The Perfect Liquid Extinguishes Jets Nearly Perfectly

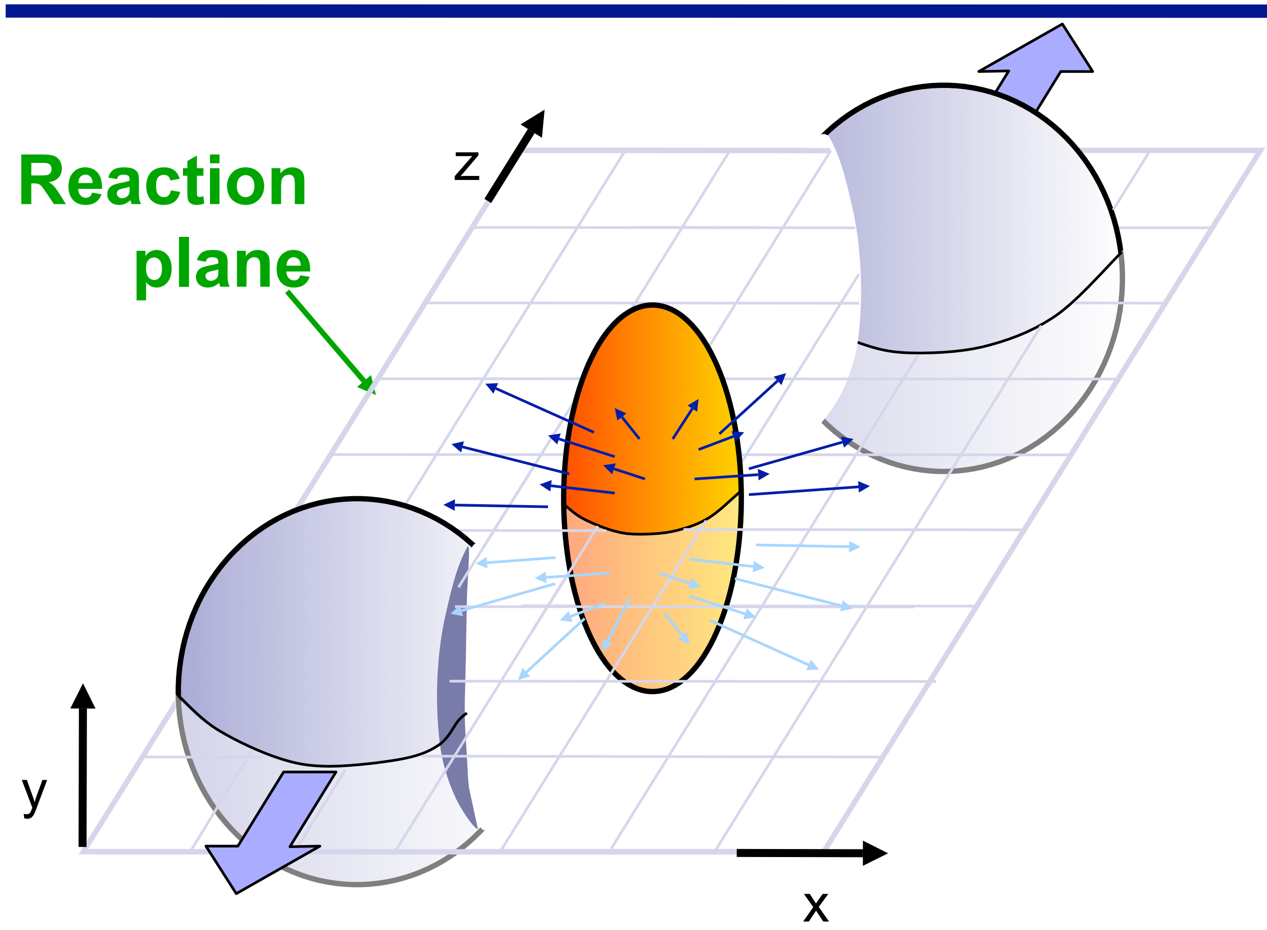
Berndt Mueller

30th Int. School on Nuclear Physics
Erice/Sicily: Sept. 16-24, 2008

An apology:

This is NOT a talk for the experts

Collision Geometry: Elliptic Flow

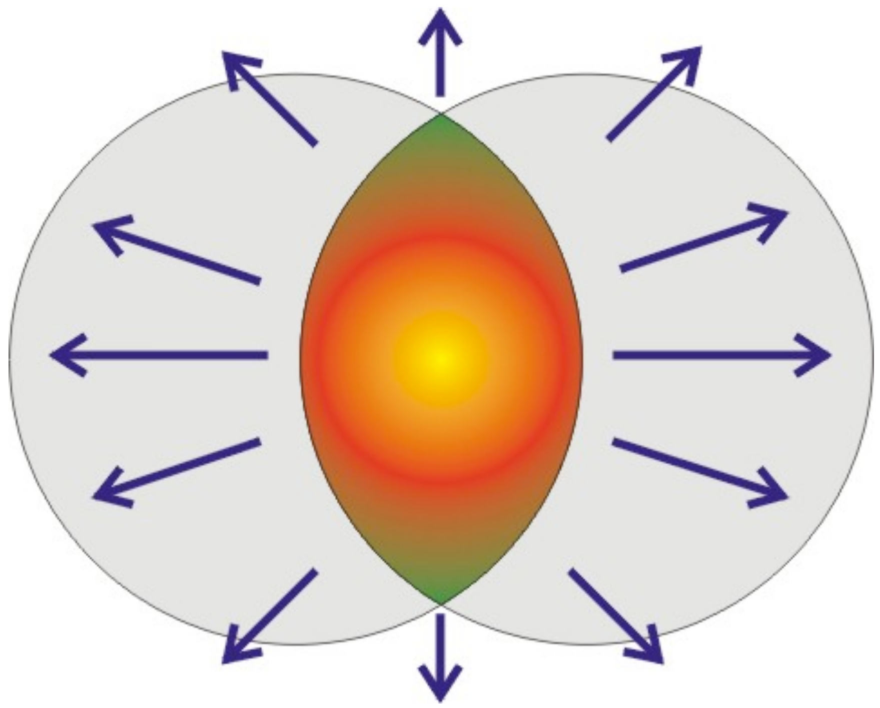


- Bulk evolution described by relativistic fluid dynamics,
- assumes that the medium is in local thermal equilibrium,
- but no details of how equilibrium was reached.
- **Input: $\varepsilon(x,\tau_i)$, $P(\varepsilon)$, (η,etc.).**

