The High Density Symmetry Energy
in Relativistic Heavy Ion Collisions

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Int. School of Nuclear Physics, „Heavy Ion Collisions from the Coulomb Barrier to the Quark-Gluon Plasma“,
Erice, Sicily, 16-24. 9. 2008

discuss:

1. Uncertainity of symmetry energy
2. test of symmetry energy in heavy ion collisions

   [- low densities (Fermi energies) → talk by Massimo Di Toro]

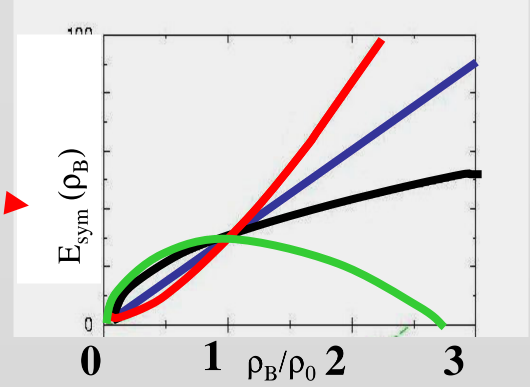
   - high density (rel. energies) → here
3. models for the symmetry energy
4. Transport calculations and search for observables sensitive to the symmetry energy:

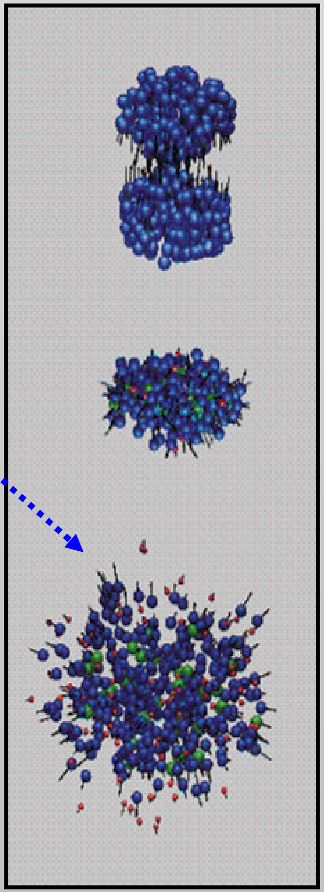
   proton-neutron differential flow,

   meson production: ratios π^+/π^- and K^0/K^+

   threshold and mean field effects, models for K-potential
5. Present status: high density symmetry energy almost not constrained by data,

   but planned experiments should improve situation: CHIMERA@GSI, RIKEN, FAIR (R^3B)



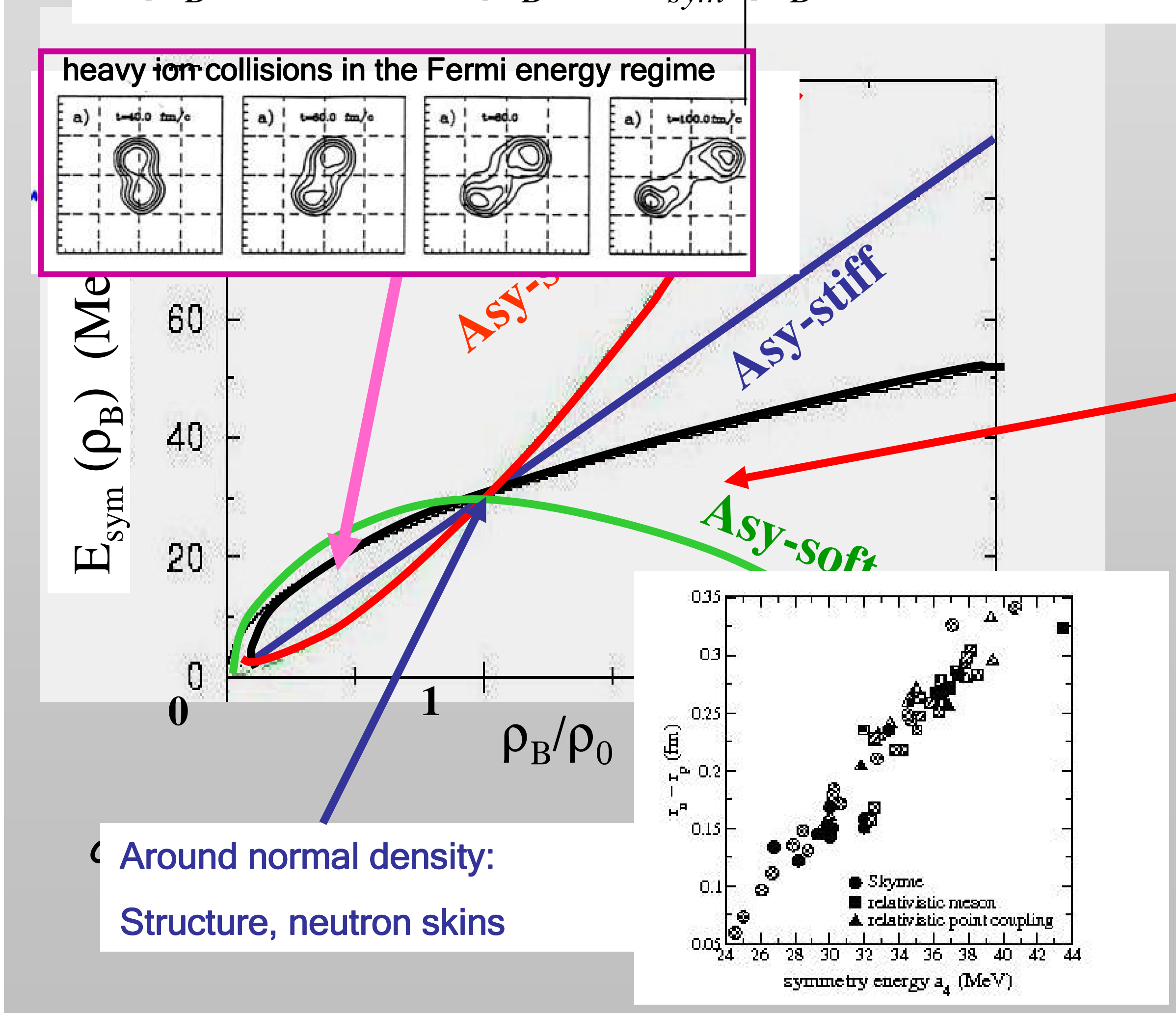


Symmetry Energy: Bethe-Weizsäcker Massenformel

$$E(A,Z) = a_v A - a_s A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_I (N-Z)^2/A + \delta_{pair}$$

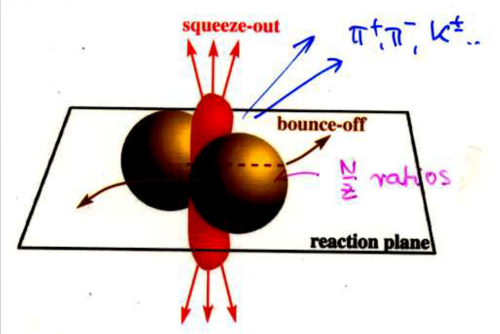
$$E(\rho_B, I)/A = E(\rho_B) + E_{sym}(\rho_B) I^2 + O(I^4) + \dots$$

$$I = \frac{N-Z}{N+Z}$$



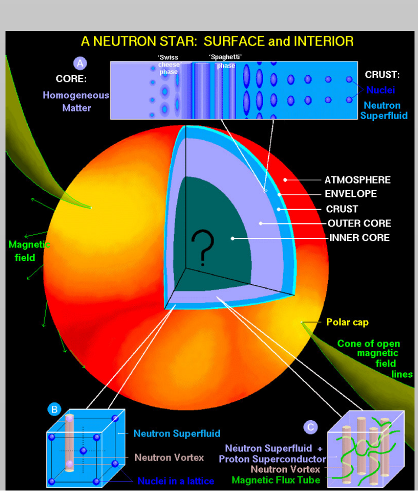
Around normal density:

Structure, neutron skins

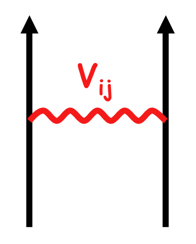


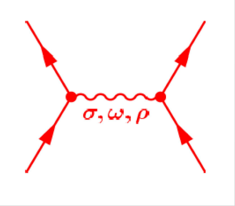
High density: HIC at relativistic energies (differential flow, particle production)

neutron stars



Theoretical Description of Nuclear Matter





Non-relativistic:

Hamiltonian $H = \Sigma T_i + \Sigma V_{ij}$; V nucleon-nucleon interaction

Relativistic:

Hadronic Lagrangian $L(\psi;\sigma,\omega,\pi,\eta,\delta,\ldots)$

ψ, nucleon, resonances; σ,ω, π,.... mesons

	phenomenological (fitted to nucl. matter)	**microscopic** (based on realistic NN interactions
non-relativistic (Schrödinger)	**Skyrme-type**	**Brueckner-HF (BHF)** (+ 3-body forces)
Relativistic (Quantumhadrodyn.)	**RMF**	**Dirac-Brueckner HF (DB)**

Density functional theory; EFT

Hadronic field theory → Quantenhadrodynamics (QHD)

Extensions of simplest σω-model necessary; many choices

$$L=\overline{\Psi}\left[i\gamma_\mu\left(\partial^\mu+ig_\omega\omega^\mu+ig_\rho\frac{\vec{\tau}}{2}\vec{b}^\mu\right)-\left(m-g_\sigma\sigma-g_\delta\frac{\vec{\tau}}{2}\vec{\delta}\right)\right]\Psi+L^{mes}$$

isovector mesons: symmetry energy
T. Gaitanos, et al., NPA732 (2004) 24

non-linear meson self-interactions
G.Lalazissis et al., PRC55 (1997) 540

$$L^{mes}=\frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma-m_\sigma^2\sigma^2\right)-\frac{b_3}{3}\sigma^3-\frac{b_4}{4}\sigma^4$$

density dependent coupling vertices
S. Typel, HHW, NPA656 (1999) 331

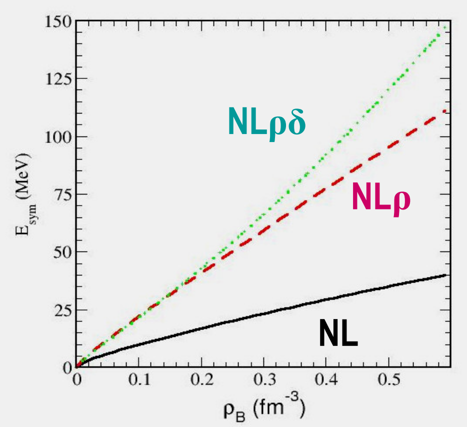
$\Gamma_\omega(\hat{\rho})$ $\Gamma_\rho(\hat{\rho})$ ...

