

Bulk viscosity and freeze-out in heavy ion collisions

Based on: 0707.4405,0805.0442

(Both published, PRC)

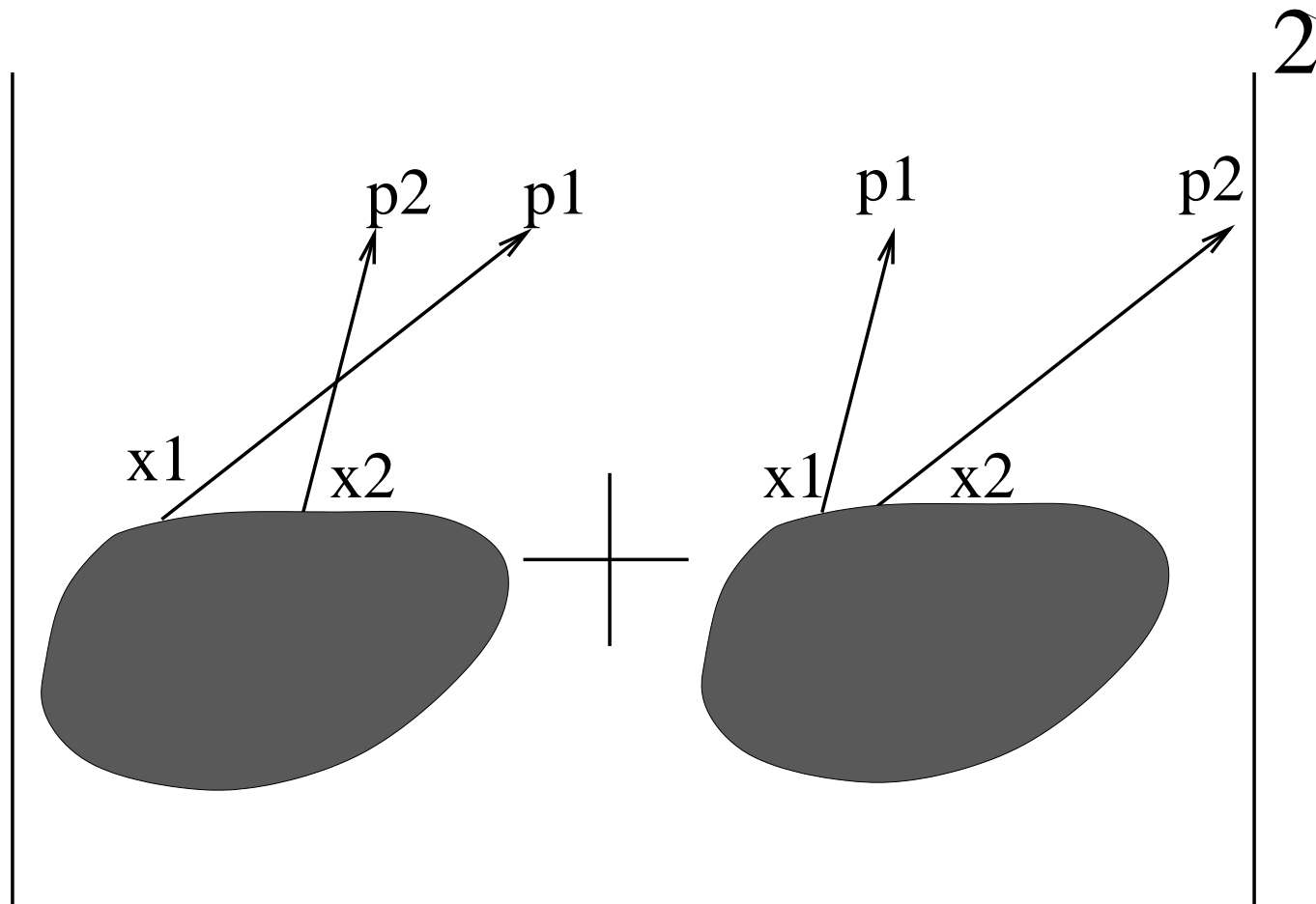
Giorgio Torrieri (With I. Mishustin, B. Tomasik)



Outline

- HBT: What is it, why its relevant, and how it does not fit
- Bulk viscosity in QCD: A short introduction
- A guess of what it means for freezeout
- Stability of hydrodynamics with a QCD-inspired bulk viscosity
- Some discussion

HBT: classical source emitting quantum free particles



$$\Psi(x_{1,2}, p_{1,2}) = \frac{1}{\sqrt{2}} \left(S(x_1, p_1) S(x_2, p_2) e^{i(p_1 x_1 + p_2 x_2)} \pm S(x_2, p_1) S(x_1, p_2) e^{i(p_2 x_1 + p_1 x_2)} \right)$$

Measurement of $C(p_1, p_2)$ gives handle on $S(x, p)$

$$C(p_1, p_2) \sim |\tilde{S}(p_1 - p_2, p_2)|^2$$

Where the momentum correlation coefficient $C(p_1, p_2)$ is

$$C(p_1, p_2) = \frac{\rho(p_1, p_2) - \rho(p_1)\rho(p_2)}{\rho(p_1)\rho(p_2)}$$

And $\tilde{S}(k, q) = \int d^4x S(x, q) e^{ikx}$

What is $S(x, p)$? Assuming at a “critical” $\Sigma^\mu = (t_f, \vec{x}_f)$ (defined by t, l_{mfp}, \dots) mean free path goes from 0 (ideal hydro) to ∞ (free particles), by Energy, momentum and entropy conservation

$$S(x, p) = d\Sigma_\mu p^\mu \frac{1}{e^{u_\mu p^\mu / T} \pm 1}$$

Hydro plus a freeze-out criterion (usually, $T = 100$ MeV or so. NOT the same as QGP-HG) gives u^μ, Σ_μ

This of course is a very rough approximation. “doing better” is model dependent. This is why signatures insensitive to freeze-out (as $v_2, \text{jet suppression are thought to be}$) are highly considered. But we should still get a qualitative effect from formation of a new state of matter

Usually $\tilde{S}(q, p) \sim \underline{\text{Gaussian}} \Rightarrow$ parametrization in terms of $R_{out}, R_{side}, R_{long}$

$$S(\underbrace{k}_{p_1+p_2}, \underbrace{q}_{p_1-p_2}) \simeq N(k) \exp [R_o^2(k)q_o^2 + R_s^2(k)q_s^2 + R_l^2(k)q_l^2 + R_{ij}(k)q_iq_j]$$

S.Pratt, PRD33, 1314 (1986), G. F. Bertsch, NPA498, 173c (1989).

"long" Beam direction (\vec{z})

"out" $(\vec{p}_1 + \vec{p}_2) \times \vec{z}$

"side" "out" \times "long"

$k_{side} = 0$ by construction

This parametrization is useful because...

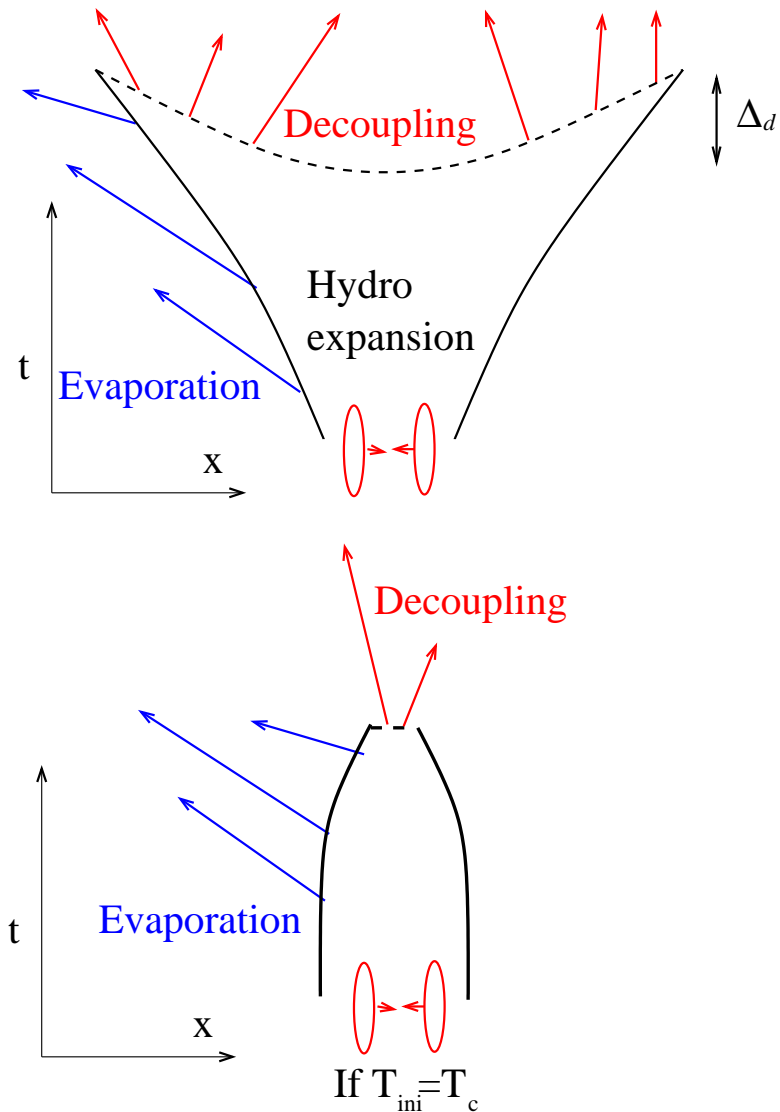
If

$$\langle (\Delta x^\mu)^2 \rangle (p) = \int d^4x S(x, p) (x - \langle x \rangle)^2$$

then

$$R_o^2 = \left\langle \left(\Delta r - \frac{k_o}{k_0} \Delta t \right)^2 \right\rangle$$
$$R_s^2 = \langle (\Delta r)^2 \rangle$$

Comparing R_0 and $R_s \rightarrow$ emission time. This was “the” signature for deconfinement, as it probed softness of EOS!