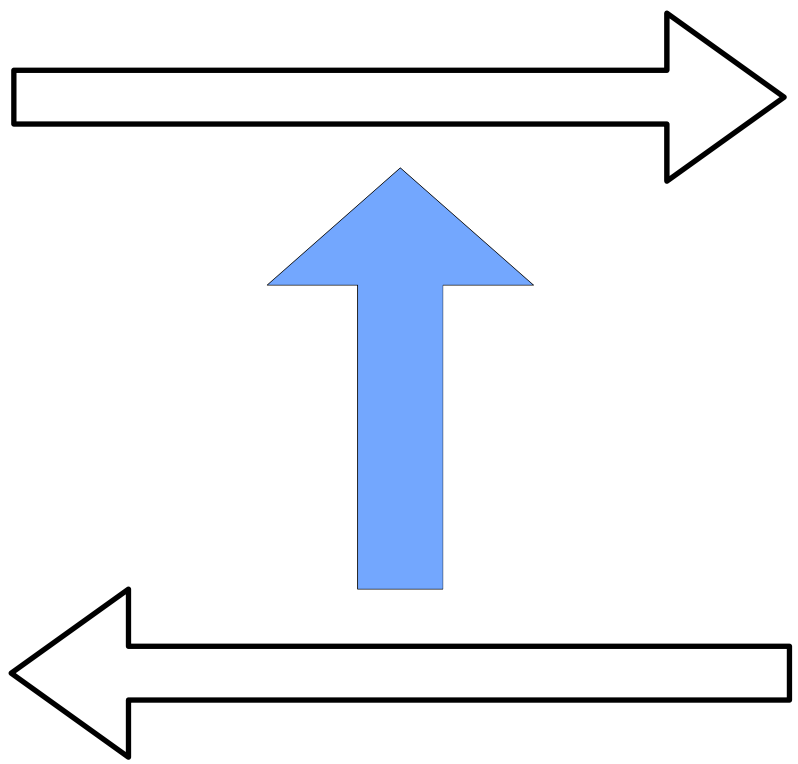
Elliptic flow (v_2):

- Gradients of almond-shape surface will lead to preferential expansion in the reaction plane
- Anisotropy of emission is quantified by 2nd Fourier coefficient of angular distribution: v_2

- prediction of fluid dynamics



Shear viscosity



Scales: *Homogeneity length* R

Mean free path λ_f

Thermal wavelength T^{-1}

Knudsen number: $K = \lambda_f / R$

Momentum transport along flow gradient:

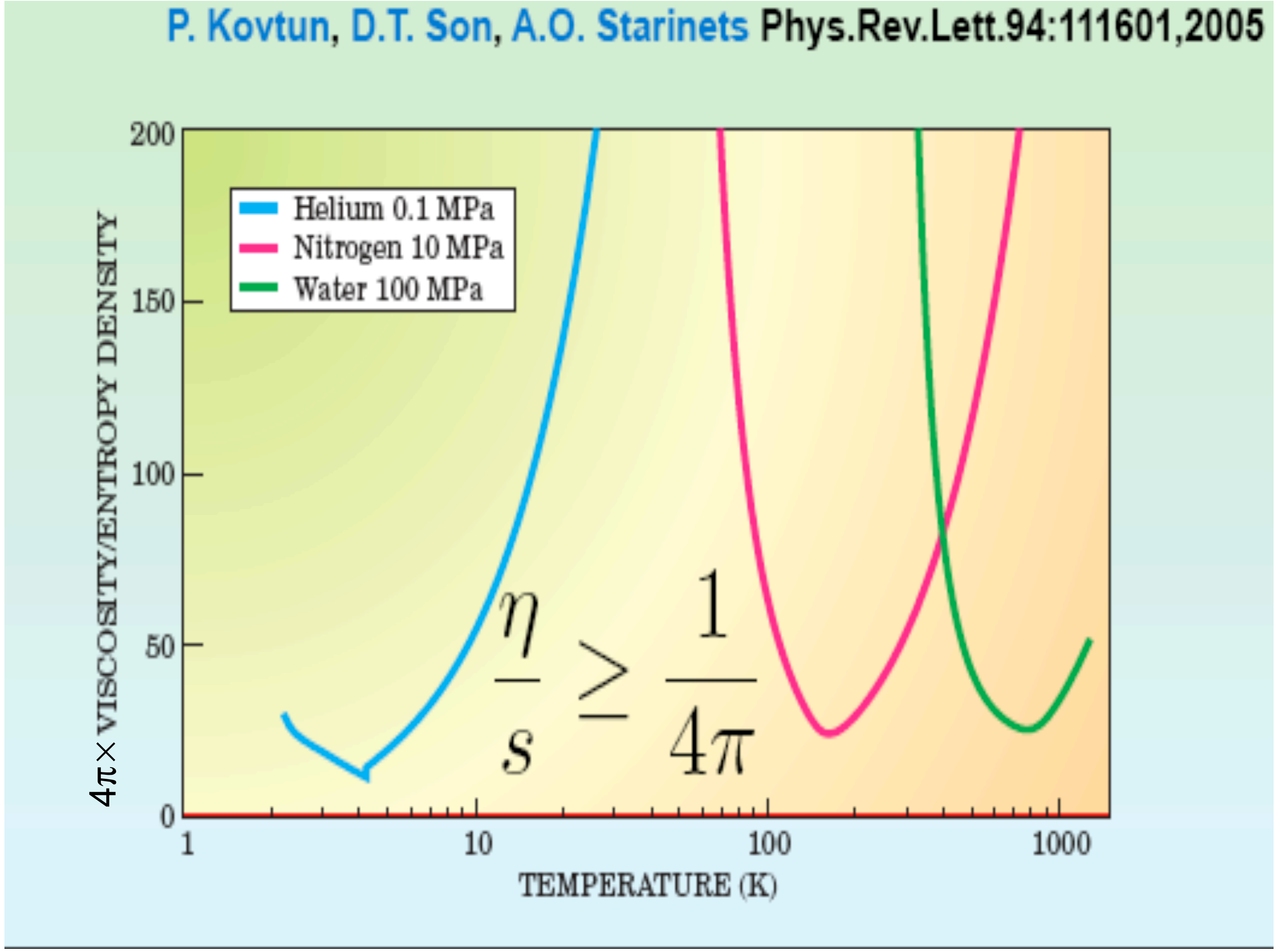
$$\eta \approx \frac{4}{15} n \bar{p} \lambda_f = \frac{4T}{5\sigma} \approx \frac{s}{5} T \lambda_f = \frac{\varepsilon + p}{5} \lambda_f \quad \rightarrow \quad \frac{\eta}{R} \approx (\varepsilon + p) K$$

Viscosity

- Kinetic theory:
  - Increasing interaction ➡ decreasing mean free path ➡ diminishing ability to transport momentum via particles ➡ decreasing shear viscosity
- Counter-intuitive:
  - Why then is honey highly viscous?
- Transforming structure ➡ alternative mechanisms:
  - As interaction grows, eventually the material's structure rearranges, and new momentum transport mechanisms with larger mean free path take over, e.g. waves (in solids); momentum transport along molecular chains (in polymers).