Connection to DBHF

Full Lorentz structure:

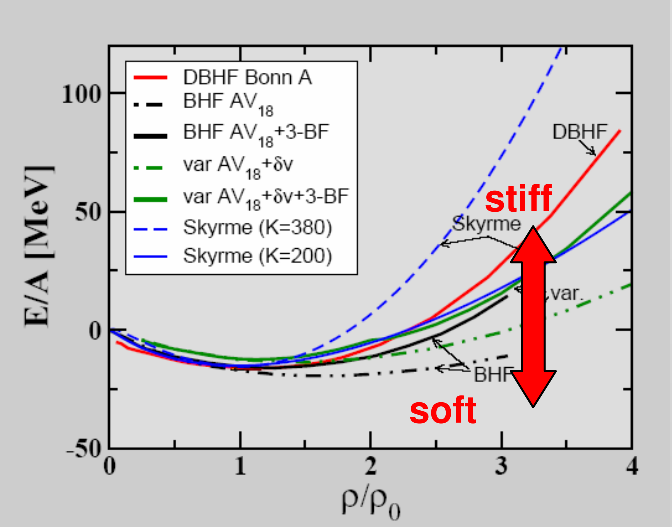
	isoscalar	isovector
scalar	σ	δ
vector	ω	ρ

cancellation!

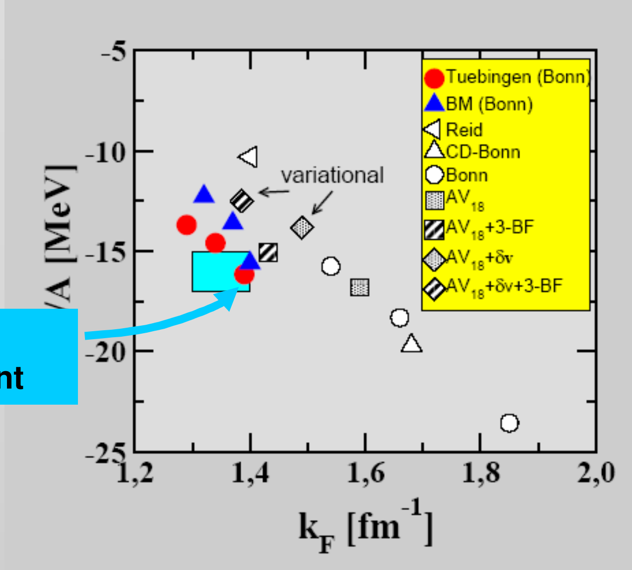


$$E_{sym}=\frac{1}{6}\frac{k_F^2}{E_F^{*2}}+\frac{1}{2}\left[f_\rho-f_\delta\left(\frac{M^*}{E^*}\right)^2\right]\rho_B$$

EOS for Symmetric Nuclear Matter



Chr. Fuchs, H.H. Wolter EPJ A30 (2006) 5

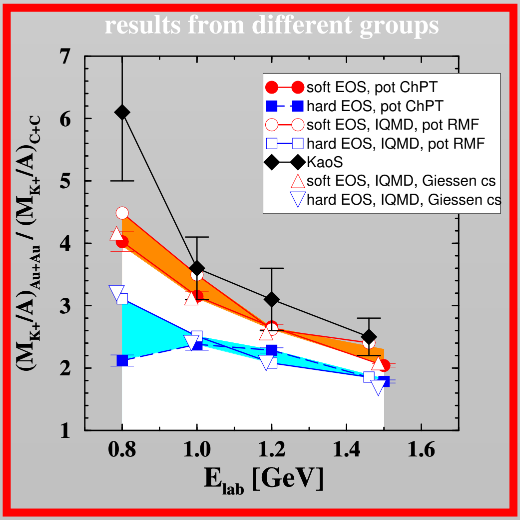


Empirical saturation point

→ for BHF to describe saturation, 3-body forces are neccessary

→ Microscopic EOS‘s are soft!

Constraint from ratio of kaon production in heavy and light system



Chr. Fuchs, et al., PRL 86 (2001)

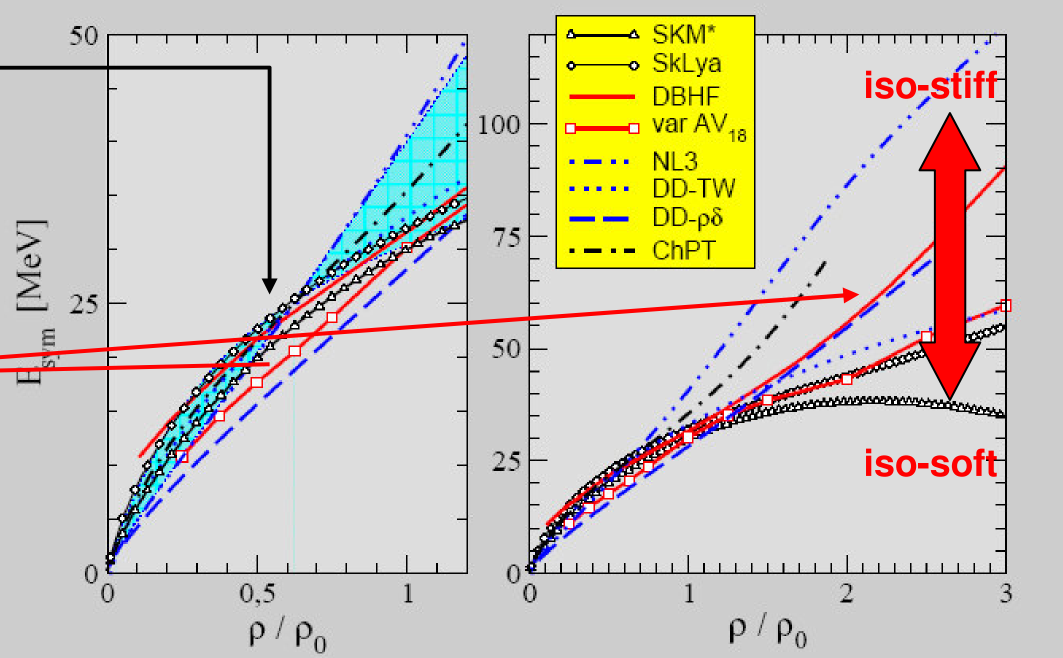
Fairly well fixed:
what about the symmetry energy?

