The High Density Symmetry Energy in Relativistic Heavy Ion Collisions

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Int. School of Nuclear Physics, „Heavy Ion Collisions from the Coulomb Barrier to the Quark-Gluon Plasma“, Erice, Sicily, 16-24. 9. 2008
discuss:

1. Uncertainty of symmetry energy
2. test of symmetry energy in heavy ion collisions
   - low densities (Fermi energies) → talk by Massimo Di Toro
   - high density (rel. energies) → here
3. models for the symmetry energy
4. Transport calculations and search for observables sensitive to the symmetry energy:
   proton-neutron differential flow,
   meson production: ratios $\pi^+/\pi^-$ and $K^0/K^+$
   threshold and mean field effects, models for K-potential
5. Present status: high density symmetry energy almost not constrained by data,
   but planned experiments should improve situation: CHIMERA@GSI, RIKEN, FAIR (R$^3$B)
Symmetry Energy: Bethe-Weizsäcker Massenformel

\[ E(A, Z) = a_v A - a_s A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_I (N - Z)^2 / A + \delta_{\text{pair}} \]

\[ E(\rho_B, I) / A = E(\rho_B) + E_{\text{sym}}(\rho_B) I^2 + O(I^4) + \ldots \]

\[ I = \frac{N - Z}{N + Z} \]

Heavy ion collisions in the Fermi energy regime

Around normal density: Structure, neutron skins

High density: HIC at relativistic energies (differential flow, particle production)

Neutron stars
**Theoretical Description of Nuclear Matter**

**Non-relativistic:**
Hamiltonian $H = \sum T_i + \sum V_{ij}$; $V$ nucleon-nucleon interaction

**Relativistic:**
Hadronic Lagrangian $L(\psi; \sigma, \omega, \pi, \eta, \delta, ...)$
$
\psi$, nucleon, resonances; $\sigma, \omega, \pi$,.... mesons

<table>
<thead>
<tr>
<th></th>
<th>phenomenological</th>
<th>microscopic</th>
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<tbody>
<tr>
<td></td>
<td>(fitted to nucl. matter)</td>
<td>(based on realistic NN interactions)</td>
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<tr>
<td>non-relativistic (Schrödinger)</td>
<td>Skyrme-type</td>
<td>Brueckner-HF (BHF) (+ 3-body forces)</td>
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<tr>
<td>Relativistic (Quantumhadrodyn.)</td>
<td>RMF</td>
<td>Dirac-Brueckner HF (DB)</td>
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Density functional theory; EFT
Hadronic field theory $\rightarrow$ Quantenhadrodynamics (QHD)

Extensions of simplest $\sigma\omega$-model necessary; many choices

$$L = \overline{\Psi} \left[ i \gamma_\mu \left( \partial^\mu + ig_\omega \omega^\mu + ig_\rho \frac{\tau^2 b^\mu}{2} \right) - \left( m - g_\sigma \sigma - g_\delta \frac{\tau^2 \delta}{2} \right) \right] \Psi + L^{mes}$$

isovector mesons: symmetry energy

non-linear meson self-interactions
G. Lalazissis et al., PRC55 (1997) 540

density dependent coupling vertices
S. Typel, HHW, NPA656 (1999) 331

Full Lorentz structure:

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<th>isoscalar</th>
<th>isovector</th>
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<tr>
<td>scalar</td>
<td>$\sigma$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>vector</td>
<td>$\omega$</td>
<td>$\rho$</td>
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Cancellation!

$$L^{mes} = \frac{1}{2} \left( \partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{b_3}{3} \sigma^3 - \frac{b_4}{4} \sigma^4$$

Connection to DBHF

$$E_{sym} = \frac{1}{6} \frac{k_F^2}{E_F} + \frac{1}{2} f_\rho - f_\delta \left( \frac{M^*}{E^*} \right)^2 \rho_B$$
EOS for Symmetric Nuclear Matter

→ for BHF to describe saturation, 3-body forces are necessary

→ Microscopic EOS’s are soft!

Constraint from ratio of kaon production in heavy and light system

Fairly well fixed: what about the symmetry energy?


Chr. Fuchs, et al., PRL 86 (2001)

Constraint from ratio of kaon production in heavy and light system

results from different groups

Chr. Fuchs, et al., PRL 86 (2001)
The Nuclear Symmetry Energy in different Models

empirical asy-EOS’s cross at about

\[ \rho \approx 0.6 \rho_0 \]

microscopic asy-EOS’s soft at low densities but stiff at high densities


many more in: B.A. Li, Phys. Rep. 08

Momentum dependence of mean fields

\[ U_{\text{opt}} = -\Sigma_S + \frac{E}{M} \Sigma_V + \frac{\Sigma_S^2 - \Sigma_V^2}{2M} \]

Schrödinger Equivalent Optical Potential

Isoscalar Potential

Isovector (Lane) Potential

Connected to splitting of proton/neutron effective masses

Heavy Ion Collisions with particle production:

→ Coupled relativistic transport eqs. with elastic and inelastic collision terms

Rel. Transport eq. (RBUU, RLV, RQMD,..)

\[
\left[ \frac{p_i^* \mu}{m_i} \frac{\partial}{\partial \mu} + \left( \frac{p_i^*}{m_i} F_{i}^{\mu \nu} + \partial_{\mu} m_i^* \right) \frac{\partial}{\partial \mu} \left( p_i^* \right) \right] f_i(x, p^*) = I_{i}^{\text{coll}}
\]

\[ i = p, n, \Delta^{(-,0,+;++)} , \vec{\pi}^{\pm} (\text{free}), K^{(0,+)} \text{(see below)} \]

Elastic baryon-baryon coll.: NN↔NN (in-medium \( \sigma_{NN} \)), NΔ↔NΔ, ΔΔ↔ΔΔ

Inelastic baryon-baryon coll. (hard \( \Delta \)-production): NN↔NΔ, NN↔ΔΔ

Inelastic baryon-meson coll. (soft \( \Delta \)-production): Nπ↔Δ

Channels with strangeness (perturbative kaon production):
Baryon-Baryon : BB→BYK (B=N,Δ^{0,+;++}, Y=Λ,Σ^{0}, K=K^{0,+})
Pion-Baryon : \( \pi B \rightarrow Y K \)
Kaon-Baryon : BK→BK (elastic, isospin exchange)
No channels with antistrangeness (K⁻)
Probes of the ASY-EOS in Heavy Ion Collisions

- High density/energy
  - n/p ratios
  - p/n differential flow
  - pions ratios
  - kaon ratios
  - neutron stars
- Low density/energy
  - fragments, p/n ratios
  - isospin diffusion
  - isoscaling
  - migration/fractionation.
  - collective excitations
  - phase transitions

Covered in talk by M. Di Toro

To be discussed (partially) here
Asymmetric matter: Differential directed and elliptic flow

$^{132}\text{Sn} + ^{132}\text{Sn} @ 1.5\text{ AGeV } b=6\text{fm}$

Proton-neutron differential flow

$$F_{n,p}^y(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} (p_i^y w_i),$$

$$w_i = +1(-1)\text{ for neutron (proton)}$$

and analogously for elliptic flow

Difference at high $p_t$, $\leftrightarrow$ first stage, dynamical boosting of vector contribution

$$\frac{dP_{p}^{*}}{d\tau} - \frac{dP_{n}^{*}}{d\tau} \simeq 2 \left[ \gamma f_{\rho} - f_{\delta} \right] \nabla \rho_{3} = \frac{4}{\rho_{B}} \mathcal{E}_{sym}^{*} \nabla \rho_{3}$$

Asymmetric matter: Differential directed and elliptic flow

$^{132}\text{Sn} + ^{132}\text{Sn} \rightarrow 1.5 \text{ AGeV \ b}=6\text{fm}$

**Proton-neutron differential flow**

$F_{n-p}^x(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} (p_i^x w_i)$,

$w_i = +1(-1)$ for neutron (proton)

and analogously for elliptic flow

**Differential directed flow**

**Elliptic flow**

D. Lambrecht et al., ZPA 350: 115-120 (1994)

Sensitivity of particle production to the symmetry energy

1. Mean field effect: $U_{\text{sym}}$ more repulsive for neutrons, and more for asystiff
   $\rightarrow$ pre-equilibrium emission of neutron,
   reduction of asymmetry of residue

2. Threshold effect, in medium effective masses:
   $\rightarrow m^{*}_N, m^{*}_\Delta$, contribution of symmetry energy
   $m^{*}_K$, models for K-potentials

3. Discuss in particular ratios: $\pi^-/\pi^+$ and $K^0/K^+$, to enhance effect of symmetry energy

consider only $K^+$ and $K^0$, since antistrange kaons have a very different dynamics (talk by H. Oeschler)
Pion and Kaon production in “open” system (HIC)...

\[ NN \rightarrow N\Delta \rightarrow NYK \]
\[ \pi N \rightarrow YK \]

mean field effects: n/p repulsion
threshold effects: \( \Delta \) and K self energies

\[ \text{Au+Au@1AGeV} \]

\[ \text{Pions: compensation} \]
- direct early production: high density phase
- isovector channel effects

G. Ferini et al., PRL 97 (2006) 202301
Pion production: Au+Au, semicentral

Equilibrium production (box results)

\[
\frac{\pi^-}{\pi^+} = \frac{\sigma_{\pi^-}^{abs}}{\sigma_{\pi^+}^{abs}} \exp\left[\frac{2(\mu^- - \mu^+)}{T}\right]
\]

\(\sim 5\) (NLρ) to 10 (NLρδ)

disagreement in magnitude, particularly at low energies, (also in other calc.), but better at midrapidity (high density), where Kaons are produced.