Real materials

Temperature dependence of the shear viscosity of typical fluids:

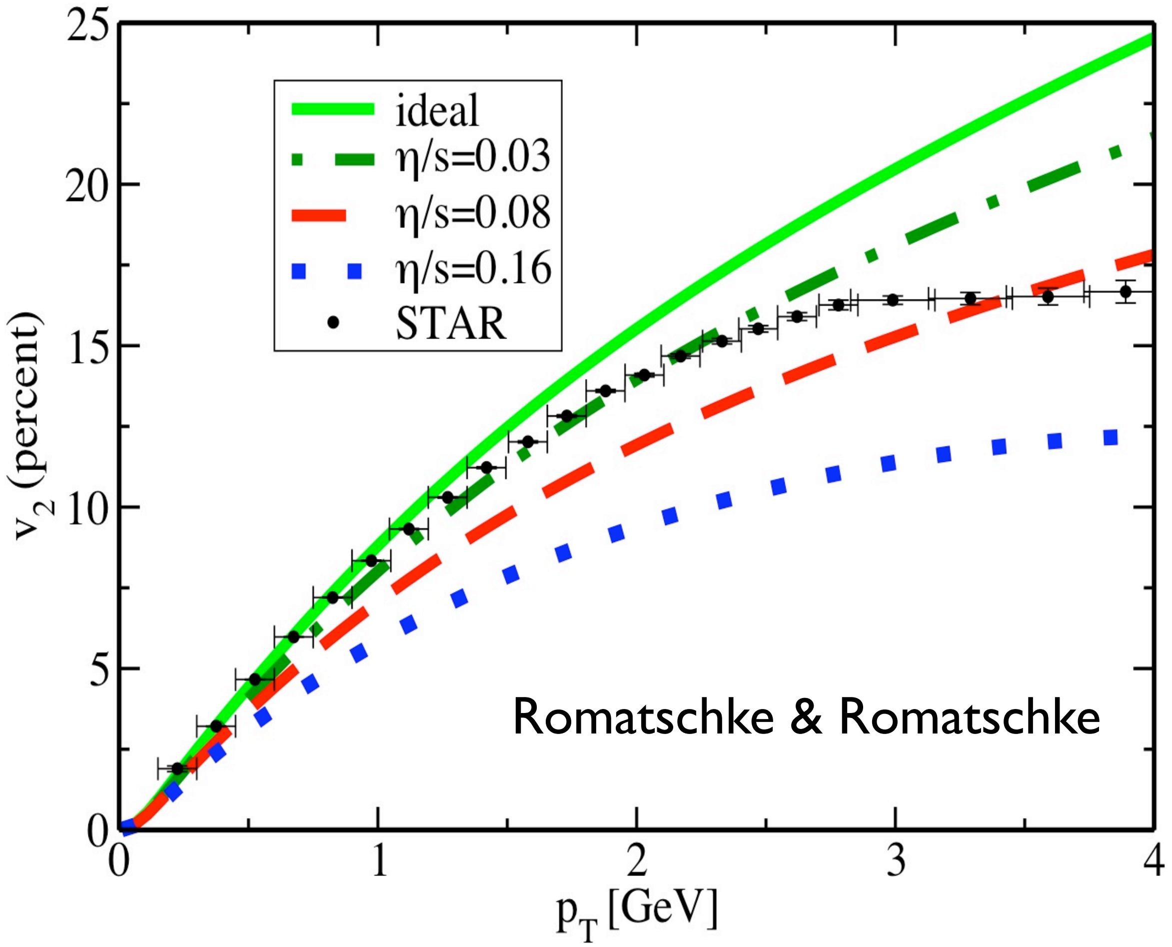


Elliptic flow "measures" η_{QGP}

Relativistic viscous hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{with} \quad T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \Pi^{\mu\nu}$$

$$\tau_\Pi \left[\frac{d\Pi^{\mu\nu}}{d\tau} + \left(u^\mu \Pi^{\nu\lambda} + u^\nu \Pi^{\mu\lambda} \right) \frac{du^\lambda}{d\tau} \right] = \eta \left(\partial^\mu u^\nu + \partial^\nu u^\mu - \text{trace} \right) - \Pi^{\mu\nu}$$



Boost invariant hydrodynamics with $T_0\tau_0$ ~ 1 requires $\eta/s \leq 0.1$.

Bound may be relaxed by "sharper" shape of initial energy density (CGC initial conditions).

The QGP is an almost perfect liquid.

BJ's critique: Theory should be compared with identified particle (π) elliptic flow data.

Flow & equilibration

Q: Does hydrodynamic flow imply local equilibration?

A: Local equilibrium and small velocity gradients (compared with mean-free path) imply the validity of hydrodynamics.

But not the reverse! - Collectivity in plasmas can be caused by the action of fields (magnetohydrodynamics):

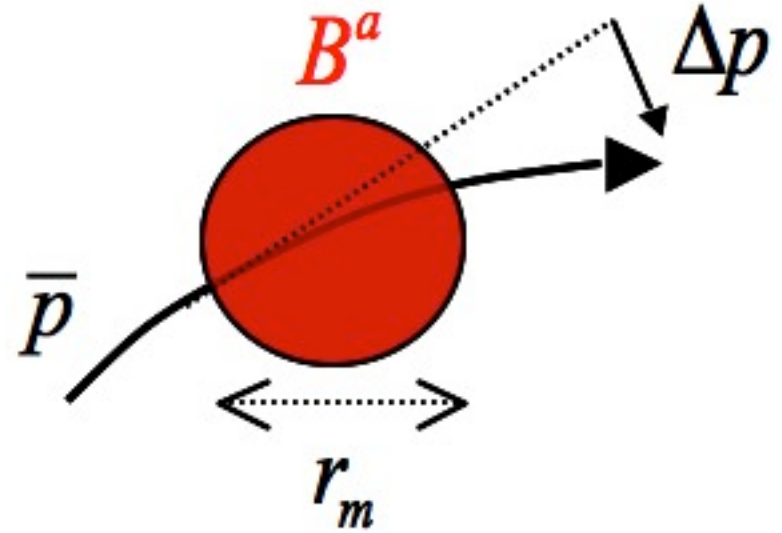
$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = \frac{1}{\rho + p}\left[-\nabla\left(p + \tfrac{1}{2}B^2\right) + \left(\vec{B}\cdot\nabla\right)\vec{B} + \eta\nabla^2\vec{v}\right]$$

Magnetic fields can also reduce the *shear viscosity* in "turbulent" plasmas (anomalous viscosity).