The Nuclear Symmetry Energy in different Models

empirical asy-EOS‘s cross at about

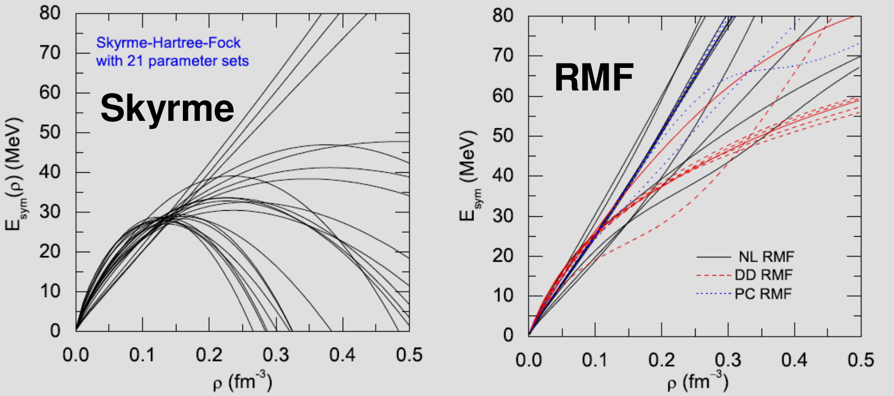
$$\rho \approx 0.6\,\rho_0$$

microscopic asy-EOS`s soft at low densities but stiff at high densities

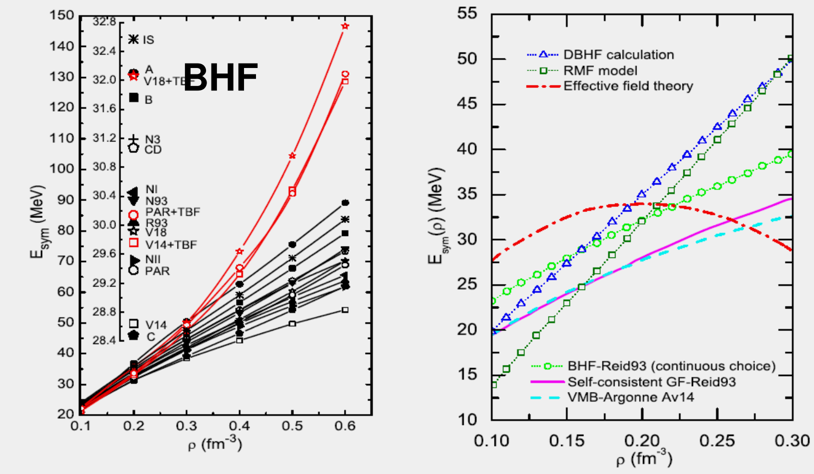


C. Fuchs, H.H. Wolter, EPJA 30(2006)5,(WCI book)

many more in: B.A. Li, Phys. Rep. 08



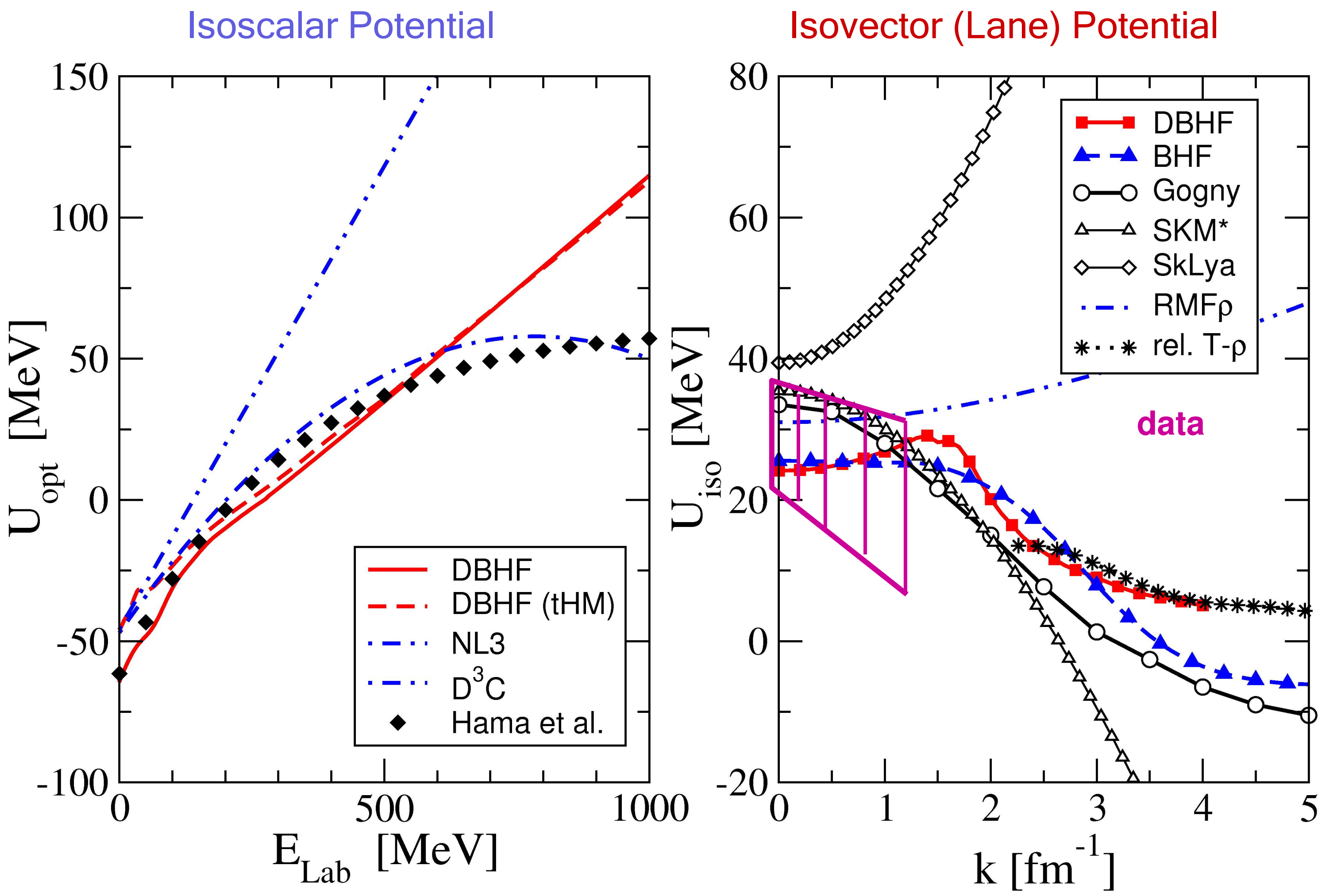
B.A. Li, Phys. Rep. (2008) in print



Momentum dependence of mean fields

$$U_{\mathrm{opt}} = -\Sigma_S + \frac{E}{M}\Sigma_V + \frac{\Sigma_S^2 - \Sigma_V^2}{2M} \; .$$

Schrödinger Equivalent Optical Potential


Isoscalar Potential

Isovector (Lane) Potential

Connected to splitting of proton/neutron effective masses

C. Fuchs, H.H. Wolter, EPJA 30(2006)5

Heavy Ion Collisions with particle production:

→ Coupled relativistic transport eqs. with elastic and inelastic collision terms

Rel. Transport eq. (RBUU, RLV, RQMD,..)

$$\left[\frac{p_i^{*\mu}}{m_i^*}\partial_\mu + \left(\frac{p_{\nu i}^*}{m_i^*}F_i^{\mu\nu} + \partial^\mu m_i^*\right)\partial_\mu^{(p^*)}\right] f_i(x,p^*) = I_i^{coll}$$

$$i = p,n,\Delta^{(-,0,+,++)}, \pi^\pm(free), K^{(0,+)}(see\ below)$$

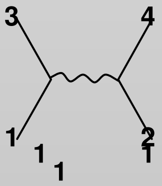
$$p_i^{*\mu} = p_i^\mu - \Sigma_i^\mu$$

$$m_i^* = m_i - \Sigma_{s,i}$$

$$F^{\mu\nu} = \partial^\mu\Sigma^\nu - \partial^\nu\Sigma^\mu$$

	isoscalar	isovector
Scalar	σ(NL)	δ
vector	ω	ρ

cancellation!

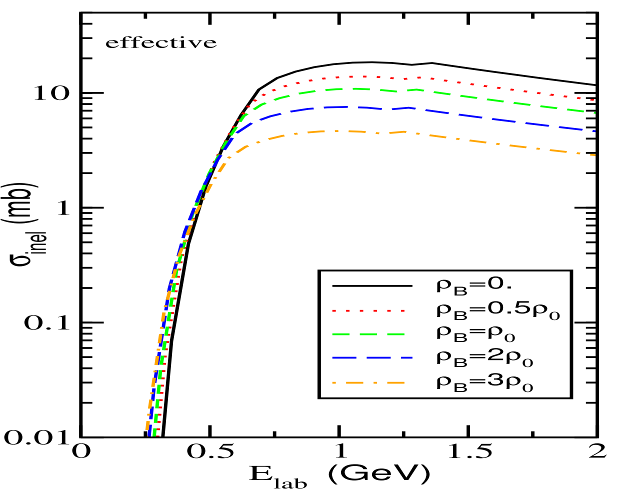


$$I_i^{(coll)} = \sum_{j=elast,inelast}\left[\frac{1}{(2\pi)^{3/2}}\iiint dp_2 dp_3 dp_4\; v_{12}\left.\frac{d\sigma^{(ij)}}{d\Omega}\right|_{12\to 34}\delta(enrgy\ conserv.)\;\left[(1-f)(1-f_2)f_3f_4 - f f_2(1-f_3)(1-f_4)\right]\right]$$

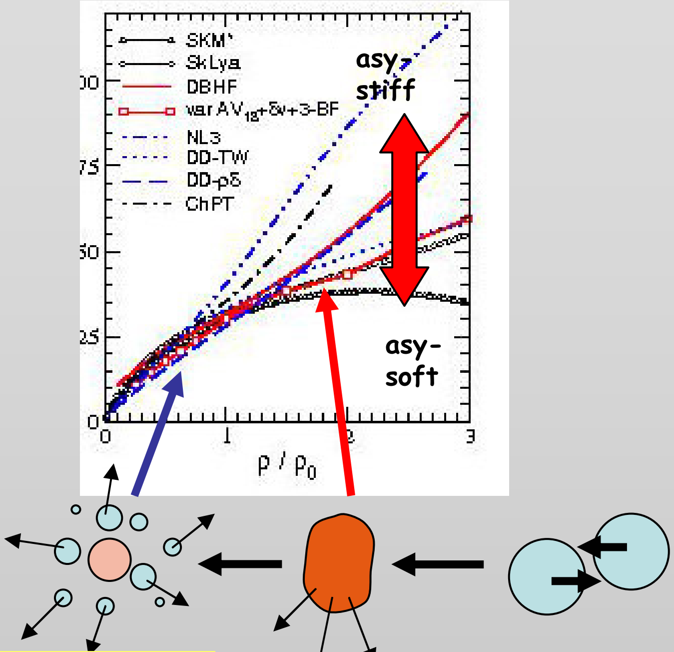
Elastic baryon-baryon coll.: NN↔NN (in-medium σ_{NN}), NΔ↔NΔ, ΔΔ↔ΔΔ

Inelastic baryon-baryon coll, (*hard* Δ-production): NN↔NΔ, NN↔ΔΔ
Inelastic baryon-meson coll. (*soft* Δ-production): Nπ↔Δ

Channels with strangeness (perturbative kaon production):
Baryon-Baryon : BB→BYK (B=N,$\Delta^{\pm,0,++}$, Y=Λ,$\Sigma^{\pm,0}$, K=$K^{0,+}$)
Pion-Baryon : πB→YK
Kaon-Baryon : BK→BK (elastic, isospin exchange)
No channels with antistrangeness (K^-)



Probes of the ASY-EOS in Heavy Ion Collisions



Covered in talk by M. Di Toro

Low density/energy

- **fragments, p/n ratios**
- **isospin diffusion**
- **isoscaling**
- **migration/fractionat.**
- **collective excitations**
- **phase transitions**

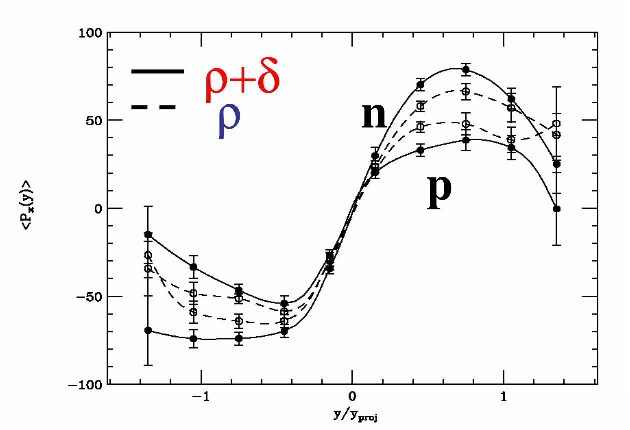
High density/energy

- **n/p ratios**
- **p/n differential flow**
- **pions ratios**
- **kaon ratios**
- **neutron stars**