W. Reisdorf et al. NPA781 (2007) 459
Transverse Pion Flows

W. Reisdorf et al. NPA781 (2007) 459

Antiflow: Decoupling of the Pion/Nucleon flows

Simulations: V. Prassa Sept. 07

OK general trend. but:
- smaller flow for both $\pi^-$ and $\pi^+$
- not much dependent on iso-EoS
Strangeness ratio: Infinite Nuclear Matter vs. HIC

\[ \alpha_{\text{Au}} \approx 0.2 \]

Au+Au@1AGeV (HIC)

Density & asymmetry of the K-source

\[ N/Z_{\text{Au}} \approx 1.5 \]

Pre-equilibrium emission (mainly of neutrons) reduced asymmetry of source for kaon production \( \rightarrow \) reduces sensitivity relative to equilibrium (box) calculation.
Kaon ratios: comparison with experiment

G. Ferini, et al., NPA762(2005) 147 and nucl-th/0607005

Data (Fopi)
X. Lopez, et al. (FOPI), PRC 75 (2007)

Comparision to FOPI data

\[
\frac{(K^+/K^0)_{Ru}^0}{{(K^+/K^0)_{Zr}^0}}
\]

- sensitivity reduced in collisions of finite nuclei
- single ratios more sensitive
- enhanced in larger systems

\[
\frac{(Ru+Ru)}{(Zr+Zr)}
\]

equilibrium (box) calculations
finite nucleus calculations
Effect of kaon potentials

In-medium Klein-Gordon eq. for Kaon propagation:

\[ \left( \partial_\mu + i V_\mu \right)^2 + m^*_K \phi_K(x) = 0 \]

Two models for medium effects tested:

**Chiral perturbation (Kaplan, Nelson et al.)**

\[ m^+_K = \sqrt{m^2_K - \frac{\Sigma_{KN}}{f^2_\pi} \rho_3 \pm \frac{C}{f^2_\pi} \rho_{33} + V_\mu V^*_\mu} \]  
(upper sign, \(K^+\))

\[ V^\mu = \frac{3}{8f^2_\pi} \tilde{j}_\mu \pm \frac{1}{8f^2_\pi} \tilde{j}_{\mu 3} \]

**One-Boson Exchange (Schaffner-Bielich et al.)**

\[ m^*_K = \sqrt{m^2_K + \frac{m_K}{3} \left( g_{\sigma N} \sigma \pm f_\delta \rho_{33} \right)} \]

\[ V^\mu = \frac{1}{3} \left( f^* \tilde{j}_\mu \pm f_\rho \tilde{j}^{\mu}_3 \right) \]

Isospin-dependence

ChPT

In-medium K energy (k=0)

\[ E_K(k) = k_0 = \sqrt{k^2 + m^*_K^2 + V_0} \]

Splitting for \(K^{0,+}\)

for NLρ and NLρδ

OBE
Kaon production (absolute yields)

Sensitivity to:
1. in-medium cross section: $\sigma_{\text{free}}$ vs. $\sigma_{\text{eff}}$
2. K- potential: ChPT vs. OBE
3. Isospin-dep- (ID) K-potential

Ni+Ni, E=1.93 AGeV, b<4 fm, rapidity distrib.

- Influence of all ingredients substantial
- rather good description with OBE potential and $\sigma_{\text{eff}}$

→ consider ratios to minimize influence of ingredients
Kaon ratios: test of "robustness" against variation of K-potential and $\sigma_{\text{inmed}}$

Influence of $\sigma_{\text{inmed}}$: small

Influence of K-pot substantial, part. with isospin-dep. Part substantial!

A$\bar{u}$+Au, b=0 fm
Tolman-Oppenheimer-Volkov equation to determine mass of neutron star

Proton fraction and direct URCA

- $\beta$–equilibrium and charge neutrality: $y = \frac{N}{Z} = y(\varepsilon_{\text{sym}})$
- direct URCA process: $p \rightarrow n + e^+ + \nu_e$
  threshold: $y \approx 11\%$, fast neutrino cooling

Heaviest observed neutron star (now retracted)

Typical neutron stars

Onset of direct URCA

Forbidden by Direct URCA constraint

Summary and Conclusions:

- While the EOS of symmetric NM is fairly well determined, the density (and momentum) dependence of the symmetry energy is still rather uncertain (but important for exotic nuclei, neutron stars and supernovae)
- Can be investigated in HIC both at low densities (Fermi energy regime, isospin transport) and high densities (relativistic collisions, flow, particle production)
- Nucleon and light cluster pre-equilibrium emission and flow are directly sensitive to the symmetry energy.
- Pion and Subthreshold Kaon production are a promising signal, in particular when considering ratios $\pi^-/\pi^+$ and $K^0/K^+$
  The kaon signal is robust with respect to the in-medium cross sections, but influenced by the model for the Kaon potential
- Effects scale with the asymmetry - thus reactions with RIB are very important