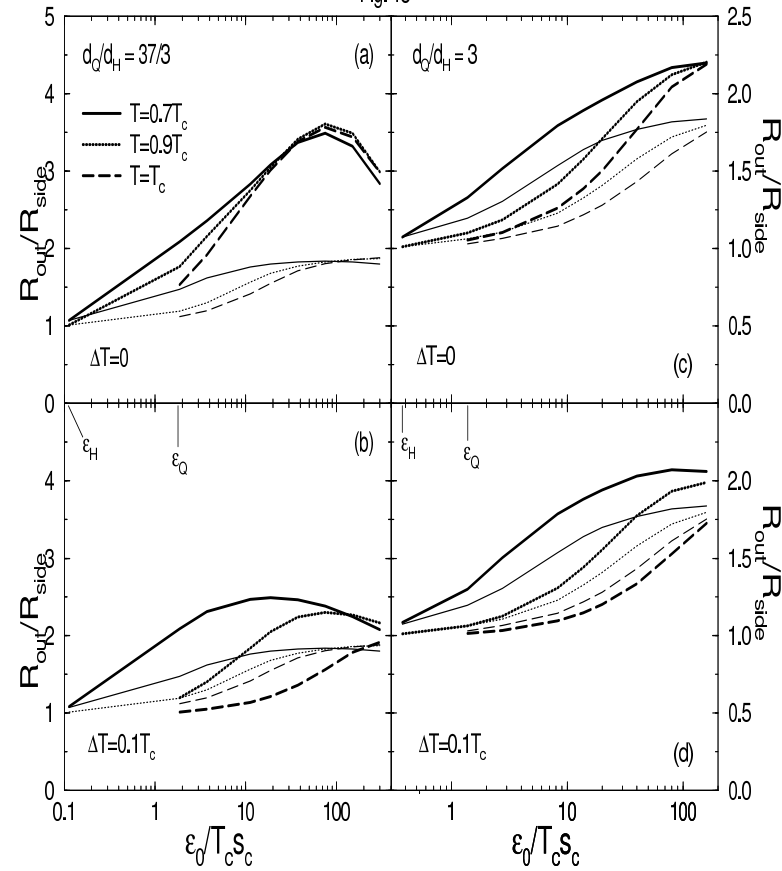


Rischke and Gyulassy

Many HIC detectors
OPTIMIZED for HBT
on basis of such papers

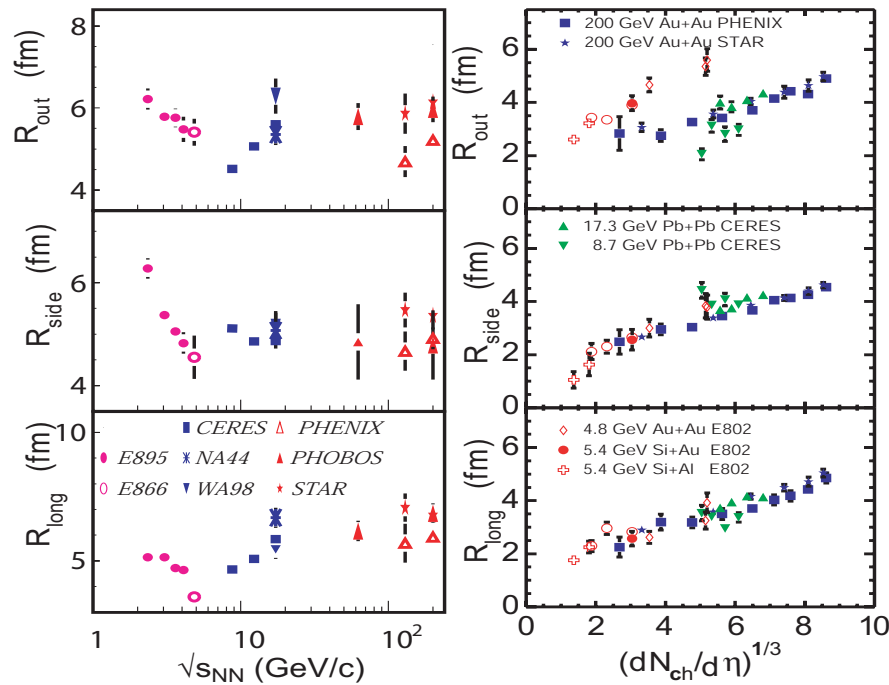
nucl-th/9606039

Fig. 18



QCD EoS not first order, but ANY softening of EoS, \rightarrow More evaporation (Unless $T_0 \gg T_c$)

Motivation: The HBT puzzle I: Why no effect of transition on R_o/R_s



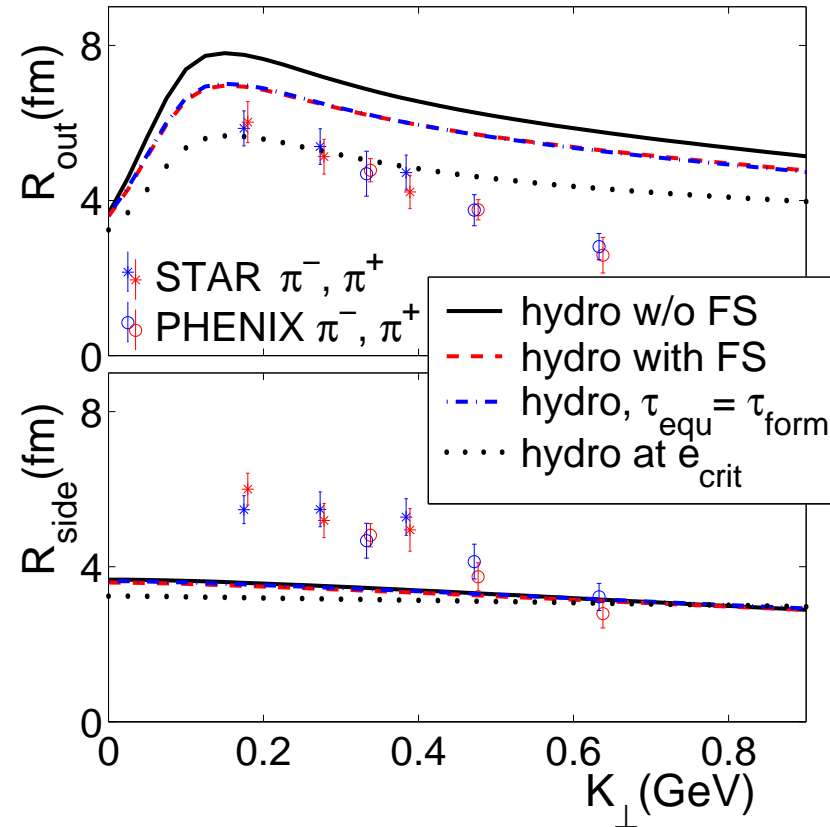
All radii are constant with energy

(Scale well with $(dN/dy)^{1/3}$)

M. Lisa
nucl-th/0701058

No first order phase transition, but why no signal at all?

And scaling with $dN/dy^{1/3}$ implies sudden break-up. not compatible with hydro+constant T f.o., as larger fireballs freeze-out more slowly



$R_o/R_s = 1$, does not fit hydro: Even for no phase transition, R_o/R_s should increase with \sqrt{s} . Unless...

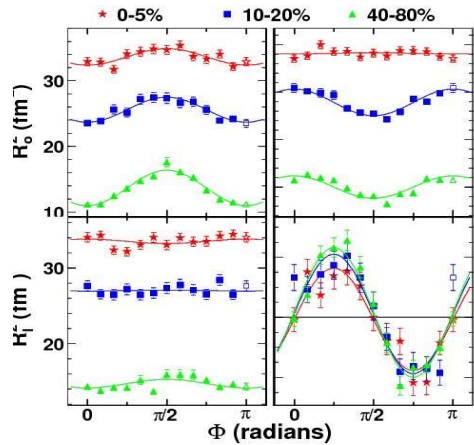
Does this mean:

(a) HBT is complicated (Gaussian approximation, homogeneity regions, deviation from chaoticity, resonances, reinteractions,...) let's not care too much if we get it wrong.

S.Pratt,WPCF2008: HBT puzzle nearly solved by interplay of pre-existing flow, correct (soft) EoS and viscosity

(b) Our physics understanding is basically correct. But something is missing that would allow us to understand freeze-out.

(c) Panic! We don't have a clue! (whole model wrong)



Heinz and Kolb, nucl-th/0208047

AZIMUTHAL dependence of HBT radii works with same parameters as v_2 .

This suggests hydro basically OK

UP TO absolute freeze-out criterion

WHY NOT (c) (Dont panic!)

Absolute value of HBT has been described in a plethora of "Hydro-inspired" fit models

"Blast wave"

Flow+Sudden freezeout
(II lab frame)
put artificially

"Buda-Lund"

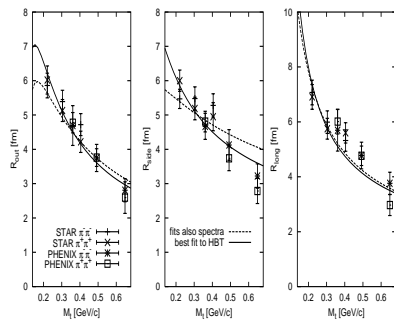
Hot ($>T_c$) core
+Colder halo

"Krakow model"

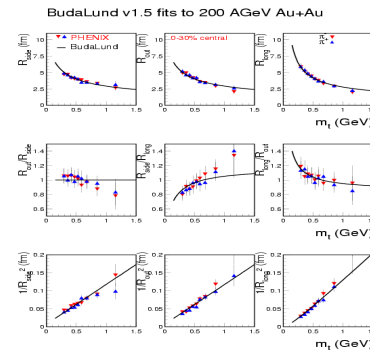
Hubble expansion and high
(chemical) T freeze-out

Many more.

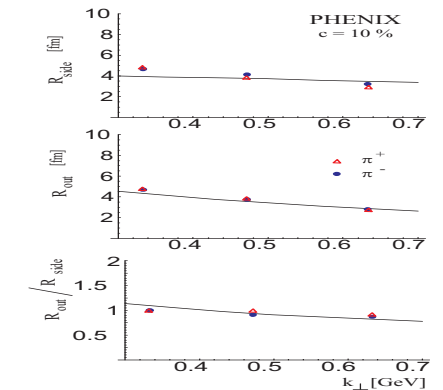
These are FITS
but they suggest
where to look
for explanations



Frodermann et al, nucl-th/0602023

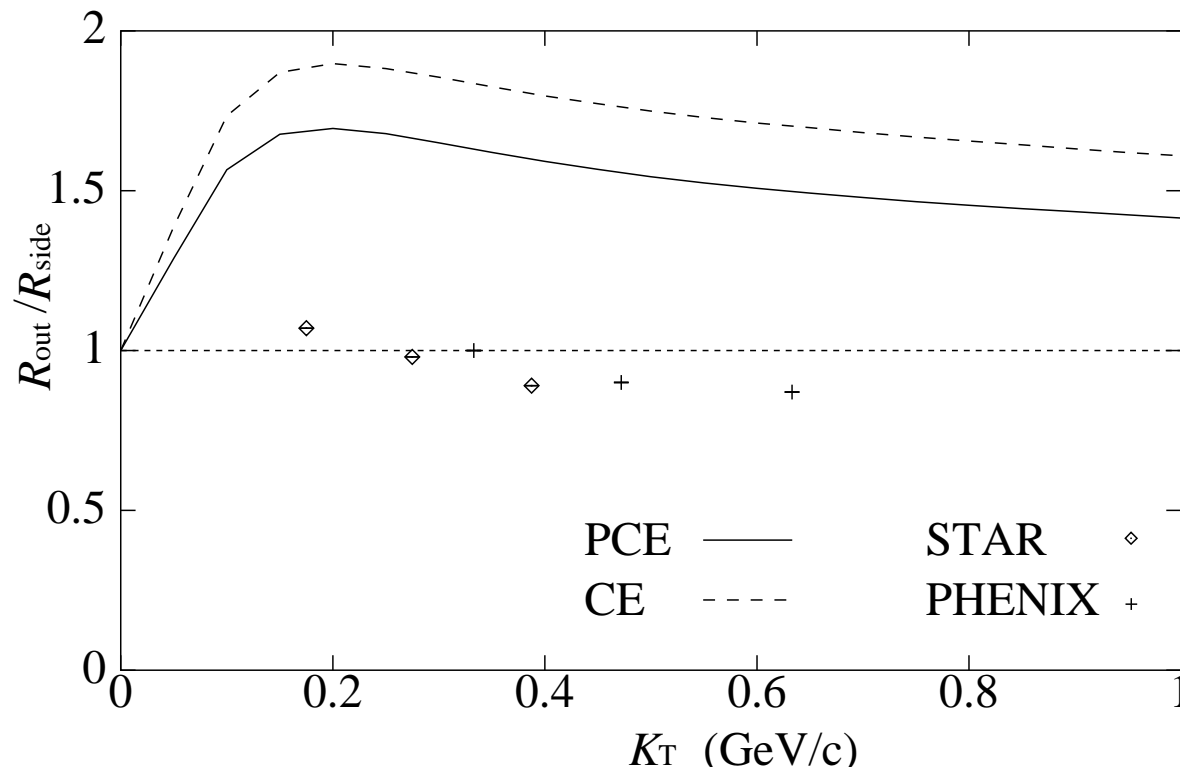


Csorgo et al, nucl-th/0510027



Baran et al, nucl-th/0212053

Why not (a) (Dont get complacent)