Anomalous viscosity

Momentum change per domain:

$$\Delta p \approx g Q^a B^a r_m$$



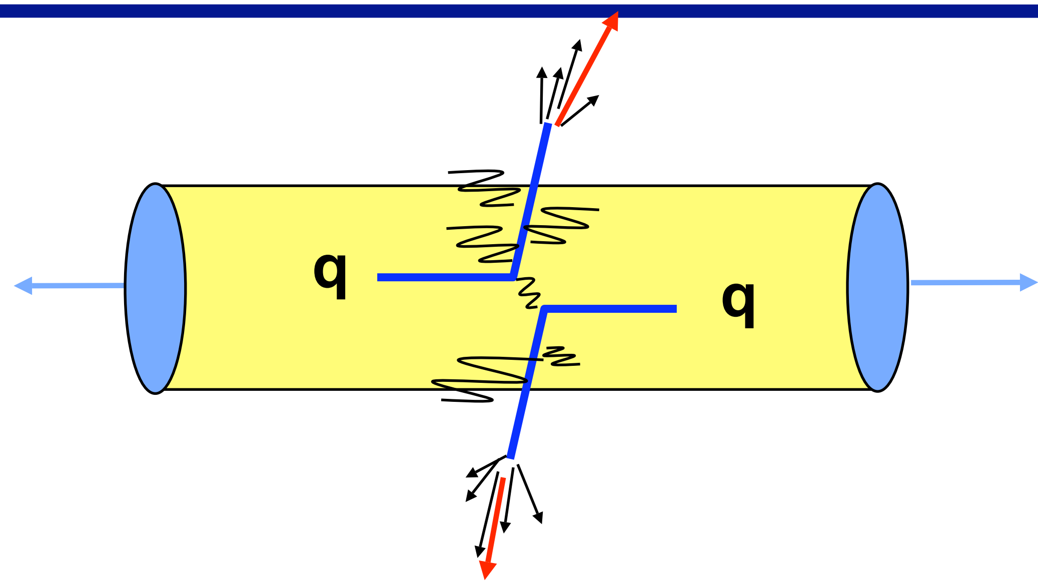
Effective mean-free path:

$$\lambda_f \approx r_m \left\langle \frac{\bar{p}^2}{(\Delta p)^2} \right\rangle \approx \frac{\bar{p}^2}{g^2 Q^2 \langle B^2 \rangle r_m}$$

Anomalous shear viscosity: $\eta_A \approx \dfrac{4 n \bar{p}^3}{15 g^2 Q^2 \langle B^2 \rangle r_m} \approx \dfrac{9 s T^3}{5 g^2 Q^2 \langle B^2 \rangle r_m}$

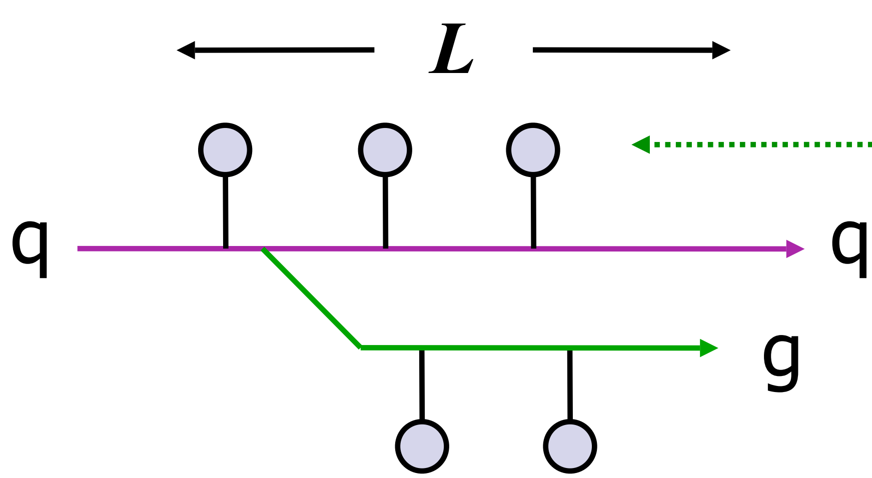
Can occur at RHIC either in "glasma" phase or during free streaming longitudinal expansion due to plasma instabilities.

Radiative energy loss



Radiative energy loss:

$$\Delta E \sim \rho L^2 \left\langle k_T{}^2 \right\rangle$$



Scattering centers = color charges

Density of scattering centers

$$\hat{q} = \rho \int k^2 \, dk^2 \, \frac{d\sigma}{dk^2} = \rho\sigma \left\langle k_T^2 \right\rangle$$

Scattering power of the QCD medium:

Range of color force

Connecting $\hat{q}$ with η

Hard partons probe the medium via transverse momentum exchange:

$$\hat{q} = \rho \int k^2 \, dk^2 \, \frac{d\sigma}{dk^2}$$

If kinetic theory applies, thermal partons can be described as quasi-particles that experience the same medium. Then the shear viscosity is:

$$\eta \approx \frac{4}{15} \rho \, \bar{p} \, \lambda_f = \frac{4\bar{p}}{15\sigma_{\rm tr}}$$

In QCD, small angle scattering dominates; then

$$\sigma_{\rm tr} \approx \frac{4}{\hat{s}} \int dk_T^2 \, k_T^2 \, \frac{d\sigma}{dk_T^2} = \frac{4\hat{q}}{\hat{s}\rho}$$

Majumder, BM, Wang
PRL 99: 192301 (2007)

Now: $\bar{p} \approx 3T, \quad \hat{s} \approx 18T^2, \quad s \approx 4\rho \quad \rightarrow \quad \frac{\eta}{s} \approx c \frac{T^3}{\hat{q}}$ with c ≈ 1 - 1.25

"Turbulent" QGP

$$\langle F^a_{\mu\nu}(x)\rangle = 0 \qquad \langle F^a_{\mu\nu}(x)F^b_{\alpha\beta}(y)\rangle = \frac{\langle E^2+B^2\rangle}{6(N_c^2-1)}(\delta_{\mu\alpha}\delta_{\nu\beta}-\delta_{\mu\beta}\delta_{\nu\alpha})\delta^{ab}C(x-y)$$

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\nabla_r+\vec{F}\cdot\nabla_p\right]f(\vec{r},\vec{p},t)=C[f] \qquad \text{with} \qquad \vec{F}=gQ^a(\vec{E}^a+\vec{v}\times\vec{B}^a)$$

$$\left[\frac{\partial}{\partial t}+\vec{v}\cdot\nabla_r-\nabla_p D(\vec{p},t)\nabla_p\right]\bar{f}=C[\bar{f}] \qquad \text{with} \qquad D_{ij}=\int_{-\infty}^{t}dt'\,\langle F_i(\bar{r}(t'),t')F_j(\bar{r}(t),t)\rangle\,.$$

$$\frac{dp_\mu(\tau)}{d\tau} = g\tau^a F^a_{\mu\nu}\big(x(\tau)\big)u^\nu(\tau)$$

$$\langle p^\mu(x_1^-)p^\nu(x_2^-)\rangle = p^\mu(0)p^\nu(0)+g^2C_R\int_0^{x_1^-}dy_1^-\int_0^{x_2^-}dy_2^-\,\langle F_a^{\mu+}(y_1^+)F_a^{\nu+}(y_2^-)\rangle$$

$$\langle p_T(x^-)^2\rangle = p^\mu(0)^2+x^-\boxed{g^2C_R\int_0^{x^-}dy^-\,\langle F_a^{\mu+}(y^+)F_a^{\nu+}(0)\rangle} = \hat{q}$$