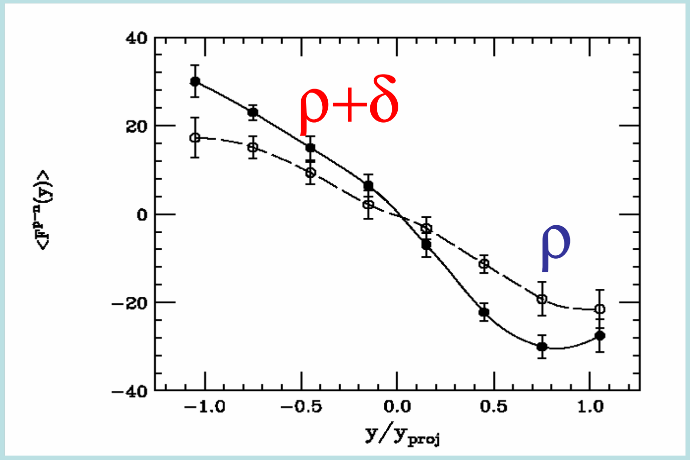
To be discussed (partially) here

Asymmetric matter: Differential directed and elliptic flow

^{132}Sn + ^{132}Sn @ 1.5 AGeV b=6fm



differential directed flow



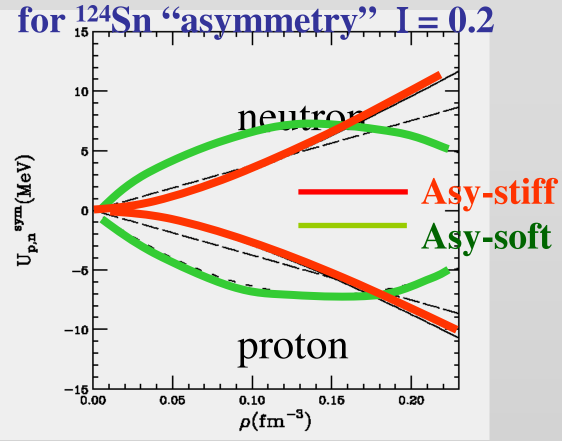
Proton-neutron differential flow

$$F^{x}_{n-p}(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} (p_i^x w_i),$$

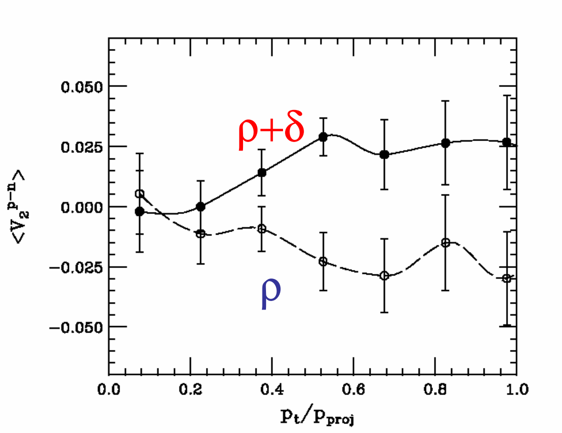
$$w_i = +1(-1)\ for\ neutron\ (proton)$$

and analogously for elliptic flow

for ^{124}Sn “asymmetry” I = 0.2



differential elliptic flow



- Difference at high p_t ⟺ first stage , dynamical boosting of vector contribution

$$\frac{d\vec{p}^{\,*}_p}{d\tau} - \frac{d\vec{p}^{\,*}_n}{d\tau} \simeq 2\left[\gamma f_\rho - \frac{f_\delta}{\gamma}\right]\vec{\nabla}\rho_3 = \frac{4}{\rho_B} E^*_{sym} \vec{\nabla}\rho_3$$

T. Gaitanos, M. Di Toro, et al., PLB562(2003)

Asymmetric matter: Differential directed and elliptic flow

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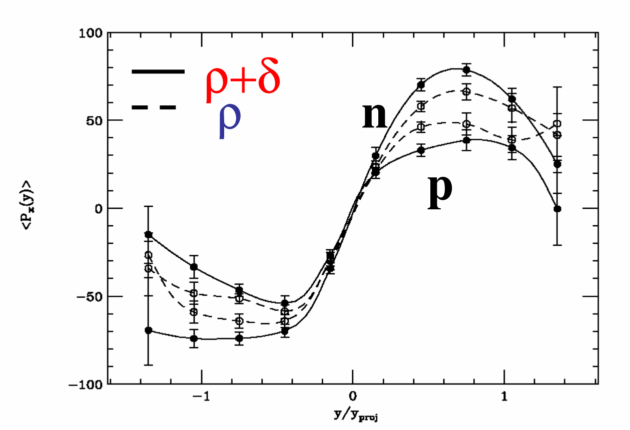
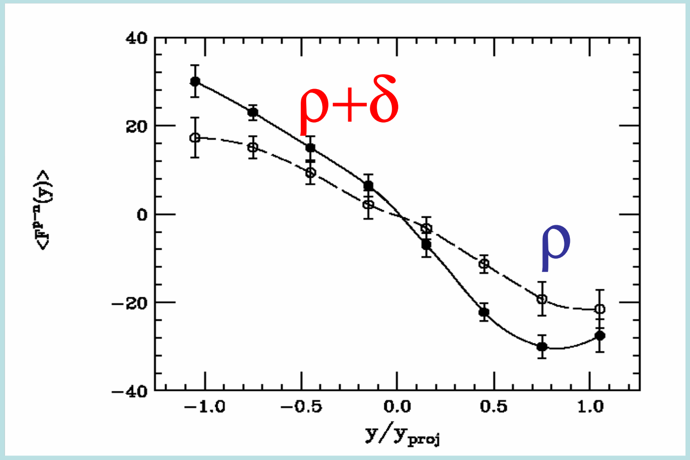


differential directed flow



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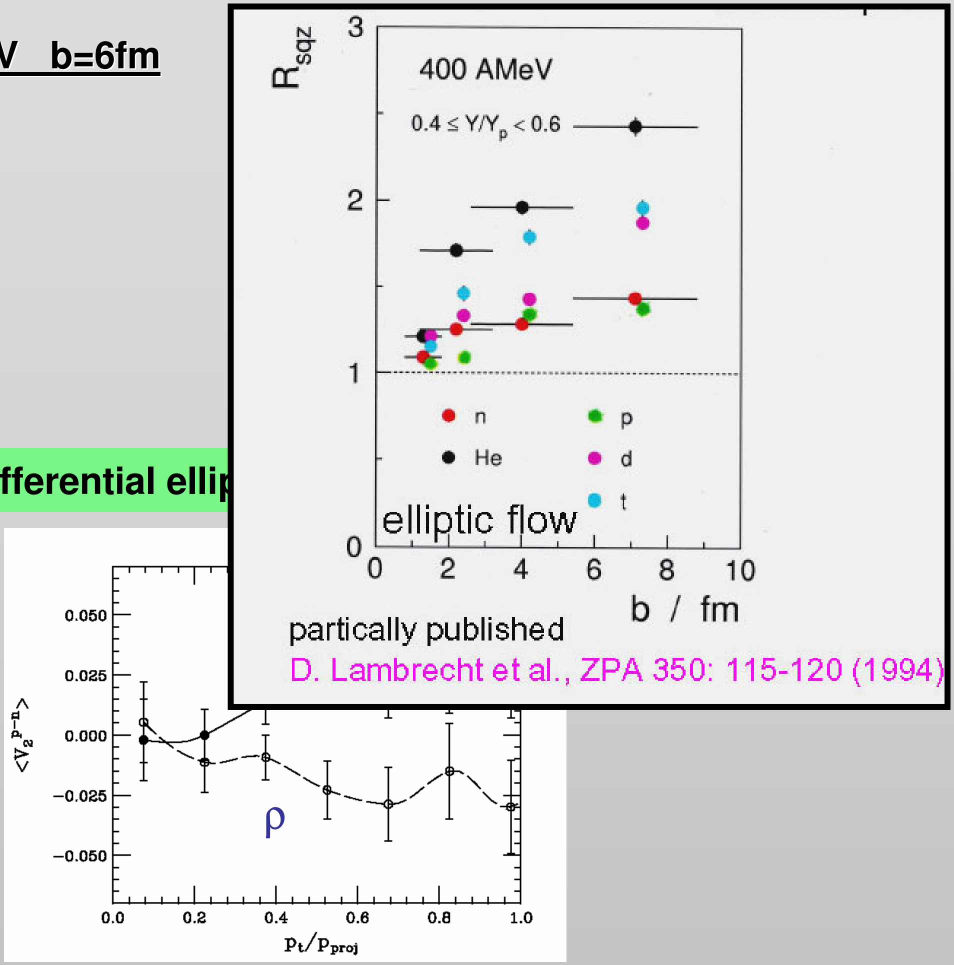
and analogously for elliptic flow

differential ellip



partically published

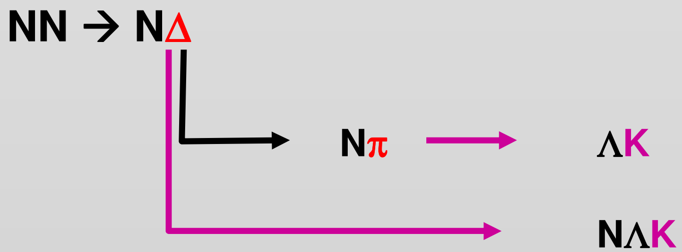
D. Lambrecht et al., ZPA 350: 115-120 (1994)

- Difference at high p_t ⇔ first stage , dynamical boosting of vector contribution

$$\frac{d\vec{p}_p^{\,*}}{d\tau} - \frac{d\vec{p}_n^{\,*}}{d\tau} \simeq 2\left[\gamma f_\rho - \frac{f_\delta}{\gamma}\right]\vec{\nabla}\rho_3 = \frac{4}{\rho_B}E^*_{sym}\vec{\nabla}\rho_3$$