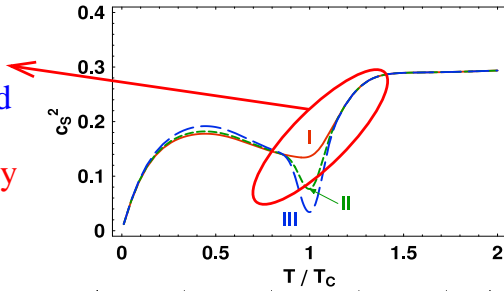
Hirano et. al.
nucl-th/0208068

Also Teaney,
Shuryak
Soff
Bleicher
Dumitru
Bass
...

All "most obvious Improvements" (3D, hadronic afterburner) spoil the fit.
Only "good" fit so far by Krakow group (0808.3663).

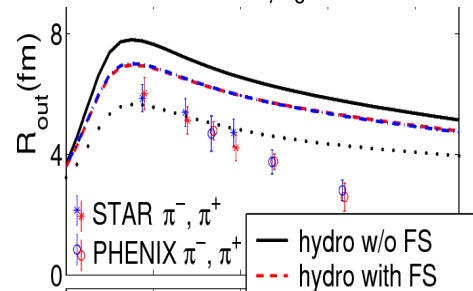
What did Chojnacki et al (arXiv:0808.3663) do differently?

There should be
STRUCTURE
when energy scanned
But R_{out}/R_{side} flat,
radii scale with dN/dy



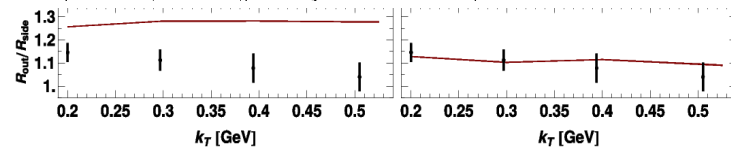
I Smooth cross-over
of speed of sound
Lattice gives a large
systematic error

But not enough!
Romatschke et al also
has this



Simultaneous
freezeout at
high T
(150–170 MeV)
(Here from U. Heinz)

Big effect, but why?
If freeze-out Temperature high
($\sim T_c$) there SHOULD be
reinteraction



Gaussian initial conditions
($\sim 20\%$ correction)

Progress... but f.o. condition has to be justified, and what about SCALING?

What we might be missing... Bulk viscosity.

In hydrodynamic system, bulk viscosity is a kind of flow-dependent force counteracting pressure

$$p \rightarrow p - \underbrace{\zeta}_{\text{Bulk viscosity}} \partial_\mu u^\mu$$

Its a “stickiness” of inter-molecular forces, related to conformal transformations

$$\underbrace{\hat{\lambda}}_{\vec{x} \rightarrow \lambda \vec{x}, \vec{p} \rightarrow \lambda^{-1} \vec{p}} T_{\mu\nu}|_{hydro} \rightarrow T_{\mu\nu}|_{hydro} (p - \lambda \partial_\mu u^\mu)$$

So if EOS conformally invariant $\zeta = 0!!!!$

Does this apply to QCD? Lagrangian nearly conformally invariant

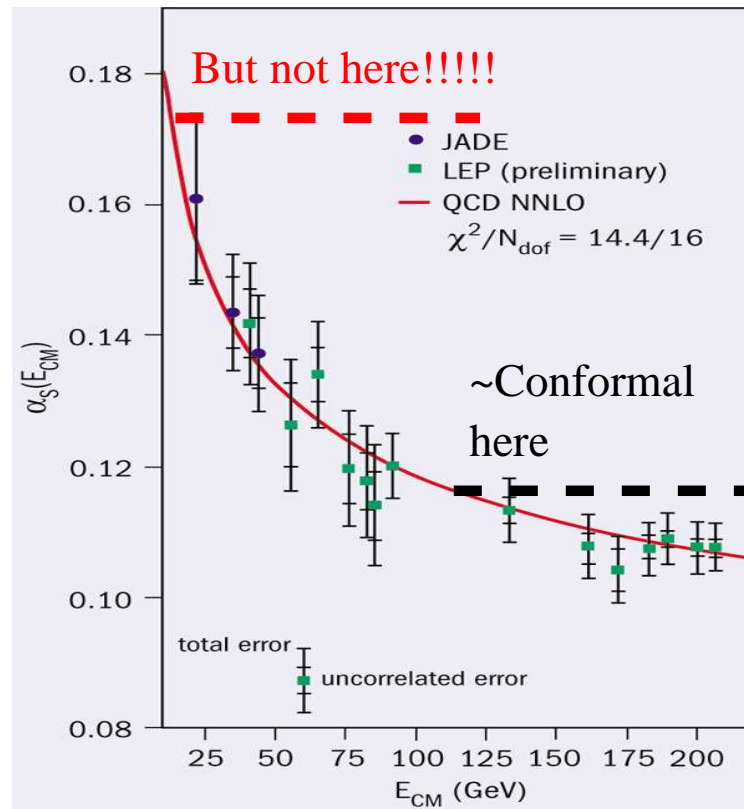
$$L_{QCD} = -\frac{1}{4} \underbrace{F_{\mu\nu}^i F_i^{\mu\nu}}_{Invariant} + \underbrace{\partial_\mu q_f - i \overbrace{g_s}^{Watch!} A_\mu^j \hat{t}^j q_f}_{Invariant} - \underbrace{m_f q_f \bar{q}_f}_{m_f \ll T}$$

Perturbatively

$$\frac{\zeta}{\eta} \sim 10^{-5}$$

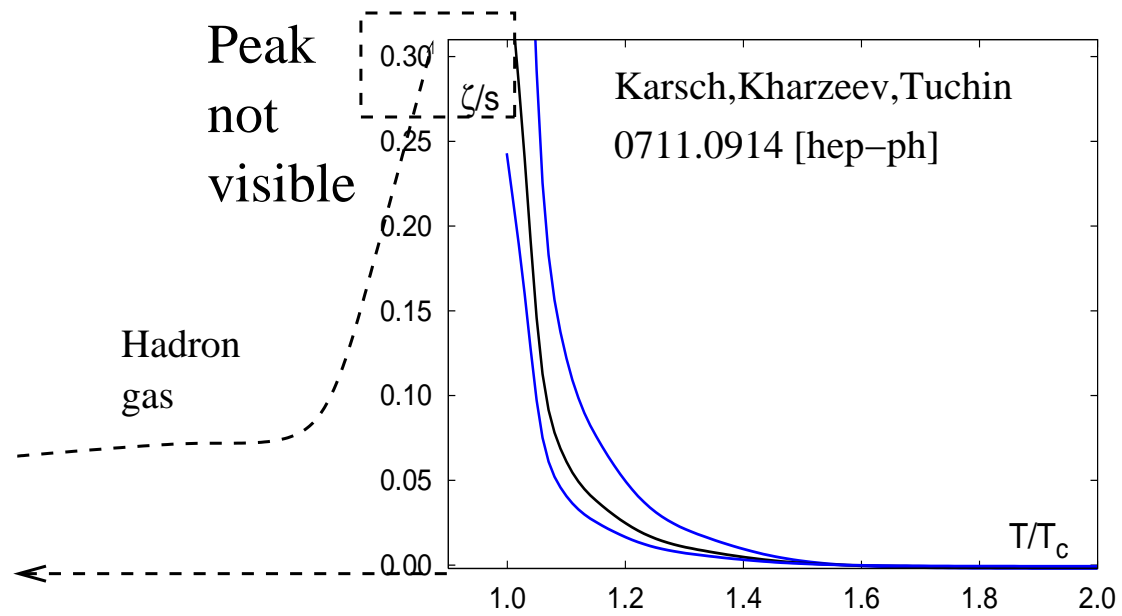
(Dogan, Arnold, Moore)

But quantum corrections break conformal symmetry.



Conformal anomaly small at weak coupling, diverges at strong coupling.

What does bulk viscosity do in this regime?



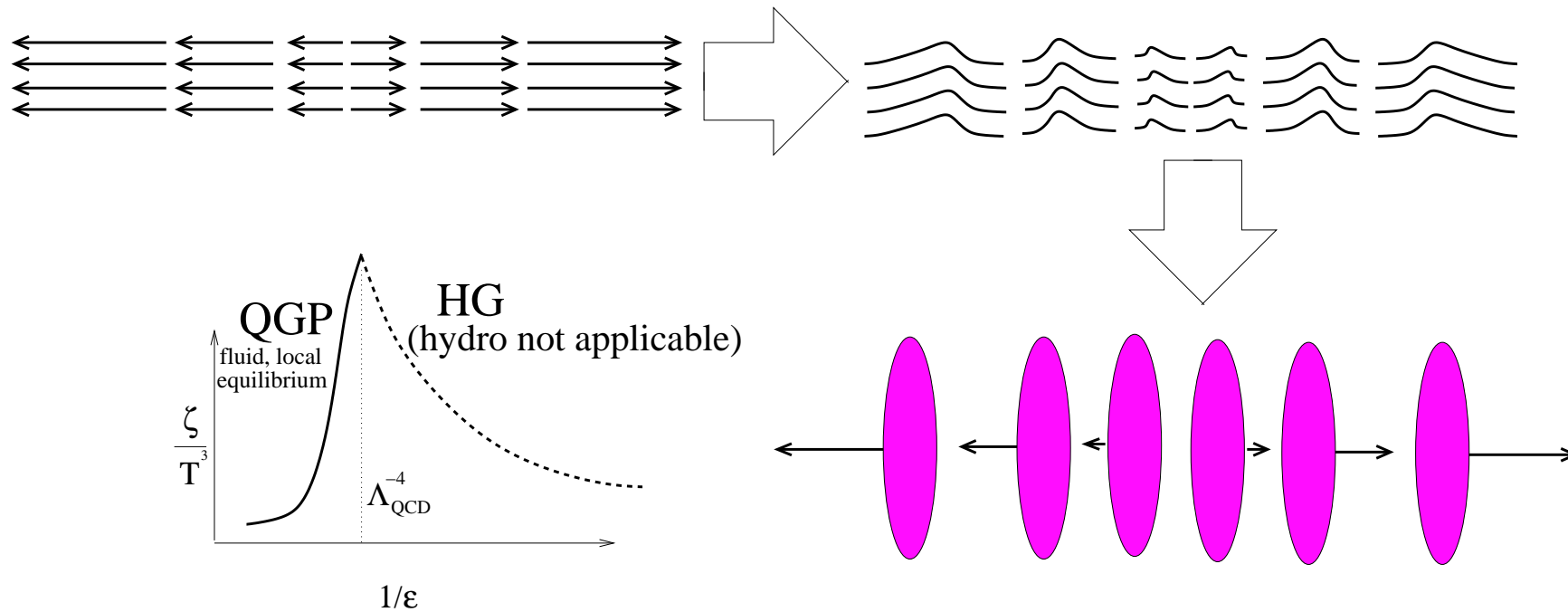
Lattice evidence, and an educated guess from QCD says it peaks, maybe even diverges

So ζ should dominate strong coupling dynamics. How?