Anomalous viscosity

Take moments of $\left[\frac{\partial}{\partial t}+\frac{p}{E_p}\cdot\nabla_r-\nabla_p\cdot D(p)\cdot\nabla_p\right]\bar{f}(r,p,t)=C[\bar{f}]$ with $(p_z)^2$

$$D_{ij}(p)=\int_{-\infty}^{t} dt'\left\langle F_i^a(\bar{r}(t'),t')U_{ab}(\bar{r},r)F_j^b(r,t)\right\rangle$$

$$\vec{F}^a = g\left(\vec{E}^a+\vec{v}\times\vec{B}^a\right) = \text{color force}$$

$$\frac{1}{\eta}=O(1)\frac{N_c}{N_c^2-1}\frac{\langle F^2\rangle\tau_m}{sT^3}+O\left(10^{-2}\right)\frac{g^4\ln g^{-1}}{T^3}\equiv\frac{1}{\eta_A}+\frac{1}{\eta_C}$$

$$\langle F^2\rangle\tau_m\equiv\int_0^\infty dt\left\langle F_i^+(x+t,t)F^{+i}(x,0)\right\rangle=\hat{q}$$

Again: $\frac{\eta_A}{s}\approx 1.25\frac{T^3}{\hat{q}}$

M. Asakawa, S.A. Bass, BM,

PRL 96: 252301 (2006)

Prog.Theor. Phys. 116: 725 (2006)

Weak vs. strong coupling

$$\frac{\eta_A}{s} \approx 1.25 \frac{T^3}{\hat{q}}$$ holds generally under two conditions ("weak coupling"):

1. Medium is described by nearly massless quasi-particles with the same properties as the high-energy ("jet") modes;
2. Interactions are dominated by small-angle scattering.

The relation fails at strong coupling, e.g. strongly coupled *N*=4 SYM theory, or when thermal quasi-particles have different quantum numbers (pion gas).

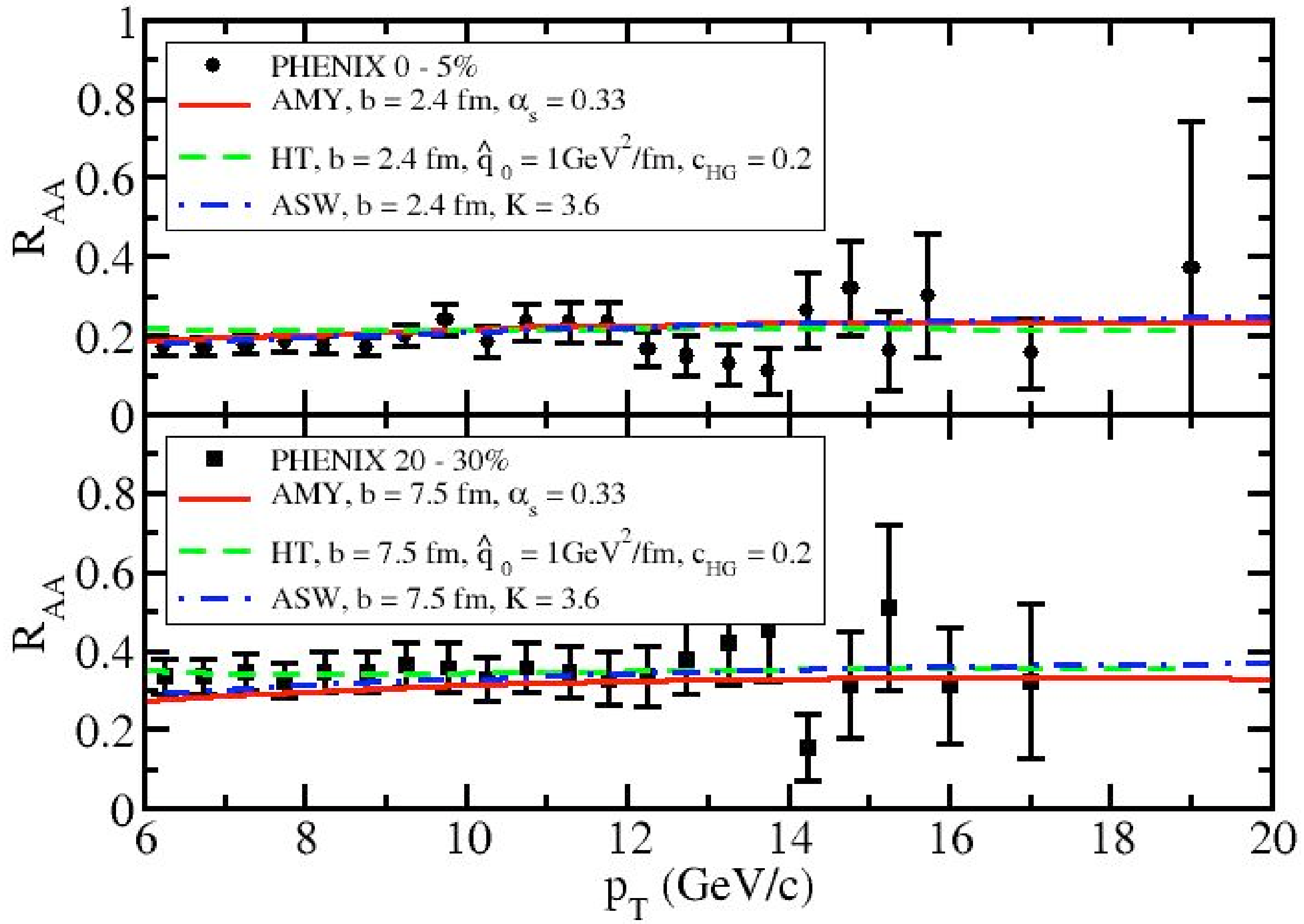
N=4 SYM theory for $g^2N_c \to \infty$: $$\frac{\eta}{s} \to \frac{1}{4\pi}$$

but: $$\hat{q} \approx 7.53\sqrt{g^2N_c}\,T^3 \quad \to \quad 1.25\frac{T^3}{\hat{q}} \approx \frac{1}{6\sqrt{g^2N_c}} \ll \frac{\eta}{s}$$