T. Gaitanos, M. Di Toro, et al., PLB562(2003)

Sensitivity of particle production to the symmetry energy



$NN \rightarrow N\Delta$

$N\pi \rightarrow \Lambda K$

$N\Lambda K$

consider only K^+ and K^0, since antistrange kaons have a very different dynamics (talk by H. Oeschler)

1. Mean field effect: U_{sym} more repulsive for neutrons, and more for asystiff

→ pre-equilibrium emmission of neutron,

reduction of asymmetry of residue

2. Threshold effect, in medium effective masses:

→ m^*_{N}, m^*_{Δ}, contribution of symmetry energy

m^*_K, models for K-potentials

3. Discuss in particular ratios: π^-/π^+ and K^0/K^+, to enhance effect of symmetry energy

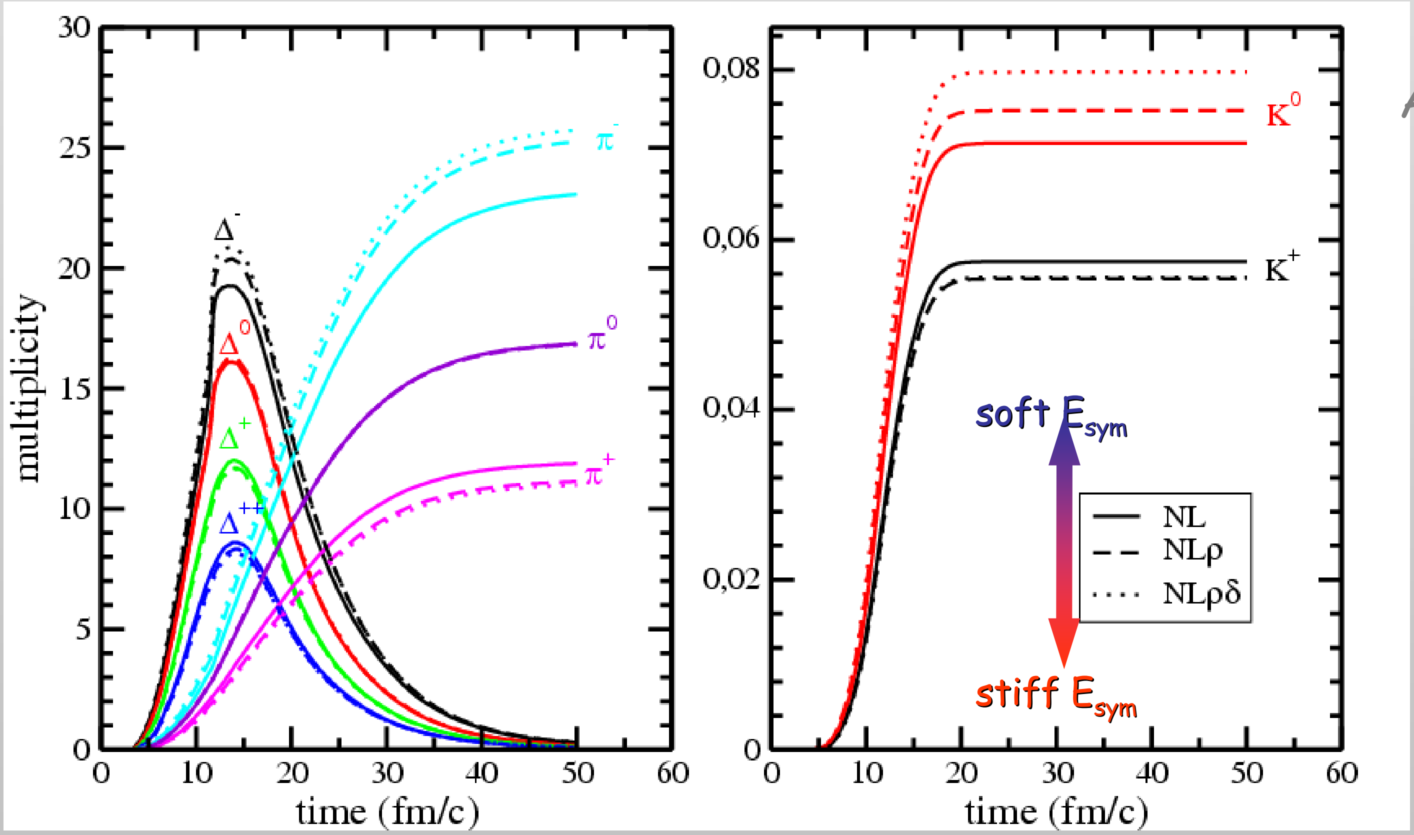
Pion and Kaon production in “open” system (HIC)…

$NN \rightarrow N\Delta \rightarrow NYK$

$\hookrightarrow \pi N \rightarrow YK$

mean field effects: n/p repulsion

threshold effects: Δ and K self energies



Au+Au@1AGeV

Pions: compensation

Kaons:
- **direct early production: high density phase**
- **isovector channel effects**

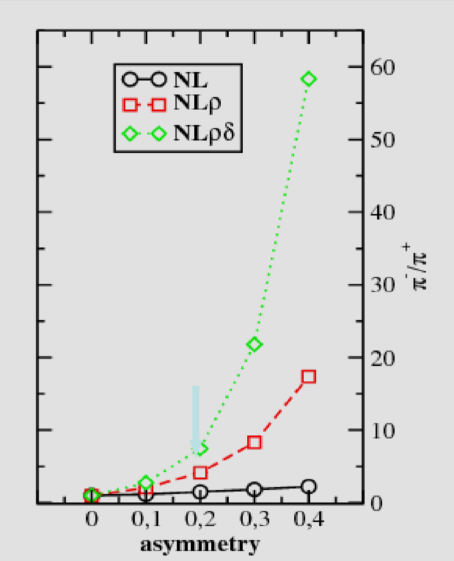
G.Ferini et al.,PRL 97 (2006) 202301

Pion production: Au+Au, semicentral

$$\frac{\pi^-}{\pi^+}$$

Ferini, NPA762(2005) 147

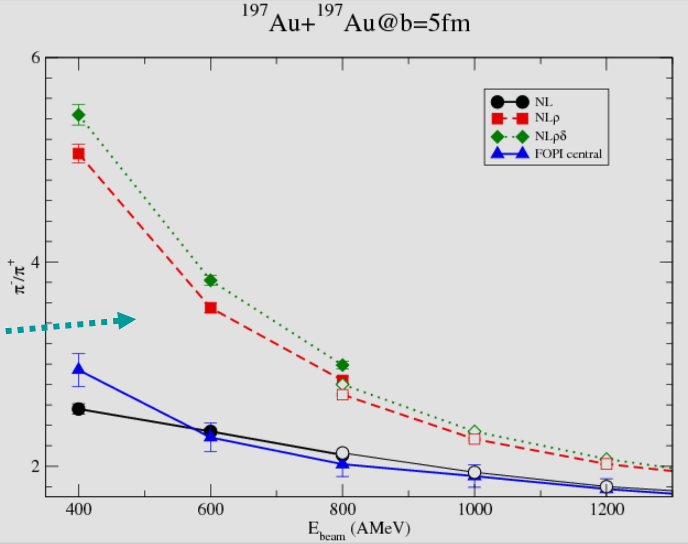
**Equilibrium production
(box results)**



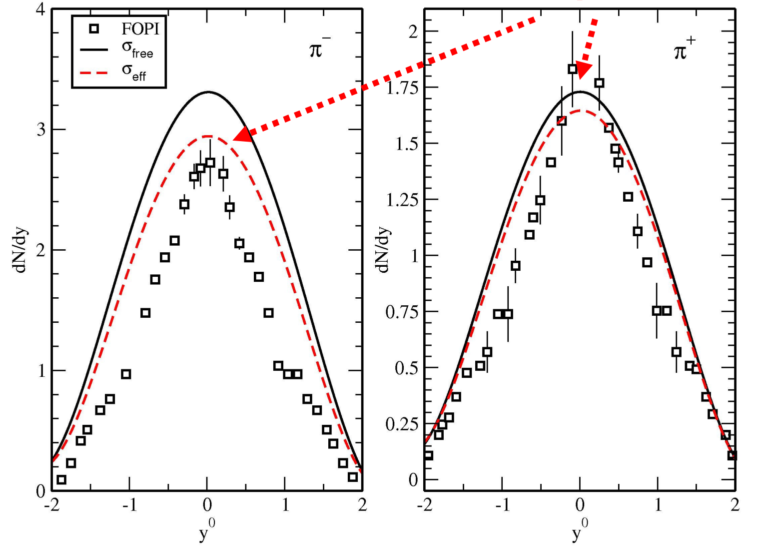
$$\frac{\pi^-}{\pi^+} = \frac{\sigma^{abs}_{\pi-}}{\sigma^{abs}_{\pi+}} \exp\left[2(\mu_n-\mu_p)/T\right]$$

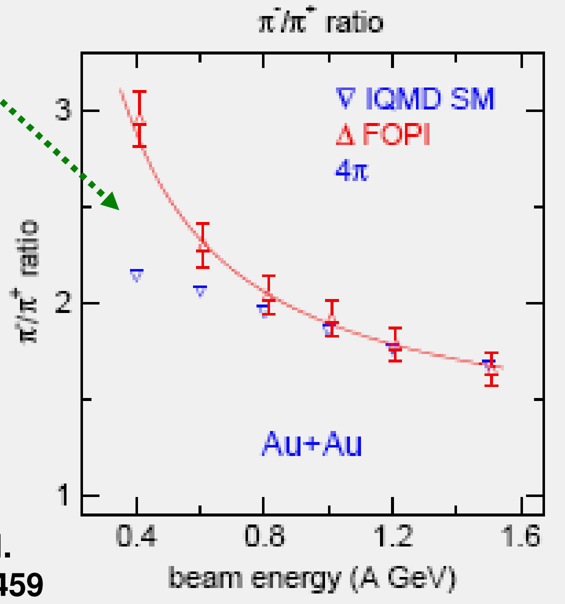
~ 5 (NLρ) to 10 (NLρδ)

Finite nucleus simulation:



disagreement in magnitude, particularly at low energies, (also in other calc.), but better at midrapidity (high density), where Kaons are produced.