Oil2Honey

ζ diverging \rightarrow Hydro not a good description anymore

But then All we have is guessing. So lets guess!



What I will say in the next pages of mathematics can be easily "conceptually" visualized: Imagine a homogeneously expanding gas, that at a certain point, turns into "glue" (with the inertial forces still in place). The homogeneous expansion continues being a solution. but its obvious that a filamentation of the medium occurs

A more formal derivation

relativistic hydrodynamics. General hydrodynamics ($Q = 0$): Conservation of energy-momentum

$$\partial_\mu T^{\mu\nu} = 0$$

local thermalization

$$T_{\mu\nu} = \left(\underbrace{p}_{\text{pressure}} + \underbrace{\rho}_{\text{energy}} \right) u_\mu \underbrace{u_\nu}_{\text{flow}} - p g_{\mu\nu} + \Pi_{\mu\nu}$$

together with Equation of state, closed system (solvable for all initial conditions). Deviations encoded in $\Pi_{\mu\nu}$. First order (Navier-Stokes)

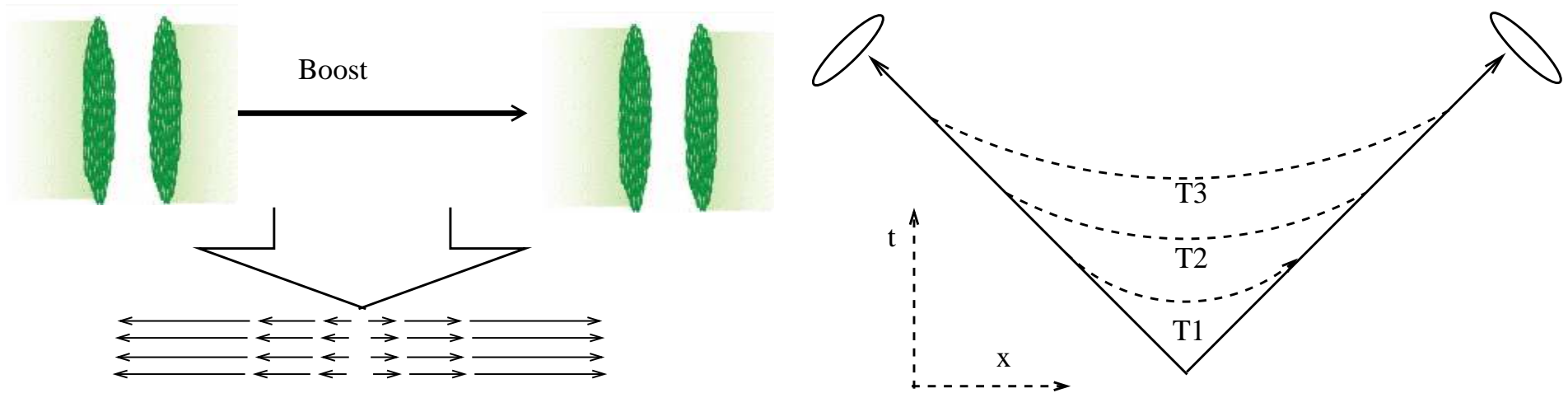
$$\Pi_{\mu\nu} = \eta \partial_{\langle\nu} u_{\mu\rangle} + \zeta \partial_\alpha u^\alpha u_\mu u_\nu$$

Only bulk viscosity just means “effective pressure”

$$P \rightarrow P - \zeta \partial_\alpha u^\alpha$$

Boost invariance (“Bjorken” hydrodynamics)

Basic idea: If $\sqrt{s} \gg m$ System invariant under boosts



$$v_z = \frac{z}{t}, \quad e = e\left(\tau = \sqrt{z^2 - t^2}\right), \quad \partial_\alpha u^\alpha = \frac{1}{\tau}$$

Easily generalized to N-Hubble dimensions, $\vec{v} \propto \vec{x}/t$, M homogeneous dimensions

The M-dimensional Hubble expansion of a viscous fluid homogeneous in $N - M$ dimensions ($N = 3, M = 1 \sim$ heavy ions, $N = M = 3 \sim$ cosmology with flat space) obeys

$$\tau^{-M} \frac{d(\tau^M s)}{d\tau} = \frac{Ms}{R\tau}$$

Where τ is the proper time, s is the entropy and the Reynolds number R is

$$R^{-1} = \frac{2(1 - M/N)\eta + M\zeta}{Ts\tau}$$

Where η is the shear and ζ is the bulk viscosity. **Linearized hydrodynamics:**

$$s(\tau) = s_0(\tau) + \delta s(\tau, y) e^{iky}$$

$$y = y_{spacetime} + \delta y e^{iky}$$

We assume the bulk viscosity to be peaked around T_c

.An ansatz compatible with lattice QCD is

$$\zeta = s \left(z_{pQCD} + z_0 \exp \left[-\frac{(T - T_c)^2}{2\sigma^2} \right] \right)$$

where $\zeta_{PQCD} = 10^{-3}s$. The shear viscosity (indistinguishable from bulk viscosity in 1D) is given in the strongly coupled limit as

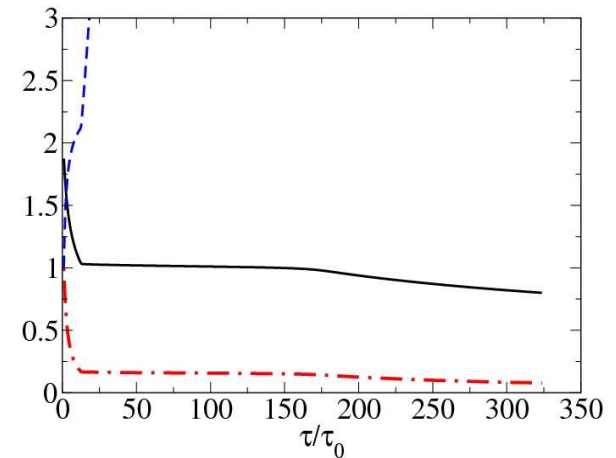
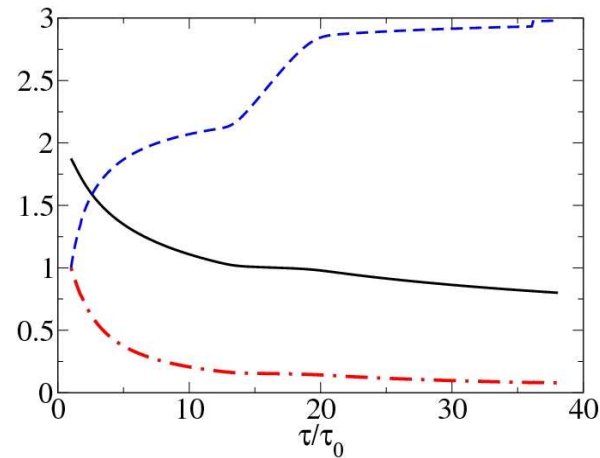
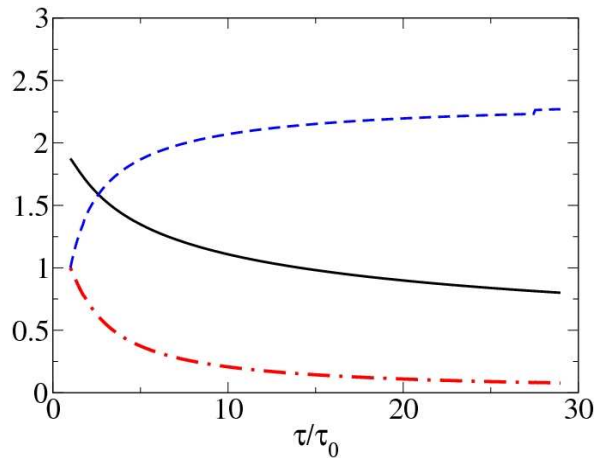
$$\eta = s/4\pi$$

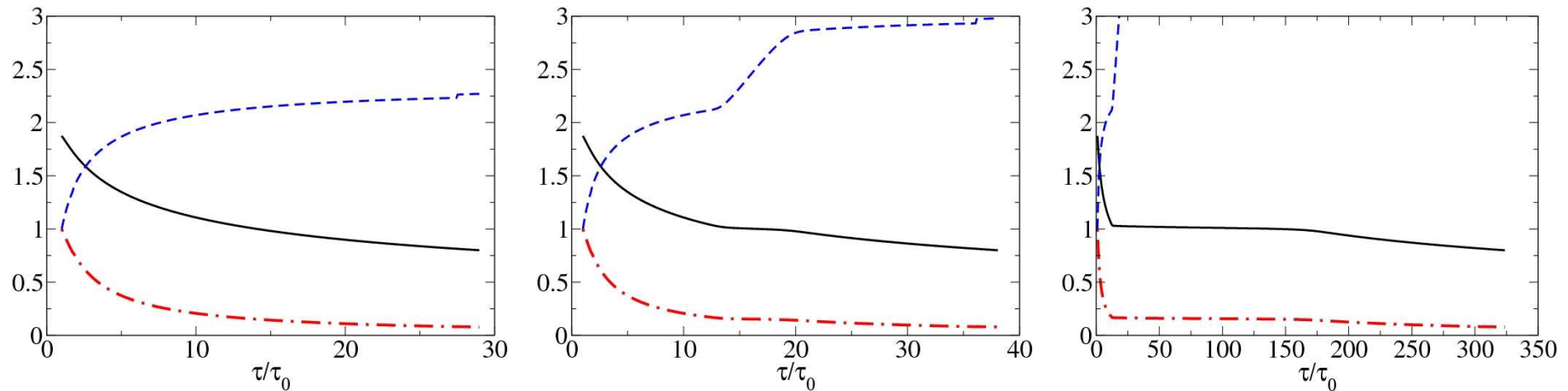
The results are qualitatively independent of N, M , EoS ($s(T) \sim T^3$ or lattice).

