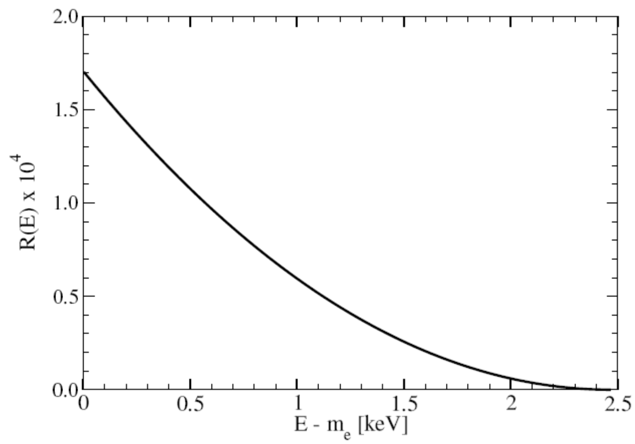
Rhenium beta decay

The contribution of the partial rates to the total rate is not equal

$$\Gamma_S = \int_{m_e}^{E_0} dE \frac{d\Gamma_S}{dE} \qquad \Gamma_P = \int_{m_e}^{E_0} dE \frac{d\Gamma_P}{dE} \qquad \Rightarrow \qquad \Gamma_S / \Gamma_P = 1.011 \times 10^{-4}$$

We define ratio of these two terms

$$R = \frac{d\Gamma_S}{dE} / \frac{d\Gamma_P}{dE}$$

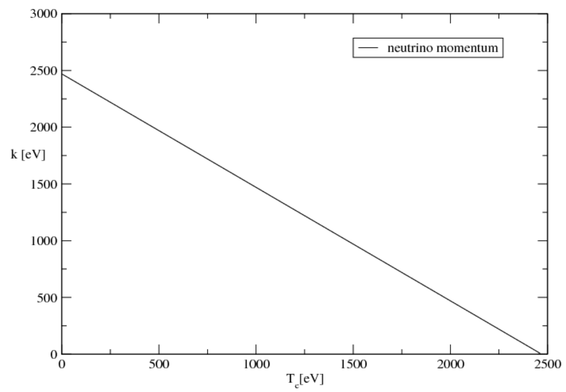


The electron $P_{3/2}$ decay rate channel is dominant $\Rightarrow$ important !
This is recently confirmed by MARE experimental results:
Arnaboldi et al.: PRL **96**, 042503 (2006)

Rhenium beta decay

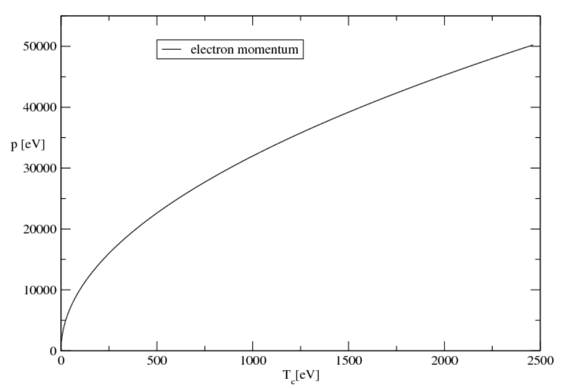
Neglecting the Coulomb interaction we set $F_k \rightarrow 1$ and from additive term originating from P waves of leptons we have only ~ (p^2+k^2)

$$k^{max} = 2.47 keV$$



The kinematics is enhancing the contribution of the electron P wave to the total decay rate

$$p^{max} \cong 50 keV$$



For the enhancement within the Fermi function see talk of K. Muto

Rhenium beta decay

Decay rate could be factorized the way to see connection with allowed beta decay rate

$$\frac{d\Gamma}{dE} = \frac{G_F^2 V_{ud}^2}{2\pi^3} \left| M \right|^2 pEF_0(Z,E)(E_0 - E)\sqrt{(E_0 - E)^2 - m_\nu^2}$$