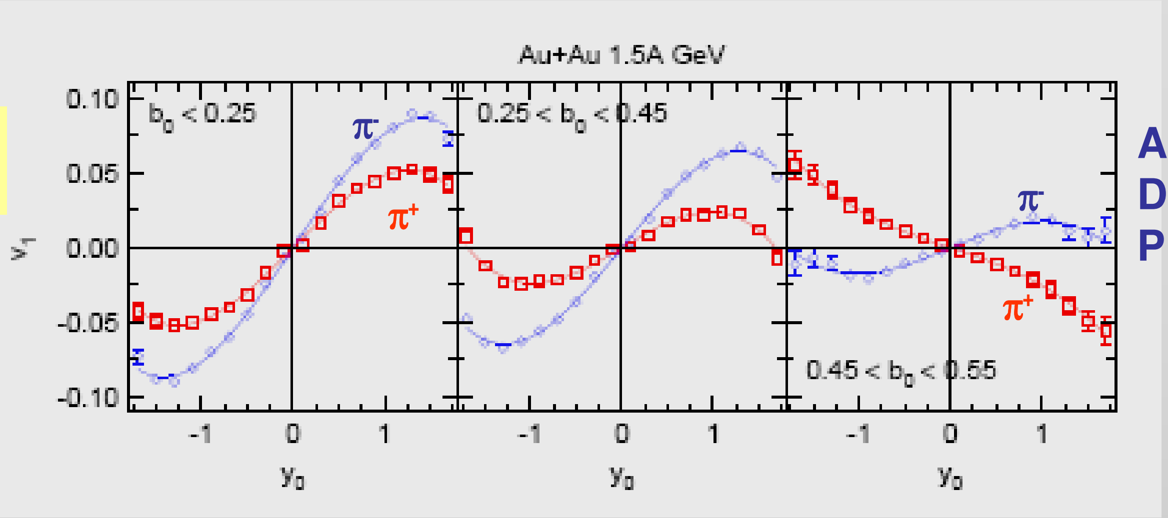




**W.Reisdorf et al.
NPA781 (2007) 459**

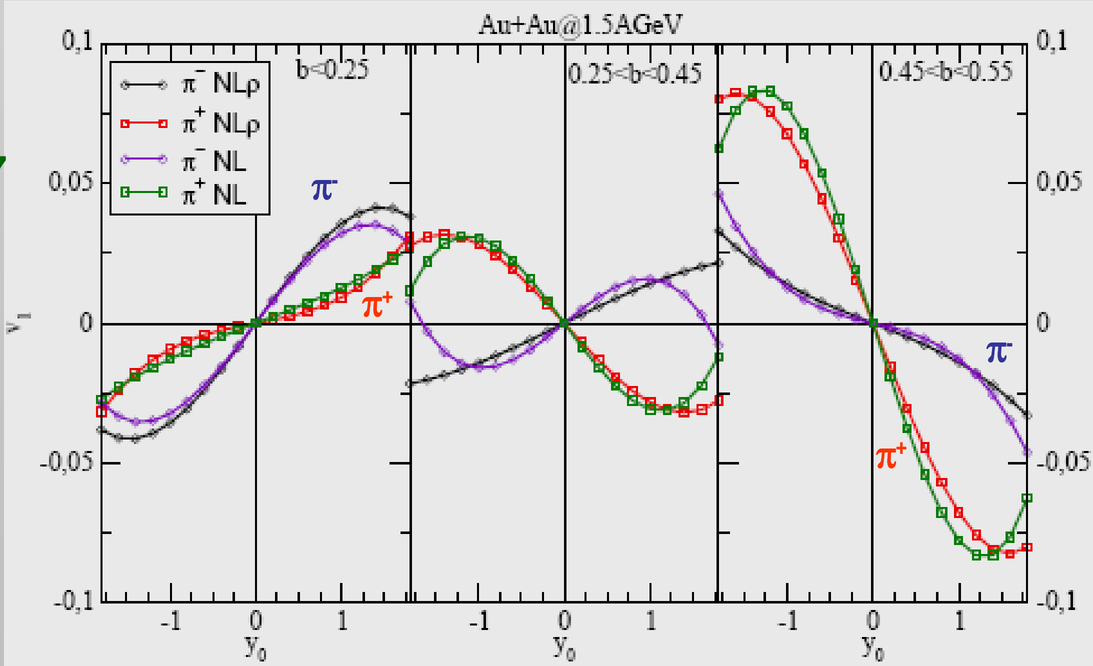
Transverse Pion Flows

W.Reisdorf et al.
NPA781 (2007) 459



Antiflow:
Decoupling of the Pion/Nucleon flows

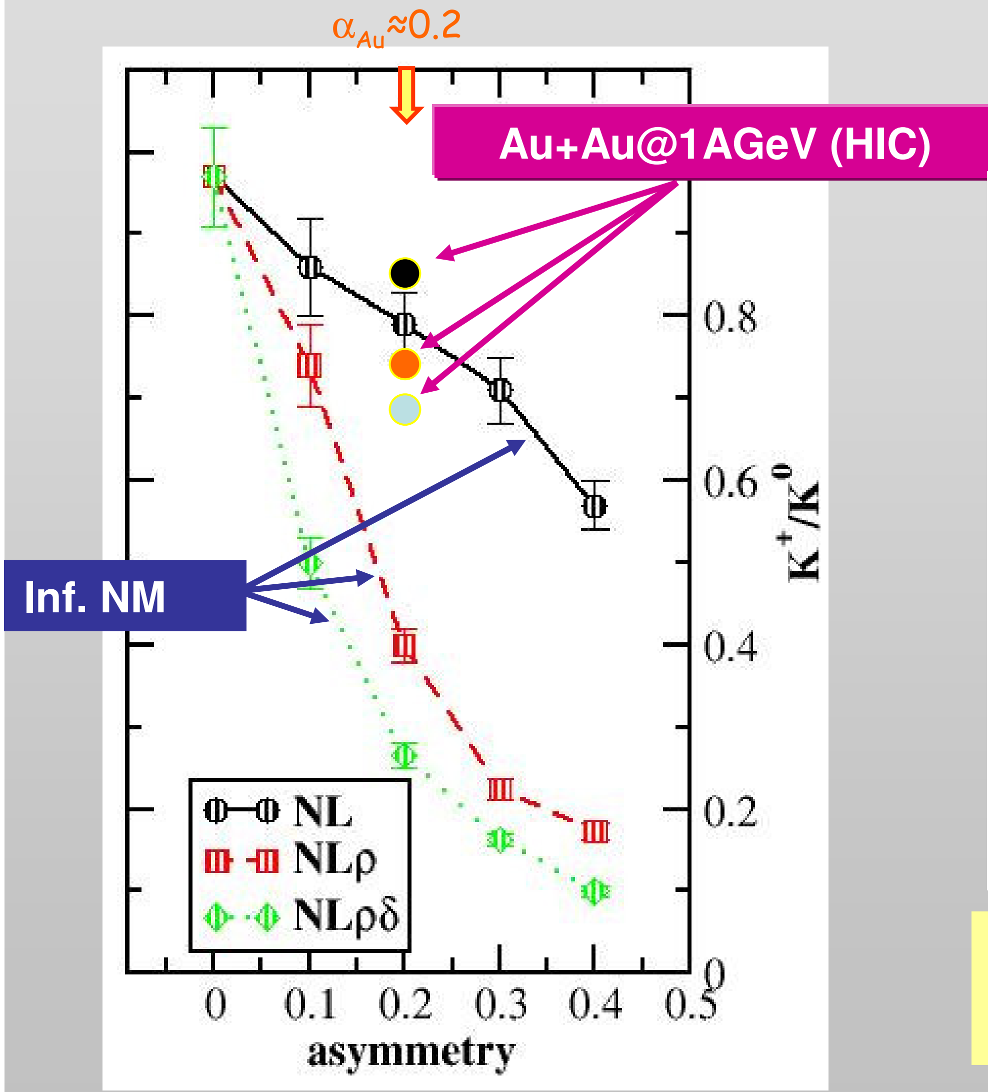
Simulations:
V.Prassa Sept.07



OK general trend.
but:
- **smaller flow for both**
  π- and π+
- **not much dependent on iso-EoS**

Strangeness ratio : Infinite Nuclear Matter vs. HIC

G. Ferini, et al., NPA762(2005) 147 and nucl-th/0607005



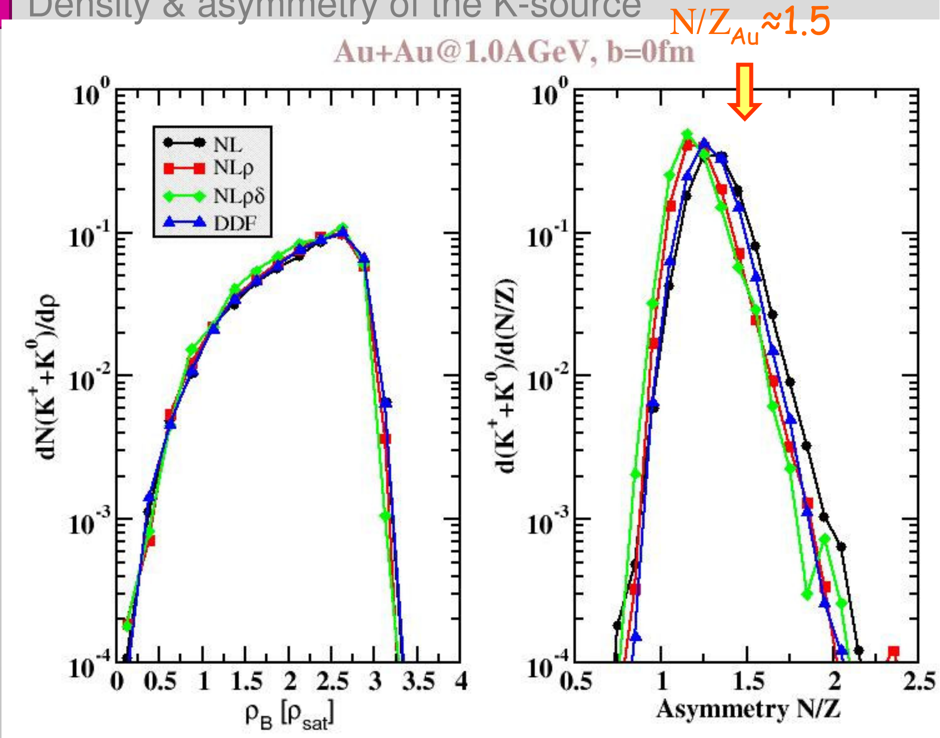
α_{Au}≈0.2

Au+Au@1AGeV (HIC)