Results: Background Solution evolution

Diagram shows Entropy density (Red dot-dashed, Normalized by $\tau = \tau_0$), total entropy in one unit of rapidity (Blue dashed, normalized), and temperature (Black solid, normalized by T_c) for $N=3, M=1$.

Panels show no peak ($z_0 = 0$), a small peak ($z_0 = 1$) and a dominant peak ($z_0 = 10$). In all cases $\sigma = 0.01T_c$.





When viscosity has a peak, temperature and entropy density encounter a plateau. During this plateau (whose duration increases rapidly with the peak amplitude, further expansion is dissipated away, and the co-moving temperature and entropy content of the system is nearly static. Total entropy, consequently, increases rapidly.

As the next sections show, in this plateau the solution is unstable against small perturbations

Stability analysis in hydrodynamics

This analysis was pioneered by Kouno et al, PRD 41, 2903 (1990). Expand perturbations around the **background solution** of the 1D Hydrodynamic equation (in Rapidity, and time) and **instabilities** $x_{1,2}$ in $s(\tau), v$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \delta s/s_0 \\ y_{flow} - y_{spacetime} \end{pmatrix} e^{iky_{spacetime}}$$

Equation of motion for instabilities is of the form

$$\tau \frac{\partial}{\partial \tau} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A_{11}(\tau, k, R) & A_{12}(\tau, k, R) \\ A_{21}(\tau, k, R) & A_{22}(\tau, k, R) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

where $A_{ij}(\tau)$ are expressions in terms of the background solution (too long to quote here, see Kouno's paper).

Stability determined by the equation of motion governing the modulus $\vec{x}\vec{x}^T$

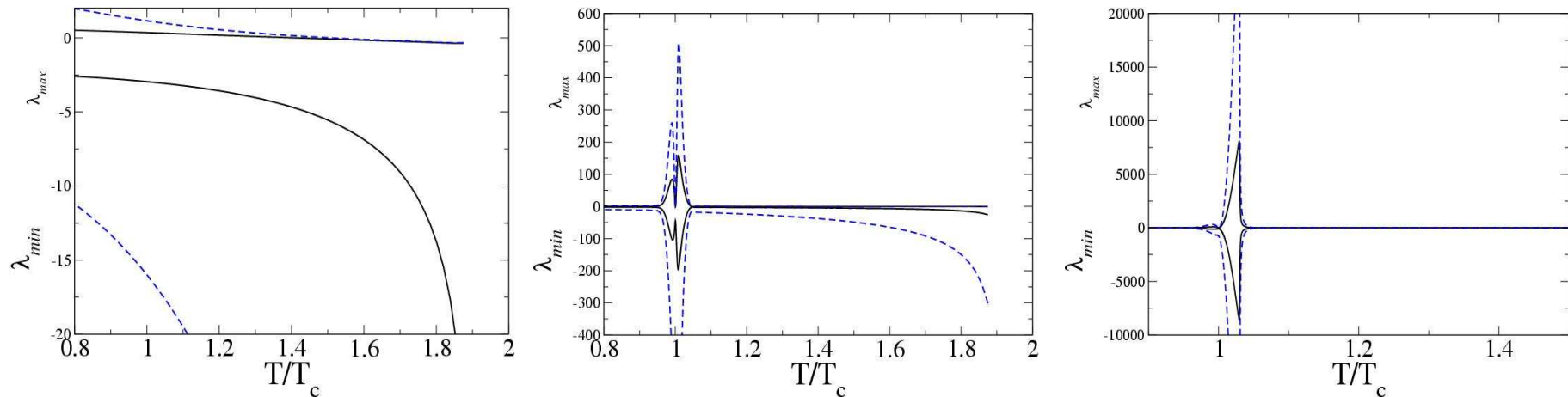
$$\tau \frac{\partial}{\partial \tau} \vec{x}\vec{x}^T = \vec{x} \left(\vec{A}^T + \vec{A} \right) \vec{x}$$

The stability is determined by the eigenvalues of $\left(\vec{A}^T + \vec{A} \right)$ ($\lambda_{min,max}$).

$$\lambda_{min} \vec{x}\vec{x}^T < \tau \frac{\partial}{\partial \tau} \vec{x}\vec{x}^T < \lambda_{max} \vec{x}\vec{x}^T$$

If $\lambda_{min,max} > 0$, solution unstable. If $\lambda_{min,max} < 0$ solution stable. If one is positive, the other negative, there is one unstable and one stable mode.

Results: Eigenvalues The plot below shows the Eigenvalues of the system for $k=4$ (black line) and 8 (blue line).



Independently of k , away from T_c the system is either stable or slightly unstable. Towards T_c , however, the modulus of both Eigenvalues increases throughout the peak. **This means perturbations in the unstable Eigenmode can grow to a large (compared to the background solution) value in a short ($\sim fm$) time, destroying the background solution**

What if there are stable and unstable solutions?

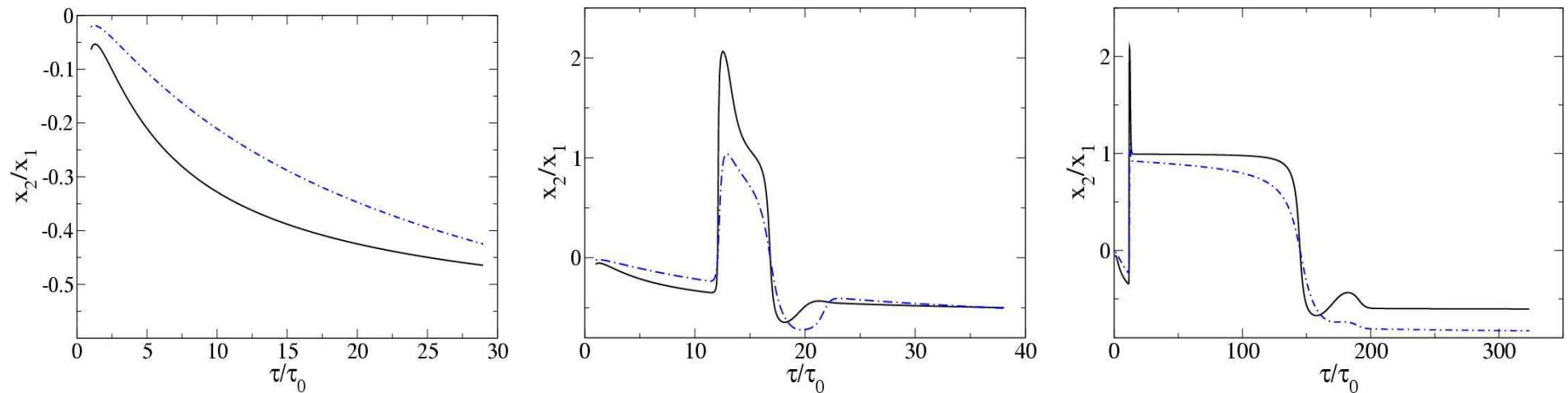
On one end One growing unstable mode is enough to destroy the symmetry of the system

On the other the time-evolution of $A_{i,j}(\tau)$ could well prevent the unstable mode from growing by rotating the Eigenvectors. The complete equation of motion for \vec{x} , needs to be integrated to fully take this effect into account.

We need to solve the complete equation of motion to understand the behaviour of the instabilities

Results: Eigenvectors

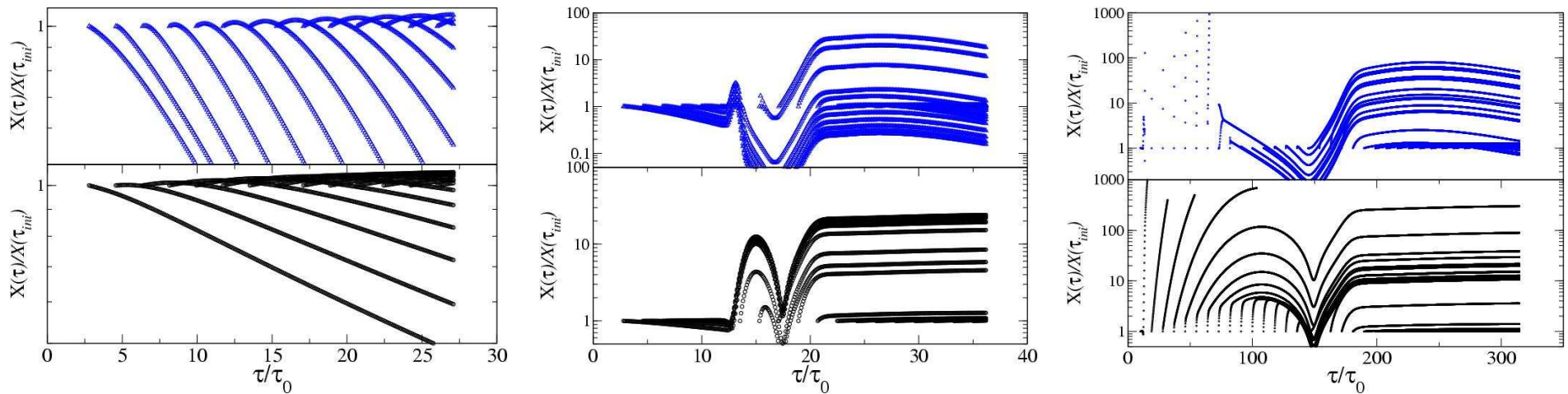
Plot shows the evolution of x_2/x_1 for the unstable Eigen-mode with τ

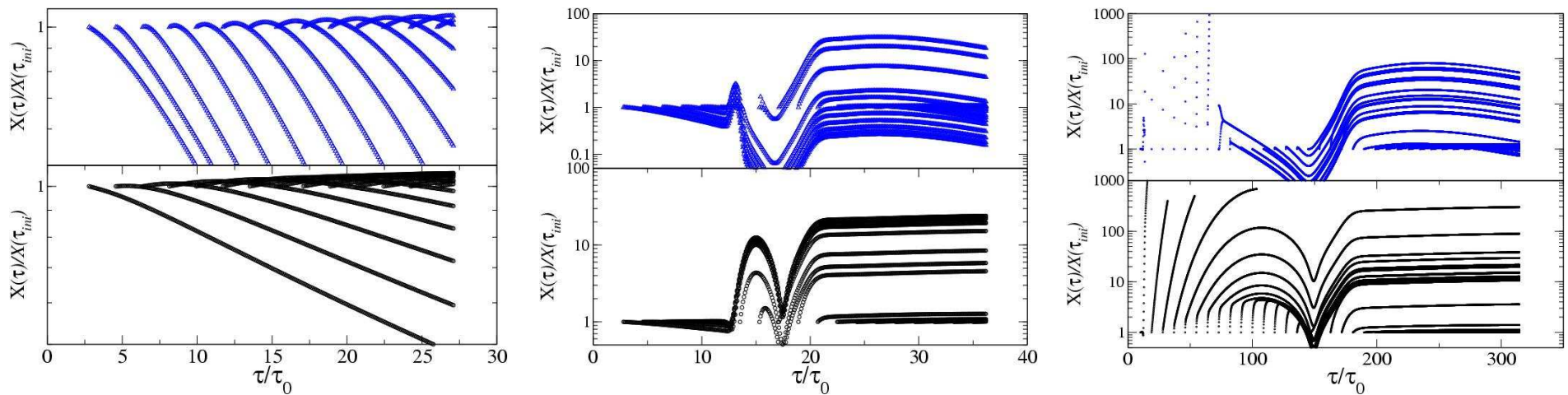


When there is no viscosity peak, direction of unstable Eigenvector rotates, cutting off growth of unstable modes. Viscosity peak, however, “freezes” direction of Eigenvectors allowing instabilities to grow