$\hat{q}$ in QCD

3-*D* ideal hydrodynamics with radiative energy loss only

S.A. Bass et al. - arXiv:0808.0908



$\hat{q}$ (GeV^2/fm) for gluons

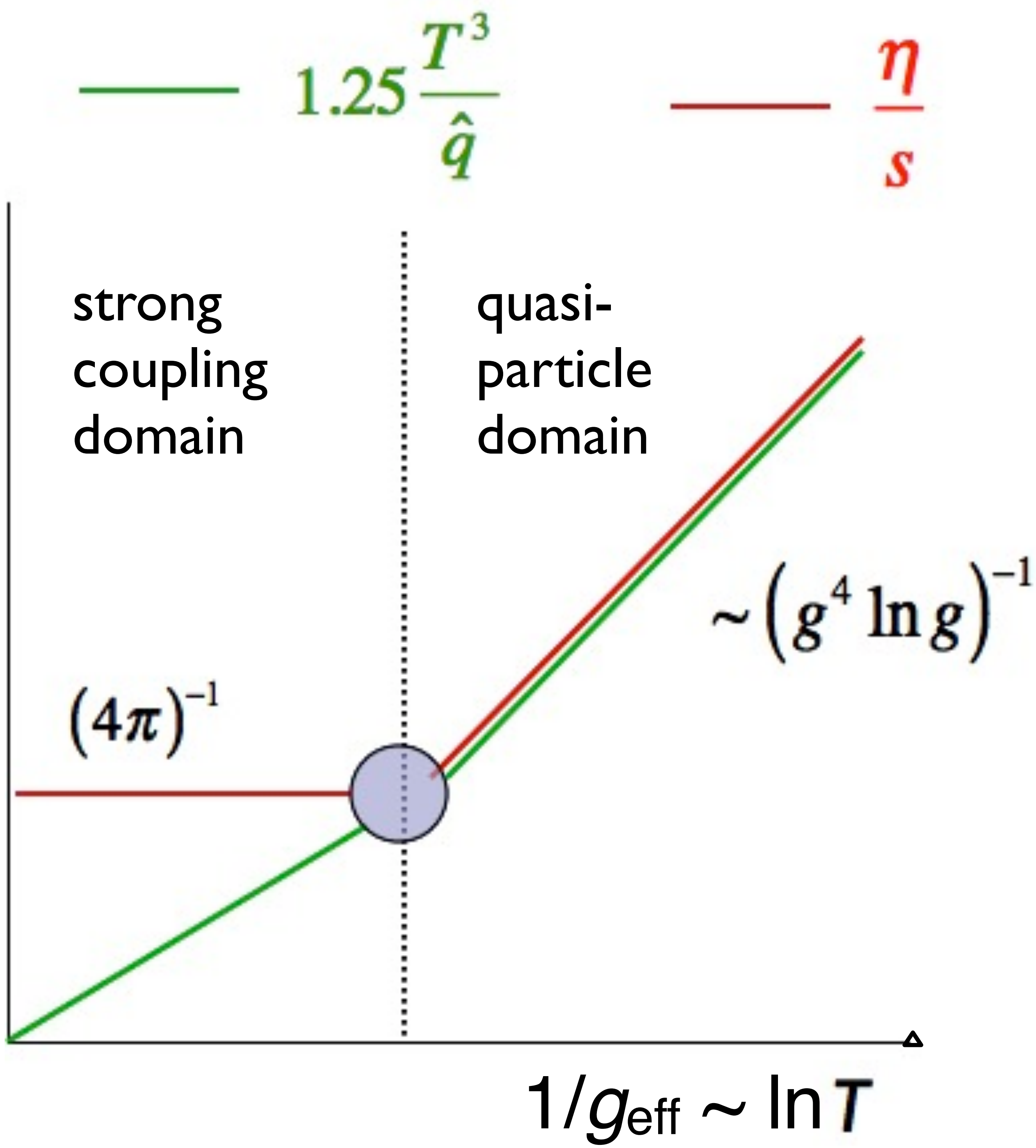
scaling	ASW	HT	AMY
T	10	2.3	4.1
$\varepsilon^{3/4}$	18.5	4.5	-

Inclusion of collisional energy loss in AMY reduces to: $\hat{q} \approx 2.75$ GeV^2/fm.

With $T_{max} \approx 400$ MeV at 0.6 fm/c this gives:

$$1.25 \frac{T^3}{\hat{q}} \approx 0.145$$

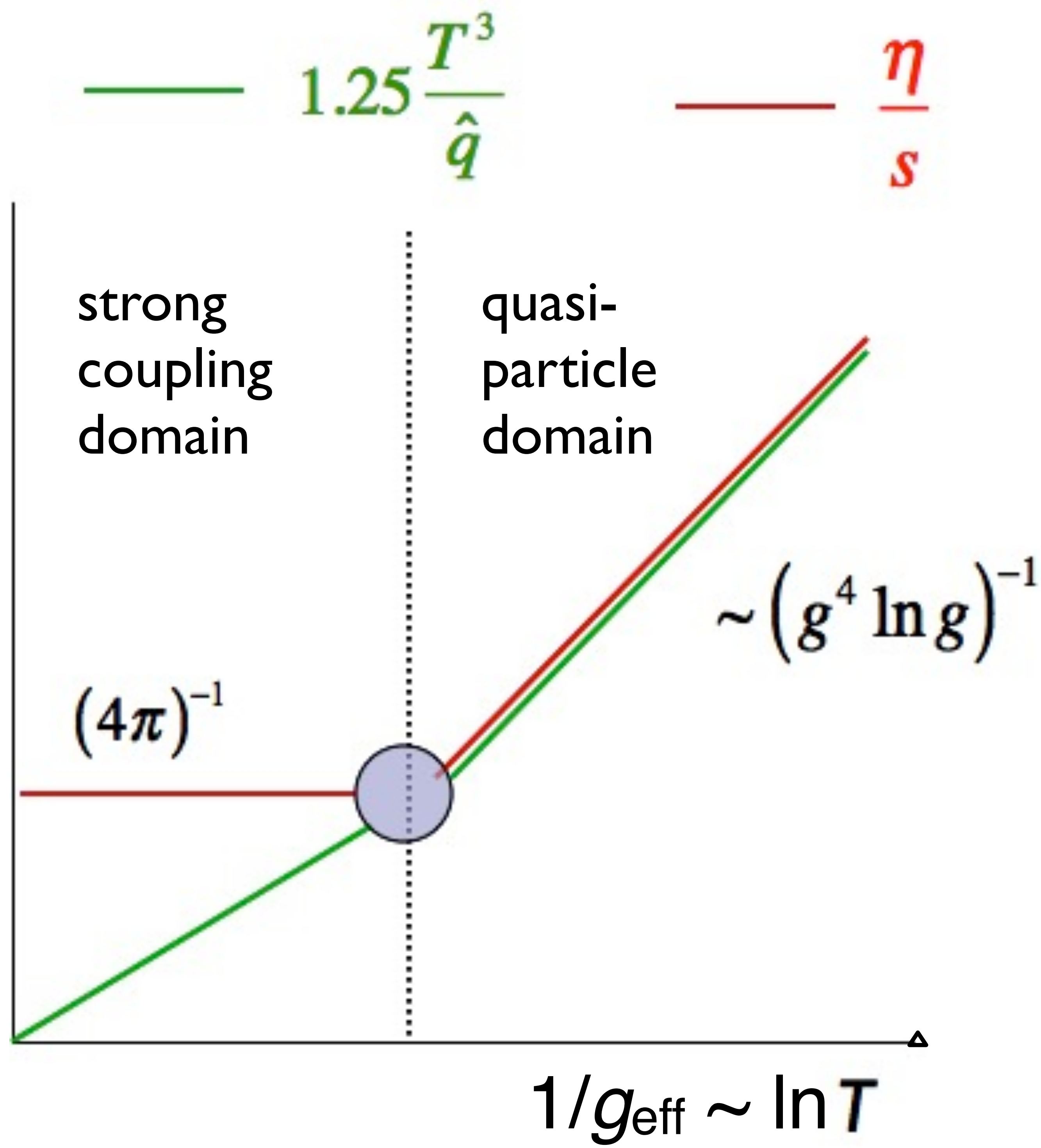
Where is RHIC?