$$\times \frac{1}{3} R^2 \left(p^2 \frac{F_1(Z,E)}{F_0(Z,E)} + k^2 \right)$$

Allowed transition. The same formula in tritium case

Term originating from the higher spherical waves of leptons

Rhenium beta decay

Neutrino momentum term could be neglected near the endpoint

$$\frac{1}{3}R^2\left(p^2\frac{F_1(Z,E)}{F_0(Z,E)}+\cancel{k^2}\right)$$

Due to the small Q value compared to the electron rest mass is the remaining term in brackets practically independent on electron kinetic energy

$$p^2\frac{F_1(Z,E)}{F_0(Z,E)}\cong 1+2\frac{E-m_e}{m_e}\cong 1$$

Rhenium beta decay

As a consequence is the goal that we can define the Kurie function similar to one for the tritium decay case

$$K(E_e)/B_{\mathrm{Re}} \cong (E_0 - E_e)\sqrt[4]{1 - \frac{m_\nu^2}{(E_0 - E_e)^2}}$$

with

$$B_{\mathrm{Re}} = \frac{G_F V_{ud}}{\sqrt{2\pi^3}} \frac{g_A}{\sqrt{2J_i + 1}} \left| <^{187}Os \,||\, \sqrt{\frac{4\pi}{3}} \sum_n \tau_n^+ \frac{r_n}{R} \{\sigma_1 \otimes Y_1\}_2 \,||^{187}Re > \right|$$

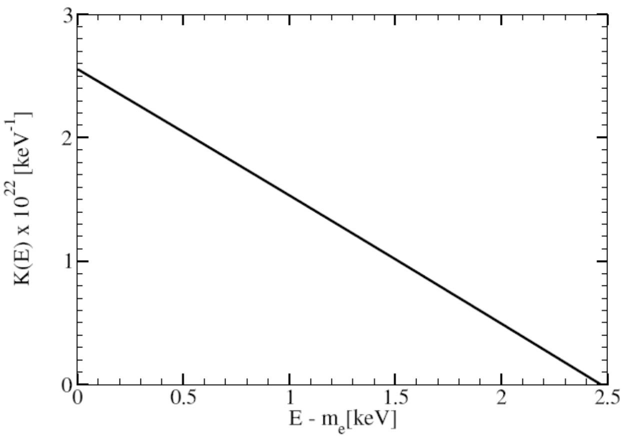
$$\times \sqrt{\frac{1}{3} R^2 p^2 \frac{F_1(Z,E)}{F_0(Z,E)}}$$

Practically constant

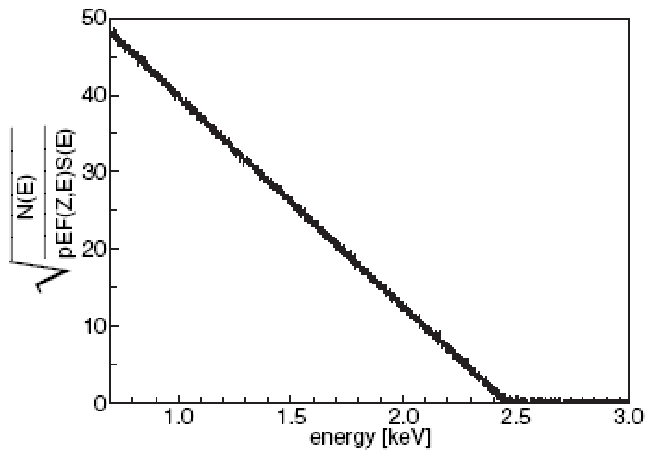
Rhenium beta decay

Even if the ^{187}Re is a first unique forbidden beta decay the Kurie plot for zero neutrino mass is linear in a very good approximation

Theory



Experiment



The $M^2 = 3.573\times10^{-4}$ is assumed from the experimental value of half-live $T_{1/2} = 4.35\times10^{10}$ y