Results: Evolution of instabilities

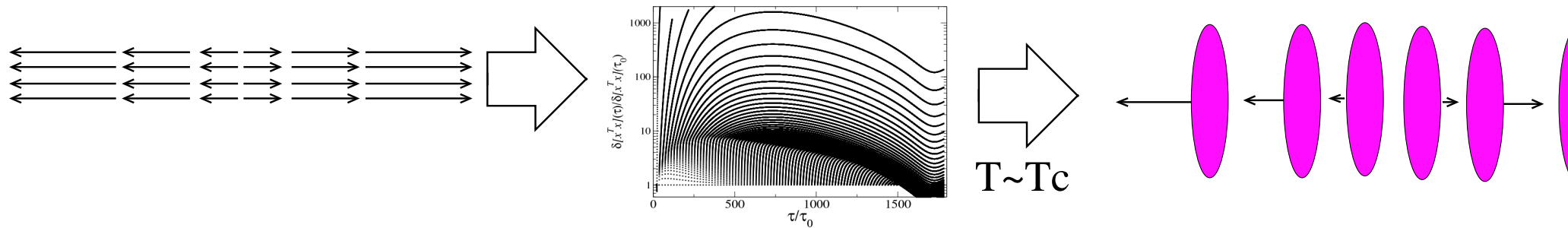
The diagram below shows the full stability analysis for the problem (Black for $k = 2$, blue for $k = 8$) At each time step, an unstable mode is generated and then integrated until the end. The graph shows $\frac{\vec{x}^T \vec{x}(\tau)}{\vec{x}^T \vec{x}(\tau_{initial})}$





If no ζ peak, the system remains stable because rotation of Eigenvectors cuts off instability growth. If viscosity has a large peak ($z_0 \sim 10$), a perturbation of $\sim 10^{-1}$ would grow to $\gg 1$ after a few fm . Note: This is not turbulence (which arises at low ζ, η and affects all solutions) but rather an instability of a specific solution

What happens to the instabilities?



Need 3D viscous (Israel-Stewart?) hydro, but a reasonable guess is that inhomogeneities become “clusters” with no internal expansion. These clusters

- Move with pre-existing flow suppressing hadronic gas phase!
- Emit particles by “evaporation” Introducing a small system size independent scale into the emission function

And THAT is the missing ingredient of the HBT puzzle.

Does Israel-Stewart matter?

Not if Relaxation time smaller than scales of expansion dynamics:

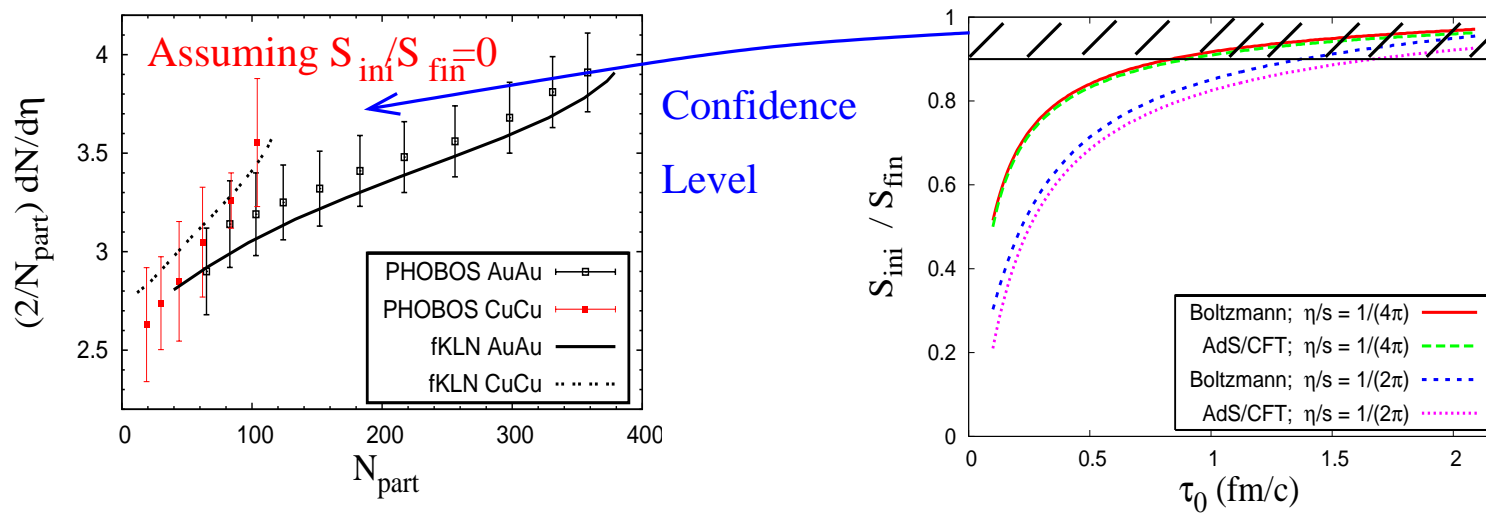
$$\beta = \frac{dT}{dt} \frac{\tau_{\Pi}}{\Delta T|_{eq}} \ll 1$$

At $\beta \sim 1$, by the time viscosity “turns on” peak is missed.

$\frac{dT}{dt}$ from dynamics, ΔT from lattice, τ_{Π} from?!?! (AdS/CFT? Hard Balls?)

Glauber,CGC etc. model dN/dy as a function of N_{part} well.
 These assume all entropy generated at beginning of collision.

Molnar,Dumitru and Nara, 0706.2203



But **viscosity** \rightarrow **Entropy generation** $\Delta S \sim \zeta (\partial_\mu u_\nu)^2$
 Any increase in viscosity generally leads to deviations from N_{part} vs dN/dy .
 Issue of all proposals of solving HBT by viscosity jump.

What about entropy?

Won't a divergence in ζ produce a lot of entropy and screw up your favourite Glauber/CGC N_{part} vs $\frac{dN}{dy}$ relation?

With clustering, no! Since

$$\partial_\mu s^\mu \sim \int dt \zeta (\partial_\mu u^\mu)^2$$

A rapid divergence in ζ followed by clustering would quickly kill $\partial_\mu u^\mu$ inside cluster, and hence entropy generation.

Scenario superficially similar to clustering due to first order phase transition

"bubbling" Is thermodynamic in origin ("bag" pressure, requires a first order transition)

oil2honey Does not require first order transition, bubbles are blobs of "hot" matter, hydrodynamic in origin, no "bag constant", its role taken by ζ , but only "turns on" with expansion

QCD very different from "everyday" fluids: these are either viscous and compressible or inviscid and incompressible, since inter-molecular distance controls both.

Cluster size

Something like Λ_{QCD}^{-1} ?

but then what distinguishes “clusters” from “Hadrons”?

Mass should certainly be of the order of high mass resonances (\sim Hagedorn?)

Clusters: High temperature “incoherent” states

Hadrons: (including resonances) Pure states

evolution between them impossible (non-unitary) without decoherence.

Hagedorn picture?

Clusters and HBT

Two step freeze-out:

Cluster formation \sim Cooper-Frye at $T \sim 170$ MeV, with pre-existing hydrodynamic flow u^μ .

Cluster decay After $\tau \sim \Lambda_{QCD}$. Each cluster a Gaussian emission source

$$S(x', p') \sim \frac{1}{\tau} e^{-E'/T} e^{-(t'^2 + x'^2 + y'^2 + z'^2)/(2\tau^2)}$$

$$x' \rightarrow u^\mu \tau + \Lambda_\nu^\mu(u^\mu) x^\nu \quad p'^\mu \rightarrow \Lambda_\nu^\mu(u^\mu) x^\nu$$

Hope: Hydro output for $u^\mu, d\Sigma_\mu$ at $T=170$ MeV+ "sensible" τ would describe HBT data.

How Clusters might solve HBT

$R_o/R_s = 1$ Cluster small, so cluster decay fast independently of size of the system

Near independence of R_o/R_s with \sqrt{s} Cluster formation (above deconfinement) in the whole region of $R_o/R_s = 1$. If this is true, QGP formed considerably below RHIC. **Not proven, but smoothness of all soft signals interesting in this respect**

$R_{o,s,l}$ scaling with $(dN/dy)^{1/3}$ Interplay of cluster size with number of clusters ($\sim (dN/dy)^{1/3}$)

A lot of work needs to be done (by me!) here!

Further phenomenology proposals

- Direct test with Kolmogorov-Smirnov test (With B. Tomasik)
- Electromagnetic probes (could " ρ " continuum seen by NA60 be clusters?)
- Resonances (Enhanced due to "Hagedorn" decay chain of clusters)
- Fluctuations (Ratio fluctuations enhanced w.r.t. statistical models)
- Cosmological implications

Conclusions and outlook

- HBT interferometry still not properly explained by hydrodynamics
- QCD predicts sharp rise of ζ near T_c , might have a role in solving HBT puzzle
- Instabilities might arise and drive freeze-out
- This idea still needs to be connected to experimental data.