Where is RHIC?

$$1.25\frac{T^3}{\hat{q}} \approx \begin{cases} 0.145 & (\text{AMY, HT}) \\ 0.03-0.04 & (\text{ASW}) \end{cases}$$

at $T \approx 400$ MeV



Where is RHIC?

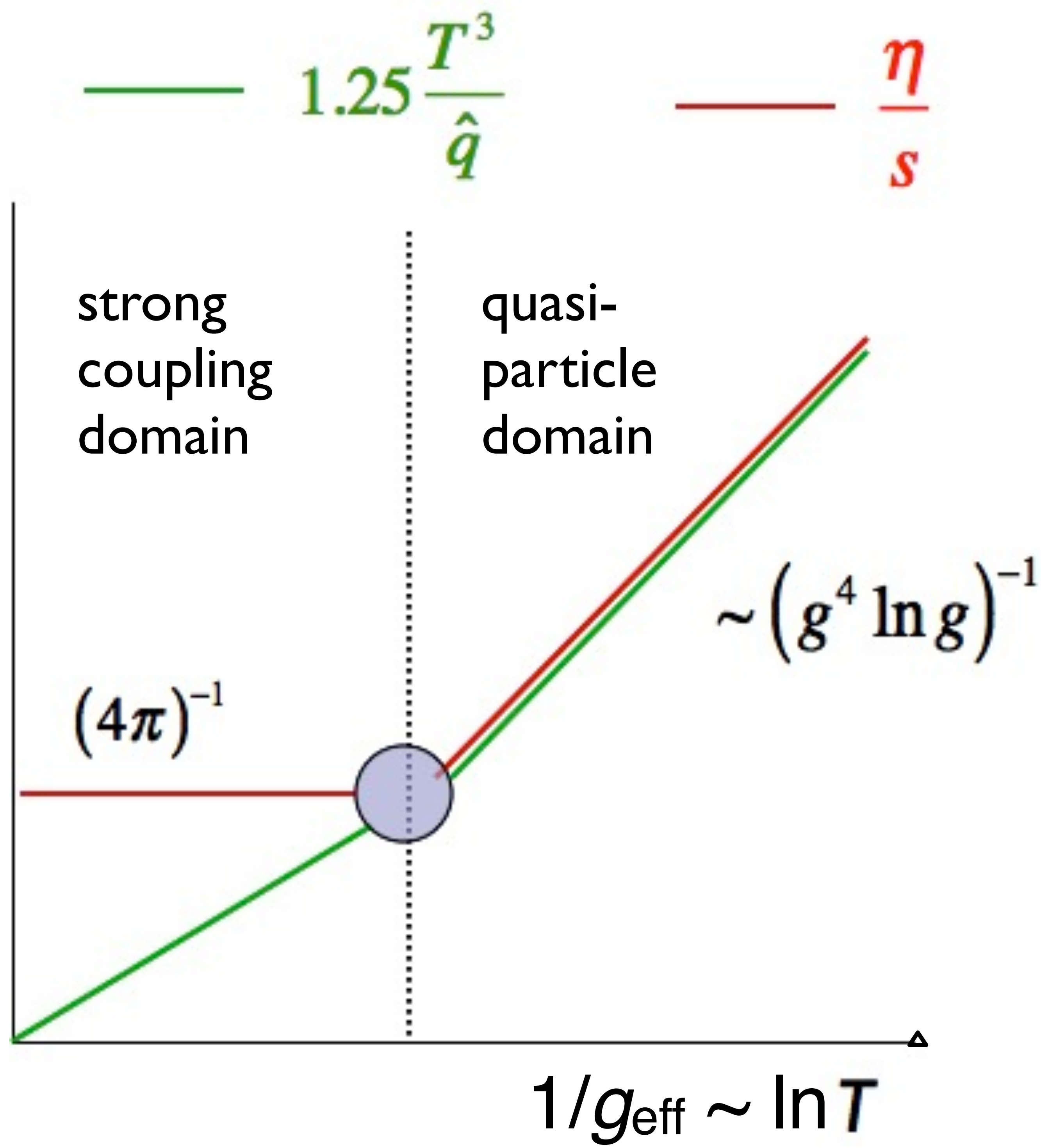
$$1.25\frac{T^3}{\hat{q}} \approx \begin{cases} 0.145 & \text{(AMY, HT)} \\ 0.03-0.04 & \text{(ASW)} \end{cases}$$

at $T \approx 400$ MeV

$$\frac{\eta}{s} = 0.134(33) \quad \text{(H.Meyer)}$$

at $T = 1.65\,T_c \approx 300$ MeV

(quenched QCD)



Where is RHIC?

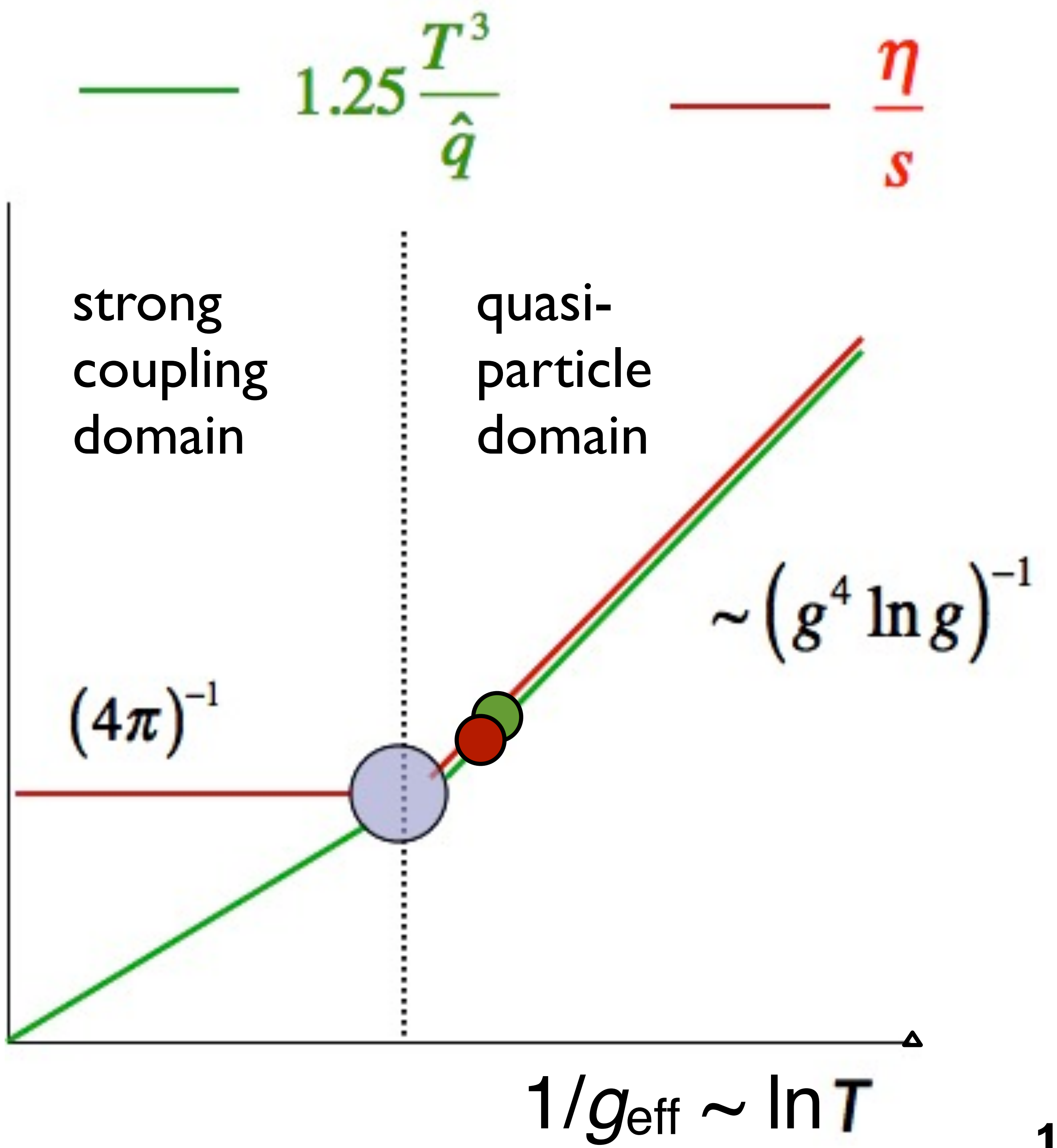
$$1.25\frac{T^3}{\hat{q}} \approx \begin{cases} 0.145 & \text{(AMY, HT)} \\ 0.03 - 0.04 & \text{(ASW)} \end{cases}$$