Inf. NM

Density & asymmetry of the K-source

N/Z_{Au}≈1.5

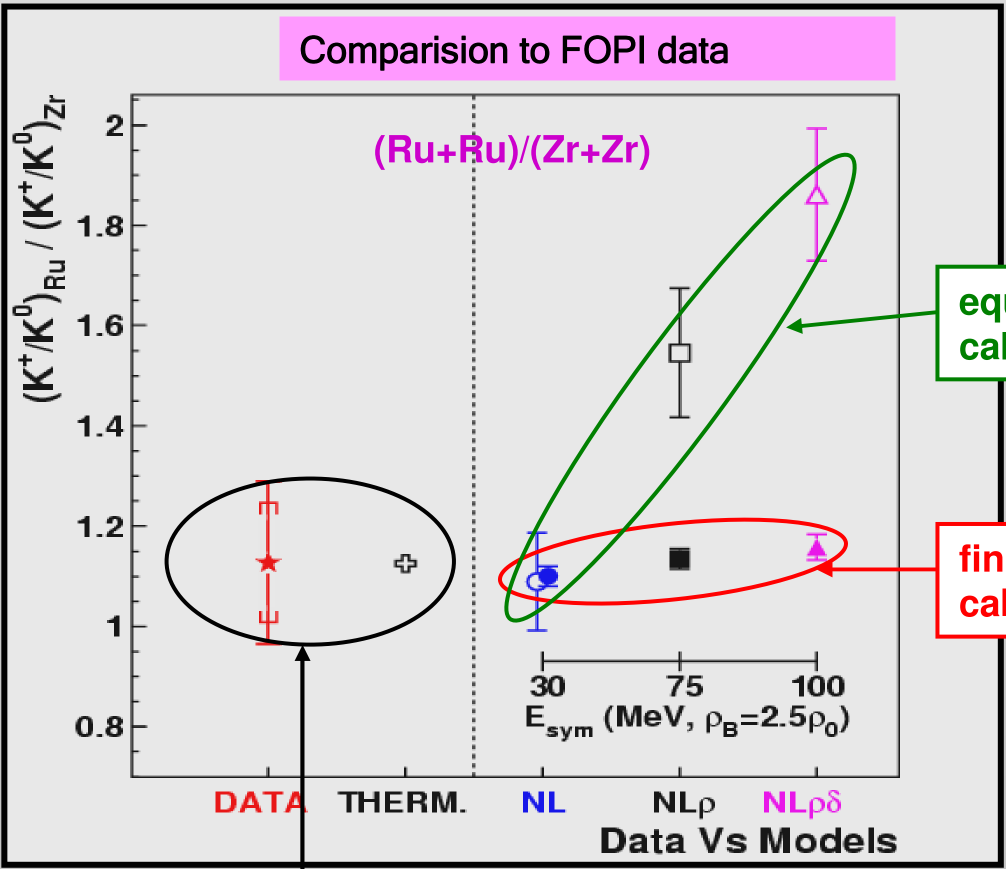


NL→ DDF→NLρ→NLρδ :
more neutron escape and more n→p *transformation*
(less asymmetry in the source)

Pre-equilibrium emission (mainly of neutrons) reduced asymmetry of source for kaon production → reduces sensitivity relative to equilibrium (box) calculation

Kaon ratios: comparison with experiment

G. Ferini, et al., NPA762(2005) 147 and nucl-th/0607005



Comparision to FOPI data

equilibrium (box) calculations

finite nucleus calculations

Data (Fopi)

X. Lopez, et al. (FOPI), PRC 75 (2007)

- **sensitivity reduced in collisions of finite nuclei**
- **single ratios more sensitive**
- **enhanced in larger systems**

Effect of kaon potentials

In-medium Klein-Gordon eq. for Kaon propagation:

$$\left[(\partial_\mu + iV_\mu)^2 + m_K^{*2}\right]\phi_K(x) = 0$$

Two models for medium effects tested:

Chiral perturbation (Kaplan, Nelson et al.)

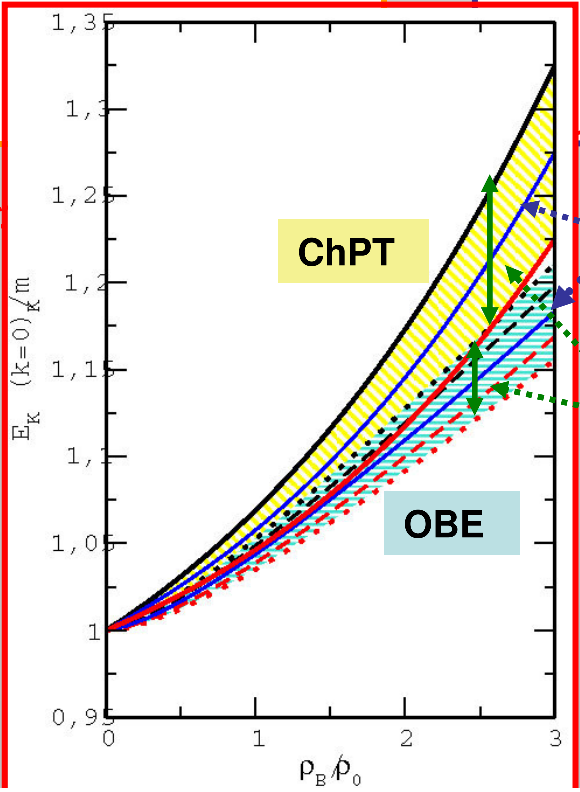
$$m_K^* = \sqrt{m_K^2 - \frac{\Sigma_{KN}}{f_\pi^2}\rho_s \mp \frac{C}{f_\pi^2}\rho_{s3} + V_\mu V^\mu} \qquad (upper\ sign,\ K^+)$$

$$V_\mu = \frac{3}{8f_\pi^{*2}} j_\mu \pm \frac{1}{8f_\pi^{*2}} j_{\mu 3}$$

One-Boson Exchange (Schaffner-Bielich et al.)

$$m_K^* = \sqrt{m_K^2 + \frac{m_K}{3}\left(g_{\sigma N}\sigma \mp f_\delta \rho_{S3}\right)}$$

$$V^\mu = \frac{1}{3}\left(f_\omega^* j^\mu \pm f_\rho j_3^\mu\right)$$



Isospin-dependence

In-medium K energy (k=0)

$$E_K(\mathbf{k}) = k_0 = \sqrt{\mathbf{k}^2 + m_K^{*2}} + V_0$$

Splitting for $K^{0,+}$

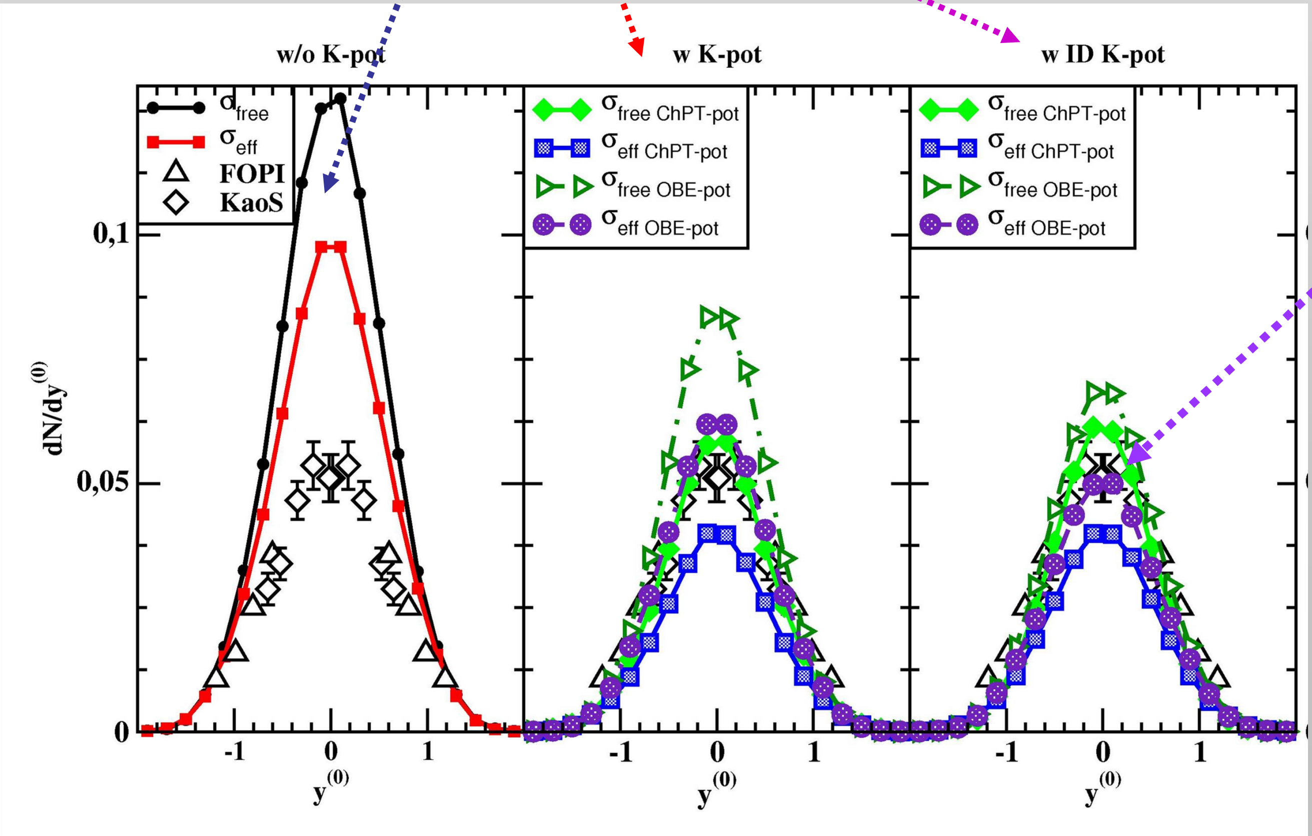
for NLρ and NLρδ

Kaon production (absolute yields)