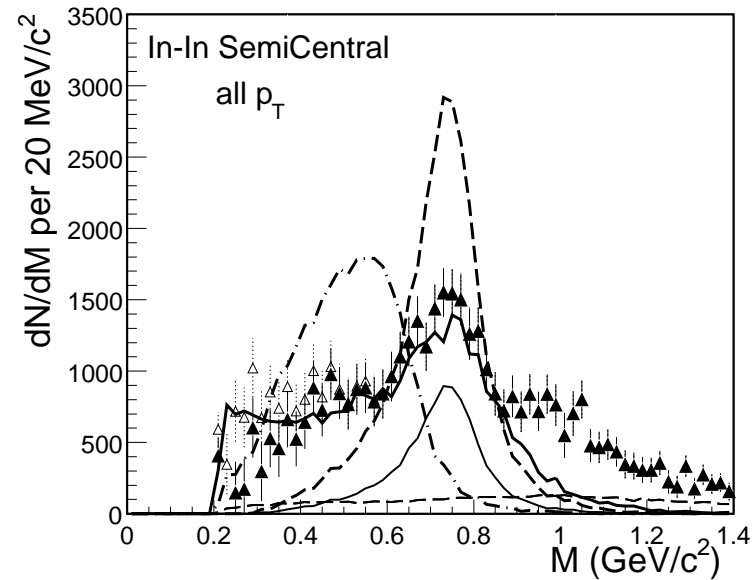
Lots of work and new ideas needed here

BACKUP SLIDES

Photon observables

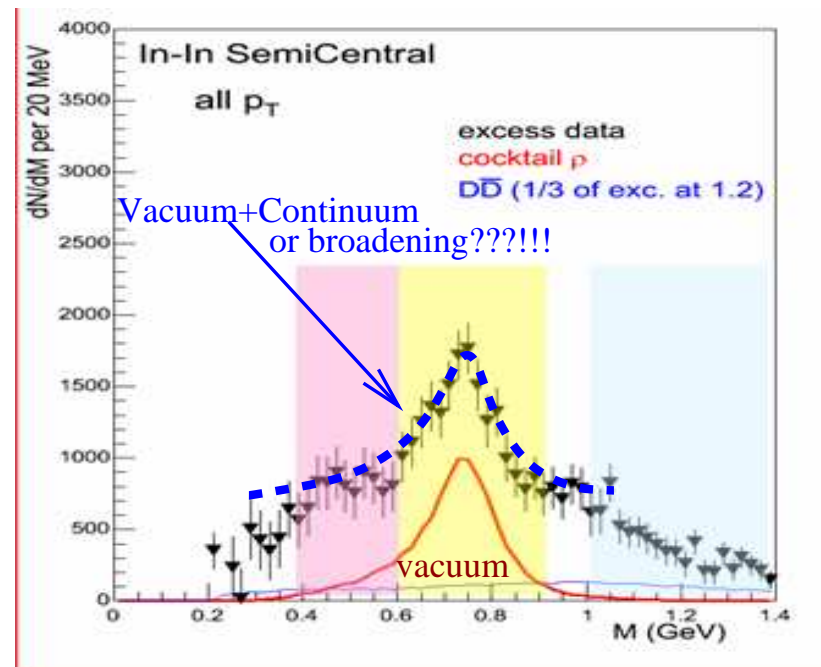
Sensitive to all freeze-out stages

Idea: Broadening of, e.g., the ρ can give information about microscopic properties and dynamics between chemical and thermal freeze-out.



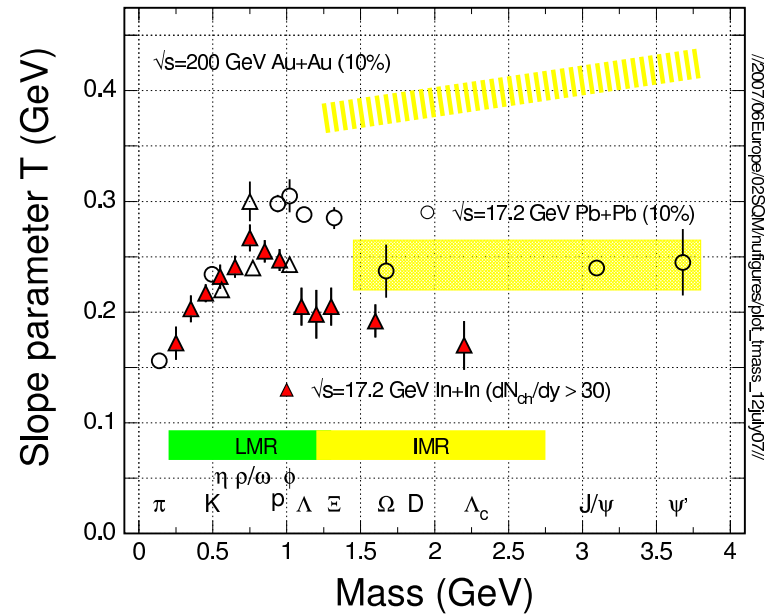
SPS In-In: The ρ has been broadened, presumably by hadronic interactions

Or is it?



What we see is an unbroadened “vacuum” ρ on top of a continuum. It could be that a model with broadening explains this after some parameters are tuned, but no confirmation that continuum is related to ρ .

Temperature of the continuum: New surprises.



In continuum mass dependence of slope nearly the same as hadrons.

But hydro tells us transverse flow created throughout collision

Are $\mu_{continuum}$ emitted at last moment? And what happens at $m = 1$ GeV

Beyond heavy ion collisions...

Could such viscosity-driven instabilities play a wider role than in heavy ion collisions?

The Universe Expands through a very similar equation to the 3D Bjorken one

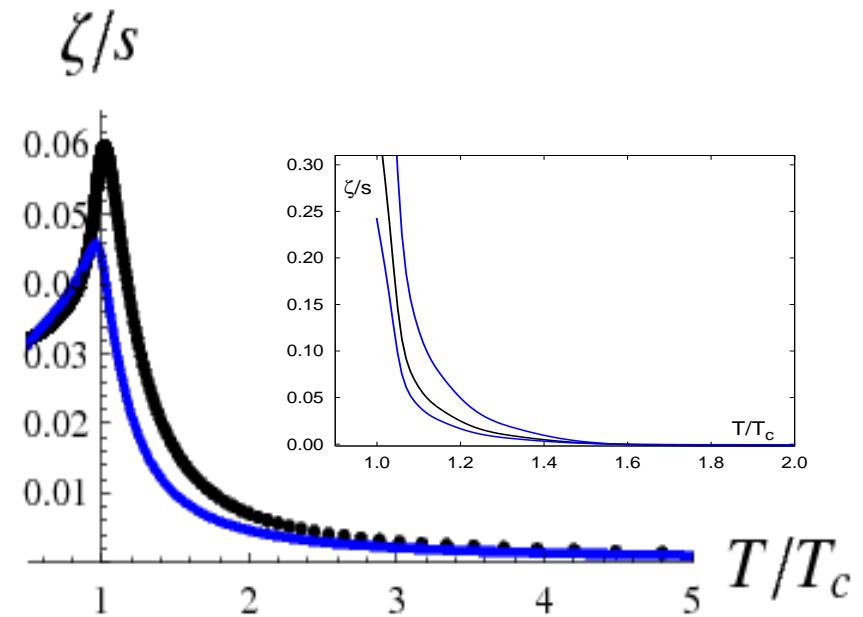
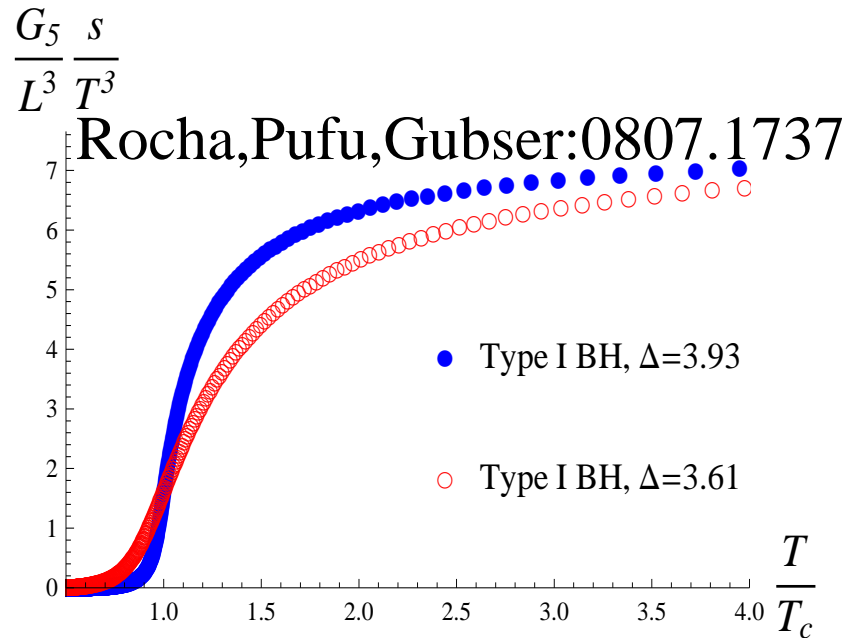
$$\tau^{-M} \frac{d(\tau^M s)}{d\tau} = \frac{Ms}{R\tau}, M = N = 3$$

Gravity should make system less stable (Jeans)

QCD transition Happens very late ($\tau \sim 10^{20}$), so a lot stabler. But this behaviour of viscosity might be universal for all asymptotically free theories. Similar behaviour during some GUT-scale transition?

Instabilities Must by causality be small, and unless distorted by inflation will dissipate. Might play a role of “hot-spots” for baryogenesis

Divergence is stronger than expected from just EoS



Gubser et.al. use AdS+Scalar to calculate ζ/s from strongly coupled theory where s/T^3 fit lattice. Peak not as strong as Khazeev et al. **Why?**

S. Jeon (hep-ph/9409250): $\eta \sim n \langle p \rangle \tau_{elastic}$, $\zeta \sim n \langle p \rangle \tau_{inelastic}$
 ($\tau_{a \rightarrow b}$: Equilibration timescale for $a \rightarrow b$)

If $\tau_{elastic} = \tau_{inelastic}$, at weak coupling (Boltzmann eq.)

$$\zeta = \eta \left(\frac{1}{3} - c_s^2 \right)$$

But in QCD...

Confinement introduces a string tension. Even if small, All elastic processes become inelastic.

χ -symmetry restoration gives rise to a $\langle q\bar{q} \rangle, \langle G_{\mu\nu} G^{\mu\nu} \rangle$. At $T = T_c$, this can be interpreted as a divergence of $\tau_{X \rightarrow X+q\bar{q}, gg}$. (NB: Inelastic) τ related to correlation lengths, which diverge in 2nd order transition.

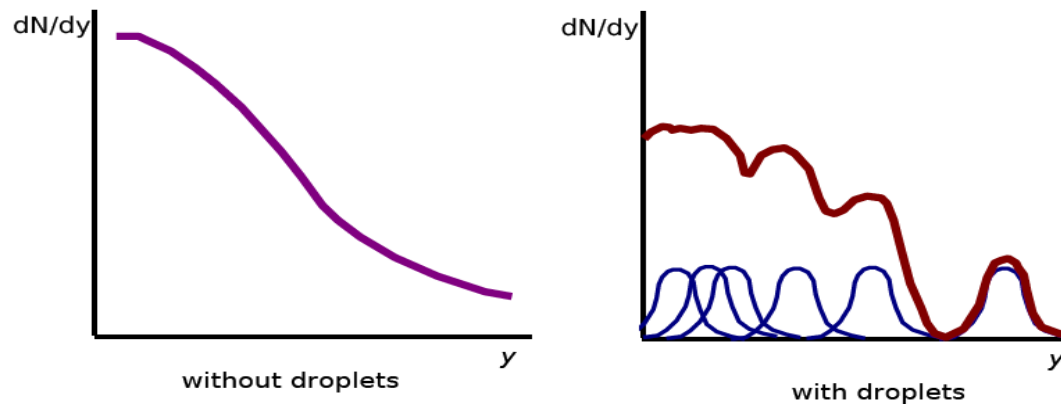
Rocha et al's calculation fails to reproduce QCD behaviour because these effects not present, evident as in their theory $\zeta \simeq \eta \left(\frac{1}{3} - c_s^2 \right)$