Arnaboldi et al.:
PRL **96**, 042503 (2006)

Kurie plots for rhenium and tritium beta decay

We now introduce the variable $y=E_e^{max}-E_e$ instead of E_e and recall the beta strengths for the rhenium and tritium

$$B_{\mathrm{Re}} = \frac{G_F V_{ud}}{\sqrt{2\pi^3}} \frac{g_A}{\sqrt{2J_i+1}} \left| <^{187}Os \parallel \sqrt{\frac{4\pi}{3}} \sum_n \tau_n^+ \frac{r_n}{R} \{\sigma_1 \otimes Y_1\}_2 \parallel ^{187}Re > \right| \times \sqrt{\frac{1}{3} R^2 p^2 \frac{F_1(Z,E)}{F_0(Z,E)}}$$

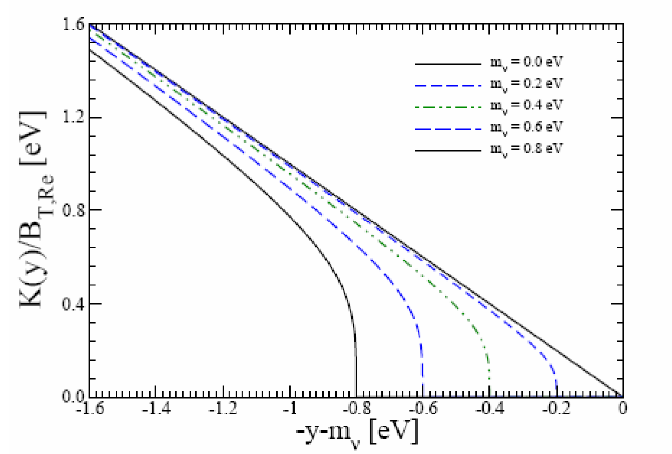
$$B_T = \frac{G_F V_{ud}}{\sqrt{2\pi^3}} \sqrt{g_V^2 + 3g_A^2}$$

Properly normalized Kurie functions become identical

$$K(E_e)/B_{\mathrm{Re}} \cong (E_0 - E_e) \sqrt[4]{1 - \frac{m_\nu^2}{(E_0 - E_e)^2}}$$