Sensitivity to: 1. in-medium cross section: σ_{free} vs. σ_{eff}

2. K- potential: ChPT vs. OBE

3. Isospin-dep- (ID) K-potential

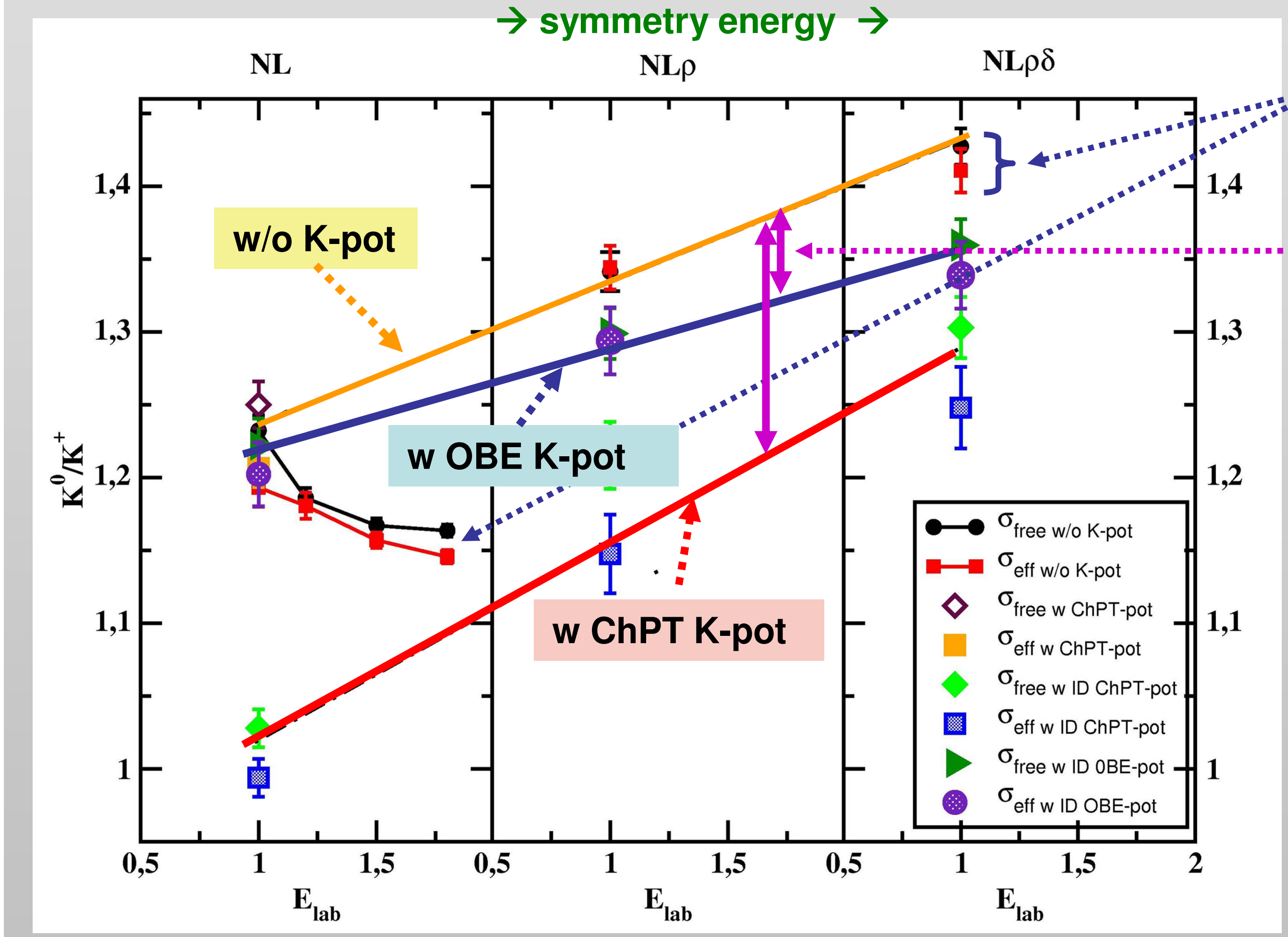


Ni+Ni, E=1.93 AGeV, b<4 fm, rapidity distrib.

- Influence of all ingredients substantial
- rather good description with OBE potential and σ_{eff}

→ consider ratios to minimize influence of ingredients

Kaon ratios: test of „robustness“ against variation of K-potential and σ_{inmed}


→ symmetry energy →

w/o K-pot

w OBE K-pot

w ChPT K-pot

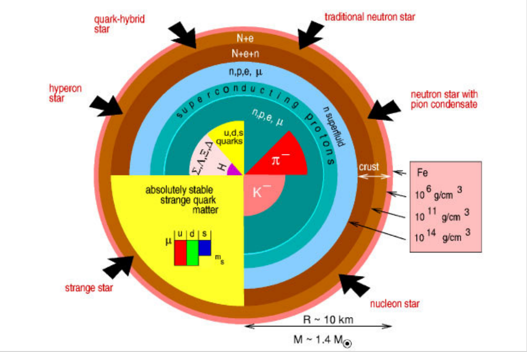
Infuence of σ_{inmed}: small

Influence of K-pot substantial, part. with isospin-dep. Part

substantial!

Aü+Au, b=0 fm

Neutron star masses and cooling and the symmetry energy

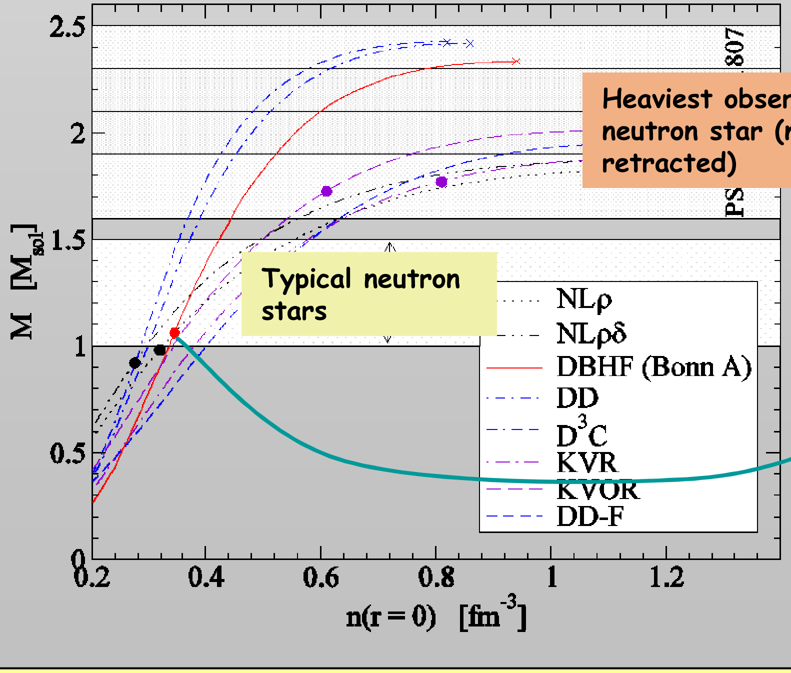


Tolman-Oppenheimer-Volkov equation to determine mass of neutron star

Proton fraction and direct URCA

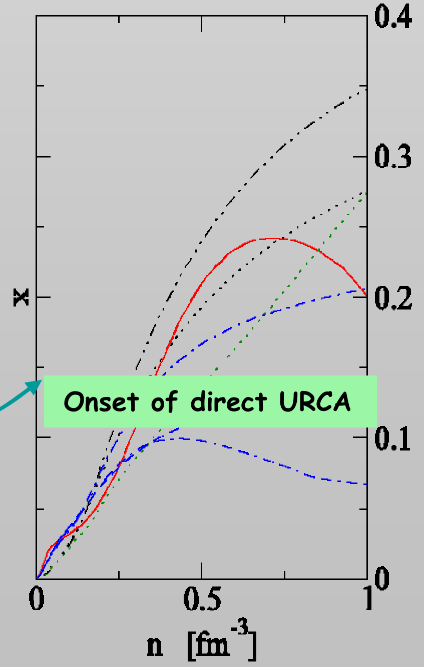
- $\beta-$equilibrium and charge neutrality : $y = \frac{N}{Z} = y(\varepsilon_{sym})$
- direct URCA process : $p \to n + e^{+} + \nu_e$

  threshold : $y \approx 11\%$, fast neutrino cooling



Heaviest observed neutron star (now retracted)

Typical neutron stars



Onset of direct URCA

Forbidden by Direct URCA constraint

Klähn, Blaschke, Typel, Faessler, Fuchs, Gaitanos, Gregorian, Trümper, Weber, Wolter, Phys. Rev. C74 (2006) 035802

Summary and Conclusions:

- While the EOS of symmetric NM is fairly well determined, the density (and momentum) dependence of the symmetry energy is still rather uncertain (but important for exotic nuclei, neutron stars and supernovae)
- Can be investigated in HIC both at low densities (Fermi energy regime, isospin transport) and high densities (relativistic collisions, flow, particle production)
- **Nucleon and light cluster pre-equilibrium emission and flow are directly sensitive to the symmetry energy.**
- Pion and Subthreshold Kaon production are a promising signal, in particular when considering ratios π^-/π^+ and K^0/K^+

  The kaon signal is robust with respect to the in-medium cross sections, but influenced by the model for the Kaon potential
- Effects scale with the asymmetry - thus reactions with RIB are very important