More direct signatures: The Kolmogorov-Smirnov test

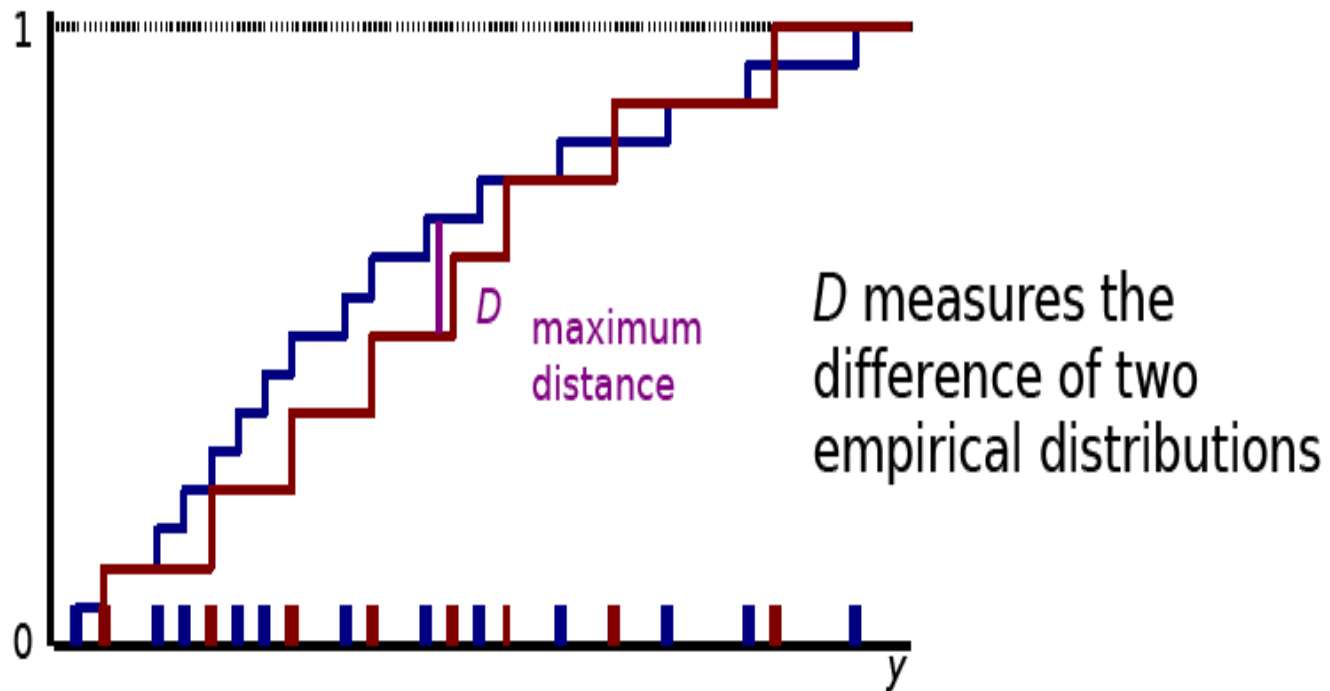
Basic idea, for any kinetic observable (y, p_T, ϕ)

A homogeneous liquid breaking up might look different event-by-event, but the distribution of any observable is the same

Clusters introduce differences between event probability-distribution functions, or event-specific correlations



This has a quantitative consequence for the cumulative distribution function
($\int_{-\infty}^x \rho(y) dy$, goes from 0 to 1 for all distributions)



If underlying distributions of empirical sets of data $(n_{1,2})$ are the same, maximum difference as $n = (n_1 n_2) / (n_1 + n_2) \rightarrow \infty$ distributed as

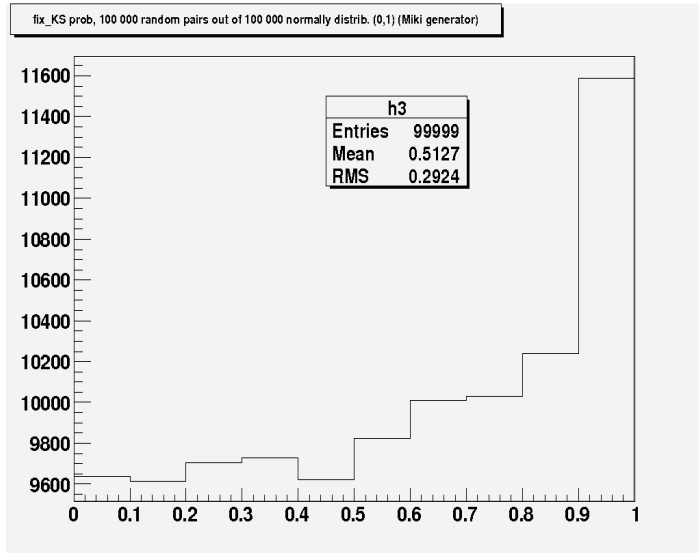
$$P(\sqrt{n}D < t) = \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2 n D^2}$$

so

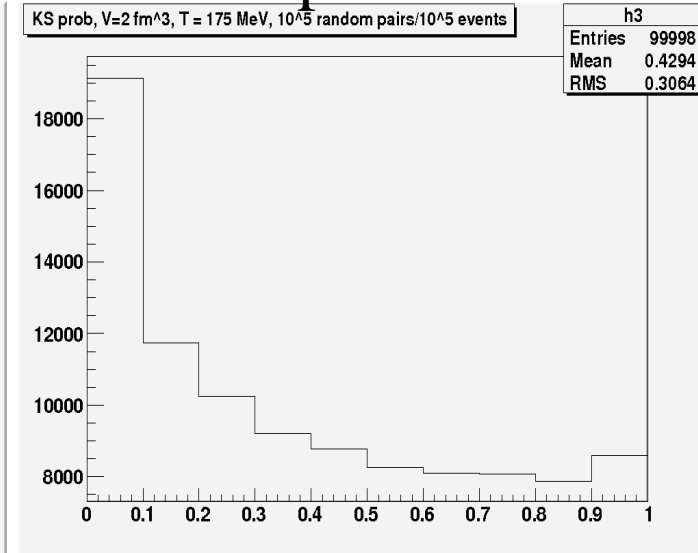
$$Q = 1 - \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2 n D^2}$$

distributed uniformly independently of underlying distributions

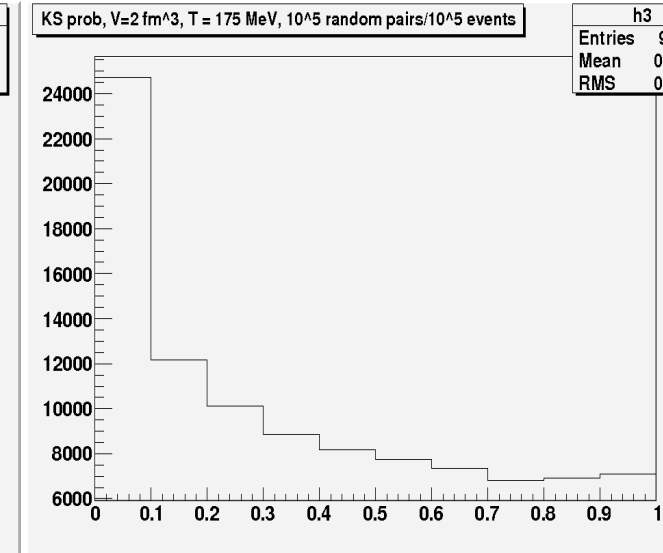
No droplets



20% droplets



All droplets

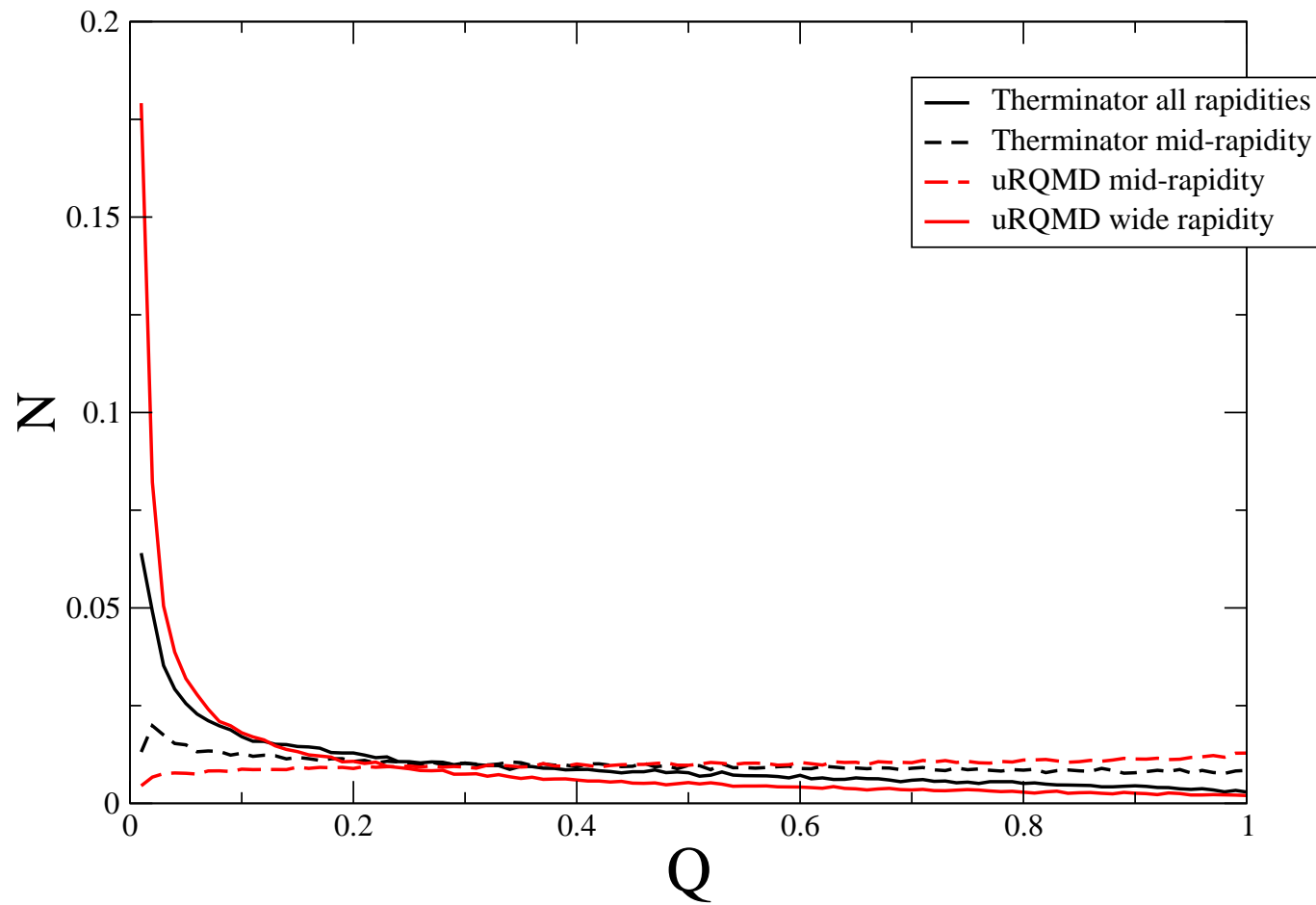


Even 20% of droplets significantly change the graph
But no resonances included here

Problem: Not the only source of inter-event correlations

- Resonances
- Initial condition fluctuations (\sim "volume fluctuations")

But worth trying with a hydrodynamic model incorporating these



Resonance correlations can be eliminated by focusing on mid-rapidity