at $T \approx 400$ MeV

$$\frac{\eta}{s} = 0.134(33) \quad \text{(H.Meyer)}$$

at $T = 1.65\,T_c \approx 300$ MeV

(quenched QCD)



Where is RHIC?

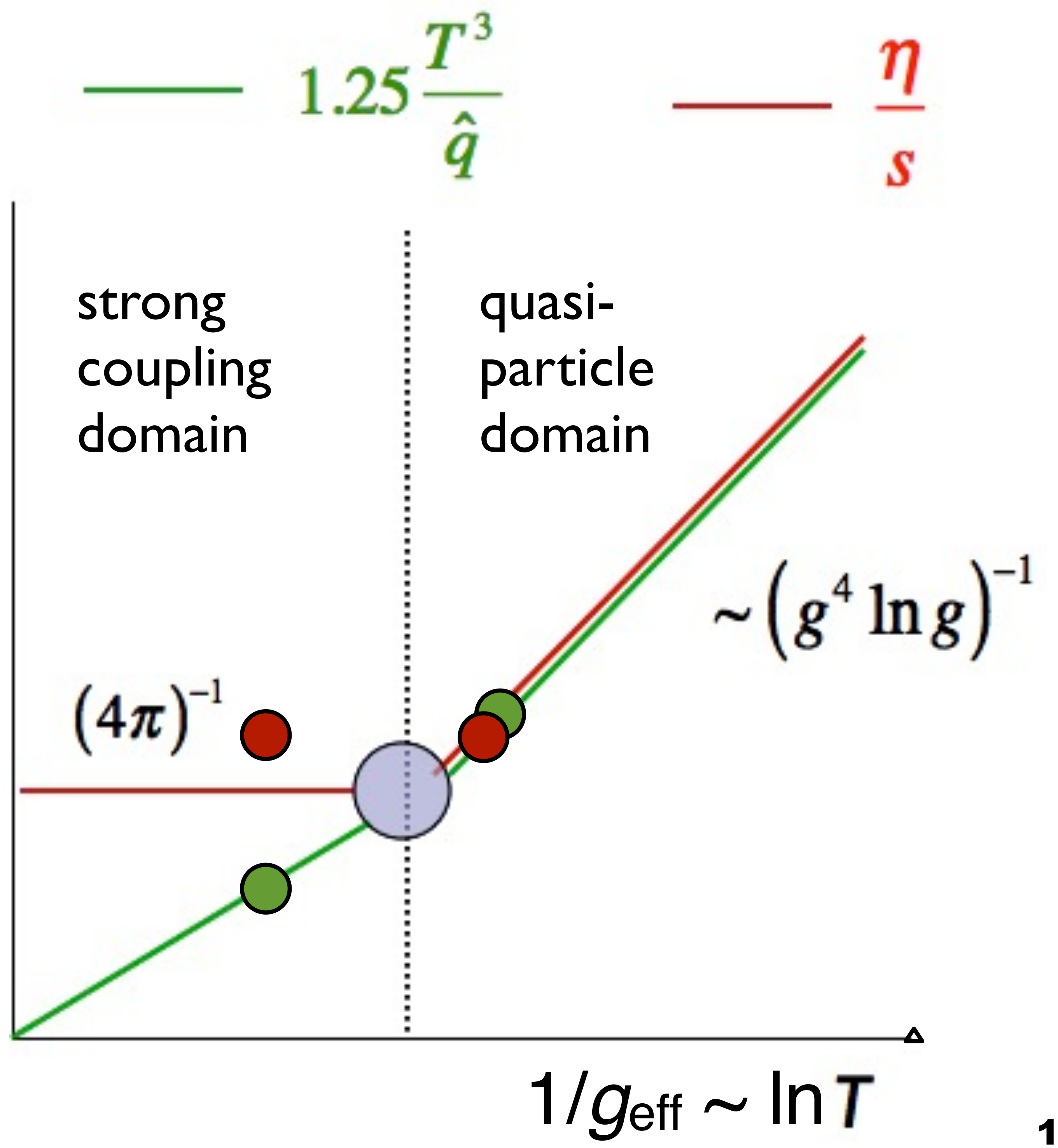
$$1.25\frac{T^3}{\hat{q}} \approx \begin{cases} 0.145 & \text{(AMY, HT)} \\ 0.03-0.04 & \text{(ASW)} \end{cases}$$

at $T \approx 400$ MeV

$$\frac{\eta}{s} = 0.134(33) \quad \text{(H.Meyer)}$$

at $T = 1.65\,T_c \approx 300$ MeV

(quenched QCD)



Summary

- Kinematic shear viscosity and radiative energy loss, both probe momentum transport in the medium. Small viscosity corresponds to large energy loss.
- A simple inverse relation holds in thermal gauge theories at weak coupling.
- At strong coupling, η/s is limited by the KSS bound, but $\hat{q}$ can become arbitrarily large.
- Existing approaches to jet quenching do not agree in their conclusions about the physical nature of the QGP formed at RHIC.
- Reliable determinations of η/s and $\hat{q}$ from RHIC (and soon LHC) data have a very high priority.

THE END