$$K(y)/B_T = \left(\sqrt{y(y+2m_\nu)}(y+m_\nu)\right)^{1/2}$$

$K(E)/B_{Re} \cong K(y)/B_T$



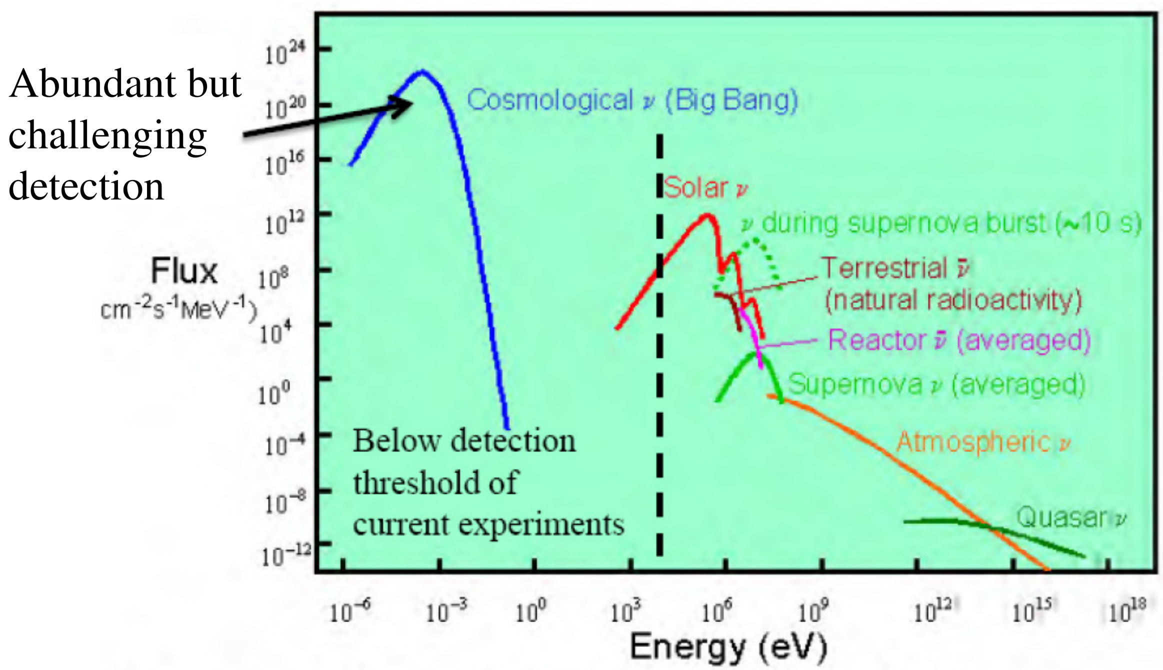
KATRIN ↔ MARE

Relic neutrinos

There are plenty of neutrinos in our Universe ~ 10^{87} per flavor

Eidelman et al.:
PLB **592**, 1 (2004)

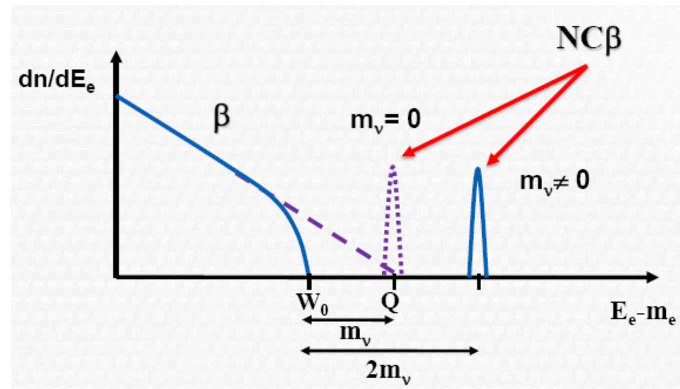
The analog of CMB is Cosmic Neutrino Background



Relic neutrinos

The neutrino capture via the beta decaying nucleus is a unique tool to detect cosmological neutrinos

$$\nu + (A,Z) \rightarrow (A,Z+1) + e^-$$



There is a gap of width $2m_\nu$ to distinguish between the beta decay and relic (low energy) neutrino capture

Relic neutrinos

The density of neutrinos <η>=56 cm^{-3}

Present neutrino temperature

$$T_\nu^{\,0} = \left(\frac{4}{11}\right)^{1/3} T_\gamma^{\,0} \approx (1.945 \pm 0.001) K \rightarrow \mathrm{k_B} T_\nu \approx (1.676 \pm 0.001) \times 10^{-4} eV$$

$$T_\gamma^{\,0} = (2.725 \pm 0.001) K = (2.348 \pm 0.001) \times 10^{-4} eV$$

Present mean momentum

$$\langle p_\nu^0 \rangle = \frac{7}{2} \frac{\zeta(4)}{\zeta(3)} T_\nu^0 \approx 3.151 T_\nu^0 \approx 5.314 \times 10^{-4} eV$$

Ref.: C.Giunti, C.W. Kim, Fundamentals of neutrino physics and astrophysics, Oxford (2007)

Relic neutrinos

The CNB neutrinos are non-relativistic and weakly clustered

If the CNB neutrinos are heavy enough $\Rightarrow$ velocities are smaller than escape velocity and they are clustered (trapped) within potential wells till present times

The expected over-densities $\eta_\nu / \langle\eta_\nu\rangle$ with respect to the average CNB neutrinos density ~ 10^3-10^4

Ref.: R. Lazauskas, P. Vogel, C. Volpe: JPG: Nucl. Part. Phys. 35, (2008)

Relic neutrinos

Neutrino capture by tritium nucleus $\nu + ^3H((1/2)^+) \rightarrow ^3He((1/2)^+) + e^-$

Assuming $M_F=1$, $M_{GT}=\sqrt{3}$ and $\eta_\nu=\langle\eta_\nu\rangle$ the capture rate

$$\Gamma^\nu(^3H) = \frac{1}{\pi} G_\beta^2 \, F_0(2,p) \, p \, p_0 \, \left(|M_F|^2 + g_A^2 \, |M_{GT}|^2\right) \, \frac{\eta_\nu}{<\eta_\nu>} \, <\eta_\nu>$$

$$\Gamma^\nu(^3H) = 4.2 \;\; 10^{-25} \;\; y^{-1}$$

$T_{1/2}=12.32$ y $\Rightarrow$

$$\frac{\Gamma^\nu(^3H)}{\Gamma^\beta(^3H)} = 7.5 \; 10^{-24}$$

KATRIN will use ~50 μg of ^3H $\Rightarrow$ number of events

$$N^\nu_{capt}(KATRIN) \approx 4.2 \; 10^{-6} \, \frac{\eta_\nu}{<\eta_\nu>} \, y^{-1}$$

Even considering clustering $\eta_\nu/\langle\eta_\nu\rangle \sim 10^3$-$10^4$ the effect is negligible

Relic neutrinos

Neutrino capture by rhenium nucleus $\nu + {}^{187}Re((5/2)^+) \to {}^{187}Os((1/2)^-) + e^-$

The capture rate $$\Gamma^\nu({}^{187}Re) = \frac{1}{\pi} G_\beta\, F_1(76,p)\, \frac{1}{3}\, (p\,R)^2\, \mathscr{B}\, p\, p_0\, \frac{\eta_\nu}{<\eta_\nu>} <\eta_\nu>$$

The beta strength $$\mathscr{B} = \frac{g_A^2}{6} |< {}^{187}Os(1/2^-) \| \sqrt{\frac{4\pi}{3}} \sum_n \tau_n^+ \frac{r_n}{R} \{\sigma_n \otimes Y_1(\Omega_{r_n})\}_2 \| {}^{187}Re(5/2^+) > |^2$$

$T_{1/2}=4.35 \times 10^{10}$ y $\Rightarrow$ $\mathscr{B} = 3.57 \times 10^{-4}$

Assuming $\eta_\nu = <\eta_\nu>$ the capture rate and the ratio of capt./emission

$$\Gamma^\nu({}^{187}Re) = 2.75\ 10^{-32}\ y^{-1}$$

$$\frac{\Gamma^\nu({}^{187}Re)}{\Gamma^\beta({}^{187}Re)} = 1.7\ 10^{-21}$$

760 g of $AgReO_4$ bolometers

$\Rightarrow$ $$N^\nu_{capt}(MARE) \simeq 7.6\ 10^{-8}\ \frac{\eta_\nu}{<\eta_\nu>}\ y^{-1}$$

>200 larger as ^{3}H

Summary

- The exact relativistic treatment of ^3H beta decay within the EPT method confirms that previously considered non-relativistic Kurie function is adequate and the recoil effect is small
- Analysis of the first unique forbidden beta decay of ^{187}Re showed that the e^- is preferably emitted in the P-wave state (in agreement with experiment)
- In a good accuracy the Kurie plot is a linear function for mass-less neutrino in first forbidden beta decay of ^{187}Re
- In the case of proper normalization of Kurie plots of ^3H & ^{187}Re they are practically identical close to the endpoint
- Unfortunately the relic neutrinos cannot be observed in KATRIN & MARE experiments even in the case of clustering of CNB, but there is a chance to put first constraint on density of neutrinos